Supplemental material

Algorithm 1 QLPO(I)

```
Index set for smaller values S = ∅
Index set for greater values G = ∅
Uniformly at random pick a pivot element p ∈ I
for i ∈ I \ {p} do
    if f_{I\{i,p}}(i) < f_{I\{i,p}}(p) then
        S = S ∪ {i}
    else
        G = G ∪ {i}
    end if
end for
if |S| > 1 then
    S_sorted ← QLPO(S)
else
    S_sorted ← S
end if
if |G| > 1 then
    G_sorted ← QLPO(G)
else
    G_sorted ← G
end if
return S_sorted ≺ {p} ≺ G_sorted
```

Cycles and their effect on TLPO and QLPO results

Instability of a learning algorithm was briefly touched already in Section 2. It was said that a learning algorithm is unstable if a small change in input changes the output a lot. The instability of a learning algorithm affects also the consistency of a tournament in TLPOCV. A tournament can be presented by a complete asymmetric directed graph where the nodes represent the data points and the directions of the edges represent which data point was predicted to have a higher value in the pairwise comparison. A tournament is said to be consistent if the corresponding graph presentation is acyclic. There is at least one circular triad in the TLPO graph if the tournament is inconsistent (Montoya Perez et al. 2019). An example of a tournament graph containing one cycle is presented together with corresponding tournament scores in Fig. 6. Now, a strict total order on the sample cannot be determined because there are equal tournament scores for three data points. This means that in this case $\mathrm{AUC}_{\mathrm{TLPO}} \neq \mathrm{AUC}_{\mathrm{LPO}}$ if the labels of the data points forming the cycle are not equal.

The number of comparisons in the QLPOCV worst-case scenario is equal to the number of comparisons in TLPOCV, but on average the comparisons in QLPOCV form a subset of comparisons in TLPOCV. This also implies that if the comparisons in QLPOCV are presented as a graph the same way as for TLPOCV, the QLPO graph is a subgraph of the TLPO graph - the nodes are same but there are fewer edges. In the example, for which the TLPO graph is presented in Fig. 6, there are three possible orders that the QLPO algorithm can return: $5 \prec 4 \prec 2 \prec 1 \prec 3$, $5 \prec 4 \prec 3 \prec 2 \prec 1$, and $5 \prec 4 \prec 1 \prec 3 \prec 2$, as is demonstrated in Fig. 7. The order returned by QLPO depends on the pivot elements, which are selected uniformly at random. This means that there are some, possibly different, probabilities for the orders. In this example, all options are equally probable. Obviously $\mathrm{AUC}_{\mathrm{QLPO}}$ depends on the returned order and thus also on the randomly chosen pivot elements. Based on a tiny example, $\mathbb{E}\left[\mathrm{AUC}_{\mathrm{QLPO}}\right]$ seems to be equal to $\mathrm{AUC}_{\mathrm{LPO}}$, but this kind of results require further studying and validation.

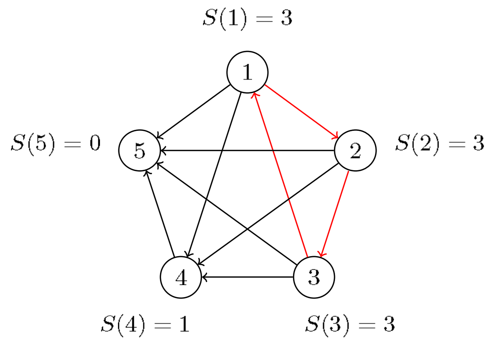

Fig. 6: TLPO graph containing a cycle, that is highlighted, and TLPO scores presented next to the nodes.

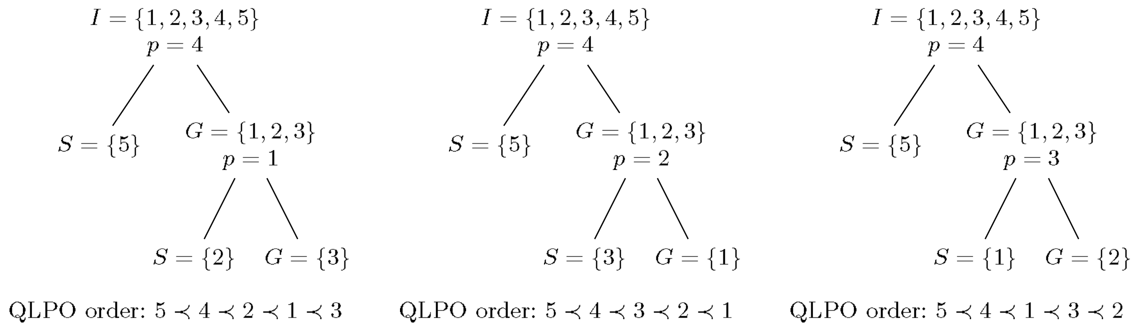

Fig. 7: Three examples of possible executions and corresponding orders returned by QLPO for the example presented for TLPO in Fig. 6.