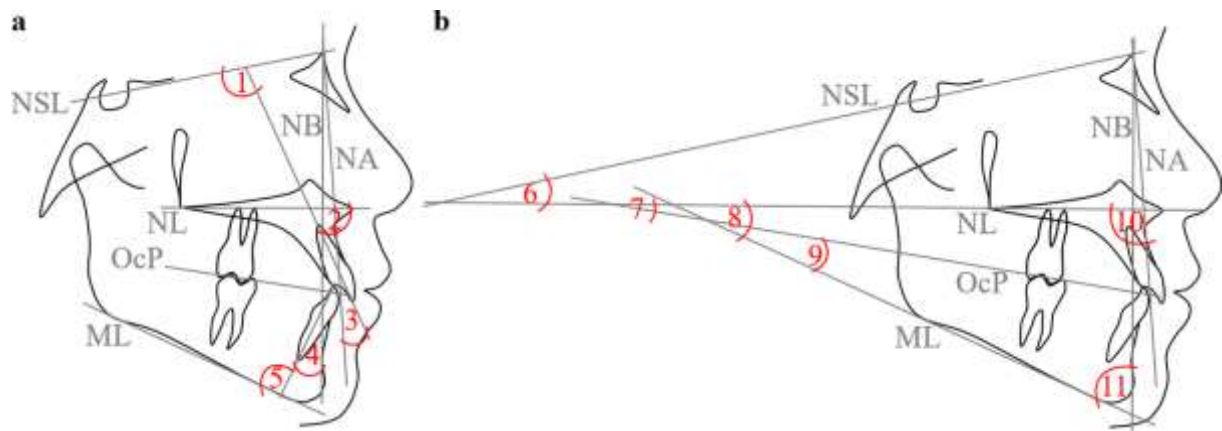


Supplementary Figure 1 Conversion formulas for calculating incisor inclination applying geometry. T0 = before treatment, T1 = after treatment



Cephalometric angular measurements of the incisors' inclination (a) and the reference planes (b). 1 = $\angle 1/\text{NSL}$, 2 = $\angle 1/\text{NL}$, 3 = $\angle 1/\text{NA}$, 4 = $\angle 1/\text{NB}$, 5 = $\angle 1/\text{ML}$, 6 = $\angle \text{NL}/\text{NSL}$, 7 = $\angle \text{OcP}/\text{NL}$, 8 = $\angle \text{ML}/\text{NL}$, 9 = $\angle \text{OcP}/\text{ML}$, 10 = $\angle \text{NL}/\text{NA}$, 11 = $\angle \text{NB}/\text{ML}$

Inclination of the upper incisors

$$\angle 1/\text{NL}_{\text{T0}} = 180^\circ - \angle 1/\text{NSL}_{\text{T0}} - \angle \text{NL}/\text{NSL}_{\text{T0}}$$

$$\angle 1/\text{NL}_{\text{T0}} = 180^\circ - \angle \text{NL}/\text{NA}_{\text{T0}} - \angle 1/\text{NA}_{\text{T0}}$$

$$\angle 1/\text{NSL}_{\text{T0}} = 180^\circ - \angle 1/\text{NL}_{\text{T0}} - \angle \text{NL}/\text{NSL}_{\text{T0}}$$

$$\angle 1/\text{NSL}_{\text{T0}} = \angle \text{NL}/\text{NA}_{\text{T0}} + \angle 1/\text{NA}_{\text{T0}} - \angle \text{NL}/\text{NSL}_{\text{T0}}$$

$$\angle 1/\text{NA}_{\text{T0}} = 180^\circ - \angle 1/\text{NL}_{\text{T0}} - \angle \text{NL}/\text{NA}_{\text{T0}}$$

$$\angle 1/\text{NA}_{\text{T0}} = -\angle \text{NL}/\text{NA}_{\text{T0}} + \angle 1/\text{NSL}_{\text{T0}} + \angle \text{NL}/\text{NSL}_{\text{T0}}$$

Changes of the upper incisors' inclination

$$\Delta \angle 1/\text{NL} = -\Delta \angle 1/\text{NSL}$$

$$\Delta \angle 1/\text{NA}_{\text{T0}} = -\Delta \angle 1/\text{NL}_{\text{T0}}$$

$$\angle 1/\text{NL}_{\text{T1}} = \angle 1/\text{NL}_{\text{T0}} - (-\Delta \angle 1/\text{NL})$$

$$\angle 1/\text{NSL}_{\text{T1}} = 180^\circ - \angle 1/\text{NL}_{\text{T0}} + (-\Delta \angle 1/\text{NL}) - \angle \text{NL}/\text{NSL}_{\text{T0}}$$

$$\angle 1/\text{NSL}_{\text{T1}} = \angle 1/\text{NSL}_{\text{T0}} + \Delta \angle 1/\text{NSL}$$

$$\Delta \angle 1/\text{NSL} = -\Delta \angle 1/\text{NL}$$

$$\Delta \angle 1/\text{NSL} = \Delta \angle 1/\text{NA}$$

$$\angle 1/\text{NA}_{\text{T1}} = 180^\circ - \angle 1/\text{NL}_{\text{T0}} + (-\Delta \angle 1/\text{NL}) - \angle \text{NL}/\text{NA}_{\text{T0}}$$

$$\angle 1/\text{NA}_{\text{T1}} = \angle 1/\text{NA}_{\text{T0}} + \Delta \angle 1/\text{NA}$$

$$\Delta \angle 1/\text{NA} = -\Delta \angle 1/\text{NL}$$

$$\Delta \angle 1/\text{NA} = \Delta \angle 1/\text{NSL}$$

Inclination of the lower incisors

$$\angle 1/ML_{T0} = \angle NB/ML_{T0} + \angle 1/NB_{T0}$$

$$\angle 1/NB_{T0} = \angle 1/ML_{T0} - \angle NB/ML_{T0}$$

Changes of the lower incisors' inclination

$$\Delta \angle 1/NB = \Delta \angle 1/ML$$

$$\angle 1/NL_{T1} = \angle 1/NL_{T0} - (-\Delta \angle 1/NL)$$

$$\angle 1/ML_{T1} = \angle 1/ML_{T0} + \Delta \angle 1/ML$$

$$\angle 1/NB_{T1} = \angle 1/ML_{T0} + \Delta \angle 1/ML - \angle NB/ML_{T0}$$

$$\angle 1/NB_{T1} = \angle 1/NB_{T0} + \Delta \angle 1/NB$$

$$\Delta \angle 1/NB = \Delta \angle 1/ML$$

Interincisal angle

$$\angle 1/1_{T0} = 180^\circ - \angle ML/NL_{T0} - \angle 1/ML_{T0} + \angle 1/NL_{T0}$$

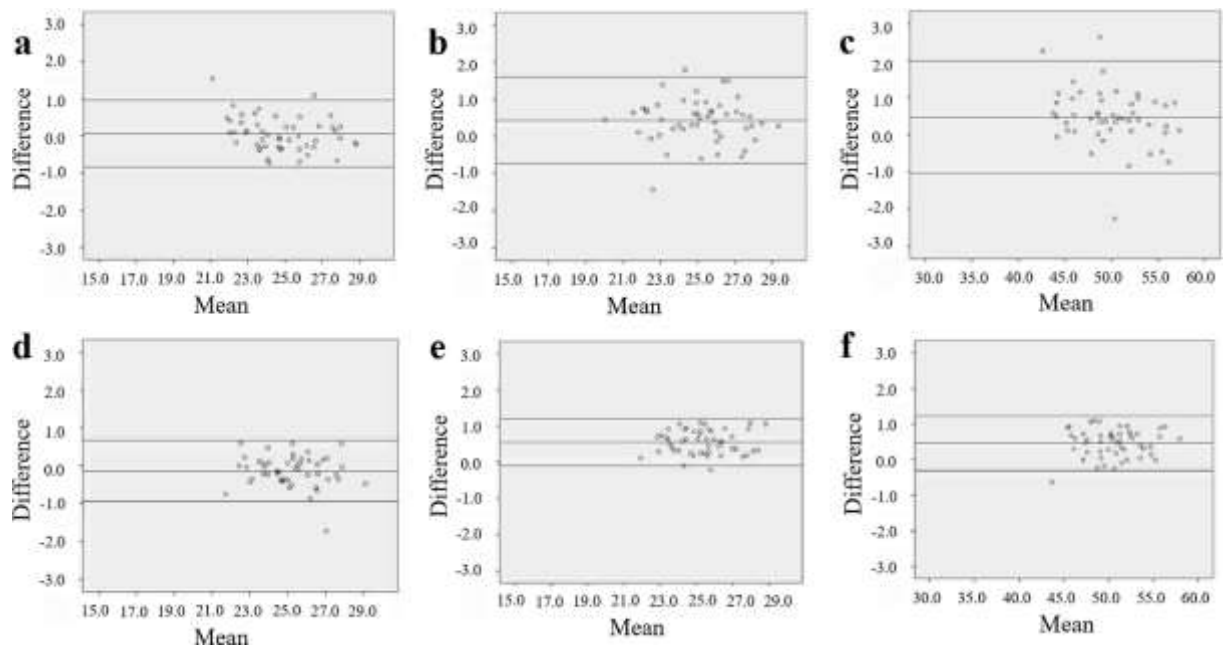
$$\angle 1/1_{T0} = 180^\circ - (\angle OcP/ML_{T0} + \angle OcP/NL_{T0}) - \angle 1/ML_{T0} + \angle 1/NL_{T0}$$

Changes of the interincisal angle

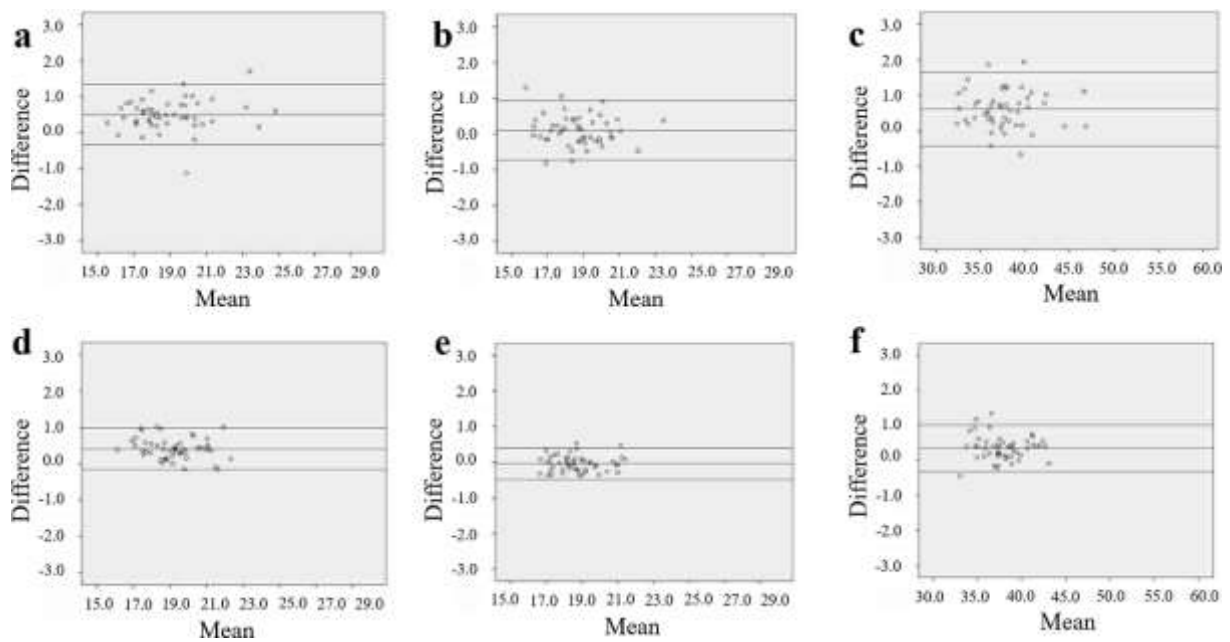
$$\angle 1/1_{T1} = 180^\circ - \angle ML/NL_{T0} - \angle 1/ML_{T0} - \Delta \angle 1/ML + \angle 1/NL_{T0} - (-\Delta \angle 1/NL)$$

$$\Delta \angle 1/1 = -(-\Delta \angle 1/NL + \Delta \angle 1/ML)$$

Supplementary Figure 2 Bland-Altman plots showing the difference between calculated and measured arch length (AL) in the upper jaw. **a:** right AL at T0, **b:** left AL at T0, **c:** total AL at T0, **d:** right lower AL at T1, **e:** left lower AL at T1, **f:** total AL at T1. Each point represents the mean (x-value) and difference (y-value) between measured and calculated AL of one arch



Supplementary Figure 3 Bland-Altman plots showing the difference between calculated and measured arch length (AL) in the lower jaw. **a:** right AL at T0, **b:** left AL at T0, **c:** total AL at T0, **d:** right lower AL at T1, **e:** left lower AL at T1, **f:** total AL at T1. Mean refers to the mean value of measured and calculated arch length of the same cast. Each point represents the mean (x-value) and difference (y-value) between measured and calculated AL of one arch



Supplementary Table 1 Impact of errors in placement of the occlusal place (OcP). Variations of the occlusal plane (OcP) of up to 6° result in changes of the arch length (AL), that do not exceed 0.1 mm and hence the potential error induced by false OcP-identification is not relevant for the clinical validation of the mathematical-geometrical model presented

Parameters: ML-NL = 25°, $\Delta\alpha_1/\text{NL} = 3^\circ$, $\Delta\alpha_1/\text{ML} = 3^\circ$, $c(lj) = 20.3\text{mm}$, $c(uj) = 22.2\text{mm}$, $\alpha_1/\text{ML} = 92^\circ$, $\alpha_1/\text{NL} = 70^\circ$, $aaw(uj, lj) = 38.2\text{mm}$, $L(lj) = 16.4\text{mm}$, $L(uj) = 18.4\text{mm}$, $AL_{T0}(lj) = 54.86\text{mm}$, $AL_{T0}(uj) = 58.00\text{mm}$

OcP-ML [°]; OcP-NL [°]	AL _{T1} (lj) [mm]	$\Delta AL(lj)$ [mm]	AL _{T1} (uj) [mm]	$\Delta AL(uj)$ [mm]	$\Delta v(\text{total})$ [mm]
17; 8	56.41	1.55	59.63	1.63	0.94
16.5; 8.5	56.42	1.56	59.62	1.62	0.94
16; 9	56.42	1.56	59.62	1.61	0.94
15.5; 9.5	56.43	1.57	59.61	1.60	0.94
15; 10	56.43	1.57	59.60	1.60	0.94
14.5; 10.5	56.43	1.58	59.59	1.60	0.94
14; 11	56.44	1.58	59.58	1.58	0.94
13.5; 11.5	56.44	1.59	59.57	1.57	0.94
13; 12	56.45	1.59	59.56	1.56	0.94
12.5; 12.5	56.45	1.59	59.55	1.55	0.94
12; 13	56.46	1.6	59.55	1.54	0.94
11.5; 13.5	56.46	1.6	59.54	1.53	0.94
11; 14	56.46	1.61	59.53	1.52	0.94