

Supplementary Material: Models for Autoregressive Processes of Bounded Counts: How Different Are They?

Hee-Young Kim^{*†} Christian H. Weiß^{‡§} Tobias A. Möller[‡]

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Acronyms

DGP = data-generating process
BAR model = binomial autoregressive model
bvAR model = binomial-variation AR model
BARCH model = binomial AR conditional heteroscedasticity model
max-BAR model = max-operator binomial AR model
logit-BARCH model = binomial logit-ARCH model
ML estimation = maximum likelihood estimation
AIC (BIC) = Akaike's (Bayesian) information criterion
BID = binomial dispersion index
(P)ACF = (partial) autocorrelation function

^{*}Division of Economics and Statistics, National Statistics, Korea University, Sejong, South Korea.

[†]E-Mail: starlike.kim@gmail.com

[‡]Helmut Schmidt University, Department of Mathematics and Statistics, Hamburg, Germany.

[§]E-Mail: weissc@hsu-hh.de

S.1 Using Information Criteria for Model Selection

Upper part: Numbers of times each of the candidate models BAR(1), BAR(2), bvAR(1), bvAR(2), BARCH(1), and BARCH(2) were selected for a given type of DGP out of 1,000 replications, using the information criteria AIC or BIC. Numbers of correct identifications are highlighted in *italic font*.

Lower part: Average distance between the i th model's AIC (BIC) and the respective minimal AIC (BIC).

BAR(1)		Model identification by AIC					Model identification by BIC						
Scenario	T	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)
1	100	594	87	66	5	223	25	659	20	71	0	243	7
	250	635	101	11	2	222	29	723	12	14	0	242	9
	500	697	96	0	0	181	26	789	5	0	0	205	1
	1000	761	104	0	0	113	22	837	6	0	0	137	0
2	100	836	125	0	0	37	2	930	27	0	0	42	1
	250	869	127	0	0	4	0	980	16	0	0	4	0
	500	864	136	0	0	0	0	985	15	0	0	0	0
	1000	853	147	0	0	0	0	986	14	0	0	0	0
3	100	582	88	45	1	245	39	650	16	50	0	276	8
	250	623	90	5	0	251	31	702	10	5	0	277	6
	500	660	126	3	0	187	24	766	13	3	0	218	0
	1000	758	113	0	0	112	17	864	7	0	0	127	2
4	100	837	125	0	0	37	1	933	29	0	0	38	0
	250	838	160	0	0	2	0	979	19	0	0	2	0
	500	878	121	0	0	1	0	987	12	0	0	1	0
	1000	885	115	0	0	0	0	993	7	0	0	0	0

BAR(1)		Mean of $AIC_i - AIC_{\min}$					Mean of $BIC_i - BIC_{\min}$						
1	100	0.60	2.20	4.90	6.73	1.20	2.85	0.43	4.64	4.73	9.17	1.03	5.29
	250	0.65	2.21	11.65	13.50	1.86	3.48	0.44	5.52	11.44	16.81	1.65	6.79
	500	0.55	2.18	22.70	24.63	2.95	4.60	0.37	6.21	22.51	28.66	2.77	8.63
	1000	0.48	2.09	44.70	46.68	4.97	6.60	0.28	6.79	44.50	51.39	4.77	11.31
2	100	0.34	2.00	19.37	21.32	9.16	10.92	0.16	4.43	19.19	23.74	8.98	13.35
	250	0.23	1.89	48.02	50.00	22.05	23.80	0.04	5.22	47.83	53.33	21.86	27.13
	500	0.25	1.89	95.56	97.53	42.30	43.97	0.03	5.88	95.34	101.52	42.07	47.96
	1000	0.27	1.85	190.87	192.87	85.19	86.86	0.02	6.50	190.62	197.53	84.94	91.52
3	100	0.60	2.16	5.65	7.47	1.22	2.87	0.42	4.58	5.48	9.90	1.04	5.30
	250	0.60	2.28	12.78	14.68	1.68	3.37	0.43	5.63	12.61	18.03	1.51	6.72
	500	0.63	2.20	25.68	27.53	2.74	4.35	0.40	6.18	25.44	31.51	2.50	8.33
	1000	0.52	2.06	51.22	53.14	4.77	6.34	0.28	6.74	50.99	57.81	4.54	11.01
4	100	0.30	1.94	20.55	22.37	9.30	11.07	0.13	4.37	20.38	24.80	9.12	13.50
	250	0.29	1.83	50.59	52.33	21.45	23.13	0.04	5.10	50.34	55.60	21.20	26.40
	500	0.24	1.90	100.55	102.40	41.32	43.04	0.02	5.89	100.33	106.40	41.10	47.03
	1000	0.21	1.88	200.78	202.70	82.78	84.52	0.01	6.59	200.58	207.41	82.57	89.23

Scenario	Model identification by AIC							Model identification by BIC						
	bvAR(1)	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)	
<i>T</i>														
5	100	65	16	733	97	71	18	68	9	812	23	83	5	
	250	10	4	818	135	29	4	13	1	925	28	33	0	
	500	0	0	877	120	2	1	0	0	987	11	2	0	
	1000	0	0	889	111	0	0	0	0	992	8	0	0	
6	100	9	1	816	138	32	4	10	0	930	24	35	1	
	250	0	0	829	168	2	1	0	0	980	18	2	0	
	500	0	0	850	150	0	0	0	0	987	13	0	0	
	1000	0	0	842	158	0	0	0	0	993	7	0	0	
7	100	58	16	758	84	68	16	68	2	830	15	81	4	
	250	11	0	836	126	25	2	11	0	940	22	26	1	
	500	0	0	884	113	2	1	0	0	986	11	2	1	
	1000	0	0	862	138	0	0	0	0	988	12	0	0	
8	100	3	1	843	130	20	3	4	0	941	33	21	1	
	250	0	0	861	139	0	0	0	0	984	16	0	0	
	500	0	0	877	123	0	0	0	0	988	12	0	0	
	1000	0	0	877	123	0	0	0	0	995	5	0	0	
Mean of $AIC_i - AIC_{\min}$							Mean of $BIC_i - BIC_{\min}$							
5	100	8.38	9.82	0.58	2.21	6.56	8.11	8.18	12.23	0.39	4.62	6.36	10.53	
	250	21.01	22.21	0.43	1.90	16.43	17.84	20.77	25.48	0.19	5.18	16.19	21.12	
	500	42.62	43.63	0.22	1.87	33.40	34.82	42.42	47.64	0.02	5.89	33.20	38.84	
	1000	87.97	88.47	0.19	1.84	69.02	70.39	87.79	93.20	0.01	6.57	68.84	75.11	
6	100	32.25	33.13	0.35	2.05	19.82	21.32	32.07	35.56	0.18	4.48	19.65	23.75	
	250	83.32	83.17	0.25	1.90	50.79	52.20	83.10	86.47	0.03	5.20	50.57	55.50	
	500	165.07	163.46	0.28	1.91	99.55	100.90	164.84	167.44	0.05	5.89	99.32	104.88	
	1000	335.05	330.78	0.26	1.93	202.17	203.53	334.83	335.46	0.04	6.62	201.95	208.21	
7	100	11.51	13.04	0.56	2.35	9.72	11.40	11.36	15.49	0.41	4.80	9.56	13.85	
	250	29.95	31.10	0.37	2.00	25.29	26.80	29.74	34.42	0.16	5.32	25.09	30.11	
	500	58.98	59.70	0.20	1.88	50.11	51.57	58.80	63.73	0.02	5.91	49.93	55.60	
	1000	118.42	118.45	0.28	1.81	100.57	101.81	118.16	123.10	0.02	6.46	100.31	106.46	
8	100	46.43	46.94	0.36	1.97	33.05	34.40	46.23	49.35	0.16	4.38	32.85	36.81	
	250	120.77	120.22	0.23	1.86	86.74	88.06	120.57	123.55	0.03	5.19	86.54	91.39	
	500	243.21	240.63	0.22	1.91	173.34	174.72	243.02	244.65	0.02	5.93	173.15	178.74	
	1000	489.71	484.08	0.19	1.93	348.93	350.44	489.53	488.80	0.01	6.66	348.75	355.17	

BARCH(1)		Model identification by AIC					Model identification by BIC						
Scenario	T	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)
9													
	100	353	54	107	14	413	59	391	12	121	1	466	9
	250	292	49	38	6	538	77	325	8	42	1	615	9
	500	226	46	7	0	612	109	263	3	7	0	718	9
	1000	144	41	0	0	718	97	179	1	0	0	815	5
10													
	100	116	24	37	3	700	120	128	7	39	1	802	23
	250	33	9	4	0	816	138	42	1	4	0	941	12
	500	8	0	0	0	855	137	8	0	0	0	981	11
	1000	0	0	0	0	841	159	0	0	0	0	983	17
11													
	100	376	70	64	9	401	80	417	22	75	3	465	18
	250	301	52	14	2	536	95	341	9	16	0	615	19
	500	219	42	2	0	629	108	251	4	3	0	731	11
	1000	169	30	0	0	698	103	196	0	0	0	802	2
12													
	100	151	28	18	1	703	99	166	7	19	0	783	25
	250	31	10	0	0	837	122	33	5	0	0	950	12
	500	3	0	0	0	848	149	3	0	0	0	985	12
	1000	0	0	0	0	856	144	0	0	0	0	991	9
Mean of $AIC_i - AIC_{\min}$													
BARCH(1)		Mean of $AIC_i - AIC_{\min}$					Mean of $BIC_i - BIC_{\min}$						
9	100	1.10	2.78	4.23	6.02	0.81	2.50	0.95	5.23	4.08	8.47	0.65	4.95
	250	1.66	3.24	9.53	11.28	0.79	2.35	1.44	6.54	9.31	14.58	0.57	5.66
	500	2.58	4.11	18.53	20.29	0.70	2.22	2.35	8.10	18.30	24.28	0.47	6.20
	1000	4.37	5.92	36.19	38.04	0.60	2.16	4.16	10.61	35.98	42.73	0.39	6.85
10	100	6.16	7.60	10.31	11.95	0.62	2.23	5.97	10.01	10.12	14.36	0.43	4.64
	250	14.29	15.67	25.51	27.22	0.40	2.01	14.07	18.98	25.30	30.53	0.18	5.31
	500	28.99	30.36	51.23	52.84	0.25	1.87	28.79	34.38	51.03	56.86	0.05	5.89
	1000	59.35	60.28	103.21	104.78	0.29	1.81	59.10	64.93	102.95	109.43	0.03	6.46
11	100	0.96	2.46	4.95	6.58	0.78	2.34	0.73	4.84	4.73	8.96	0.55	4.73
	250	1.57	3.08	11.62	13.28	0.84	2.35	1.32	6.35	11.37	16.55	0.59	5.62
	500	2.31	3.90	21.98	23.70	0.66	2.24	2.08	7.88	21.74	27.68	0.42	6.22
	1000	3.92	5.59	43.41	45.22	0.60	2.27	3.74	10.31	43.23	49.94	0.42	7.00
12	100	5.57	7.08	13.41	14.46	0.62	2.29	5.40	9.51	13.23	16.89	0.45	4.71
	250	14.19	15.59	33.36	33.64	0.30	1.95	14.00	18.92	33.17	36.97	0.11	5.28
	500	28.86	29.95	67.61	66.46	0.26	1.75	28.62	33.93	67.37	70.43	0.02	5.72
	1000	58.39	59.27	135.22	132.02	0.25	1.76	58.16	63.95	134.99	136.70	0.02	6.44

BAR(2)		Model identification by AIC					Model identification by BIC						
Scenario	T	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)
13	100	256	474	21	27	71	151	423	303	44	14	117	99
	250	52	790	1	7	9	141	193	638	11	3	41	114
	500	6	917	0	1	0	76	33	888	0	1	7	71
	1000	0	990	0	0	0	10	0	990	0	0	0	10
14	100	273	698	1	0	5	23	508	459	2	0	16	15
	250	61	936	0	0	0	3	194	804	0	0	1	1
	500	0	1000	0	0	0	0	21	979	0	0	0	0
	1000	0	1000	0	0	0	0	0	1000	0	0	0	0
15	100	248	530	22	22	55	123	436	333	38	13	111	69
	250	54	804	1	2	6	133	181	662	3	2	42	110
	500	2	923	0	0	1	74	17	905	0	0	4	74
	1000	0	970	0	0	0	30	0	970	0	0	0	30
16	100	291	694	0	0	6	9	519	459	0	0	14	8
	250	52	948	0	0	0	0	163	837	0	0	0	0
	500	1	999	0	0	0	0	12	988	0	0	0	0
	1000	0	1000	0	0	0	0	0	1000	0	0	0	0

BAR(2)		Mean of $AIC_i - AIC_{\min}$					Mean of $BIC_i - BIC_{\min}$						
13	100	3.61	0.87	7.81	7.64	4.36	2.51	2.23	2.08	6.42	8.86	2.97	3.72
	250	9.62	0.36	19.93	17.57	11.25	4.14	6.61	0.86	16.91	18.08	8.24	4.64
	500	21.26	0.19	41.80	35.28	24.26	7.80	17.13	0.28	37.68	35.37	20.14	7.89
	1000	42.51	0.01	84.56	71.41	48.68	15.03	37.60	0.01	79.65	71.41	43.77	15.03
14	100	4.14	0.51	22.95	23.47	13.89	12.90	2.59	1.57	21.40	24.53	12.34	13.96
	250	10.80	0.10	58.95	58.30	35.42	30.58	7.69	0.50	55.84	58.71	32.31	30.98
	500	23.99	0.00	119.34	116.19	71.43	59.62	19.81	0.04	115.16	116.23	67.25	59.66
	1000	51.07	0.00	241.46	233.23	144.60	118.52	46.16	0.00	236.55	233.23	139.69	118.52
15	100	3.52	0.83	8.37	8.39	4.23	2.52	2.10	2.02	6.96	9.58	2.82	3.71
	250	9.73	0.32	21.57	19.42	11.20	4.09	6.68	0.79	18.51	19.89	8.15	4.56
	500	21.15	0.17	45.04	39.16	23.74	7.37	16.97	0.21	40.86	39.20	19.55	7.40
	1000	44.72	0.11	92.75	79.17	49.74	14.82	39.81	0.11	87.85	79.17	44.84	14.82
16	100	4.22	0.49	24.01	24.07	13.63	12.77	2.71	1.58	22.50	25.16	12.12	13.87
	250	11.81	0.06	62.02	59.89	34.97	30.26	8.67	0.44	58.87	60.27	31.82	30.64
	500	25.73	0.00	125.32	119.37	70.46	59.14	21.54	0.03	121.13	119.40	66.27	59.16
	1000	53.39	0.00	252.25	238.80	141.75	116.52	48.48	0.00	247.34	238.80	136.84	116.52

Scenario	Model identification by AIC				Model identification by BIC									
	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bvAR(1)	bvAR(2)	BARCH(1)	BARCH(2)		
17	100	5	31	207	674	32	51	16	18	357	526	52	31	
	250	0	0	33	957	1	9	0	0	95	894	4	7	
	500	0	0	2	998	0	0	0	0	10	990	0	0	
	1000	0	0	0	1000	0	0	0	0	0	1000	0	0	
18	100	1	6	228	744	8	13	1	0	414	561	15	9	
	250	0	0	31	968	0	1	0	0	94	905	0	1	
	500	0	0	1	999	0	0	0	0	11	989	0	0	
	1000	0	0	0	1000	0	0	0	0	0	1000	0	0	
19	100	7	18	162	755	27	31	12	10	287	630	48	13	
	250	0	1	13	982	0	4	0	1	64	931	1	3	
	500	0	0	0	999	0	1	0	0	3	996	0	1	
	1000	0	0	0	1000	0	0	0	0	0	1000	0	0	
20	100	1	2	159	830	4	4	1	1	283	705	9	1	
	250	0	0	12	987	0	1	0	0	27	972	1	0	
	500	0	0	0	1000	0	0	0	0	0	1000	0	0	
	1000	0	0	0	1000	0	0	0	0	0	1000	0	0	
bvAR(2)	Mean of $AIC_i - AIC_{\min}$						Mean of $BIC_i - BIC_{\min}$							
	17	100	20.32	14.71	7.67	0.71	17.13	12.90	18.58	15.57	5.92	1.57	15.39	13.77
	250	54.70	37.47	19.51	0.06	47.11	34.42	51.41	51.41	37.70	16.22	0.29	43.82	34.65
	500	112.10	75.21	41.63	0.00	97.07	69.33	107.91	107.91	75.24	37.44	0.03	92.88	69.35
	1000	225.71	151.71	82.02	0.00	195.69	140.55	220.80	220.80	151.71	77.11	0.00	190.78	140.55
18	100	49.39	38.25	8.53	0.54	33.65	28.82	47.68	47.68	39.14	6.82	1.43	31.94	29.71
	250	132.35	98.77	22.65	0.06	89.54	74.03	129.03	129.03	98.98	19.33	0.27	86.23	74.23
	500	271.56	199.16	48.46	0.00	185.11	151.67	267.37	267.37	199.18	44.27	0.02	180.92	151.69
	1000	545.25	399.27	97.44	0.00	370.45	301.72	540.34	540.34	399.27	92.54	0.00	365.54	301.72
19	100	27.60	20.83	9.76	0.55	24.31	20.29	25.71	25.71	21.55	7.88	1.27	22.42	21.01
	250	74.69	53.87	25.95	0.03	66.35	52.75	71.31	71.31	54.01	22.57	0.17	62.97	52.89
	500	151.48	107.18	54.21	0.00	135.86	107.31	147.27	147.27	107.18	50.00	0.01	131.64	107.31
	1000	306.53	214.39	111.27	0.00	274.45	214.25	301.62	301.62	214.39	106.37	0.00	269.55	214.25
20	100	71.93	56.31	11.93	0.30	53.17	47.88	69.93	69.93	56.92	9.93	0.91	51.17	48.48
	250	191.66	145.89	32.94	0.01	142.94	126.25	188.21	188.21	145.96	29.49	0.09	139.49	126.32
	500	389.01	296.54	64.74	0.00	288.88	254.85	384.79	384.79	296.54	60.52	0.00	284.66	254.85
	1000	789.58	597.40	135.05	0.00	588.39	516.85	784.68	784.68	597.40	130.14	0.00	583.49	516.85

BARCH(2)		Model identification by AIC						Model identification by BIC					
Scenario	T	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)
21	100	173	214	54	41	141	377	270	118	91	14	270	237
	250	27	209	6	9	37	712	90	160	17	5	154	574
	500	0	142	0	2	1	855	10	130	1	1	33	825
	1000	0	52	0	0	0	948	0	52	0	0	0	948
22	100	84	76	20	6	288	526	112	48	33	3	510	294
	250	10	17	0	1	68	904	15	13	1	1	245	725
	500	1	4	0	0	4	991	2	3	0	0	43	952
	1000	0	0	0	0	0	1000	0	0	0	0	1	999
23	100	165	205	40	20	177	393	263	111	59	10	329	228
	250	30	216	3	8	32	711	86	165	12	2	157	578
	500	1	132	0	0	2	865	11	123	0	0	25	841
	1000	0	44	0	0	0	956	1	43	0	0	0	956
24	100	80	87	8	3	276	546	114	41	16	1	532	296
	250	3	37	0	0	55	905	20	24	0	0	235	721
	500	0	2	0	0	3	995	0	2	0	0	33	965
	1000	0	0	0	0	0	1000	0	0	0	0	0	1000

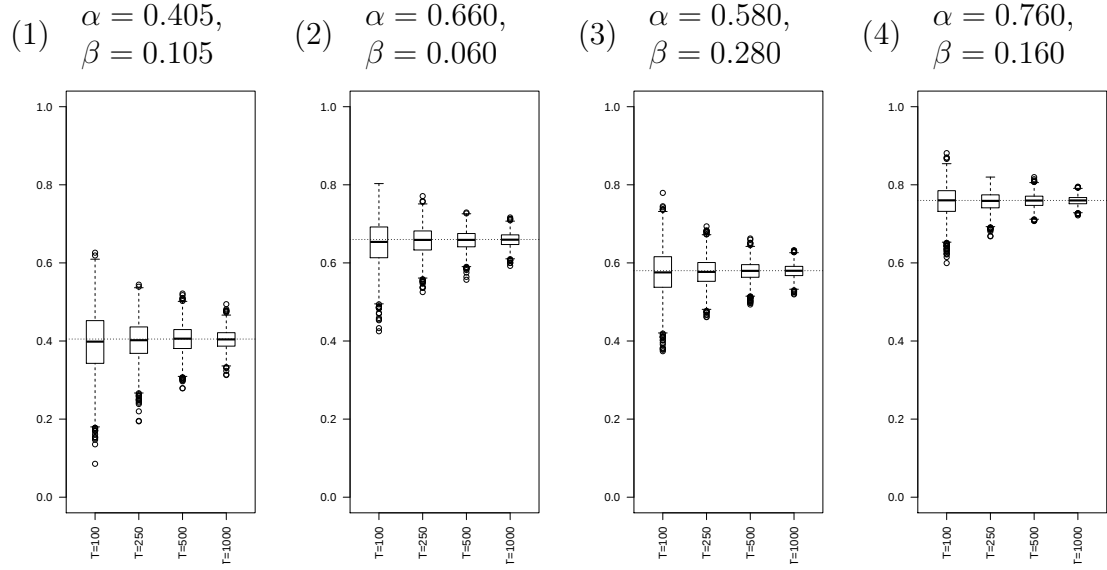
BARCH(2)		Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$							
		BAR(1)	BAR(2)	bVAR(1)	bVAR(2)	BARCH(1)	BARCH(2)	BAR(1)	BAR(2)				
21	100	3.71	1.81	6.44	6.12	3.28	1.15	2.42	3.12	5.15	7.43	1.98	2.46
	250	10.33	2.80	16.88	13.90	8.85	0.53	7.38	3.37	13.93	14.46	5.90	1.10
	500	21.84	5.32	34.72	27.00	18.58	0.35	17.70	5.39	30.57	27.07	14.44	0.42
	1000	46.48	10.58	71.91	54.50	40.02	0.16	41.57	10.58	67.00	54.50	35.12	0.16
22	100	8.21	6.20	12.01	11.49	2.97	0.96	6.98	7.58	10.78	12.87	1.74	2.34
	250	24.03	16.08	30.88	27.01	8.26	0.15	21.10	16.66	27.94	27.60	5.32	0.73
	500	51.67	33.01	66.56	56.51	19.13	0.01	47.54	33.09	62.43	56.60	15.00	0.09
	1000	107.26	68.84	135.70	114.19	39.77	0.00	102.36	68.84	130.79	114.19	34.86	0.00
23	100	3.76	1.93	7.17	6.98	3.21	1.14	2.50	3.27	5.91	8.32	1.95	2.48
	250	10.38	2.91	18.55	15.58	8.82	0.55	7.40	3.46	15.57	16.13	5.85	1.10
	500	21.97	5.26	38.19	30.91	18.84	0.31	17.82	5.33	34.05	30.98	14.69	0.38
	1000	46.63	10.80	79.21	62.34	40.03	0.10	41.72	10.80	74.30	62.34	35.12	0.10
24	100	8.25	6.16	15.09	13.49	2.92	0.91	7.02	7.53	13.86	14.87	1.68	2.29
	250	24.89	16.24	40.57	32.99	8.68	0.21	21.88	16.75	37.56	33.50	5.68	0.72
	500	52.94	33.95	84.81	67.42	19.05	0.02	48.78	34.00	80.65	67.48	14.89	0.07
	1000	107.61	68.42	171.52	135.31	39.53	0.00	102.71	68.42	166.61	135.31	34.62	0.00

S.2 Estimation Performance for Adequate Models

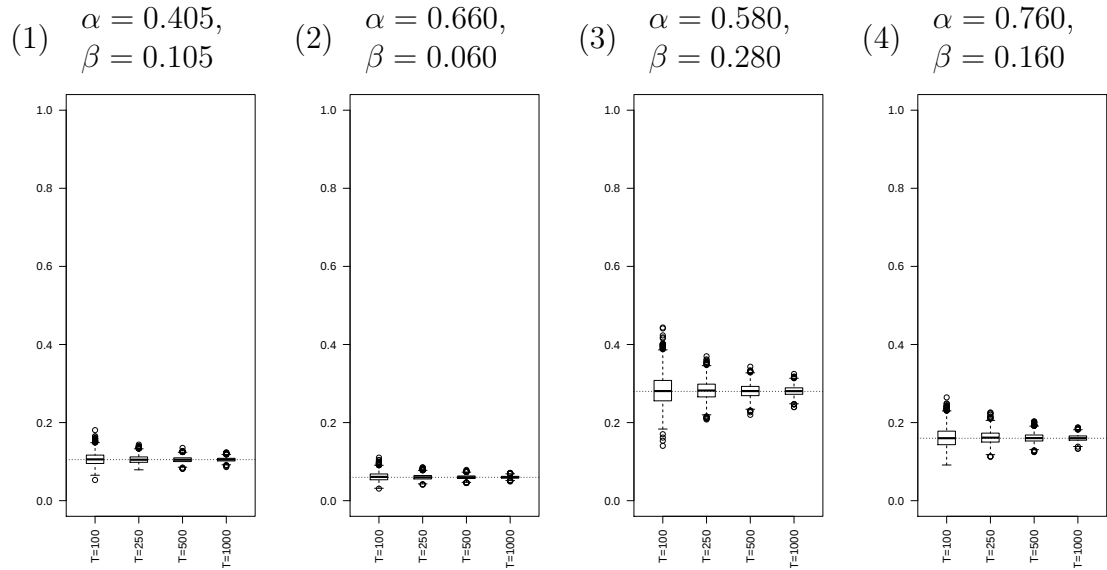
Boxplots of the ML estimates from the simulation study (1,000 replications) for the respective model parameters. The numbers in parentheses provide the scenario according to Tables 1 and 2 in the main manuscript.

BAR(1) model

Estimation of α :



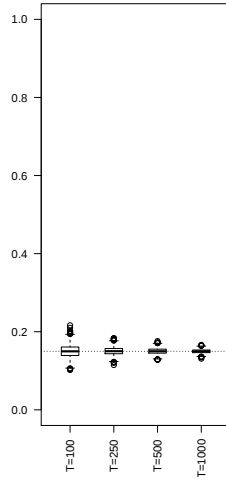
Estimation of β :



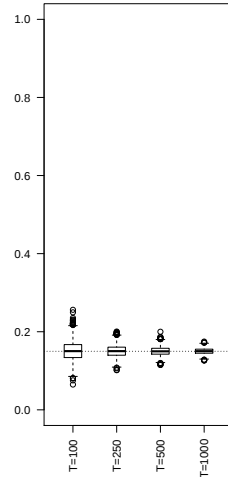
bvAR(1) model

Estimation of π :

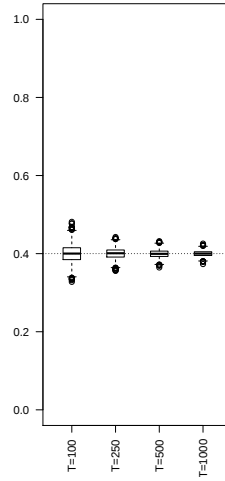
(5) $\pi = 0.15$,
 $\phi_1 = 0.3$



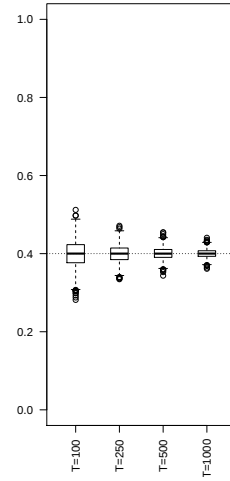
(6) $\pi = 0.15$,
 $\phi_1 = 0.6$



(7) $\pi = 0.4$,
 $\phi_1 = 0.3$

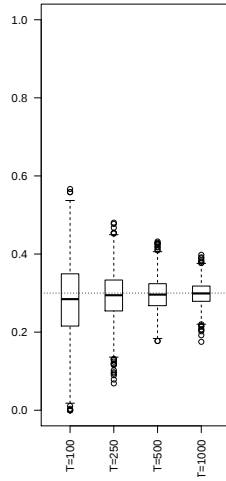


(8) $\pi = 0.4$,
 $\phi_1 = 0.6$

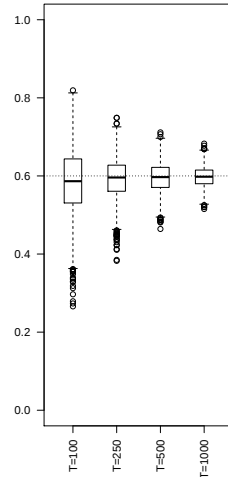


Estimation of ϕ_1 :

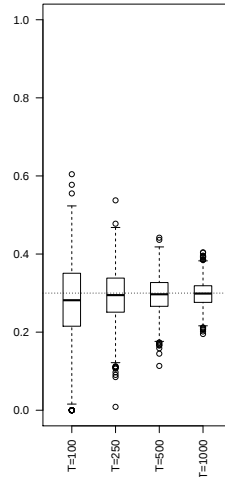
(5) $\pi = 0.15$,
 $\phi_1 = 0.3$



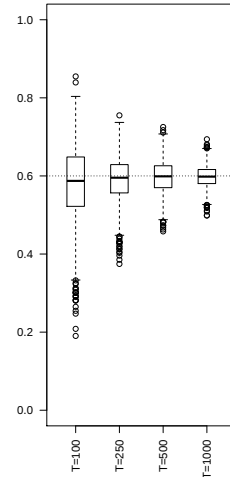
(6) $\pi = 0.15$,
 $\phi_1 = 0.6$



(7) $\pi = 0.4$,
 $\phi_1 = 0.3$



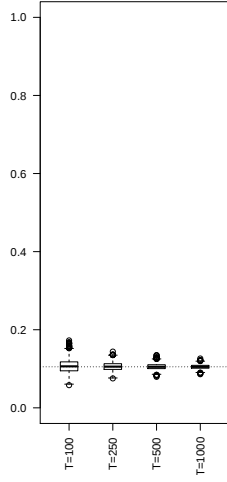
(8) $\pi = 0.4$,
 $\phi_1 = 0.6$



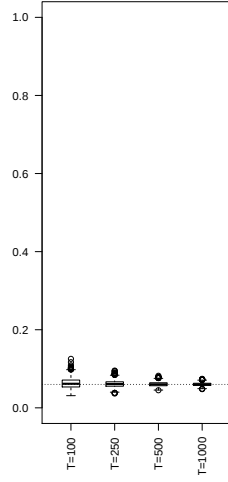
BARCH(1) model

Estimation of α_0 :

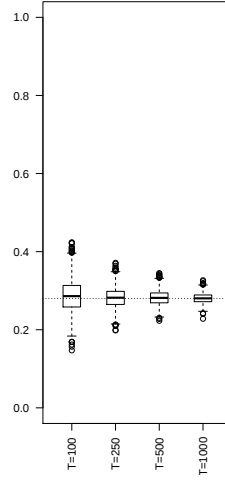
(9) $\alpha_0 = 0.105,$
 $\alpha_1 = 0.3$



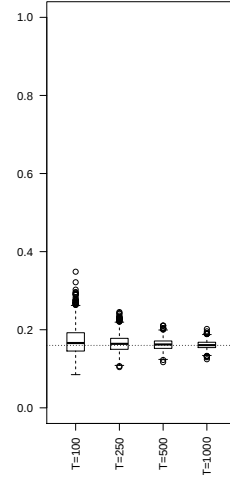
(10) $\alpha_0 = 0.060,$
 $\alpha_1 = 0.6$



(11) $\alpha_0 = 0.280,$
 $\alpha_1 = 0.3$

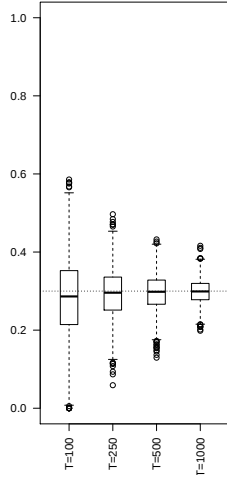


(12) $\alpha_0 = 0.160,$
 $\alpha_1 = 0.6$

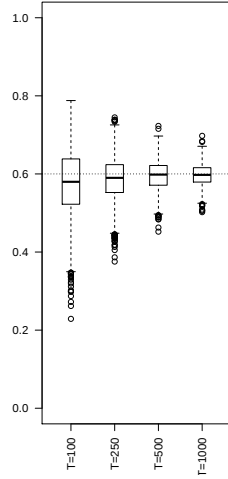


Estimation of α_1 :

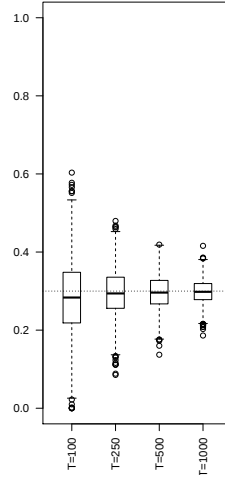
(9) $\alpha_0 = 0.105,$
 $\alpha_1 = 0.3$



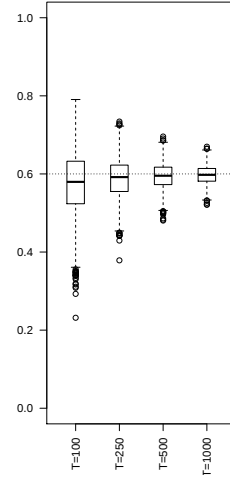
(10) $\alpha_0 = 0.060,$
 $\alpha_1 = 0.6$



(11) $\alpha_0 = 0.280,$
 $\alpha_1 = 0.3$



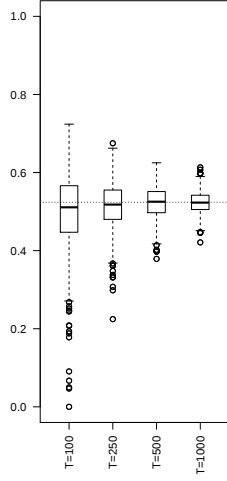
(12) $\alpha_0 = 0.160,$
 $\alpha_1 = 0.6$



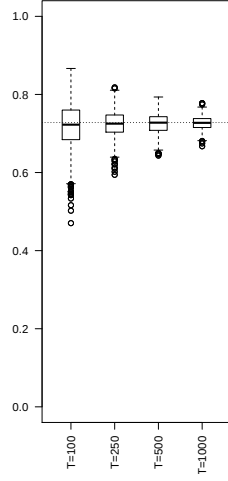
BAR(2) model

Estimation of α :

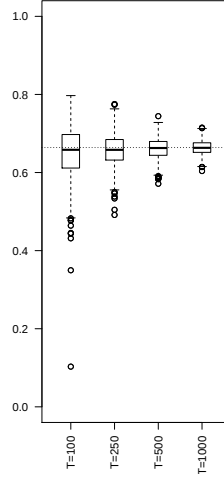
$$(17) \quad \begin{aligned} \alpha &= 0.524, \\ \beta &= 0.084, \\ \phi_1 &= 0.545 \end{aligned}$$



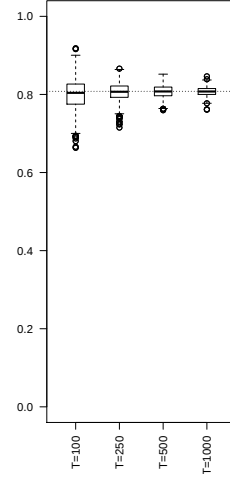
$$(18) \quad \begin{aligned} \alpha &= 0.728, \\ \beta &= 0.048, \\ \phi_1 &= 0.706 \end{aligned}$$



$$(19) \quad \begin{aligned} \alpha &= 0.664, \\ \beta &= 0.224, \\ \phi_1 &= 0.545 \end{aligned}$$

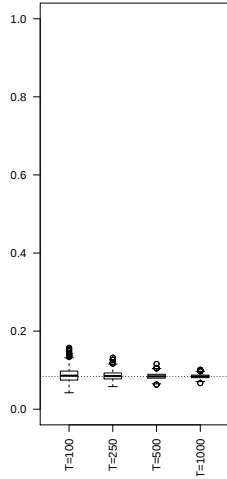


$$(20) \quad \begin{aligned} \alpha &= 0.808, \\ \beta &= 0.128, \\ \phi_1 &= 0.706 \end{aligned}$$

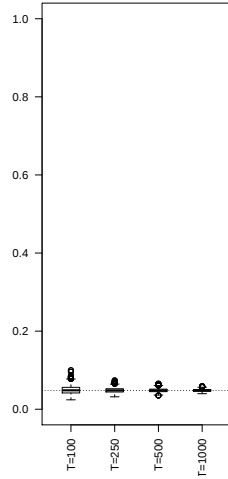


Estimation of β :

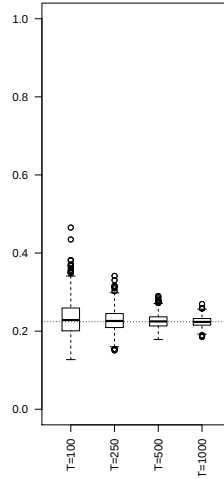
$$(17) \quad \begin{aligned} \alpha &= 0.524, \\ \beta &= 0.084, \\ \phi_1 &= 0.545 \end{aligned}$$



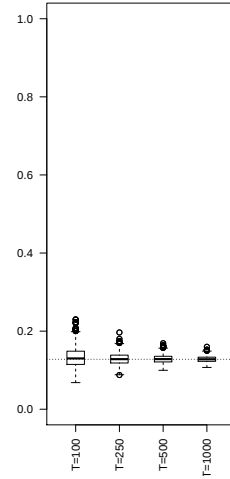
$$(18) \quad \begin{aligned} \alpha &= 0.728, \\ \beta &= 0.048, \\ \phi_1 &= 0.706 \end{aligned}$$



$$(19) \quad \begin{aligned} \alpha &= 0.664, \\ \beta &= 0.224, \\ \phi_1 &= 0.545 \end{aligned}$$

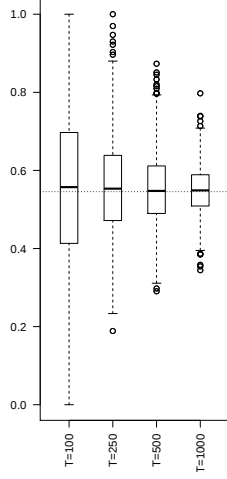


$$(20) \quad \begin{aligned} \alpha &= 0.808, \\ \beta &= 0.128, \\ \phi_1 &= 0.706 \end{aligned}$$

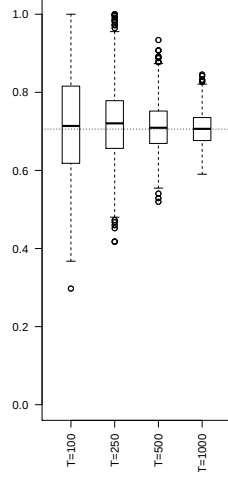


Estimation of ϕ_1 :

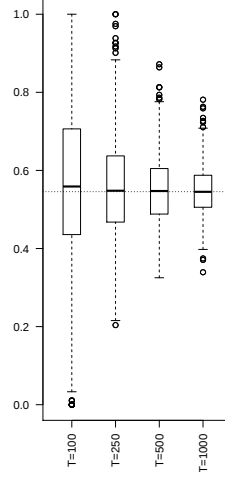
$$(17) \quad \begin{aligned} \alpha &= 0.524, \\ \beta &= 0.084, \\ \phi_1 &= 0.545 \end{aligned}$$



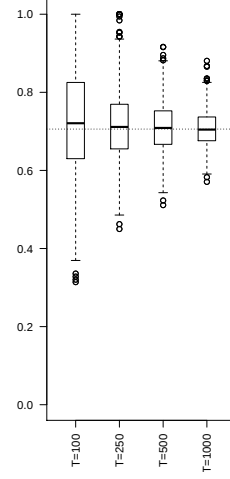
$$(18) \quad \begin{aligned} \alpha &= 0.728, \\ \beta &= 0.048, \\ \phi_1 &= 0.706 \end{aligned}$$



$$(19) \quad \begin{aligned} \alpha &= 0.664, \\ \beta &= 0.224, \\ \phi_1 &= 0.545 \end{aligned}$$



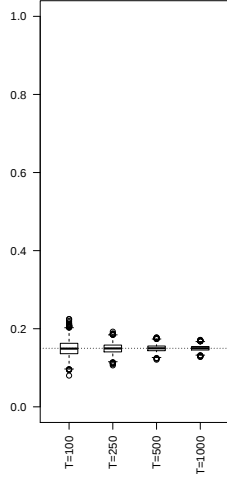
$$(20) \quad \begin{aligned} \alpha &= 0.808, \\ \beta &= 0.128, \\ \phi_1 &= 0.706 \end{aligned}$$



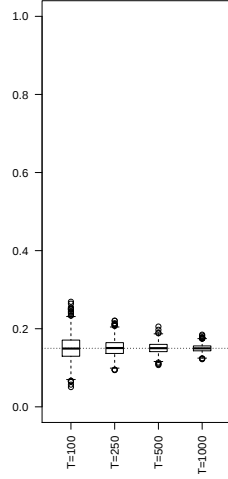
bvAR(2) model

Estimation of π :

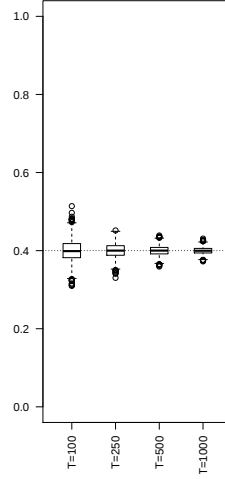
$$(21) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$



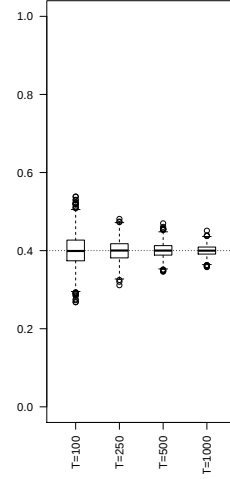
$$(22) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$



$$(23) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$

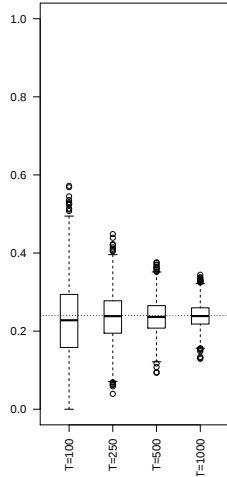


$$(24) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$

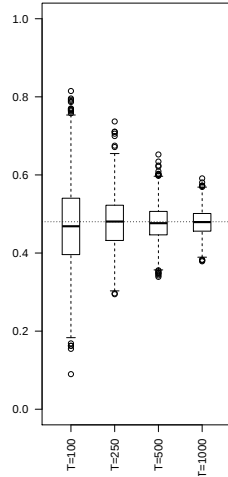


Estimation of ϕ_1 :

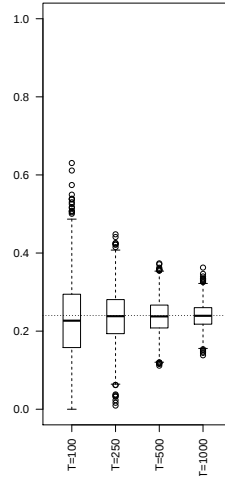
$$(21) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$



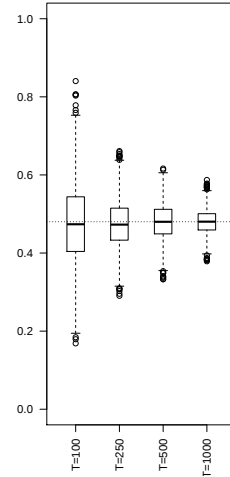
$$(22) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$



$$(23) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$

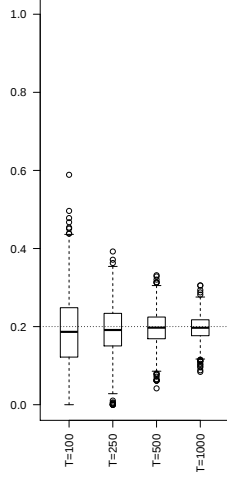


$$(24) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$

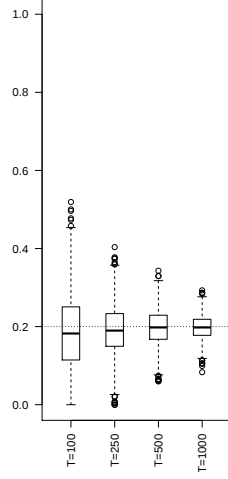


Estimation of ϕ_2 :

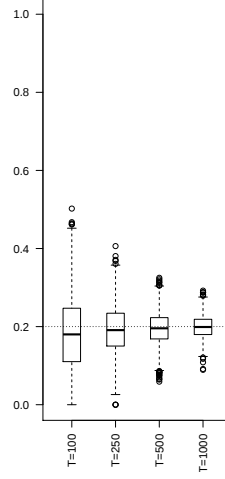
$$(21) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$



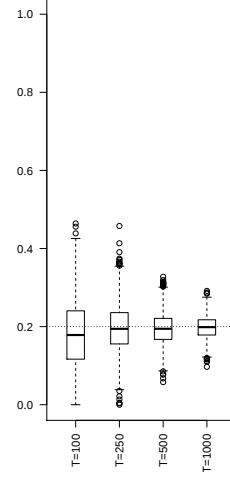
$$(22) \quad \begin{aligned} \pi &= 0.15, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$



$$(23) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.24, \\ \phi_2 &= 0.2 \end{aligned}$$



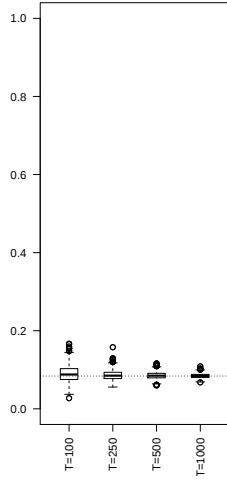
$$(24) \quad \begin{aligned} \pi &= 0.40, \\ \phi_1 &= 0.48, \\ \phi_2 &= 0.2 \end{aligned}$$



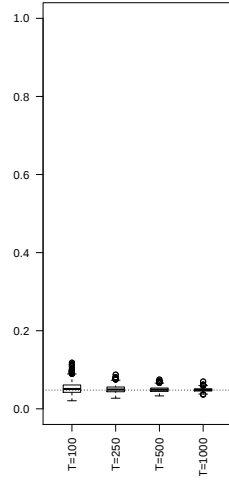
BARCH(2) model

Estimation of α_0 :

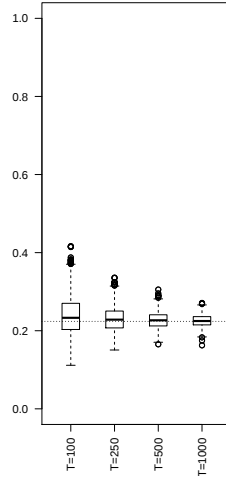
$$(25) \quad \begin{aligned} \alpha_0 &= 0.084, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



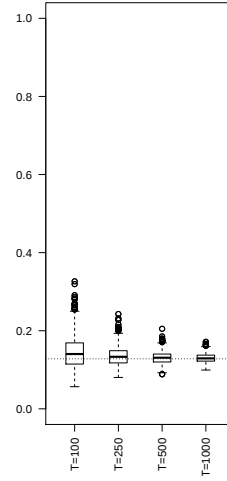
$$(26) \quad \begin{aligned} \alpha_0 &= 0.048, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(27) \quad \begin{aligned} \alpha_0 &= 0.224, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$

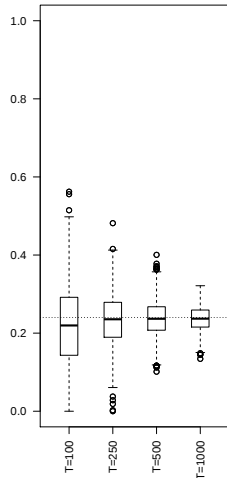


$$(28) \quad \begin{aligned} \alpha_0 &= 0.128, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$

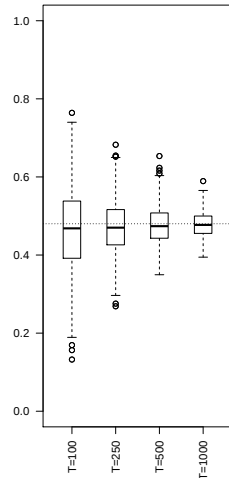


Estimation of α_1 :

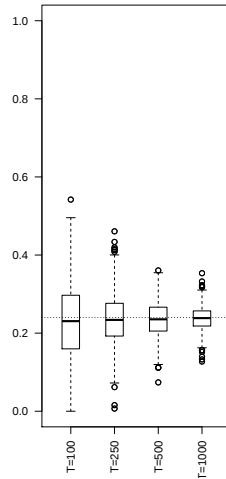
$$(25) \quad \begin{aligned} \alpha_0 &= 0.084, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



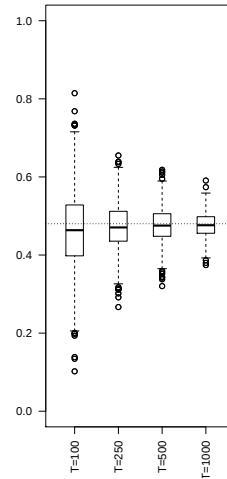
$$(26) \quad \begin{aligned} \alpha_0 &= 0.048, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(27) \quad \begin{aligned} \alpha_0 &= 0.224, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$

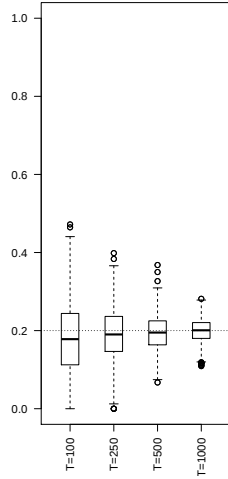


$$(28) \quad \begin{aligned} \alpha_0 &= 0.128, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$

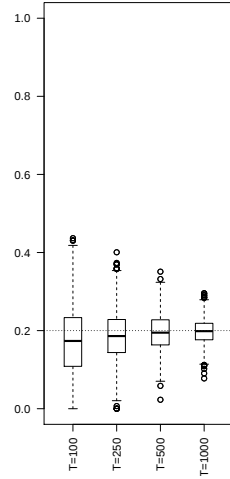


Estimation of α_2 :

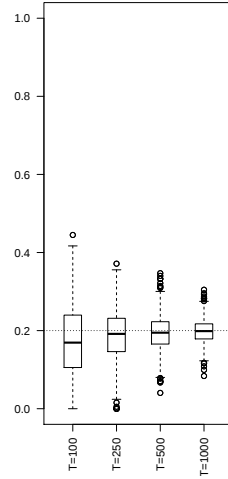
$$(25) \quad \begin{aligned} \alpha_0 &= 0.084, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



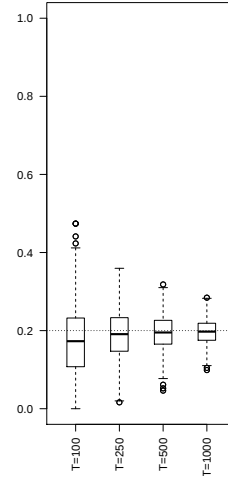
$$(26) \quad \begin{aligned} \alpha_0 &= 0.048, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(27) \quad \begin{aligned} \alpha_0 &= 0.224, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(28) \quad \begin{aligned} \alpha_0 &= 0.128, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



S.3 Properties of (Mis)Fitted Models

The six candidate models were fitted in each simulation run, and for each fitted model, the corresponding theoretical mean, BID, and ACF at lags 1–3 were computed. The following tables display the mean quantities for the generated 1,000 replications.

Scenarios 1 & 2:		True DGP is BAR(1) with mean $\mu = 1.5$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.000</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>1.500</i>	<i>1.000</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	1.504	1	0.290	0.093	0.032	1.506	1	0.589	0.351	0.212
	BAR(2)	1.504	1	0.287	0.117	0.046	1.506	1	0.587	0.368	0.231
	bvAR(1)	1.511	1.214	0.184	0.044	0.012	1.498	1.819	0.479	0.243	0.129
	bvAR(2)	1.513	1.227	0.181	0.058	0.016	1.498	1.833	0.478	0.249	0.135
	BARCH(1)	1.504	1.091	0.283	0.090	0.031	1.504	1.455	0.572	0.335	0.200
	BARCH(2)	1.504	1.095	0.283	0.118	0.046	1.504	1.461	0.572	0.356	0.222
250	BAR(1)	1.499	1	0.295	0.090	0.029	1.498	1	0.597	0.358	0.215
	BAR(2)	1.499	1	0.295	0.110	0.039	1.498	1	0.596	0.369	0.228
	bvAR(1)	1.504	1.216	0.193	0.041	0.010	1.492	1.856	0.504	0.259	0.136
	bvAR(2)	1.504	1.223	0.191	0.049	0.012	1.492	1.859	0.504	0.261	0.137
	BARCH(1)	1.499	1.089	0.293	0.089	0.028	1.498	1.473	0.590	0.351	0.211
	BARCH(2)	1.499	1.091	0.293	0.110	0.040	1.498	1.475	0.590	0.366	0.227
500	BAR(1)	1.500	1	0.300	0.091	0.028	1.497	1	0.598	0.358	0.215
	BAR(2)	1.500	1	0.299	0.104	0.036	1.497	1	0.597	0.365	0.224
	bvAR(1)	1.506	1.222	0.200	0.042	0.009	1.490	1.856	0.509	0.261	0.135
	bvAR(2)	1.506	1.225	0.199	0.045	0.010	1.490	1.857	0.508	0.261	0.136
	BARCH(1)	1.500	1.090	0.298	0.091	0.028	1.497	1.474	0.594	0.355	0.212
	BARCH(2)	1.500	1.091	0.298	0.104	0.036	1.497	1.475	0.594	0.366	0.225
1000	BAR(1)	1.501	1	0.299	0.090	0.027	1.501	1	0.599	0.359	0.216
	BAR(2)	1.501	1	0.298	0.099	0.033	1.501	1	0.599	0.365	0.222
	bvAR(1)	1.507	1.220	0.199	0.041	0.009	1.493	1.865	0.513	0.265	0.137
	bvAR(2)	1.507	1.221	0.199	0.042	0.009	1.493	1.864	0.513	0.265	0.137
	BARCH(1)	1.501	1.088	0.298	0.090	0.027	1.501	1.478	0.598	0.358	0.215
	BARCH(2)	1.501	1.089	0.298	0.100	0.033	1.501	1.478	0.598	0.365	0.223

Scenarios 3 & 4:		True DGP is BAR(1) with mean $\mu = 4.0$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.000</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>4.000</i>	<i>1.000</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	4.002	1	0.292	0.094	0.033	4.000	1	0.595	0.358	0.217
	BAR(2)	4.002	1	0.289	0.119	0.048	4.000	1	0.594	0.374	0.235
	bvAR(1)	3.999	1.224	0.189	0.049	0.014	3.973	2.323	0.587	0.366	0.239
	bvAR(2)	3.999	1.238	0.185	0.062	0.019	3.989	2.420	0.592	0.383	0.258
	BARCH(1)	4.002	1.091	0.283	0.090	0.031	3.999	1.456	0.574	0.337	0.201
	BARCH(2)	4.002	1.095	0.283	0.118	0.046	3.999	1.460	0.574	0.356	0.222
250	BAR(1)	3.998	1	0.294	0.090	0.028	4.002	1	0.596	0.356	0.214
	BAR(2)	3.998	1	0.294	0.108	0.038	4.002	1	0.595	0.367	0.226
	bvAR(1)	3.994	1.223	0.196	0.044	0.011	3.971	2.260	0.603	0.372	0.234
	bvAR(2)	3.995	1.229	0.195	0.051	0.013	3.976	2.297	0.606	0.381	0.245
	BARCH(1)	3.998	1.088	0.292	0.089	0.028	4.002	1.466	0.588	0.348	0.208
	BARCH(2)	3.998	1.090	0.292	0.108	0.039	4.002	1.469	0.588	0.362	0.224
500	BAR(1)	4.003	1	0.298	0.090	0.028	3.999	1	0.599	0.359	0.216
	BAR(2)	4.003	1	0.298	0.103	0.035	3.999	1	0.598	0.367	0.225
	bvAR(1)	4.000	1.227	0.203	0.044	0.010	3.965	2.276	0.615	0.382	0.240
	bvAR(2)	4.000	1.231	0.202	0.048	0.012	3.967	2.296	0.617	0.388	0.246
	BARCH(1)	4.003	1.088	0.296	0.090	0.028	3.999	1.475	0.595	0.355	0.213
	BARCH(2)	4.003	1.089	0.296	0.104	0.036	3.999	1.476	0.595	0.366	0.225
1000	BAR(1)	4.004	1	0.299	0.090	0.027	4.003	1	0.599	0.359	0.216
	BAR(2)	4.003	1	0.299	0.100	0.033	4.003	1	0.599	0.365	0.222
	bvAR(1)	4.000	1.226	0.204	0.043	0.009	3.972	2.264	0.617	0.382	0.238
	bvAR(2)	4.000	1.228	0.203	0.045	0.010	3.973	2.274	0.618	0.385	0.241
	BARCH(1)	4.003	1.088	0.298	0.090	0.027	4.003	1.474	0.596	0.356	0.213
	BARCH(2)	4.003	1.088	0.298	0.100	0.034	4.003	1.474	0.596	0.363	0.222

Scenarios 5 & 6:		True DGP is bvAR(1) with mean $\mu = 1.5$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.370</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>1.500</i>	<i>2.174</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	1.496	1	0.234	0.063	0.019	1.501	1	0.457	0.215	0.104
	BAR(2)	1.498	1	0.232	0.088	0.032	1.510	1	0.457	0.245	0.130
	bvAR(1)	1.502	1.358	0.281	0.089	0.030	1.506	2.163	0.584	0.348	0.211
	bvAR(2)	1.504	1.383	0.279	0.109	0.040	1.511	2.200	0.583	0.361	0.226
	BARCH(1)	1.496	1.089	0.272	0.086	0.030	1.492	1.452	0.563	0.327	0.196
	BARCH(2)	1.496	1.092	0.272	0.113	0.044	1.493	1.457	0.563	0.345	0.214
250	BAR(1)	1.502	1	0.245	0.064	0.017	1.504	1	0.471	0.224	0.108
	BAR(2)	1.502	1	0.245	0.087	0.029	1.513	1	0.470	0.254	0.135
	bvAR(1)	1.503	1.366	0.293	0.090	0.028	1.506	2.165	0.593	0.354	0.213
	bvAR(2)	1.504	1.381	0.292	0.102	0.035	1.507	2.188	0.593	0.362	0.223
	BARCH(1)	1.502	1.089	0.289	0.089	0.029	1.502	1.466	0.584	0.345	0.207
	BARCH(2)	1.501	1.091	0.289	0.108	0.039	1.501	1.469	0.584	0.359	0.222
500	BAR(1)	1.501	1	0.248	0.063	0.017	1.502	1	0.476	0.228	0.110
	BAR(2)	1.502	1	0.248	0.084	0.027	1.510	1	0.476	0.258	0.138
	bvAR(1)	1.502	1.367	0.296	0.089	0.028	1.501	2.167	0.596	0.356	0.214
	bvAR(2)	1.502	1.378	0.295	0.098	0.033	1.502	2.182	0.596	0.362	0.220
	BARCH(1)	1.501	1.088	0.294	0.089	0.028	1.503	1.475	0.592	0.353	0.212
	BARCH(2)	1.501	1.089	0.294	0.104	0.036	1.502	1.476	0.592	0.363	0.223
1000	BAR(1)	1.501	1	0.251	0.064	0.016	1.502	1	0.478	0.229	0.110
	BAR(2)	1.501	1	0.251	0.082	0.026	1.510	1	0.478	0.259	0.139
	bvAR(1)	1.500	1.370	0.299	0.090	0.027	1.502	2.170	0.598	0.358	0.215
	bvAR(2)	1.500	1.377	0.298	0.096	0.031	1.502	2.179	0.598	0.361	0.219
	BARCH(1)	1.501	1.089	0.298	0.090	0.028	1.501	1.477	0.596	0.356	0.214
	BARCH(2)	1.501	1.089	0.298	0.101	0.034	1.501	1.477	0.596	0.363	0.222

Scenarios 7 & 8:		True DGP is bvAR(1) with mean $\mu = 4.0$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.370</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>4.000</i>	<i>2.174</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	4.004	1	0.236	0.064	0.019	3.999	1	0.461	0.219	0.107
	BAR(2)	4.003	1	0.234	0.090	0.033	4.003	1	0.459	0.248	0.133
	bvAR(1)	4.004	1.357	0.280	0.089	0.030	4.000	2.162	0.581	0.346	0.211
	bvAR(2)	4.003	1.376	0.279	0.103	0.038	4.003	2.190	0.581	0.356	0.221
	BARCH(1)	4.004	1.091	0.276	0.089	0.031	4.002	1.468	0.568	0.333	0.201
	BARCH(2)	4.003	1.095	0.276	0.117	0.046	4.004	1.463	0.567	0.351	0.220
250	BAR(1)	4.007	1	0.247	0.064	0.017	3.995	1	0.472	0.225	0.109
	BAR(2)	4.008	1	0.248	0.089	0.030	3.998	1	0.471	0.254	0.135
	bvAR(1)	4.004	1.367	0.293	0.090	0.029	3.999	2.166	0.592	0.354	0.213
	bvAR(2)	4.005	1.380	0.293	0.100	0.034	4.000	2.183	0.593	0.360	0.220
	BARCH(1)	4.007	1.090	0.292	0.090	0.029	3.994	1.468	0.586	0.347	0.208
	BARCH(2)	4.007	1.092	0.292	0.110	0.040	3.994	1.469	0.585	0.359	0.222
500	BAR(1)	3.997	1	0.250	0.064	0.017	4.004	1	0.478	0.230	0.111
	BAR(2)	3.998	1	0.251	0.087	0.028	4.007	1	0.477	0.259	0.139
	bvAR(1)	3.997	1.368	0.297	0.090	0.028	4.005	2.174	0.597	0.358	0.216
	bvAR(2)	3.997	1.376	0.296	0.096	0.031	4.006	2.185	0.597	0.362	0.220
	BARCH(1)	3.998	1.089	0.296	0.090	0.028	4.004	1.477	0.594	0.355	0.213
	BARCH(2)	3.998	1.090	0.296	0.105	0.037	4.004	1.477	0.593	0.364	0.225
1000	BAR(1)	4.001	1	0.251	0.064	0.017	4.002	1	0.479	0.231	0.111
	BAR(2)	4.001	1	0.252	0.086	0.027	4.005	1	0.478	0.260	0.139
	bvAR(1)	4.001	1.369	0.298	0.090	0.027	4.003	2.172	0.598	0.358	0.215
	bvAR(2)	4.001	1.374	0.298	0.094	0.030	4.003	2.179	0.598	0.361	0.218
	BARCH(1)	4.001	1.089	0.298	0.090	0.028	4.001	1.478	0.597	0.357	0.215
	BARCH(2)	4.001	1.089	0.298	0.102	0.034	4.001	1.478	0.597	0.364	0.223

Scenarios 9 & 10:		True DGP is BARCH(1) with mean $\mu = 1.5$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.088</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>1.500</i>	<i>1.479</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	1.499	1	0.266	0.079	0.025	1.504	1	0.484	0.239	0.120
	BAR(2)	1.499	1	0.263	0.101	0.037	1.506	1	0.482	0.256	0.136
	bvAR(1)	1.498	1.224	0.193	0.047	0.013	1.417	1.807	0.480	0.241	0.125
	bvAR(2)	1.499	1.242	0.189	0.065	0.019	1.419	1.841	0.477	0.258	0.140
	BARCH(1)	1.499	1.091	0.282	0.090	0.031	1.500	1.463	0.575	0.339	0.204
	BARCH(2)	1.499	1.095	0.283	0.117	0.045	1.501	1.468	0.575	0.358	0.223
250	BAR(1)	1.500	1	0.274	0.078	0.023	1.500	1	0.492	0.244	0.122
	BAR(2)	1.500	1	0.274	0.095	0.032	1.500	1	0.491	0.258	0.136
	bvAR(1)	1.496	1.229	0.203	0.045	0.011	1.410	1.811	0.491	0.245	0.124
	bvAR(2)	1.497	1.239	0.200	0.055	0.015	1.411	1.834	0.489	0.257	0.136
	BARCH(1)	1.500	1.090	0.294	0.090	0.029	1.499	1.467	0.588	0.348	0.208
	BARCH(2)	1.500	1.091	0.294	0.110	0.040	1.498	1.470	0.588	0.362	0.224
500	BAR(1)	1.499	1	0.276	0.078	0.022	1.502	1	0.497	0.248	0.124
	BAR(2)	1.499	1	0.276	0.091	0.029	1.503	1	0.495	0.259	0.135
	bvAR(1)	1.495	1.229	0.204	0.044	0.010	1.408	1.822	0.498	0.250	0.126
	bvAR(2)	1.496	1.235	0.203	0.051	0.012	1.408	1.839	0.497	0.259	0.135
	BARCH(1)	1.499	1.089	0.297	0.090	0.028	1.502	1.477	0.596	0.356	0.214
	BARCH(2)	1.499	1.090	0.297	0.105	0.036	1.502	1.479	0.596	0.367	0.226
1000	BAR(1)	1.500	1	0.277	0.078	0.022	1.496	1	0.497	0.248	0.124
	BAR(2)	1.500	1	0.277	0.087	0.027	1.497	1	0.496	0.258	0.134
	bvAR(1)	1.496	1.230	0.207	0.044	0.009	1.402	1.823	0.500	0.251	0.127
	bvAR(2)	1.496	1.234	0.206	0.048	0.011	1.402	1.835	0.499	0.257	0.133
	BARCH(1)	1.500	1.089	0.298	0.090	0.027	1.496	1.477	0.597	0.358	0.215
	BARCH(2)	1.500	1.089	0.299	0.101	0.034	1.496	1.478	0.598	0.366	0.224

Scenarios 11 & 12:		True DGP is BARCH(1) with mean $\mu = 4.0$ and lag-1 ACF									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.088</i>	<i>0.300</i>	<i>0.090</i>	<i>0.027</i>	<i>4.000</i>	<i>1.479</i>	<i>0.600</i>	<i>0.360</i>	<i>0.216</i>
100	BAR(1)	3.998	1	0.269	0.081	0.026	4.004	1	0.490	0.244	0.123
	BAR(2)	3.998	1	0.267	0.103	0.038	4.004	1	0.488	0.263	0.141
	bvAR(1)	3.993	1.236	0.198	0.052	0.015	3.906	2.015	0.537	0.301	0.175
	bvAR(2)	3.994	1.254	0.194	0.068	0.021	3.926	2.147	0.544	0.342	0.217
	BARCH(1)	3.998	1.090	0.281	0.088	0.030	4.004	1.454	0.574	0.336	0.200
	BARCH(2)	3.998	1.093	0.281	0.116	0.044	4.003	1.457	0.574	0.356	0.221
250	BAR(1)	4.001	1	0.277	0.080	0.024	4.004	1	0.496	0.247	0.124
	BAR(2)	4.000	1	0.277	0.098	0.033	4.003	1	0.494	0.261	0.137
	bvAR(1)	3.993	1.241	0.210	0.049	0.012	3.902	2.016	0.551	0.309	0.175
	bvAR(2)	3.994	1.252	0.208	0.060	0.017	3.915	2.131	0.558	0.348	0.217
	BARCH(1)	4.001	1.089	0.294	0.090	0.029	4.004	1.467	0.589	0.349	0.208
	BARCH(2)	4.000	1.091	0.294	0.110	0.040	4.004	1.468	0.588	0.362	0.223
500	BAR(1)	4.002	1	0.278	0.079	0.023	4.002	1	0.499	0.250	0.125
	BAR(2)	4.002	1	0.277	0.091	0.029	4.002	1	0.498	0.261	0.137
	bvAR(1)	3.995	1.241	0.213	0.048	0.011	3.901	2.020	0.557	0.313	0.177
	bvAR(2)	3.995	1.248	0.212	0.055	0.014	3.913	2.134	0.565	0.353	0.219
	BARCH(1)	4.002	1.088	0.296	0.089	0.028	4.002	1.472	0.594	0.354	0.212
	BARCH(2)	4.002	1.089	0.296	0.103	0.036	4.002	1.474	0.594	0.365	0.224
1000	BAR(1)	3.999	1	0.280	0.079	0.022	3.999	1	0.500	0.251	0.126
	BAR(2)	3.999	1	0.279	0.088	0.027	3.999	1	0.499	0.260	0.135
	bvAR(1)	3.992	1.243	0.216	0.048	0.011	3.893	2.025	0.560	0.315	0.178
	bvAR(2)	3.992	1.248	0.215	0.052	0.013	3.905	2.138	0.568	0.356	0.221
	BARCH(1)	3.999	1.088	0.298	0.090	0.027	3.999	1.476	0.597	0.357	0.214
	BARCH(2)	3.999	1.089	0.298	0.100	0.033	3.999	1.477	0.597	0.365	0.223

Scenarios 13 & 14:		True DGP is BAR(2) with $\mu = 1.5$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.000</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>1.500</i>	<i>1.000</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	1.499	1	0.284	0.095	0.034	1.510	1	0.595	0.360	0.221
	BAR(2)	1.497	1	0.282	0.260	0.111	1.509	1	0.589	0.478	0.339
	bvAR(1)	1.506	1.220	0.185	0.048	0.014	1.513	1.862	0.485	0.254	0.140
	bvAR(2)	1.509	1.331	0.163	0.147	0.040	1.514	2.041	0.476	0.329	0.202
	BARCH(1)	1.498	1.094	0.276	0.091	0.033	1.507	1.467	0.568	0.334	0.202
	BARCH(2)	1.497	1.137	0.276	0.252	0.107	1.508	1.526	0.568	0.452	0.316
250	BAR(1)	1.501	1	0.296	0.093	0.031	1.502	1	0.606	0.369	0.226
	BAR(2)	1.501	1	0.294	0.265	0.119	1.502	1	0.598	0.483	0.347
	bvAR(1)	1.509	1.227	0.199	0.046	0.012	1.502	1.889	0.511	0.269	0.144
	bvAR(2)	1.511	1.335	0.175	0.147	0.043	1.502	2.055	0.505	0.341	0.210
	BARCH(1)	1.501	1.090	0.289	0.090	0.029	1.501	1.475	0.588	0.350	0.211
	BARCH(2)	1.501	1.131	0.289	0.261	0.116	1.501	1.528	0.588	0.471	0.335
500	BAR(1)	1.507	1	0.302	0.094	0.030	1.502	1	0.606	0.368	0.224
	BAR(2)	1.507	1	0.300	0.271	0.124	1.502	1	0.598	0.485	0.350
	bvAR(1)	1.515	1.238	0.210	0.047	0.011	1.498	1.886	0.517	0.270	0.143
	bvAR(2)	1.518	1.353	0.186	0.155	0.048	1.498	2.065	0.511	0.349	0.217
	BARCH(1)	1.507	1.091	0.297	0.092	0.029	1.501	1.473	0.592	0.353	0.211
	BARCH(2)	1.507	1.132	0.298	0.269	0.123	1.501	1.525	0.592	0.477	0.343
1000	BAR(1)	1.499	1	0.304	0.094	0.029	1.500	1	0.606	0.368	0.224
	BAR(2)	1.500	1	0.300	0.271	0.124	1.500	1	0.598	0.487	0.352
	bvAR(1)	1.507	1.233	0.209	0.045	0.010	1.495	1.887	0.520	0.271	0.143
	bvAR(2)	1.510	1.345	0.185	0.152	0.047	1.495	2.072	0.514	0.353	0.222
	BARCH(1)	1.499	1.089	0.298	0.091	0.028	1.500	1.474	0.595	0.355	0.213
	BARCH(2)	1.499	1.129	0.298	0.269	0.123	1.500	1.526	0.595	0.482	0.348

Scenarios 15 & 16:		True DGP is BAR(2) with $\mu = 4.0$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.000</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>4.000</i>	<i>1.000</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	3.994	1	0.290	0.097	0.036	3.993	1	0.593	0.357	0.218
	BAR(2)	3.994	1	0.289	0.261	0.114	3.991	1	0.589	0.475	0.336
	bvAR(1)	3.990	1.238	0.193	0.055	0.018	3.970	2.349	0.566	0.353	0.235
	bvAR(2)	3.990	1.362	0.177	0.152	0.049	3.974	2.939	0.591	0.468	0.349
	BARCH(1)	3.994	1.095	0.278	0.092	0.033	3.992	1.439	0.558	0.322	0.191
	BARCH(2)	3.993	1.135	0.277	0.247	0.107	3.992	1.492	0.558	0.440	0.303
250	BAR(1)	3.994	1	0.294	0.092	0.030	3.995	1	0.604	0.366	0.224
	BAR(2)	3.994	1	0.294	0.266	0.119	3.995	1	0.598	0.485	0.349
	bvAR(1)	3.990	1.233	0.201	0.048	0.013	3.961	2.333	0.612	0.385	0.249
	bvAR(2)	3.990	1.354	0.183	0.153	0.047	3.973	2.888	0.642	0.508	0.384
	BARCH(1)	3.994	1.088	0.287	0.088	0.029	3.995	1.468	0.585	0.347	0.208
	BARCH(2)	3.994	1.128	0.287	0.259	0.115	3.994	1.519	0.585	0.469	0.334
500	BAR(1)	3.998	1	0.298	0.091	0.029	4.000	1	0.604	0.366	0.223
	BAR(2)	3.998	1	0.296	0.270	0.122	4.002	1	0.599	0.487	0.352
	bvAR(1)	3.994	1.235	0.207	0.047	0.011	3.972	2.309	0.618	0.388	0.247
	bvAR(2)	3.994	1.359	0.187	0.158	0.050	3.983	2.852	0.650	0.518	0.391
	BARCH(1)	3.998	1.088	0.293	0.089	0.028	4.000	1.474	0.593	0.353	0.212
	BARCH(2)	3.998	1.128	0.293	0.266	0.120	4.000	1.525	0.592	0.478	0.344
1000	BAR(1)	4.001	1	0.302	0.092	0.029	3.996	1	0.605	0.366	0.222
	BAR(2)	4.000	1	0.300	0.272	0.125	3.996	1	0.599	0.487	0.353
	bvAR(1)	3.998	1.240	0.213	0.047	0.011	3.966	2.295	0.622	0.389	0.245
	bvAR(2)	3.998	1.368	0.193	0.161	0.052	3.976	2.823	0.655	0.523	0.395
	BARCH(1)	4.001	1.089	0.298	0.090	0.028	3.996	1.474	0.595	0.356	0.213
	BARCH(2)	4.001	1.129	0.298	0.270	0.123	3.996	1.527	0.595	0.483	0.349

Scenarios 17 & 18:		True DGP is bvAR(2) with $\mu = 1.5$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.656</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>1.500</i>	<i>2.577</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	1.497	1	0.212	0.056	0.017	1.498	1	0.426	0.192	0.091
	BAR(2)	1.507	1	0.235	0.216	0.083	1.525	1	0.442	0.336	0.201
	bvAR(1)	1.510	1.396	0.301	0.102	0.037	1.554	2.169	0.580	0.346	0.212
	bvAR(2)	1.496	1.634	0.279	0.261	0.111	1.507	2.515	0.573	0.461	0.324
	BARCH(1)	1.497	1.093	0.268	0.089	0.033	1.501	1.439	0.542	0.309	0.184
	BARCH(2)	1.496	1.140	0.270	0.250	0.105	1.484	1.483	0.542	0.423	0.286
250	BAR(1)	1.494	1	0.222	0.054	0.014	1.515	1	0.452	0.209	0.099
	BAR(2)	1.506	1	0.246	0.223	0.089	1.543	1	0.465	0.360	0.223
	bvAR(1)	1.507	1.401	0.312	0.102	0.035	1.558	2.200	0.599	0.362	0.221
	bvAR(2)	1.495	1.643	0.292	0.265	0.119	1.508	2.559	0.592	0.478	0.343
	BARCH(1)	1.495	1.088	0.282	0.087	0.029	1.519	1.473	0.580	0.343	0.207
	BARCH(2)	1.494	1.130	0.284	0.257	0.113	1.509	1.523	0.580	0.461	0.326
500	BAR(1)	1.500	1	0.226	0.053	0.013	1.509	1	0.458	0.213	0.100
	BAR(2)	1.511	1	0.249	0.227	0.091	1.540	1	0.472	0.370	0.231
	bvAR(1)	1.512	1.401	0.315	0.101	0.033	1.558	2.200	0.603	0.365	0.222
	bvAR(2)	1.501	1.646	0.294	0.268	0.121	1.506	2.569	0.595	0.484	0.349
	BARCH(1)	1.501	1.087	0.289	0.087	0.027	1.514	1.474	0.588	0.350	0.210
	BARCH(2)	1.500	1.128	0.291	0.264	0.119	1.508	1.527	0.589	0.475	0.340
1000	BAR(1)	1.498	1	0.229	0.054	0.013	1.501	1	0.463	0.215	0.101
	BAR(2)	1.510	1	0.253	0.229	0.092	1.531	1	0.475	0.372	0.234
	bvAR(1)	1.510	1.404	0.318	0.102	0.033	1.549	2.204	0.605	0.367	0.223
	bvAR(2)	1.500	1.650	0.297	0.269	0.123	1.499	2.571	0.598	0.485	0.351
	BARCH(1)	1.499	1.087	0.294	0.088	0.027	1.506	1.477	0.594	0.355	0.213
	BARCH(2)	1.498	1.128	0.295	0.266	0.121	1.500	1.528	0.594	0.481	0.347

Scenarios 19 & 20:		True DGP is bvAR(2) with $\mu = 4.0$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.656</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>4.000</i>	<i>2.577</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	3.995	1	0.213	0.056	0.017	3.995	1	0.442	0.205	0.099
	BAR(2)	4.000	1	0.241	0.218	0.086	4.006	1	0.457	0.348	0.213
	bvAR(1)	3.998	1.372	0.286	0.094	0.034	4.015	2.112	0.565	0.330	0.198
	bvAR(2)	3.993	1.626	0.278	0.255	0.110	4.001	2.528	0.577	0.461	0.326
	BARCH(1)	3.995	1.094	0.269	0.090	0.033	4.009	1.483	0.563	0.332	0.203
	BARCH(2)	3.994	1.135	0.267	0.243	0.101	4.001	1.510	0.558	0.434	0.300
250	BAR(1)	4.003	1	0.228	0.056	0.015	4.004	1	0.456	0.212	0.100
	BAR(2)	4.007	1	0.255	0.231	0.094	4.016	1	0.470	0.368	0.230
	bvAR(1)	4.003	1.384	0.302	0.096	0.032	4.017	2.102	0.575	0.335	0.197
	bvAR(2)	4.000	1.648	0.293	0.266	0.120	3.998	2.554	0.588	0.477	0.342
	BARCH(1)	4.003	1.093	0.290	0.092	0.031	4.007	1.481	0.586	0.350	0.212
	BARCH(2)	4.003	1.133	0.289	0.260	0.116	4.001	1.521	0.581	0.466	0.331
500	BAR(1)	4.003	1	0.230	0.055	0.014	4.006	1	0.465	0.218	0.103
	BAR(2)	4.008	1	0.258	0.234	0.096	4.018	1	0.478	0.374	0.235
	bvAR(1)	4.003	1.380	0.303	0.094	0.030	4.021	2.122	0.584	0.343	0.202
	bvAR(2)	4.000	1.646	0.295	0.267	0.121	4.007	2.570	0.596	0.483	0.349
	BARCH(1)	4.004	1.089	0.293	0.090	0.028	4.007	1.491	0.597	0.360	0.219
	BARCH(2)	4.004	1.128	0.292	0.263	0.119	4.005	1.530	0.592	0.477	0.343
1000	BAR(1)	3.999	1	0.234	0.056	0.014	3.999	1	0.467	0.219	0.103
	BAR(2)	4.004	1	0.262	0.239	0.099	4.012	1	0.480	0.378	0.239
	bvAR(1)	4.000	1.384	0.307	0.095	0.030	4.016	2.121	0.586	0.344	0.202
	bvAR(2)	3.998	1.657	0.299	0.272	0.125	4.001	2.576	0.599	0.486	0.352
	BARCH(1)	3.999	1.091	0.300	0.092	0.029	4.000	1.491	0.601	0.363	0.220
	BARCH(2)	3.999	1.131	0.299	0.271	0.125	3.998	1.530	0.596	0.483	0.349

Scenarios 21 & 22:		True DGP is BARCH(2) with $\mu = 1.5$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>1.500</i>	<i>1.128</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>1.500</i>	<i>1.532</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	1.508	1	0.249	0.074	0.024	1.494	1	0.471	0.229	0.114
	BAR(2)	1.505	1	0.233	0.216	0.078	1.490	1	0.454	0.326	0.191
	bvAR(1)	1.504	1.220	0.186	0.047	0.014	1.413	1.789	0.468	0.233	0.122
	bvAR(2)	1.504	1.326	0.161	0.147	0.039	1.410	1.980	0.453	0.323	0.193
	BARCH(1)	1.508	1.090	0.268	0.087	0.031	1.491	1.444	0.558	0.323	0.193
	BARCH(2)	1.506	1.133	0.268	0.250	0.106	1.492	1.503	0.561	0.443	0.308
250	BAR(1)	1.498	1	0.265	0.075	0.022	1.492	1	0.481	0.234	0.116
	BAR(2)	1.496	1	0.249	0.223	0.087	1.489	1	0.463	0.335	0.201
	bvAR(1)	1.494	1.235	0.205	0.048	0.012	1.398	1.810	0.488	0.244	0.124
	bvAR(2)	1.495	1.344	0.178	0.153	0.045	1.394	2.008	0.473	0.338	0.206
	BARCH(1)	1.498	1.090	0.289	0.089	0.029	1.491	1.456	0.578	0.340	0.202
	BARCH(2)	1.498	1.132	0.290	0.264	0.119	1.490	1.514	0.581	0.466	0.331
500	BAR(1)	1.503	1	0.268	0.074	0.021	1.506	1	0.487	0.238	0.117
	BAR(2)	1.501	1	0.252	0.223	0.089	1.501	1	0.467	0.342	0.208
	bvAR(1)	1.497	1.237	0.210	0.047	0.011	1.416	1.817	0.495	0.248	0.125
	bvAR(2)	1.497	1.349	0.182	0.155	0.047	1.408	2.028	0.479	0.349	0.215
	BARCH(1)	1.503	1.089	0.294	0.089	0.028	1.506	1.464	0.588	0.348	0.207
	BARCH(2)	1.502	1.129	0.295	0.266	0.121	1.505	1.523	0.591	0.478	0.344
1000	BAR(1)	1.501	1	0.269	0.073	0.020	1.505	1	0.490	0.241	0.119
	BAR(2)	1.499	1	0.252	0.227	0.091	1.500	1	0.470	0.345	0.211
	bvAR(1)	1.496	1.237	0.211	0.046	0.010	1.409	1.824	0.500	0.251	0.127
	bvAR(2)	1.496	1.352	0.182	0.159	0.049	1.402	2.036	0.484	0.353	0.219
	BARCH(1)	1.501	1.087	0.296	0.089	0.027	1.506	1.471	0.594	0.354	0.211
	BARCH(2)	1.501	1.129	0.297	0.272	0.124	1.504	1.529	0.596	0.484	0.350

Scenarios 23 & 24:		True DGP is BARCH(2) with $\mu = 4.0$, $\rho_{\text{part}}(2) = 0.2$ and									
T	Properties	$\rho(1) = 0.3$					$\rho(1) = 0.6$				
		Mean	BID	ACF1	ACF2	ACF3	Mean	BID	ACF1	ACF2	ACF3
	True DGP	<i>4.000</i>	<i>1.128</i>	<i>0.300</i>	<i>0.272</i>	<i>0.125</i>	<i>4.000</i>	<i>1.532</i>	<i>0.600</i>	<i>0.488</i>	<i>0.354</i>
100	BAR(1)	3.993	1	0.258	0.077	0.025	4.000	1	0.478	0.234	0.117
	BAR(2)	3.995	1	0.243	0.212	0.080	3.999	1	0.460	0.326	0.192
	bvAR(1)	3.986	1.244	0.199	0.055	0.018	3.900	1.985	0.521	0.289	0.168
	bvAR(2)	3.988	1.367	0.180	0.157	0.049	3.923	2.412	0.534	0.414	0.285
	BARCH(1)	3.993	1.093	0.278	0.090	0.032	3.998	1.440	0.561	0.324	0.192
	BARCH(2)	3.995	1.134	0.278	0.246	0.108	3.996	1.489	0.560	0.440	0.304
250	BAR(1)	4.001	1	0.266	0.075	0.022	3.991	1	0.486	0.238	0.118
	BAR(2)	4.001	1	0.249	0.221	0.087	3.990	1	0.467	0.337	0.204
	bvAR(1)	3.992	1.248	0.213	0.052	0.014	3.884	1.986	0.541	0.299	0.168
	bvAR(2)	3.992	1.374	0.190	0.163	0.052	3.902	2.427	0.559	0.441	0.308
	BARCH(1)	4.001	1.089	0.289	0.089	0.029	3.990	1.459	0.583	0.343	0.204
	BARCH(2)	4.001	1.130	0.289	0.261	0.117	3.990	1.512	0.583	0.468	0.333
500	BAR(1)	3.992	1	0.269	0.074	0.021	4.001	1	0.491	0.242	0.120
	BAR(2)	3.992	1	0.251	0.223	0.089	3.999	1	0.471	0.342	0.208
	bvAR(1)	3.984	1.249	0.218	0.051	0.013	3.891	2.007	0.552	0.308	0.173
	bvAR(2)	3.983	1.375	0.195	0.164	0.053	3.910	2.468	0.572	0.457	0.323
	BARCH(1)	3.992	1.088	0.293	0.089	0.028	4.001	1.471	0.592	0.352	0.211
	BARCH(2)	3.992	1.128	0.293	0.266	0.120	4.000	1.524	0.592	0.479	0.345
1000	BAR(1)	4.000	1	0.270	0.074	0.020	3.995	1	0.492	0.243	0.120
	BAR(2)	3.999	1	0.252	0.225	0.090	3.994	1	0.472	0.343	0.209
	bvAR(1)	3.993	1.250	0.221	0.050	0.012	3.885	2.004	0.554	0.308	0.172
	bvAR(2)	3.992	1.380	0.197	0.167	0.055	3.903	2.459	0.574	0.459	0.324
	BARCH(1)	4.000	1.088	0.296	0.089	0.027	3.996	1.471	0.594	0.354	0.211
	BARCH(2)	4.000	1.128	0.296	0.270	0.123	3.996	1.523	0.594	0.482	0.347

S.4 Further Analyses: BARCH(p) Model Order Selection

Left part: Numbers of times each of the candidate models BARCH(1), BARCH(2), and BARCH(3) were selected for a given type of DGP out of 1,000 replications, using the information criteria AIC or BIC. Numbers of correct identifications are highlighted in italic font.

Right part: Average distance between the i th model's AIC (BIC) and the respective minimal AIC (BIC).

Scenario	T	Model identification by						Mean of					
		AIC			BIC			AIC _{i} – AIC _{min}			BIC _{i} – BIC _{min}		
		BARCH(1)	BARCH(2)	BARCH(3)	BARCH(1)	BARCH(2)	BARCH(3)	BARCH(1)	BARCH(2)	BARCH(3)	BARCH(1)	BARCH(2)	BARCH(3)
BARCH(1)													
9	100	856	90	54	974	22	4	0.27	1.92	3.55	0.04	4.29	8.53
	250	849	96	55	981	16	3	0.30	1.90	3.60	0.03	5.15	10.37
	500	841	115	44	987	12	1	0.33	1.93	3.62	0.02	5.83	11.73
	1000	845	105	50	986	14	0	0.34	1.92	3.54	0.03	6.52	13.05
10	100	873	82	45	970	25	5	0.28	2.05	3.85	0.05	4.43	8.84
	250	811	139	50	978	20	2	0.36	1.87	3.61	0.04	5.08	10.34
	500	817	123	60	987	13	0	0.36	1.89	3.56	0.02	5.77	11.66
	1000	821	112	67	995	5	0	0.34	1.97	3.69	0.01	6.54	13.16
BARCH(2)													
21	100	382	524	94	659	328	13	2.58	0.73	2.34	1.26	2.01	6.23
	250	73	801	126	267	712	21	8.39	0.37	1.90	5.21	0.72	5.77
	500	4	873	123	30	958	12	19.18	0.23	1.82	14.81	0.08	5.89
	1000	0	857	143	0	994	6	38.86	0.26	1.77	33.71	0.01	6.43
22	100	343	536	121	629	346	25	2.91	0.78	2.33	1.45	1.92	6.07
	250	66	800	134	265	724	11	8.47	0.29	1.93	5.32	0.67	5.84
	500	5	849	146	33	950	17	19.32	0.29	1.83	14.91	0.10	5.85
	1000	0	847	153	0	986	14	39.83	0.30	1.73	34.65	0.02	6.36
BARCH(3)													
25	100	213	363	424	504	332	164	4.67	1.94	1.14	2.24	2.12	3.92
	250	8	223	769	93	455	452	16.63	4.46	0.32	11.11	2.46	1.84
	500	0	39	961	2	203	795	35.68	10.18	0.04	27.75	6.46	0.54
	1000	0	5	995	0	24	976	76.68	21.42	0.01	66.92	16.56	0.06
26	100	125	442	433	383	426	191	6.29	1.76	1.02	3.56	1.63	3.49
	250	5	218	777	50	490	460	19.81	4.57	0.30	14.20	2.48	1.73
	500	0	34	966	0	210	790	45.42	10.59	0.04	37.46	6.85	0.51
	1000	0	2	998	0	28	972	94.49	21.81	0.00	84.73	16.96	0.06

S.5 Further Analyses: Max-BAR(p) Model

Upper part: Numbers of times each of the candidate models BARCH(1), BARCH(2), max-BAR(1), and max-BAR(2) were selected for a given type of DGP out of 1,000 replications, using the information criteria AIC or BIC. Numbers of correct identifications are highlighted in italic font.

Lower part: Average distance between the i th model's AIC (BIC) and the respective minimal AIC (BIC).

Scenario	T	Model identification by AIC				Model identification by BIC			
		BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)	BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)
9	100	<i>812</i>	109	68	11	<i>906</i>	17	75	2
	250	<i>860</i>	129	10	1	<i>973</i>	16	10	1
	500	<i>854</i>	146	0	0	<i>989</i>	11	0	0
	1000	<i>869</i>	131	0	0	<i>994</i>	6	0	0
10	100	<i>850</i>	147	3	0	<i>970</i>	27	3	0
	250	<i>856</i>	144	0	0	<i>987</i>	13	0	0
	500	<i>863</i>	137	0	0	<i>989</i>	11	0	0
	1000	<i>841</i>	159	0	0	<i>983</i>	17	0	0
11	100	<i>783</i>	127	74	16	<i>880</i>	34	85	1
	250	<i>854</i>	140	2	4	<i>972</i>	22	5	1
	500	<i>857</i>	143	0	0	<i>984</i>	16	0	0
	1000	<i>868</i>	132	0	0	<i>997</i>	3	0	0
12	100	<i>874</i>	120	6	0	<i>965</i>	29	6	0
	250	<i>869</i>	131	0	0	<i>985</i>	15	0	0
	500	<i>851</i>	149	0	0	<i>988</i>	12	0	0
	1000	<i>856</i>	144	0	0	<i>991</i>	9	0	0

Scenario	T	Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
		BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)	BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)
9	100	<i>0.37</i>	2.06	7.93	9.83	<i>0.23</i>	4.52	7.79	12.29
	250	<i>0.26</i>	1.83	21.23	23.17	<i>0.06</i>	5.15	21.02	26.48
	500	<i>0.24</i>	1.76	43.70	45.72	<i>0.02</i>	5.75	43.49	49.71
	1000	<i>0.21</i>	1.76	89.29	91.40	<i>0.01</i>	6.47	89.10	96.11
10	100	<i>0.22</i>	1.82	38.54	40.44	<i>0.03</i>	4.24	38.36	42.86
	250	<i>0.24</i>	1.85	98.84	100.79	<i>0.03</i>	5.16	98.63	104.11
	500	<i>0.22</i>	1.83	202.45	204.78	<i>0.02</i>	5.85	202.26	208.80
	1000	<i>0.29</i>	1.81	411.00	413.40	<i>0.03</i>	6.46	410.74	418.05
11	100	<i>0.44</i>	2.01	8.16	9.88	<i>0.24</i>	4.42	7.96	12.28
	250	<i>0.30</i>	1.82	22.51	24.35	<i>0.07</i>	5.10	22.28	27.63
	500	<i>0.25</i>	1.83	46.19	48.15	<i>0.03</i>	5.82	45.96	52.14
	1000	<i>0.18</i>	1.85	95.21	97.27	<i>0.00</i>	6.58	95.04	102.00
12	100	<i>0.23</i>	1.90	42.03	43.95	<i>0.08</i>	4.34	41.87	46.40
	250	<i>0.20</i>	1.85	113.04	114.98	<i>0.02</i>	5.18	112.86	118.32
	500	<i>0.26</i>	1.74	231.43	233.38	<i>0.01</i>	5.71	231.19	237.35
	1000	<i>0.25</i>	1.76	467.23	469.27	<i>0.02</i>	6.44	467.00	473.94

Scenario	T	Model identification by AIC				Model identification by BIC			
		BARC(1)	BARC(2)	max-BAR(1)	max-BAR(2)	BARC(1)	BARC(2)	max-BAR(1)	max-BAR(2)
21	100	311	597	74	18	541	357	93	9
	250	65	932	0	3	262	736	2	0
	500	1	999	0	0	47	952	1	0
	1000	0	1000	0	0	0	1000	0	0
22	100	375	619	6	0	645	349	6	0
	250	74	926	0	0	262	738	0	0
	500	4	996	0	0	45	955	0	0
	1000	0	1000	0	0	1	999	0	0
23	100	343	588	47	22	586	334	73	7
	250	67	930	3	0	254	741	5	0
	500	3	997	0	0	36	964	0	0
	1000	0	1000	0	0	0	1000	0	0
24	100	373	624	2	1	667	330	2	1
	250	62	938	0	0	256	744	0	0
	500	3	997	0	0	33	967	0	0
	1000	0	1000	0	0	0	1000	0	0

BARC(2)		Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
21	100	2.81	0.68	10.31	11.58	1.55	2.02	9.05	12.93
	250	8.40	0.07	30.57	31.82	5.44	0.63	27.61	32.38
	500	18.23	0.00	64.59	65.89	14.10	0.08	60.46	65.97
	1000	39.87	0.00	133.89	135.07	34.96	0.00	128.98	135.07
22	100	2.59	0.59	38.99	40.54	1.36	1.96	37.76	41.91
	250	8.19	0.08	110.06	112.06	5.25	0.66	107.12	112.64
	500	19.13	0.00	227.08	229.23	14.99	0.08	222.94	229.31
	1000	39.77	0.00	466.86	469.14	34.86	0.00	461.96	469.14
23	100	2.81	0.73	11.50	12.87	1.58	2.11	10.28	14.25
	250	8.35	0.09	31.90	33.42	5.39	0.65	28.94	33.99
	500	18.53	0.00	67.12	68.60	14.38	0.07	62.97	68.67
	1000	39.93	0.00	139.40	141.05	35.02	0.00	134.50	141.05
24	100	2.55	0.55	44.67	46.33	1.34	1.94	43.46	47.72
	250	8.55	0.07	126.29	128.20	5.55	0.59	123.29	128.72
	500	19.03	0.00	263.16	265.15	14.87	0.06	259.00	265.20
	1000	39.53	0.00	530.40	532.47	34.62	0.00	525.50	532.47

max-BAR(1)		Model identification by AIC				Model identification by BIC			
Scenario	T	BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)	BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)
27	100	70	9	<i>743</i>	178	76	2	<i>875</i>	47
	250	24	2	<i>819</i>	155	26	0	<i>952</i>	22
	500	1	0	<i>856</i>	143	1	0	<i>984</i>	15
	1000	0	0	<i>830</i>	170	0	0	<i>992</i>	8
28	100	3	0	<i>817</i>	180	3	0	<i>953</i>	44
	250	0	0	<i>781</i>	219	0	0	<i>960</i>	40
	500	0	0	<i>799</i>	201	0	0	<i>971</i>	29
	1000	0	0	<i>818</i>	182	0	0	<i>982</i>	18
29	100	133	38	<i>678</i>	151	160	7	<i>800</i>	33
	250	114	28	<i>681</i>	177	136	2	<i>839</i>	23
	500	56	10	<i>766</i>	168	65	0	<i>921</i>	14
	1000	11	4	<i>788</i>	197	13	1	<i>972</i>	14
30	100	25	3	<i>801</i>	171	30	1	<i>923</i>	46
	250	1	1	<i>815</i>	183	1	0	<i>962</i>	37
	500	0	0	<i>819</i>	181	0	0	<i>977</i>	23
	1000	0	0	<i>790</i>	210	0	0	<i>975</i>	25

max-BAR(1)		Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
27	100	6.07	7.74	<i>0.50</i>	1.80	5.81	10.09	<i>0.24</i>	4.15
	250	12.76	14.47	<i>0.36</i>	1.78	12.51	17.74	<i>0.11</i>	5.05
	500	24.66	26.39	<i>0.26</i>	1.79	24.43	30.38	<i>0.03</i>	5.78
	1000	49.15	50.84	<i>0.30</i>	1.74	48.86	55.46	<i>0.01</i>	6.36
28	100	21.22	23.04	<i>0.36</i>	1.80	20.96	25.39	<i>0.11</i>	4.15
	250	52.43	54.16	<i>0.45</i>	1.65	52.06	57.31	<i>0.08</i>	4.80
	500	103.64	105.34	<i>0.45</i>	1.68	103.27	109.19	<i>0.08</i>	5.53
	1000	205.63	207.38	<i>0.40</i>	1.80	205.25	211.91	<i>0.03</i>	6.33
29	100	2.18	3.81	<i>0.50</i>	1.85	1.91	6.15	<i>0.23</i>	4.19
	250	3.64	5.17	<i>0.60</i>	1.84	3.32	8.37	<i>0.28</i>	5.04
	500	6.22	7.80	<i>0.42</i>	1.66	5.92	11.71	<i>0.12</i>	5.57
	1000	11.80	13.33	<i>0.41</i>	1.62	11.44	17.88	<i>0.05</i>	6.18
30	100	8.77	10.53	<i>0.40</i>	1.78	8.52	12.89	<i>0.15</i>	4.14
	250	21.03	22.77	<i>0.38</i>	1.66	20.72	25.99	<i>0.07</i>	4.88
	500	41.00	42.70	<i>0.37</i>	1.67	40.68	46.59	<i>0.04</i>	5.56
	1000	82.69	84.30	<i>0.49</i>	1.65	82.27	88.79	<i>0.07</i>	6.14

Scenario	max-BAR(2)		Model identification by AIC				Model identification by BIC			
			BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)	BARCH(1)	BARCH(2)	max-BAR(1)	max-BAR(2)
	T									
31	100		25	11	539	<i>425</i>	30	2	798	<i>170</i>
	250		1	3	318	<i>678</i>	3	0	675	<i>322</i>
	500		0	0	102	<i>898</i>	0	0	439	<i>561</i>
	1000		0	0	15	<i>985</i>	0	0	153	<i>847</i>
32	100		5	0	609	<i>386</i>	5	0	843	<i>152</i>
	250		0	0	430	<i>570</i>	0	0	789	<i>211</i>
	500		0	0	239	<i>761</i>	0	0	636	<i>364</i>
	1000		0	0	63	<i>937</i>	0	0	346	<i>654</i>
33	100		105	27	663	<i>205</i>	120	7	819	<i>54</i>
	250		56	19	661	<i>264</i>	67	3	873	<i>57</i>
	500		16	8	560	<i>416</i>	26	1	916	<i>57</i>
	1000		3	1	477	<i>519</i>	3	0	900	<i>97</i>
34	100		33	7	728	<i>232</i>	40	1	901	<i>58</i>
	250		7	0	764	<i>229</i>	8	0	942	<i>50</i>
	500		0	0	704	<i>296</i>	0	0	952	<i>48</i>
	1000		0	0	633	<i>367</i>	0	0	941	<i>59</i>

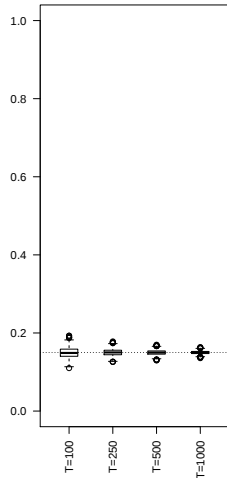
max-BAR(2)		Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
31	100	8.21	9.70	1.20	<i>1.00</i>	7.45	11.55	0.44	<i>2.85</i>
	250	19.93	20.99	2.81	<i>0.48</i>	18.22	22.81	1.11	<i>2.29</i>
	500	39.86	40.44	6.01	<i>0.11</i>	36.72	41.51	2.87	<i>1.18</i>
	1000	79.35	79.15	12.13	<i>0.01</i>	74.75	79.47	7.53	<i>0.33</i>
32	100	17.67	19.32	1.07	<i>1.11</i>	16.99	21.24	0.39	<i>3.03</i>
	250	43.44	44.96	1.85	<i>0.69</i>	42.15	47.19	0.56	<i>2.92</i>
	500	85.45	86.74	3.72	<i>0.36</i>	83.09	88.60	1.36	<i>2.22</i>
	1000	168.92	169.85	7.60	<i>0.08</i>	164.98	170.82	3.67	<i>1.05</i>
33	100	2.25	3.83	0.56	<i>1.55</i>	1.90	6.09	0.21	<i>3.81</i>
	250	4.00	5.46	0.70	<i>1.33</i>	3.48	8.46	0.18	<i>4.33</i>
	500	7.28	8.79	0.98	<i>0.98</i>	6.45	12.17	0.14	<i>4.36</i>
	1000	13.41	14.80	1.55	<i>0.77</i>	12.12	18.41	0.25	<i>4.38</i>
34	100	5.73	7.44	0.47	<i>1.49</i>	5.40	9.72	0.13	<i>3.76</i>
	250	12.43	14.07	0.51	<i>1.41</i>	12.02	17.19	0.11	<i>4.52</i>
	500	24.49	26.13	0.68	<i>1.24</i>	23.93	29.78	0.12	<i>4.89</i>
	1000	47.91	49.48	0.97	<i>1.03</i>	47.07	53.55	0.13	<i>5.10</i>

Boxplots of the ML estimates from the simulation study (1,000 replications) for the respective model parameters. The numbers in parentheses provide the scenario according to Table 4 in the main manuscript.

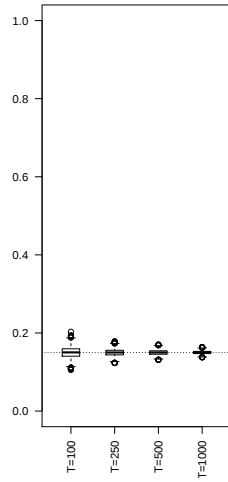
max-BAR(1) model

Estimation of π :

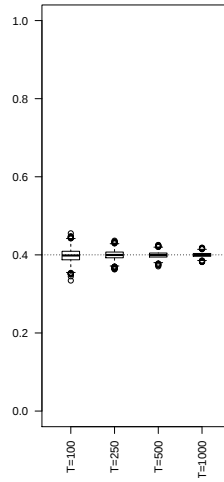
(27) $\pi = 0.15,$
 $\alpha_1 = 0.3$



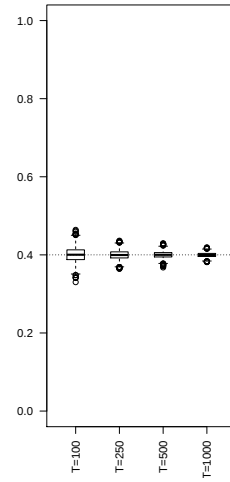
(28) $\pi = 0.15,$
 $\alpha_1 = 0.6$



(29) $\pi = 0.40,$
 $\alpha_1 = 0.3$

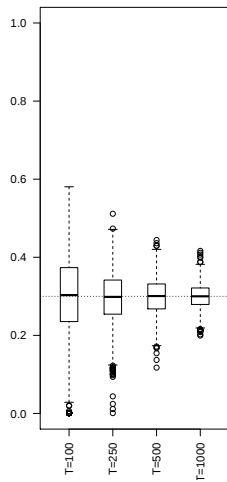


(30) $\pi = 0.40,$
 $\alpha_1 = 0.6$

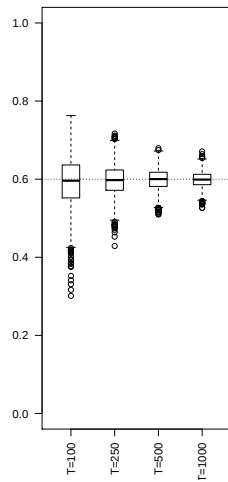


Estimation of α_1 :

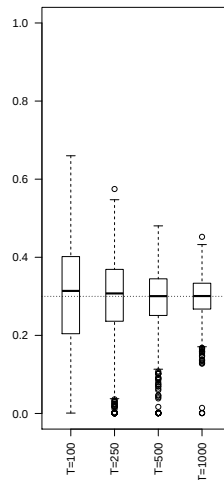
(27) $\pi = 0.15,$
 $\alpha_1 = 0.3$



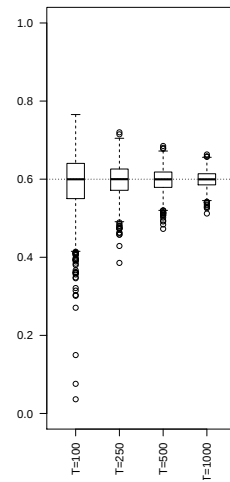
(28) $\pi = 0.15,$
 $\alpha_1 = 0.6$



(29) $\pi = 0.40,$
 $\alpha_1 = 0.3$



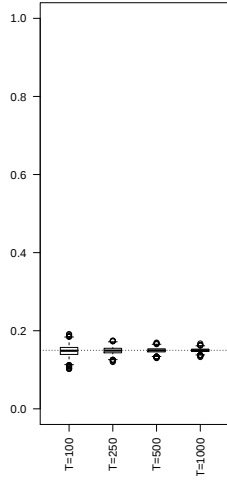
(30) $\pi = 0.40,$
 $\alpha_1 = 0.6$



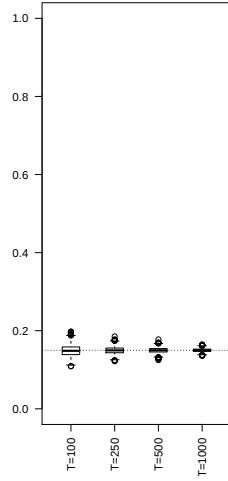
max-AR(2) model

Estimation of π :

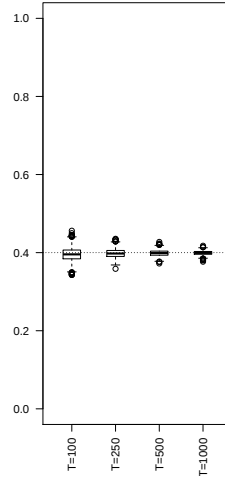
$$(31) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



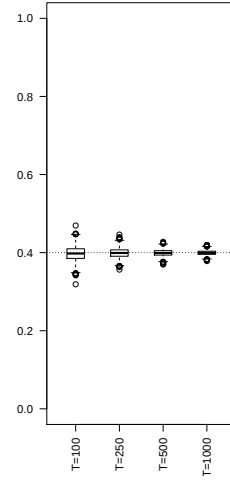
$$(32) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(33) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$

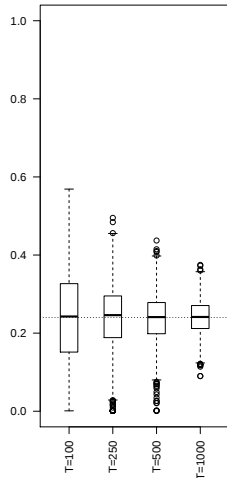


$$(34) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$

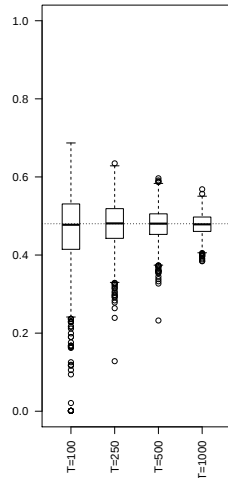


Estimation of α_1 :

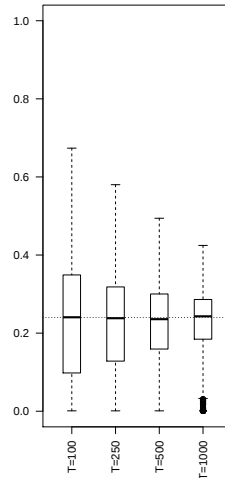
$$(31) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



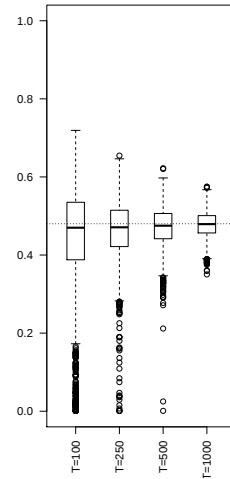
$$(32) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(33) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$

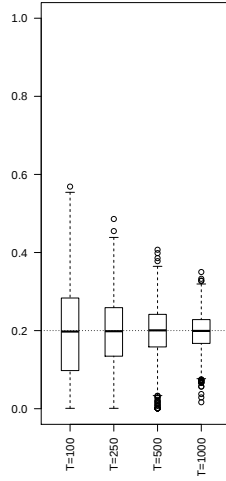


$$(34) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$

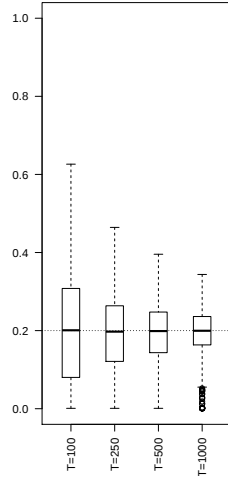


Estimation of α_2 :

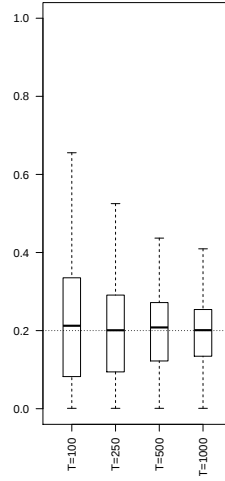
$$(31) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



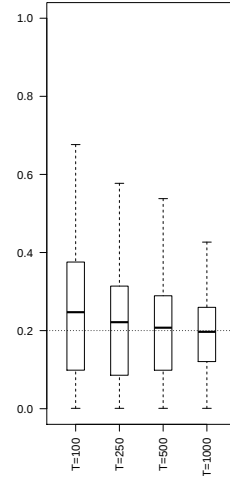
$$(32) \quad \begin{aligned} \pi &= 0.15, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(33) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.24, \\ \alpha_2 &= 0.2 \end{aligned}$$



$$(34) \quad \begin{aligned} \pi &= 0.40, \\ \alpha_1 &= 0.48, \\ \alpha_2 &= 0.2 \end{aligned}$$



S.6 Further Analyses: Logit-BARCH(p) Model

Upper part: Numbers of times each of the candidate models BARCH(1), BARCH(2), logit-BARCH(1), and logit-BARCH(2) were selected for a given type of DGP out of 1,000 replications, using the information criteria AIC or BIC. Numbers of correct identifications are highlighted in italic font.

Lower part: Average distance between the i th model's AIC (BIC) and the respective minimal AIC (BIC).

Scenario	T	Model identification by AIC				Model identification by BIC			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
9	100	<i>487</i>	55	327	131	<i>568</i>	14	381	37
	250	<i>516</i>	74	304	106	<i>618</i>	10	358	14
	500	<i>562</i>	83	240	115	<i>670</i>	8	308	14
	1000	<i>624</i>	73	218	85	<i>713</i>	8	271	8
10	100	<i>708</i>	95	111	86	<i>817</i>	18	142	23
	250	<i>788</i>	113	68	31	<i>903</i>	12	80	5
	500	<i>828</i>	142	20	10	<i>964</i>	12	23	1
	1000	<i>850</i>	148	0	2	<i>993</i>	6	1	0
11	100	<i>390</i>	45	362	203	<i>485</i>	13	452	50
	250	<i>425</i>	60	371	144	<i>514</i>	12	450	24
	500	<i>420</i>	62	372	146	<i>525</i>	2	463	10
	1000	<i>425</i>	73	338	164	<i>544</i>	4	442	10
12	100	<i>474</i>	62	332	132	<i>547</i>	19	406	28
	250	<i>530</i>	71	279	120	<i>626</i>	7	338	29
	500	<i>565</i>	101	233	101	<i>684</i>	9	293	14
	1000	<i>615</i>	101	209	75	<i>727</i>	4	260	9

Scenario	T	Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
9	100	<i>0.54</i>	2.13	0.72	1.73	<i>0.27</i>	4.47	0.46	4.07
	250	<i>0.63</i>	2.19	0.99	1.97	<i>0.33</i>	5.41	0.69	5.19
	500	<i>0.73</i>	2.26	1.41	2.37	<i>0.37</i>	6.12	1.06	6.23
	1000	<i>0.73</i>	2.32	1.94	3.00	<i>0.44</i>	6.94	1.65	7.62
10	100	<i>0.53</i>	2.09	3.09	3.92	<i>0.27</i>	4.44	2.83	6.27
	250	<i>0.38</i>	2.07	6.90	7.85	<i>0.17</i>	5.37	6.68	11.15
	500	<i>0.29</i>	1.94	13.27	14.23	<i>0.08</i>	5.94	13.06	18.24
	1000	<i>0.25</i>	1.75	26.68	27.47	<i>0.01</i>	6.42	26.44	32.14
11	100	<i>0.54</i>	2.08	0.55	1.44	<i>0.18</i>	4.33	0.19	3.68
	250	<i>0.50</i>	2.14	0.52	1.54	<i>0.18</i>	5.34	0.19	4.74
	500	<i>0.49</i>	2.08	0.51	1.57	<i>0.16</i>	5.96	0.18	5.46
	1000	<i>0.65</i>	2.08	0.73	1.62	<i>0.20</i>	6.54	0.28	6.07
12	100	<i>0.62</i>	2.29	0.80	1.84	<i>0.36</i>	4.63	0.53	4.18
	250	<i>0.71</i>	2.32	1.24	2.19	<i>0.40</i>	5.53	0.93	5.40
	500	<i>0.76</i>	2.33	1.87	2.85	<i>0.43</i>	6.23	1.55	6.74
	1000	<i>0.76</i>	2.36	2.78	3.84	<i>0.48</i>	6.98	2.50	8.47

Scenario	BARCH(2)		Model identification by AIC				Model identification by BIC			
	T		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
21	100		220	<i>419</i>	171	190	365	<i>260</i>	266	109
	250		41	<i>652</i>	22	285	165	<i>508</i>	111	216
	500		1	<i>775</i>	1	223	22	<i>751</i>	19	208
	1000		0	<i>817</i>	0	183	0	<i>817</i>	0	183
22	100		303	<i>557</i>	76	64	529	<i>308</i>	134	29
	250		52	<i>910</i>	14	24	244	<i>705</i>	37	14
	500		2	<i>987</i>	0	11	43	<i>946</i>	0	11
	1000		0	<i>998</i>	0	2	0	<i>998</i>	0	2
23	100		195	<i>318</i>	196	291	338	<i>180</i>	331	151
	250		29	<i>480</i>	35	456	125	<i>390</i>	125	360
	500		2	<i>519</i>	1	478	17	<i>507</i>	16	460
	1000		0	<i>570</i>	0	430	0	<i>570</i>	0	430
24	100		202	<i>348</i>	160	290	351	<i>204</i>	293	152
	250		44	<i>574</i>	40	342	164	<i>454</i>	106	276
	500		0	<i>688</i>	0	312	24	<i>664</i>	9	303
	1000		0	<i>773</i>	0	227	0	<i>773</i>	0	227

BARCH(2)		Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
21	100	2.94	<i>0.76</i>	3.15	1.18	1.70	<i>2.12</i>	1.91	2.54
	250	8.40	<i>0.35</i>	8.76	1.18	5.45	<i>0.92</i>	5.81	1.75
	500	19.23	<i>0.34</i>	19.88	1.98	15.10	<i>0.42</i>	15.75	2.06
	1000	39.88	<i>0.35</i>	41.01	3.30	34.97	<i>0.35</i>	36.10	3.30
22	100	2.82	<i>0.78</i>	5.46	3.87	1.60	<i>2.16</i>	4.24	5.26
	250	8.29	<i>0.17</i>	14.80	8.24	5.36	<i>0.75</i>	11.87	8.83
	500	19.11	<i>0.02</i>	32.10	16.29	14.99	<i>0.11</i>	27.97	16.38
	1000	39.13	<i>0.01</i>	63.85	31.83	34.23	<i>0.01</i>	58.94	31.83
23	100	2.67	<i>0.68</i>	2.66	0.67	1.46	<i>2.08</i>	1.45	2.07
	250	8.81	<i>0.21</i>	8.82	0.23	5.84	<i>0.76</i>	5.85	0.79
	500	18.27	<i>0.19</i>	18.29	0.25	14.12	<i>0.25</i>	14.14	0.31
	1000	40.43	<i>0.23</i>	40.53	0.42	35.53	<i>0.23</i>	35.62	0.42
24	100	2.95	<i>0.81</i>	3.09	1.01	1.69	<i>2.16</i>	1.83	2.35
	250	8.51	<i>0.51</i>	8.92	1.05	5.59	<i>1.11</i>	6.00	1.65
	500	19.03	<i>0.43</i>	19.93	1.58	14.86	<i>0.48</i>	15.76	1.63
	1000	40.56	<i>0.39</i>	42.33	2.86	35.65	<i>0.39</i>	37.42	2.86

Scenario	logit-BARCH(1) T	Model identification by AIC				Model identification by BIC			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
35	100	374	44	<i>422</i>	160	439	6	<i>516</i>	39
	250	349	54	<i>450</i>	147	423	6	<i>548</i>	23
	500	337	45	<i>470</i>	148	394	7	<i>577</i>	22
	1000	302	48	<i>497</i>	153	366	0	<i>623</i>	11
36	100	349	63	<i>406</i>	182	427	15	<i>514</i>	44
	250	273	46	<i>506</i>	175	344	4	<i>623</i>	29
	500	209	31	<i>574</i>	186	250	6	<i>724</i>	20
	1000	103	28	<i>682</i>	187	136	1	<i>850</i>	13
37	100	372	50	<i>386</i>	192	468	15	<i>470</i>	47
	250	364	47	<i>417</i>	172	450	7	<i>518</i>	25
	500	353	47	<i>443</i>	157	438	7	<i>541</i>	14
	1000	345	55	<i>429</i>	171	433	3	<i>552</i>	12
38	100	306	39	<i>489</i>	166	353	6	<i>589</i>	52
	250	244	27	<i>557</i>	172	288	3	<i>679</i>	30
	500	166	30	<i>614</i>	190	205	2	<i>770</i>	23
	1000	91	17	<i>701</i>	191	111	1	<i>870</i>	18

Scenario	logit-BARCH(1) T	Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
35	100	0.54	2.19	<i>0.51</i>	1.59	0.28	4.53	<i>0.25</i>	3.94
	250	0.71	2.30	<i>0.60</i>	1.63	0.41	5.53	<i>0.31</i>	4.86
	500	1.04	2.60	<i>0.74</i>	1.72	0.71	6.49	<i>0.41</i>	5.61
	1000	1.32	2.81	<i>0.72</i>	1.70	0.99	7.38	<i>0.39</i>	6.28
36	100	1.03	2.51	<i>0.86</i>	1.74	0.69	4.78	<i>0.52</i>	4.01
	250	1.78	3.33	<i>0.85</i>	1.79	1.43	6.50	<i>0.50</i>	4.96
	500	3.46	4.92	<i>0.81</i>	1.80	3.09	8.77	<i>0.44</i>	5.65
	1000	6.22	7.59	<i>0.64</i>	1.66	5.87	12.15	<i>0.29</i>	6.21
37	100	0.53	2.13	<i>0.52</i>	1.41	0.20	4.40	<i>0.19</i>	3.69
	250	0.51	2.08	<i>0.49</i>	1.50	0.17	5.26	<i>0.15</i>	4.69
	500	0.59	2.10	<i>0.54</i>	1.53	0.24	5.96	<i>0.19</i>	5.40
	1000	0.75	2.22	<i>0.65</i>	1.53	0.33	6.71	<i>0.23</i>	6.01
38	100	1.15	2.76	<i>0.73</i>	1.66	0.86	5.07	<i>0.43</i>	3.97
	250	1.97	3.53	<i>0.76</i>	1.78	1.65	6.73	<i>0.44</i>	4.98
	500	3.47	4.88	<i>0.79</i>	1.75	3.08	8.70	<i>0.40</i>	5.57
	1000	5.69	7.16	<i>0.66</i>	1.70	5.33	11.72	<i>0.30</i>	6.25

Scenario	logit-BARCH(2)	Model identification by AIC				Model identification by BIC			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
39	100	294	162	377	167	391	67	476	66
	250	184	253	252	311	355	91	424	130
	500	90	322	127	461	236	164	348	252
	1000	17	283	26	674	85	208	179	528
40	100	352	108	392	148	429	37	488	46
	250	299	126	360	215	394	36	515	55
	500	167	174	348	311	277	60	573	90
	1000	81	181	201	537	185	71	546	198
41	100	146	330	120	404	260	221	245	274
	250	14	401	13	572	61	364	45	530
	500	0	420	0	580	3	418	1	578
	1000	0	355	0	645	0	355	0	645
42	100	119	223	236	422	212	131	412	245
	250	11	223	33	733	47	186	177	590
	500	0	134	2	864	1	133	24	842
	1000	0	51	0	949	0	51	0	949

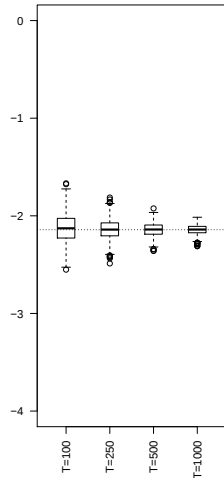
Scenario	logit-BARCH(2)	Mean of $AIC_i - AIC_{\min}$				Mean of $BIC_i - BIC_{\min}$			
		BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)	BARCH(1)	BARCH(2)	logit-BARCH(1)	logit-BARCH(2)
39	100	1.17	1.52	1.09	1.35	0.60	3.55	0.52	3.39
	250	2.46	1.18	2.35	1.04	1.14	3.39	1.03	3.24
	500	5.28	1.10	4.87	0.69	2.77	2.80	2.35	2.39
	1000	11.65	1.84	10.38	0.44	7.41	2.51	6.14	1.11
40	100	0.92	1.95	0.89	1.69	0.53	4.16	0.50	3.91
	250	1.79	2.11	1.30	1.54	1.11	4.94	0.61	4.38
	500	3.38	2.48	2.14	1.30	2.13	5.44	0.89	4.26
	1000	6.17	3.18	3.82	0.87	3.88	5.81	1.54	3.50
41	100	4.03	0.49	4.01	0.45	2.43	1.50	2.41	1.46
	250	13.29	0.28	13.23	0.18	9.98	0.49	9.91	0.39
	500	27.02	0.35	26.95	0.22	22.81	0.35	22.73	0.22
	1000	58.15	0.56	57.98	0.26	53.24	0.56	53.07	0.26
42	100	4.27	1.70	3.29	0.90	2.97	3.01	1.99	2.20
	250	12.48	2.76	9.50	0.45	9.39	3.20	6.41	0.89
	500	27.76	5.42	21.08	0.31	23.60	5.47	16.92	0.36
	1000	56.77	10.42	43.51	0.15	51.86	10.42	38.60	0.15

Boxplots of the ML estimates from the simulation study (1,000 replications) for the respective model parameters. The numbers in parentheses provide the scenario according to Table 5 in the main manuscript.

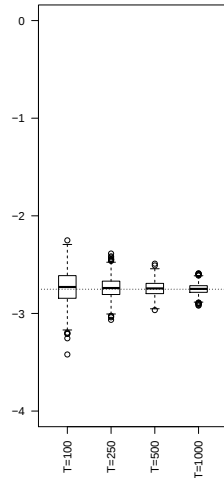
logit-BARCH(1) model

Estimation of r_0 :

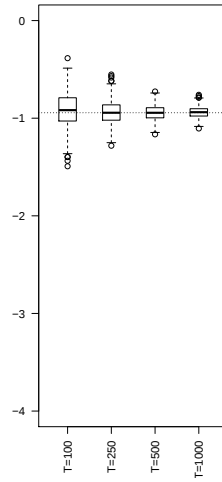
$$(35) \quad r_0 = -2.14, \\ r_1 = 0.20$$



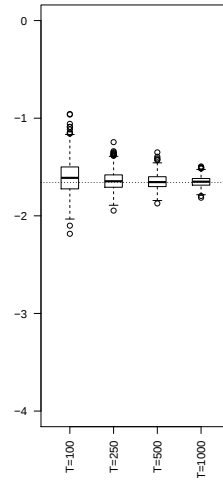
$$(36) \quad r_0 = -2.75, \\ r_1 = 0.40$$



$$(37) \quad r_0 = -0.94, \\ r_1 = 0.13$$

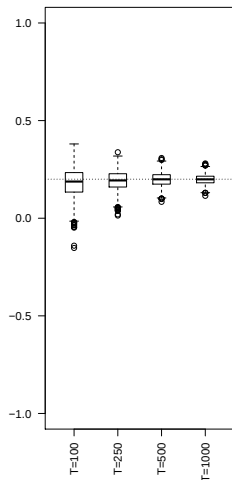


$$(38) \quad r_0 = -1.66, \\ r_1 = 0.30$$

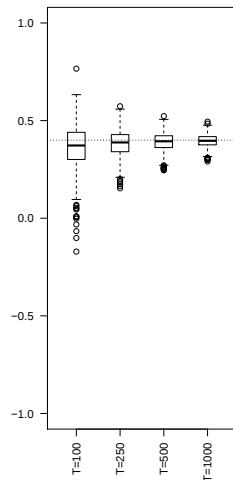


Estimation of r_1 :

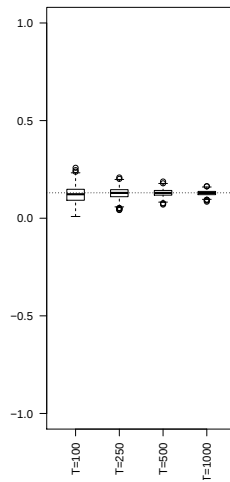
$$(35) \quad r_0 = -2.14, \\ r_1 = 0.20$$



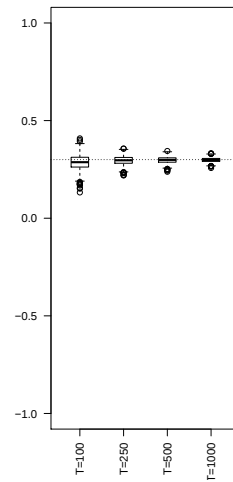
$$(36) \quad r_0 = -2.75, \\ r_1 = 0.40$$



$$(37) \quad r_0 = -0.94, \\ r_1 = 0.13$$



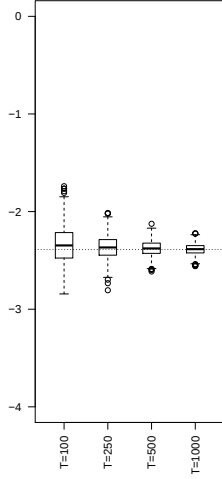
$$(38) \quad r_0 = -1.66, \\ r_1 = 0.30$$



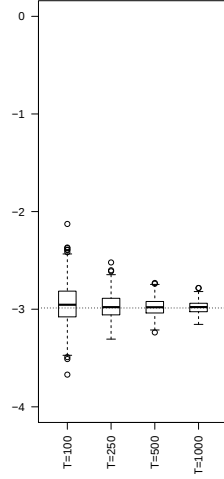
logit-BARCH(2) model

Estimation of r_0 :

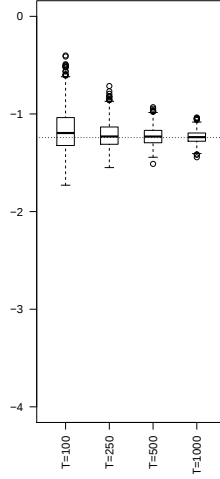
$$(39) \quad \begin{aligned} r_0 &= -2.39, \\ r_1 &= 0.20, \\ r_2 &= 0.1 \end{aligned}$$



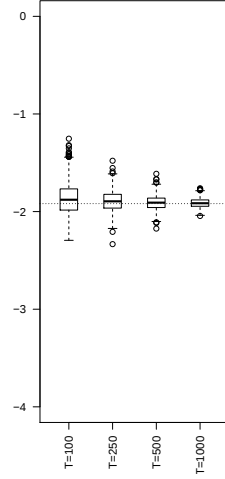
$$(40) \quad \begin{aligned} r_0 &= -2.99, \\ r_1 &= 0.35, \\ r_2 &= 0.1 \end{aligned}$$



$$(41) \quad \begin{aligned} r_0 &= -1.24, \\ r_1 &= 0.11, \\ r_2 &= 0.1 \end{aligned}$$

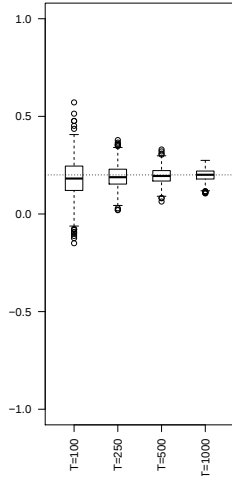


$$(42) \quad \begin{aligned} r_0 &= -1.92, \\ r_1 &= 0.25, \\ r_2 &= 0.1 \end{aligned}$$

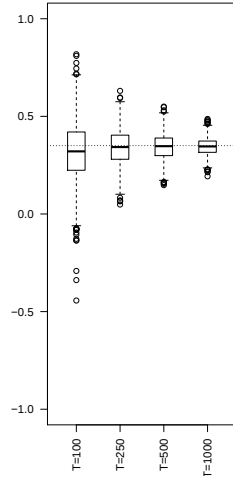


Estimation of r_1 :

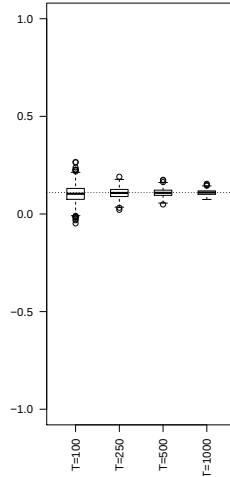
$$(39) \quad \begin{aligned} r_0 &= -2.39, \\ r_1 &= 0.20, \\ r_2 &= 0.1 \end{aligned}$$



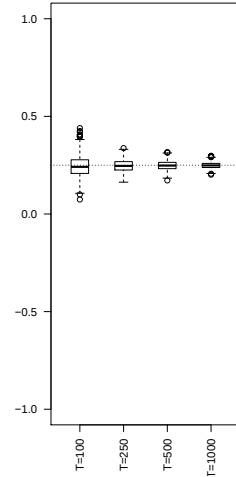
$$(40) \quad \begin{aligned} r_0 &= -2.99, \\ r_1 &= 0.35, \\ r_2 &= 0.1 \end{aligned}$$



$$(41) \quad \begin{aligned} r_0 &= -1.24, \\ r_1 &= 0.11, \\ r_2 &= 0.1 \end{aligned}$$

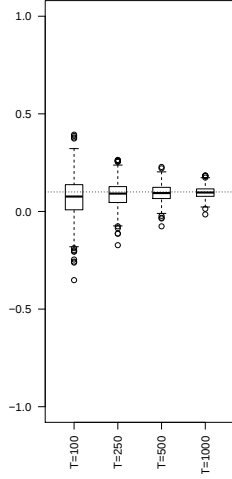


$$(42) \quad \begin{aligned} r_0 &= -1.92, \\ r_1 &= 0.25, \\ r_2 &= 0.1 \end{aligned}$$

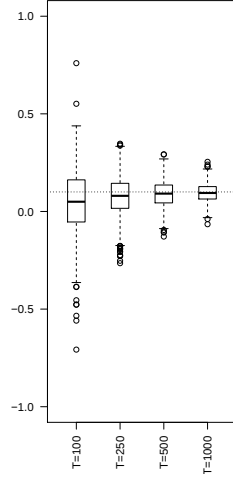


Estimation of r_2 :

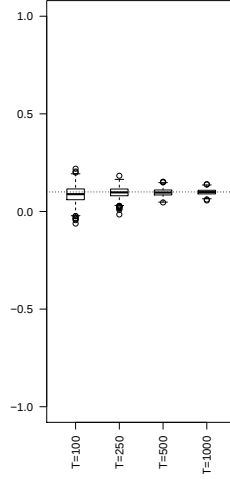
$$(39) \quad \begin{aligned} r_0 &= -2.39, \\ r_1 &= 0.20, \\ r_2 &= 0.1 \end{aligned}$$



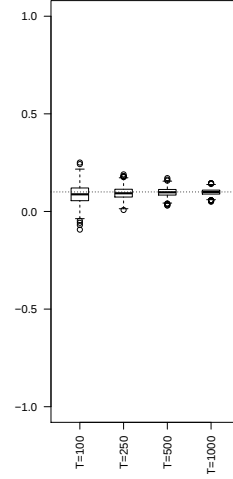
$$(40) \quad \begin{aligned} r_0 &= -2.99, \\ r_1 &= 0.35, \\ r_2 &= 0.1 \end{aligned}$$



$$(41) \quad \begin{aligned} r_0 &= -1.24, \\ r_1 &= 0.11, \\ r_2 &= 0.1 \end{aligned}$$



$$(42) \quad \begin{aligned} r_0 &= -1.92, \\ r_1 &= 0.25, \\ r_2 &= 0.1 \end{aligned}$$



S.7 Real-Data Application: Access Counts

(Conditional) ML estimates for the AR-type models fitted to the access count data, together with the corresponding maximized log-likelihood values ($\ell_{\max}$):

DGP	Parameter estimates			$\ell_{\max}$	
BAR(1)	$\hat{\alpha}$	$\hat{\beta}$		614.8	
	0.329	0.068			
bvAR(1)	$\hat{\pi}$	$\hat{\phi}_1$		623.0	
	0.094	0.219			
BARCH(1)	$\hat{\alpha}_0$	$\hat{\alpha}_1$		615.3	
	0.068	0.263			
max-BAR(1)	$\hat{\pi}$	$\hat{\alpha}_1$		624.7	
	0.078	0.254			
logit-BARCH(1)	$\hat{r}_0$	$\hat{r}_1$		615.5	
	-2.588	0.458			
BAR(2)	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\phi}_1$	609.5	
	0.404	0.061	0.638		
bvAR(2)	$\hat{\pi}$	$\hat{\phi}_1$	$\hat{\phi}_2$	621.3	
	0.095	0.186	0.076		
BARCH(2)	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	609.9	
	0.060	0.224	0.127		
max-BAR(2)	$\hat{\pi}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	623.1	
	0.075	0.234	0.084		
logit-BARCH(2)	$\hat{r}_0$	$\hat{r}_1$	$\hat{r}_2$	609.6	
	-2.694	0.389	0.243		
BAR(3)	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\phi}_1$	$\hat{\phi}_2$	607.4
	0.448	0.057	0.523	0.279	
bvAR(3)	$\hat{\pi}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	620.9
	0.095	0.180	0.072	0.017	
BARCH(3)	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	608.4
	0.057	0.210	0.113	0.066	
max-BAR(3)	$\hat{\pi}$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	622.6
	0.075	0.234	0.084	0.000	
logit-BARCH(3)	$\hat{r}_0$	$\hat{r}_1$	$\hat{r}_2$	$\hat{r}_3$	609.2
	-2.691	0.388	0.144	0.098	

Sample properties of access count data (highlighted in italic font), and the fitted models and their properties, where “(P)ACF k ” refers to the (P)ACF at lag k .

DGP	AIC	BIC	Mean	Var	$P(X=0)$	ACF1	ACF2	ACF3	ACF4	PACF1	PACF2	PACF3	PACF4
data			<i>0.552</i>	<i>0.514</i>	<i>0.560</i>	<i>0.269</i>	<i>0.201</i>	<i>0.153</i>	<i>0.057</i>	<i>0.269</i>	<i>0.139</i>	<i>0.076</i>	<i>-0.024</i>
BAR(1)	1235.4	1244.4	0.553	0.502	0.560	0.261	0.068	0.018	0.005	0.261	0.000	0.000	0.000
bvAR(1)	1251.9	1260.9	0.562	0.624	0.585	0.219	0.048	0.011	0.002	0.219	0.000	0.000	0.000
BARCH(1)	1236.4	1245.4	0.553	0.533	0.570	0.263	0.069	0.018	0.005	0.263	0.000	0.000	0.000
max-BAR(1)	1255.2	1264.2	0.556	0.440	0.532	0.164	0.026	0.004	0.001	0.164	-0.001	0.000	0.000
logit-BARCH(1)	1236.8	1245.8	0.554	0.537	0.569	0.269	0.075	0.021	0.006	0.269	0.003	0.000	0.000
BAR(2)	1228.8	1242.3	0.555	0.504	0.559	0.250	0.179	0.070	0.038	0.250	0.124	0.000	0.000
bvAR(2)	1252.4	1265.9	0.569	0.659	0.588	0.201	0.113	0.036	0.015	0.201	0.076	0.000	0.000
BARCH(2)	1229.6	1243.1	0.555	0.540	0.570	0.257	0.185	0.074	0.040	0.257	0.127	0.000	0.000
max-BAR(2)	1255.9	1269.4	0.559	0.426	0.523	0.153	0.068	0.017	0.005	0.153	0.045	0.000	0.000
logit-BARCH(2)	1228.9	1242.4	0.558	0.533	0.571	0.289	0.222	0.101	0.060	0.289	0.151	0.003	0.002
BAR(3)	1228.3	1246.3	0.559	0.507	0.556	0.245	0.178	0.141	0.067	0.245	0.126	0.077	0.000
bvAR(3)	1255.4	1273.4	0.571	0.666	0.588	0.196	0.110	0.051	0.020	0.196	0.075	0.017	0.000
BARCH(3)	1230.3	1248.3	0.556	0.541	0.570	0.250	0.182	0.132	0.065	0.250	0.127	0.066	0.000
max-BAR(3)	1258.9	1276.8	0.559	0.426	0.523	0.153	0.067	0.016	0.005	0.153	0.045	0.000	0.000
logit-BARCH(3)	1232.0	1249.9	0.559	0.554	0.570	0.277	0.212	0.096	0.057	0.277	0.147	0.006	0.003