

# Supplementary Material to “Conformal Outlier Detection for Multivariate Functional Data”

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## S1 Additional simulation results

### S1.1 Impact of the detection of training outliers

In this subsection, we investigate the impact of the training set outlier detection step. Specifically, we revisit the two simulation scenarios from Section 4 of the main manuscript and apply the alternative algorithms to evaluate the trade-offs of our filtering procedure: the clean training setting is evaluated using Algorithm 4, and the mixed training setting is evaluated using Algorithm 2.

Table S1 presents the results of applying Algorithm 4 (which includes the training outlier detection) to the clean training scenario (Simulation 1). Crucially, the results are virtually identical to those of Algorithm 2 (Table 1 in the manuscript). This indicates that our filtering procedure is sufficiently conservative; it does not inadvertently remove clean observations when no contamination is present, thus maintaining statistical power and valid FDR control.

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Table S2 shows the results of Algorithm 2 (without training outlier detection) applied to the mixed training scenario (Simulation 2). As anticipated, the method fails to detect test outliers effectively. The presence of outliers in the calibration set inflates the non-conformity scores (raising the conformal threshold), which leads to a severe loss of power. This confirms that the training outlier detection step (Algorithms 3-4) is essential when the quality of the training data cannot be guaranteed.

## S1.2 Simulations under various outlier rates

In this subsection, we compare the proposed method with existing methods across various outlier rates. In this additional analysis, we restricted our comparison to conformal outlier detection methods (MFD-agg, MFD, Esssup, EFDM). The non-conformal methods were excluded due to their excessive computational cost, which made extensive simulation unfeasible. We consider a wide range of outlier contamination rates—specifically 5%, 10%, 15%, and 25%—to demonstrate the robustness of the proposed method across different contamination levels.

Table S3 and Figure S1 summarize the results of Simulation 1 across varying outlier rates. Consistent with the main results in the manuscript, the proposed MFD-agg demonstrates robust performance and superior detection power across all tested contamination levels.

Table S4 and Figure S2 present the results for Simulation 2. Similar to the previous scenario, MFD-agg maintains robust performance across moderate contamination levels. However, when the outlier rate reaches 25%, the power of all competing methods drops to nearly zero. This drastic performance drop occurs because the contamination rate in the training set is too high. Specifically, the boxplot-based threshold used for training set cleaning (Algorithm 3) fails to identify outliers, as the high density of contaminants inflates the upper whisker. Consequently, the undetected training outliers pollute the calibration set, inflating the nonconformity scores and rendering the final test detection ineffective.

Table S1: Simulation 1 using Algorithm 1 with training outlier detection. Bold faces indicate the best power while controlling the FDR.

|           | Model 1 |               |                      | Model 2       |                      |               | Model 3       |                      |               | Model 4       |                      |  |
|-----------|---------|---------------|----------------------|---------------|----------------------|---------------|---------------|----------------------|---------------|---------------|----------------------|--|
|           | FDR     | Power         |                      | FDR           | Power                |               | FDR           | Power                |               | FDR           | Power                |  |
| MFD-agg   | marg    | 0.090 (0.049) | <b>1.000 (0.000)</b> | 0.087 (0.051) | <b>1.000 (0.000)</b> | 0.087 (0.051) | 0.087 (0.051) | <b>1.000 (0.000)</b> | 0.091 (0.052) | 0.091 (0.052) | <b>1.000 (0.000)</b> |  |
|           | simes   | 0.025 (0.027) | <b>1.000 (0.002)</b> | 0.024 (0.032) | <b>1.000 (0.000)</b> | 0.024 (0.032) | 0.024 (0.032) | <b>1.000 (0.002)</b> | 0.026 (0.032) | 0.026 (0.032) | <b>1.000 (0.000)</b> |  |
|           | asympt  | 0.025 (0.027) | <b>1.000 (0.002)</b> | 0.022 (0.026) | <b>1.000 (0.000)</b> | 0.022 (0.026) | 0.022 (0.026) | <b>1.000 (0.002)</b> | 0.024 (0.024) | 0.024 (0.024) | <b>1.000 (0.000)</b> |  |
| MFD       | marg    | 0.093 (0.056) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.081 (0.062) | 0.602 (0.157)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        |  |
|           | simes   | 0.026 (0.031) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        |  |
|           | asympt  | 0.025 (0.029) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.003 (0.016) | 0.022 (0.127)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        |  |
| Essup     | marg    | 0.301 (0.081) | 0.983 (0.022)        | 0.371 (0.092) | 0.581 (0.144)        | 0.368 (0.101) | 0.368 (0.101) | 0.571 (0.151)        | 0.360 (0.482) | 0.360 (0.482) | 0.000 (0.000)        |  |
|           | simes   | 0.162 (0.108) | 0.761 (0.384)        | 0.015 (0.075) | 0.028 (0.141)        | 0.015 (0.075) | 0.015 (0.075) | 0.030 (0.150)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        |  |
|           | asympt  | 0.164 (0.097) | 0.841 (0.287)        | 0.052 (0.132) | 0.086 (0.217)        | 0.045 (0.126) | 0.045 (0.126) | 0.074 (0.204)        | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000)        |  |
| EFDM      | marg    | 0.152 (0.131) | 0.260 (0.198)        | 0.135 (0.064) | 1.000 (0.000)        | 0.136 (0.066) | 0.136 (0.066) | 0.939 (0.055)        | 0.124 (0.060) | 0.124 (0.060) | 1.000 (0.000)        |  |
|           | simes   | 0.000 (0.000) | 0.000 (0.000)        | 0.050 (0.052) | 0.969 (0.171)        | 0.040 (0.057) | 0.040 (0.057) | 0.368 (0.454)        | 0.043 (0.044) | 0.043 (0.044) | 0.979 (0.141)        |  |
|           | asympt  | 0.000 (0.000) | 0.000 (0.000)        | 0.048 (0.049) | 0.986 (0.102)        | 0.050 (0.054) | 0.050 (0.054) | 0.635 (0.402)        | 0.043 (0.044) | 0.043 (0.044) | 0.979 (0.141)        |  |
| MS-plot   |         | 0.170 (0.047) | 1.000 (0.000)        | 0.636 (0.117) | 0.115 (0.044)        | 0.177 (0.046) | 0.177 (0.046) | 0.930 (0.040)        | 1.000 (0.000) | 1.000 (0.000) | 0.000 (0.000)        |  |
| Seq-trans |         | 0.000 (0.000) | 0.761 (0.063)        | 0.010 (0.100) | 0.011 (0.016)        | 0.001 (0.008) | 0.001 (0.008) | 0.110 (0.046)        | 0.010 (0.100) | 0.010 (0.100) | 0.001 (0.005)        |  |

Table S2: Simulation 2 using Algorithm 2 without training outlier detection. Bold faces indicate the best power while controlling the FDR.

|           | Model 1 |               |               | Model 2       |               |               | Model 3       |               |               | Model 4       |               |  |
|-----------|---------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|--|
|           | FDR     | Power         |               | FDR           | Power         |               | FDR           | Power         |               | FDR           | Power         |  |
| MFD-agg   | marg    | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | simes   | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | asympt  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
| MFD       | marg    | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | simes   | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | asympt  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
| Essup     | marg    | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | simes   | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | asympt  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
| EFDM      | marg    | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | simes   | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
|           | asympt  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |  |
| MS-plot   |         | 0.151 (0.049) | 1.000 (0.000) | 0.692 (0.150) | 0.066 (0.037) | 0.162 (0.046) | 0.162 (0.046) | 0.913 (0.045) | 1.000 (0.000) | 1.000 (0.000) | 0.000 (0.000) |  |
| Seq-trans |         | 0.000 (0.000) | 0.710 (0.077) | 0.005 (0.050) | 0.009 (0.014) | 0.003 (0.033) | 0.003 (0.033) | 0.030 (0.024) | 0.000 (0.000) | 0.000 (0.000) | 0.002 (0.006) |  |

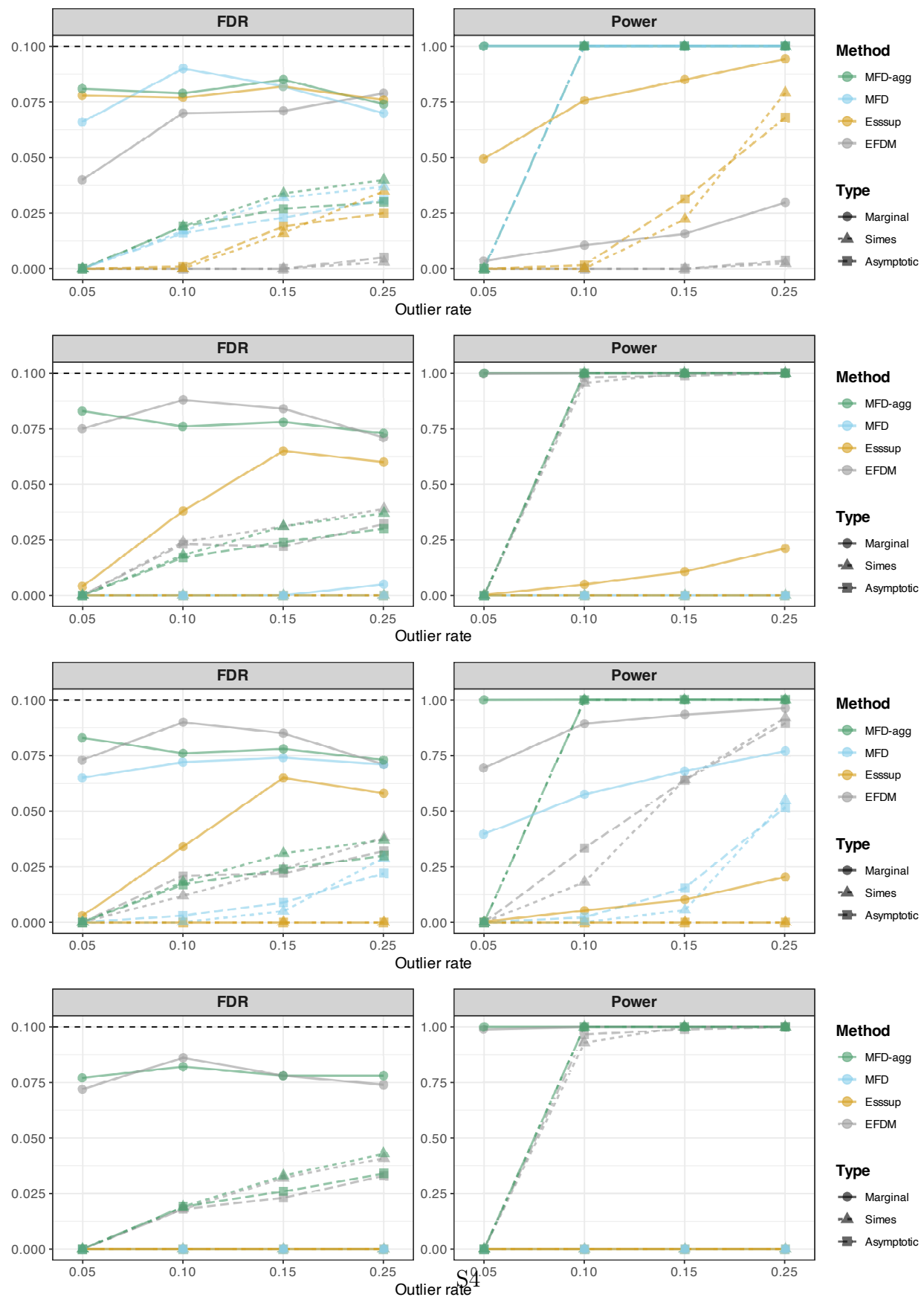


Figure S1: Simulation 1 with different outlier rates.

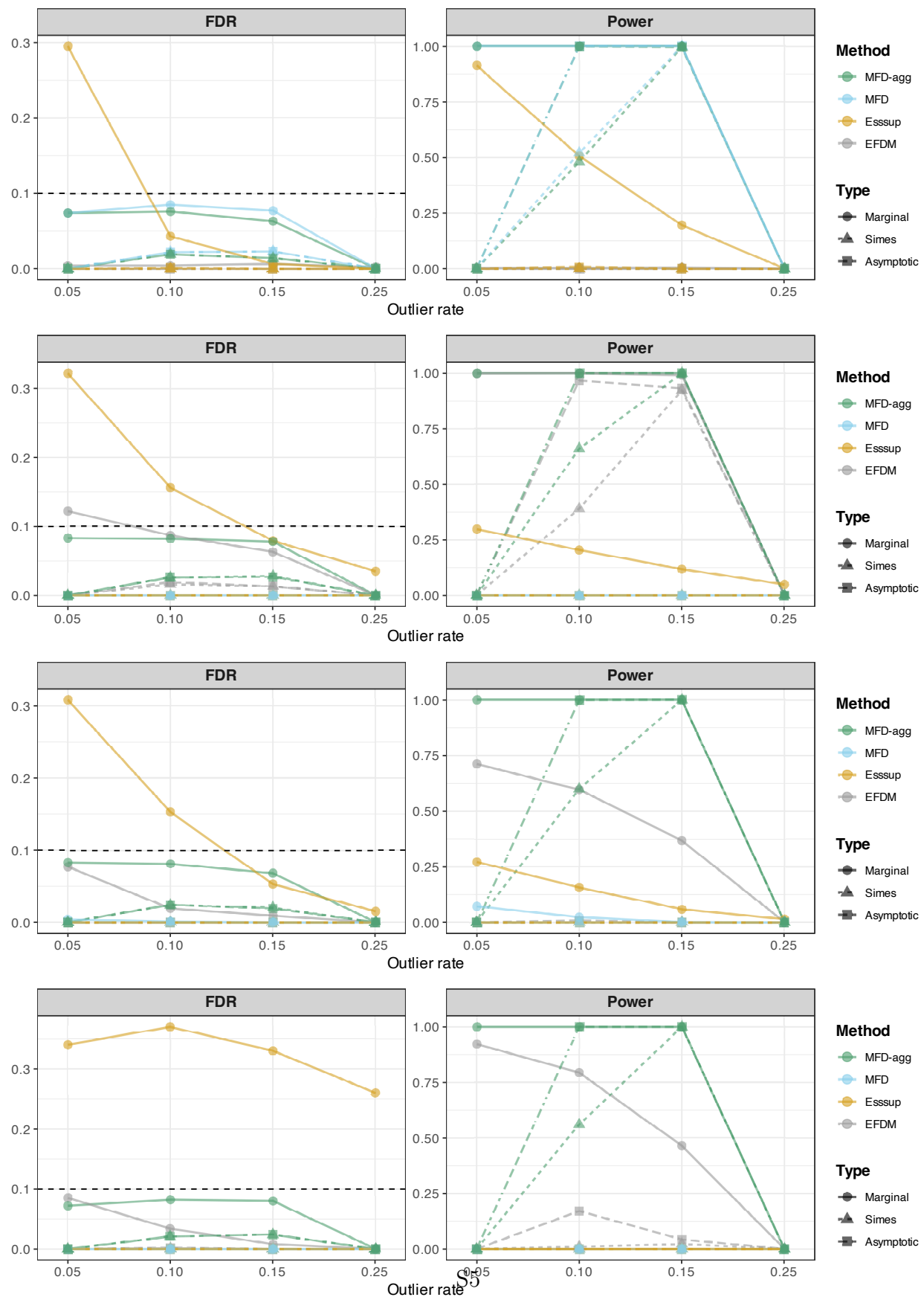


Figure S2: Simulation 2 with different outlier rates.

Table S3: Simulation 1 with different outlier rates. Bold faces indicate the best power while controlling the FDR.

| Model 1      |        |               | Model 2       |               | Model 3       |               | Model 4       |               |               |
|--------------|--------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
|              | FDR    | Power         | FDR           | Power         | FDR           | Power         | FDR           | Power         |               |
| MFD-agg      | marg   | 0.081 (0.085) | 1.000 (0.000) | 5% outliers   |               | 0.083 (0.085) | 1.000 (0.000) | 0.083 (0.085) | 0.999 (0.006) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| MFD          | marg   | 0.066 (0.064) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.065 (0.087) | 0.397 (0.258) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| Esssup       | marg   | 0.078 (0.087) | 0.494 (0.327) | 0.004 (0.040) | 0.002 (0.024) | 0.003 (0.030) | 0.003 (0.028) | 0.003 (0.030) | 0.003 (0.028) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| EFDM         | marg   | 0.040 (0.131) | 0.034 (0.105) | 0.075 (0.075) | 0.998 (0.020) | 0.073 (0.077) | 0.694 (0.306) | 0.073 (0.077) | 0.694 (0.306) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| 10% outliers |        |               |               |               |               |               |               |               |               |
| MFD-agg      | marg   | 0.079 (0.049) | 1.000 (0.000) | 0.076 (0.055) | 1.000 (0.000) | 0.076 (0.055) | 1.000 (0.000) | 0.076 (0.055) | 1.000 (0.000) |
|              | simes  | 0.019 (0.026) | 1.000 (0.002) | 0.018 (0.031) | 1.000 (0.000) | 0.018 (0.031) | 0.999 (0.003) | 0.018 (0.031) | 0.999 (0.003) |
|              | asympt | 0.019 (0.026) | 1.000 (0.002) | 0.017 (0.024) | 1.000 (0.000) | 0.017 (0.024) | 0.999 (0.003) | 0.017 (0.024) | 0.999 (0.003) |
| MFD          | marg   | 0.090 (0.057) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.072 (0.061) | 0.575 (0.169) |
|              | simes  | 0.017 (0.026) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.016 (0.025) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.003 (0.017) | 0.022 (0.128) |
| Esssup       | marg   | 0.077 (0.048) | 0.757 (0.178) | 0.038 (0.091) | 0.049 (0.111) | 0.034 (0.082) | 0.053 (0.111) | 0.034 (0.082) | 0.053 (0.111) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.001 (0.007) | 0.016 (0.110) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| EFDM         | marg   | 0.070 (0.112) | 0.106 (0.147) | 0.088 (0.055) | 1.000 (0.000) | 0.090 (0.058) | 0.894 (0.088) | 0.090 (0.058) | 0.894 (0.088) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.024 (0.031) | 0.956 (0.197) | 0.012 (0.028) | 0.181 (0.364) | 0.012 (0.028) | 0.181 (0.364) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.023 (0.028) | 0.982 (0.102) | 0.021 (0.033) | 0.333 (0.420) | 0.021 (0.033) | 0.333 (0.420) |
| 15% outliers |        |               |               |               |               |               |               |               |               |
| MFD-agg      | marg   | 0.085 (0.041) | 1.000 (0.000) | 0.078 (0.042) | 1.000 (0.000) | 0.078 (0.042) | 1.000 (0.000) | 0.078 (0.042) | 1.000 (0.000) |
|              | simes  | 0.034 (0.030) | 1.000 (0.000) | 0.031 (0.030) | 1.000 (0.000) | 0.031 (0.030) | 1.000 (0.001) | 0.031 (0.030) | 1.000 (0.001) |
|              | asympt | 0.027 (0.030) | 1.000 (0.000) | 0.024 (0.028) | 1.000 (0.000) | 0.024 (0.028) | 1.000 (0.002) | 0.024 (0.028) | 1.000 (0.002) |
| MFD          | marg   | 0.082 (0.043) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.074 (0.047) | 0.680 (0.102) | 0.074 (0.047) | 0.680 (0.102) |
|              | simes  | 0.032 (0.026) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.005 (0.017) | 0.055 (0.189) | 0.005 (0.017) | 0.055 (0.189) |
|              | asympt | 0.023 (0.025) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.009 (0.020) | 0.154 (0.264) | 0.009 (0.020) | 0.154 (0.264) |
| Esssup       | marg   | 0.082 (0.042) | 0.850 (0.112) | 0.065 (0.092) | 0.107 (0.123) | 0.065 (0.092) | 0.103 (0.123) | 0.065 (0.092) | 0.103 (0.123) |
|              | simes  | 0.016 (0.031) | 0.223 (0.356) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.019 (0.032) | 0.315 (0.355) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| EFDM         | marg   | 0.071 (0.095) | 0.157 (0.180) | 0.084 (0.046) | 1.000 (0.000) | 0.085 (0.047) | 0.935 (0.051) | 0.085 (0.047) | 0.935 (0.051) |
|              | simes  | 0.000 (0.000) | 0.000 (0.000) | 0.031 (0.026) | 1.000 (0.000) | 0.024 (0.026) | 0.642 (0.370) | 0.024 (0.026) | 0.642 (0.370) |
|              | asympt | 0.000 (0.000) | 0.000 (0.000) | 0.022 (0.023) | 0.989 (0.100) | 0.022 (0.025) | 0.639 (0.354) | 0.022 (0.025) | 0.639 (0.354) |
| 25% outliers |        |               |               |               |               |               |               |               |               |
| MFD-agg      | marg   | 0.074 (0.030) | 1.000 (0.000) | 0.073 (0.035) | 1.000 (0.000) | 0.073 (0.035) | 1.000 (0.000) | 0.073 (0.035) | 1.000 (0.000) |
|              | simes  | 0.040 (0.021) | 1.000 (0.000) | 0.037 (0.025) | 1.000 (0.000) | 0.037 (0.025) | 1.000 (0.000) | 0.037 (0.025) | 1.000 (0.000) |
|              | asympt | 0.030 (0.020) | 1.000 (0.000) | 0.030 (0.024) | 1.000 (0.000) | 0.030 (0.024) | 1.000 (0.001) | 0.030 (0.024) | 1.000 (0.001) |
| MFD          | marg   | 0.070 (0.029) | 1.000 (0.000) | 0.005 (0.055) | 0.000 (0.004) | 0.071 (0.036) | 0.770 (0.069) | 0.071 (0.036) | 0.770 (0.069) |
|              | simes  | 0.037 (0.023) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.029 (0.030) | 0.549 (0.231) | 0.029 (0.030) | 0.549 (0.231) |
|              | asympt | 0.031 (0.022) | 1.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.022 (0.023) | 0.517 (0.204) | 0.022 (0.023) | 0.517 (0.204) |
| Esssup       | marg   | 0.076 (0.032) | 0.944 (0.036) | 0.060 (0.056) | 0.211 (0.157) | 0.058 (0.053) | 0.204 (0.147) | 0.058 (0.053) | 0.204 (0.147) |
|              | simes  | 0.035 (0.025) | 0.792 (0.189) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
|              | asympt | 0.025 (0.021) | 0.680 (0.272) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| EFDM         | marg   | 0.079 (0.053) | 0.298 (0.180) | 0.071 (0.032) | 1.000 (0.000) | 0.071 (0.033) | 0.963 (0.029) | 0.071 (0.033) | 0.963 (0.029) |
|              | simes  | 0.003 (0.012) | 0.024 (0.104) | 0.039 (0.024) | 1.000 (0.000) | 0.038 (0.025) | 0.921 (0.054) | 0.038 (0.025) | 0.921 (0.054) |
|              | asympt | 0.005 (0.018) | 0.036 (0.118) | 0.032 (0.023) | 1.000 (0.000) | 0.032 (0.022) | 0.896 (0.067) | 0.032 (0.022) | 0.896 (0.067) |

Table S4: Simulation 2 with different outlier rates. Bold faces indicate the best power while controlling the FDR.

| Model 1      |        |               | Model 2              |               | Model 3              |               | Model 4              |               |                      |
|--------------|--------|---------------|----------------------|---------------|----------------------|---------------|----------------------|---------------|----------------------|
|              | FDR    | Power         | FDR                  | Power         | FDR                  | Power         | FDR                  | Power         |                      |
| MFD-agg      | marg   | 0.074 (0.073) | <b>1.000 (0.000)</b> | 0.083 (0.075) | <b>1.000 (0.000)</b> | 0.083 (0.076) | <b>1.000 (0.004)</b> | 0.072 (0.070) | <b>1.000 (0.000)</b> |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| MFD          | marg   | 0.074 (0.072) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.004 (0.022) | 0.071 (0.171)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| Essup        | marg   | 0.295 (0.113) | 0.914 (0.095)        | 0.322 (0.246) | 0.298 (0.236)        | 0.308 (0.240) | 0.272 (0.216)        | 0.340 (0.476) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| EFDM         | marg   | 0.004 (0.042) | 0.003 (0.028)        | 0.122 (0.092) | 0.999 (0.012)        | 0.077 (0.082) | 0.712 (0.240)        | 0.085 (0.089) | 0.922 (0.182)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| 10% outliers |        |               |                      |               |                      |               |                      |               |                      |
| MFD-agg      | marg   | 0.076 (0.054) | <b>1.000 (0.000)</b> | 0.082 (0.057) | <b>1.000 (0.000)</b> | 0.081 (0.058) | <b>1.000 (0.000)</b> | 0.082 (0.057) | <b>1.000 (0.000)</b> |
|              | simes  | 0.019 (0.026) | 0.480 (0.502)        | 0.026 (0.030) | <b>0.660 (0.476)</b> | 0.024 (0.030) | <b>0.600 (0.492)</b> | 0.021 (0.025) | <b>0.560 (0.499)</b> |
|              | asympt | 0.019 (0.026) | 0.999 (0.005)        | 0.026 (0.030) | <b>1.000 (0.000)</b> | 0.024 (0.030) | <b>0.999 (0.005)</b> | 0.021 (0.025) | <b>1.000 (0.000)</b> |
| MFD          | marg   | 0.085 (0.057) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.001 (0.007) | 0.023 (0.073)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | simes  | 0.022 (0.032) | <b>0.520 (0.502)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.022 (0.032) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| Essup        | marg   | 0.043 (0.042) | 0.507 (0.269)        | 0.156 (0.141) | 0.204 (0.173)        | 0.153 (0.157) | 0.157 (0.152)        | 0.370 (0.485) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.001 (0.011) | 0.008 (0.082)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| EFDM         | marg   | 0.004 (0.044) | 0.003 (0.028)        | 0.087 (0.057) | <b>1.000 (0.003)</b> | 0.019 (0.027) | 0.597 (0.189)        | 0.034 (0.041) | 0.794 (0.241)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.016 (0.025) | 0.390 (0.490)        | 0.000 (0.000) | 0.000 (0.000)        | 0.001 (0.006) | 0.010 (0.096)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.019 (0.025) | 0.968 (0.141)        | 0.000 (0.005) | 0.008 (0.082)        | 0.002 (0.010) | 0.171 (0.367)        |
| 15% outliers |        |               |                      |               |                      |               |                      |               |                      |
| MFD-agg      | marg   | 0.063 (0.044) | <b>1.000 (0.000)</b> | 0.078 (0.048) | <b>1.000 (0.000)</b> | 0.068 (0.041) | <b>1.000 (0.000)</b> | 0.080 (0.050) | <b>1.000 (0.000)</b> |
|              | simes  | 0.014 (0.024) | 0.996 (0.011)        | 0.028 (0.028) | <b>1.000 (0.000)</b> | 0.020 (0.025) | <b>1.000 (0.002)</b> | 0.024 (0.027) | <b>1.000 (0.000)</b> |
|              | asympt | 0.014 (0.023) | 0.996 (0.011)        | 0.027 (0.027) | <b>1.000 (0.000)</b> | 0.019 (0.023) | <b>1.000 (0.002)</b> | 0.024 (0.024) | <b>1.000 (0.000)</b> |
| MFD          | marg   | 0.077 (0.044) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.001 (0.015)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | simes  | 0.023 (0.024) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.023 (0.024) | <b>1.000 (0.000)</b> | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| Essup        | marg   | 0.006 (0.019) | 0.196 (0.180)        | 0.079 (0.106) | 0.118 (0.136)        | 0.053 (0.098) | 0.058 (0.089)        | 0.330 (0.473) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| EFDM         | marg   | 0.006 (0.037) | 0.004 (0.024)        | 0.063 (0.052) | 0.992 (0.020)        | 0.009 (0.018) | 0.369 (0.189)        | 0.008 (0.018) | 0.464 (0.295)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.013 (0.019) | 0.924 (0.199)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.004) | 0.022 (0.127)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.013 (0.019) | 0.933 (0.155)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.004) | 0.043 (0.170)        |
| 25% outliers |        |               |                      |               |                      |               |                      |               |                      |
| MFD-agg      | marg   | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.001 (0.009)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.001 (0.009)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.001 (0.009)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| MFD          | marg   | 0.000 (0.000) | 0.001 (0.008)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| Essup        | marg   | 0.000 (0.000) | 0.000 (0.000)        | 0.035 (0.064) | 0.049 (0.068)        | 0.015 (0.053) | 0.014 (0.041)        | 0.260 (0.441) | 0.000 (0.000)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
| EFDM         | marg   | 0.002 (0.017) | 0.001 (0.008)        | 0.000 (0.000) | 0.001 (0.010)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.001 (0.008)        |
|              | simes  | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |
|              | asympt | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        | 0.000 (0.000) | 0.000 (0.000)        |