

Appendix (Not for publication)

A Rebasing consumption forecasts data

Since the Survey of Professional Forecasters (SPF) began, there have been a number of changes of the base year in the national income and product accounts (NIPA). The forecasts for levels of consumption (SPF variable name: RCONSUM) use the base year that was in effect when the forecasters received the survey questionnaire. This appendix explains how consumption forecasts data are rebased.

Table A.1 provides the base year in effect for NIPA variables (including consumption expenditures), reproduced from Table 4 of the documentation of the Survey of Professional Forecasters (p. 23). For rebasing, we use real consumption expenditures data of different vintages from the Real-Time Data Set for Macroeconomists managed by the Federal Reserve Bank of Philadelphia. The year 1996 is used as the common base year for all consumption forecast data. The data in each window needs to be rebased by multiplying a base ratio. For instance the 1959:Q4 real consumption in the window from 1996:Q1 to 1999:Q3 is 1409.5 while it is 1469.5 in 1999:Q4 to 2003:Q4 window and hence the ratio is 1469.5/1409.5.

Table A.1: Base years and ratios for rebasing

Range of Survey Dates	Base Year	Ratio
1976:Q1 to 1985:Q4	1972	3.31
1986:Q1 to 1991:Q4	1982	1.48
1992:Q1 to 1995:Q4	1987	1.23
1996:Q1 to 1999:Q3	1992	1.04
1999:Q4 to 2003:Q4	1996	1
2004:Q1 to 2009:Q2	2000	0.94
2009:Q3 to 2013:Q2	2005	0.84
2013:Q3 to present	2009	0.79

B Cointegration between Realized Stock Prices and Fundamentals

This section tests for cointegration between realized stock prices and fundamentals with the cointegrating vector $(1, -1)$. Data on stock prices (S&P 500 index), aggregate consumption, dividend, and output for the US are downloaded from the website of Robert Shiller and the Federal Reserve Bank of St. Louis. The samples cover the same period as those of the survey forecast data. Table A.2 reports test results by applying the DF-GLS test to log stock prices to consumption (dividend or output) ratios. Panel A shows that the DF-GLS test rejects the null of unit root at the 10% level, in favor of cointegration between stock prices and consumption. Panel B (C) reports similar results for cointegration testing using stock prices and aggregate dividend (output), respectively.

Table A.2: **Tests of the cointegration with the vector $(1, -1)$ using realized data**

	Test stat.	10% critical
Panel A: <i>Realized (log) stock prices-consumption ratio</i>		
DF-GLS	-1.705*	-1.619
Panel B: <i>Realized (log) stock prices-dividend ratio</i>		
DF-GLS	-3.155*	-2.732
Panel C: <i>Realized (log) stock prices-output ratio</i>		
DF-GLS	-1.832*	-1.619

Note: *: the test statistics with one asterisk indicate that the corresponding test rejects the null of a unit root at the 10% significance level.

Sample period in Panel A and C is 1957Q1-2017Q2;

sample period in Panel B is 2003Q1 to 2015Q3.

C Sentiment-based models

In asset pricing modeling, another way to deviate from FIRE is incorporating sentiment, such as in Jin and Sui (2022) and the exchange rate model of Yu (2013). As an example, suppose the representative agent's preferences are represented by the Epstein-Zin utility (6) and the actual

exogenous driving processes are

$$\Delta c_{t+1} = \mu_c + \sigma_t \eta_{t+1}, \quad (\text{A.1})$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + \nu(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1}, \quad (\text{A.2})$$

$$\Delta d_{t+1} = \mu_d + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}. \quad (\text{A.3})$$

Comparing (A.1) - (A.3) with (7) - (10) in the long-run risks model of Bansal, Kiku and Yaron (2012), we drop the persistent component x_t in the actual exogenous driving processes (because sentiment plays the role of the persistent component x_t).²⁸

Now assume that agents have a misperception about the exogenous consumption and dividend process. They perceive consumption and dividend processes as

$$\Delta c_{t+1} = \mu_c + a_t + \sigma_t \hat{\eta}_{t+1}, \quad (\text{A.4})$$

$$a_{t+1} = \rho a_t + \varphi_e \sigma_t e_{t+1}, \quad (\text{A.5})$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + \nu(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1}, \quad (\text{A.6})$$

$$\Delta d_{t+1} = \mu_d + \phi a_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t u_{d,t+1}, \quad (\text{A.7})$$

where $(\hat{\eta}_{t+1}, e_{t+1})$ are i.i.d joint standard normal under agents' belief. a_t is an AR(1) process and does not appear in the true driving processes (called "sentiment"). When a_t is positive (negative), agents are optimistic (pessimistic). Assuming $0 < \rho < 1$. If $\rho = 1$, realized $\log(P_t/C_t)$ will have unbounded volatility (see equation (A.8) below), which is rejected by the data in the Appendix B. Thus, we require $\rho < 1$. Note that in this type of models, despite agents' misperception of the exogenous driving process, they know the equilibrium pricing function. The following result provides testable implications for the formation of stock price expectations.

Proposition 3. *Given agents' beliefs (A.4) - (A.7), agents' stock price forecasts $E_i \log P_{i+j}$ are cointegrated with their forecasts of aggregated consumption $E_k \log C_{k+l}$ with cointegrating vector $(1, -1)$ for arbitrary $i, j, k, l > 0$.*

²⁸Our proposition below is not affected by adding x_t in the exogenous driving processes, as is argued later.

Proof. Following Bansal, Kiku and Yaron (2012), the equilibrium pricing function delivers the (approximate) analytical solution for price consumption ratio as

$$\log\left(\frac{P_t}{C_t}\right) = A_0 + A_1 a_t + A_2 \sigma_t^2, \quad (\text{A.8})$$

where A_0, A_1, A_2 remain the same constant as in Bansal, Kiku and Yaron (2012). Because agent can use equation (A.8) to forecast, the trend growth rate of both stock prices forecasts and consumption forecasts are identical to μ_c (noting sentiment is a stationary process). Thus, stock price forecasts and consumption forecasts can be expressed as $E_i \log P_{i+j} = \log P_i + j\mu_c + s(i, j)$ and $E_k \log C_{k+l} = \log C_k + l\mu_c + \tilde{s}(k, l)$ where $s(i, j)$ and $\tilde{s}(k, l)$ are stationary terms and omitted. Let $E_i \log P_{i+j} - E_k \log C_{k+l} = (\log P_t - \log C_k) + (j - l)\mu_c + s(i, j) - \tilde{s}(k, l)$. It is stationary because $(\log P_t - \log C_k)$ is stationary.²⁹ $\square$

Stock price forecasts depend on current stock prices and beliefs about the growth rate of stock prices. Similarly, consumption forecasts depend on current consumption and beliefs about the growth rate of consumption. Stock price forecasts and consumption forecasts are cointegrated because (1) realized stock prices and consumption are cointegrated and (2) beliefs about consumption growth and stock price growth rates are mean-reverting. If beliefs about consumption growth rates are not mean-reverting ($\rho = 1$), price consumption ratio will have unbounded volatility given agents' knowledge of the equilibrium pricing function. If the consumption driving process (A.1) - (A.3) contains a persistent and stationary component x_t as in Bansal, Kiku and Yaron (2012), $(\log)$ price consumption ratio will be a linear function of x_t, a_t and σ_t^2 with constant coefficients. The above proposition will still hold because realized price consumption ratios are stationary and the trend growth rate of both stock prices forecasts and consumption forecasts are identical to μ_c .

We can also use the similar proof to show that agents' stock price forecasts $E_i \log P_{i+j}$ are cointegrated with their forecasts of aggregated dividend $E_k \log D_{k+l}$ with cointegrating vector $(1, -1)$ for arbitrary $i, j, k, l > 0$.

²⁹ $(\log P_t - \log C_k) = (\log P_t - \log C_t) + (\log C_t - \log C_k)$ is stationary because $(\log P_t - \log C_t)$ is stationary (see equation (A.8)) and $(\log C_t - \log C_k)$ is stationary (since $\log C_t$ is I(1)).

D Proofs

This section provides proofs for Lemmas and Propositions in Sections 3 and 4.

D.1 Proof for Lemma 1

Proof. Given (1)-(5), we have $E_t X_{t+i} = E_t X_{t+i}^P + E_t X_{t+i}^C = \mu i + X_t^P + E_t X_{t+i}^C$. $E_t X_{t+i}$ is the sum of a unit root process and a stationary process and hence a unit root process. $\square$

D.2 Proof for Lemma 2

Proof. Let

$$\begin{aligned} E_t X_{t+i} - X_k &= (E_t X_{t+i}^P + E_t X_{t+i}^C) - X_t + (X_t - X_k) \\ &= (E_t X_{t+i}^P - X_t^P) + (E_t X_{t+i}^C - X_t^C) + (X_t - X_k) \\ &= \mu i + (E_t X_{t+i}^C - X_t^C) + (X_t - X_k). \end{aligned}$$

$(E_t X_{t+i} - X_k)$ is stationary as $E_t X_{t+i}^C$, X_t^C and $(X_t - X_k)$ are stationary. $\square$

D.3 Proof for Theorem 1

Proof. Let

$$\begin{aligned} &[a_1 E_{i_1} y_{1,i_1+j_1} + a_2 E_{i_2} y_{2,i_2+j_2} + \dots + a_n E_{i_n} y_{n,i_n+j_n}] \\ &= \left[\sum_{k=1}^n a_k (E_{i_k} y_{k,i_k+j_k} - y_{k,i_k}) + \sum_{k=1}^n a_k y_{k,i_k} \right] \\ &= \left[\sum_{k=1}^n a_k (E_{i_k} y_{k,i_k+j_k} - y_{k,i_k}) + \sum_{k=1}^n a_k y_{k,i_1} + \sum_{k=1}^n a_k (y_{k,i_k} - y_{k,i_1}) \right]. \end{aligned}$$

Note Lemma 2 implies $(E_{i_k} y_{k,i_k+j_k} - y_{k,i_k})$ is stationary for $k = 1, 2, \dots, n$. In addition, the cointegration of the vector y_t yields $\sum_{k=1}^n a_k y_{k,i_1}$ is stationary and $y_{k,t} \sim I(1)$ gives $(y_{k,i_k} - y_{k,i_1})$ is stationary. Thus, we have $a_1 E_{i_1} y_{1,i_1+j_1} + a_2 E_{i_2} y_{2,i_2+j_2} + \dots + a_n E_{i_n} y_{n,i_n+j_n}$ is stationary. $\square$

D.4 Proof for Proposition 1

Proof. The RE version of the model here is a special case of Bansal et al. (2012) with setting $\rho = 0$, $\nu = 0$, $x_t = 0$, $\sigma_t = \bar{\sigma}$, $\sigma_w = 0$, $\varphi_e = 0$. From the RE version to the learning model, we replace the actual growth rate of consumption μ_c by agents' beliefs about consumption growth rate $\mu_{c,t}$ in the analytical solution. The (analytical) solution for log price consumption ratio in the learning model is

$$\log\left(\frac{P_t}{C_t}\right) = \tilde{A}_{0,t} + \tilde{A}_2 \bar{\sigma}^2, \quad (\text{A.9})$$

where $\tilde{A}_{0,t} = \frac{1}{1-\kappa_{1,t}} \left(\log \delta + \kappa_{0,t} + \left(1 - \frac{1}{\psi}\right) \mu_{c,t} + \kappa_{1,t} \tilde{A}_2 \bar{\sigma}^2 \right)$, $\tilde{A}_2 = -\frac{(\gamma-1)(1-\frac{1}{\psi})}{2}$, $\kappa_{0,t} = \log(1 + \exp(\bar{z}_t)) - \kappa_{1,t} \bar{z}_t$, $\kappa_{1,t} = \frac{\exp(\bar{z}_t)}{1+\exp(\bar{z}_t)}$, $\bar{z}_t = \tilde{A}_{0,t}(\bar{z}_t) + \tilde{A}_2 \bar{\sigma}^2$. Note that $\tilde{A}_{0,t}$ is a nonlinear function of $\mu_{c,t}$. Using Taylor expansion, the right hand side of the process (A.9) can be well approximated by a polynomial function of the AR(1) process $\mu_{c,t}$ and is again a stationary process.³⁰ Again, $(E_i \log P_{i+j} - E_k \log C_{k+l})$ is stationary because agents use equation (A.9) to forecast, then the trend growth rate of stock price forecasts and consumption forecasts are identical to each other. $\square$

D.5 Proof for Proposition 2

Proof. All agents know that (1) aggregate consumption C_t equals to Y_t ($C_t = Y_t$) each period and that consumption process is identical to output process, and (2) the mapping from output to equilibrium stock prices, i.e., $P_t = \frac{1-\omega}{\rho+\nu(1-\beta)} Y_t$ (equation (B60) in EGH), and hence, $P_t = \frac{1-\omega}{\rho+\nu(1-\beta)} C_t$. Thus, the perceived consumption process for cohort s is $dC_t/C_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t}$. Agents' heterogeneous beliefs about fundamentals Y_t (or C_t) are carried over to stock prices P_t , which reads as $dP_t/P_t = \hat{\mu}_{s,t} dt + \sigma_Y dz_{s,t}$. Thus, every cohort perceives that the growth rate of C_t and S_t are the same stochastic process despite heterogeneous beliefs about the growth rate across cohorts.

For cohort s , stock price forecasts $E_i^s \log P_{i+j}$ depends on realized stock prices P_i and perceived stock price growth rates $\hat{\mu}_{s,i}$. Similarly, consumption forecasts $E_k^s \log C_{k+l}$ depend on re-

³⁰If agents learn use least squares (i.e., $g_t = 1/t$), $\mu_{c,t}$ will converge to μ_c and $\tilde{A}_{0,t}$ will converge to a constant.

alized consumption C_k and perceived consumption growth rates $\hat{\mu}_{s,k}$. First, note $P_i = \frac{1-\omega}{\rho+\nu(1-\beta)}Y_i$ and $C_k = Y_k$. Clearly, $\log P_i$ is cointegrated with $\log C_k$ with cointegrating vector $(1, -1)$ because $\log Y_i$ and $\log Y_k$ is cointegrated with cointegrating vector $(1, -1)$. Second, Proposition 1 of EGH implies that as $t \rightarrow \infty$, $\hat{\mu}_{s,t}$ and $\hat{\mu}_{s,k}$ will converge almost surely to the true output growth rate μ_Y . So $E_i^s \log P_{i+j}$ is cointegrated with $E_k^s \log C_{k+l}$ with cointegrating vector $(1, -1)$ for every cohort s . Taking the two features together, the average stock price forecasts $\bar{E}_i \log P_{i+j}$ is cointegrated with $\bar{E}_k \log C_{k+l}$ with cointegrating vector $(1, -1)$. $\square$

E Integration properties of the forecasts

This section studies the integration properties of forecasts of the aggregate stock price index and aggregate fundamentals (aggregate consumption and dividend).

Forecasts stock price. Table A.3 reports the p-values of Augmented Dickey-Fuller (ADF) test for median forecasts of stock prices. Panel A shows that for all surveys and all forecast horizons, DF-GLS test cannot reject the null hypothesis that median stock price forecasts are I(1) at the 10% significance level.³¹ In addition, stock price forecasts are not I(2) processes as shown in Panel B. Similar results are obtained using mean stock price forecasts and reported in Table A.4. Lemma 1 suggests that forecasts of stock prices, aggregate consumption and dividend from typical FIRE stock pricing models (e.g., the long-run risks model) are I(1) but not I(2) processes, consistent with the evidence here.

Forecasts of consumption. Test results for median and mean forecasts of consumption are consistent with Lemma 1 that aggregate consumption forecasts are I(1) but not I(2) processes. Results are reported in Tables A.5 and A.6.

Forecasts of dividends. Test results for median and mean forecasts of dividends are consistent with Lemma 1 that aggregate dividend forecasts are I(1) but not I(2) processes. Results are reported in Table A.7.

³¹This result implies that this test cannot reject that stock price forecasts are I(1) at the 5% or 1% significance level either.

Table A.3: Integration properties of median stock price forecasts

P values			
Median	1Q ahead	2Q ahead	4Q ahead
Panel A: I(1) test			
<i>Livingston stock price forecasts (1981Q4-2019Q4)</i>			
Dickey–Fuller	<i>n.a</i>	0.561	0.577
<i>Shiller stock price forecasts (1999Q1-2015Q2)</i>			
Dickey–Fuller	0.692	0.752	0.712
<i>CFO stock price forecasts (2001Q4-2018Q4)</i>			
Dickey–Fuller	<i>n.a</i>	<i>n.a</i>	0.578
Panel B: I(2) test			
<i>Livingston stock price forecasts (1981Q4-2019Q4)</i>			
Dickey–Fuller	<i>n.a</i>	0.00	0.00
<i>Shiller stock price forecasts (1999Q1-2015Q2)</i>			
Dickey–Fuller	0.00	0.00	0.00
<i>CFO stock price forecasts (2001Q4-2018Q4)</i>			
Dickey–Fuller	<i>n.a</i>	<i>n.a</i>	0.00
Evidence A.1: Median stock price forecasts are I(1) but not I(2).			

Table A.4: Integration properties of mean stock price forecasts

P values			
Median	1Q ahead	2Q ahead	4Q ahead
Panel A: I(1) test			
<i>Livingston stock price forecasts (1981Q4-2019Q4)</i>			
Dickey–Fuller	<i>n.a</i>	0.581	0.607
<i>Shiller stock price forecasts (1999Q1-2015Q2)</i>			
Dickey–Fuller	0.704	0.685	0.656
Panel B: I(2) test			
<i>Livingston stock price forecasts (1981Q4-2019Q4)</i>			
Dickey–Fuller	<i>n.a</i>	0.00	0.00
<i>Shiller stock price forecasts (1999Q1-2015Q2)</i>			
Dickey–Fuller	0.00	0.00	0.00
Evidence A.2: Mean stock price forecasts are I(1) but not I(2).			

Table A.5: Integration of median SPF consumption forecasts over different periods

P values			
Median	1Q ahead	2Q ahead	4Q ahead
Panel A: SPF consumption forecasts - I(1) test			
<i>Period: 1981Q4-2019Q4</i>			
Dickey-Fuller	0.830	0.826	0.842
<i>Period: 1999Q1-2015Q2</i>			
Dickey-Fuller	0.112	0.125	0.116
<i>Period: 2001Q4-2018Q4</i>			
Dickey-Fuller	0.513	0.526	0.563
Panel B: SPF consumption forecasts - I(2) test			
<i>Period: 1981Q4-2019Q4</i>			
Dickey-Fuller	0.00	0.00	0.00
<i>Period: 1999Q1-2015Q2</i>			
Dickey-Fuller	0.00	0.00	0.00
<i>Period: 2001Q4-2018Q4</i>			
Dickey-Fuller	0.00	0.00	0.00
Evidence A.3: Median aggregate consumption forecasts are I(1) but not I(2).			

Table A.6: **Integration of mean SPF consumption forecasts over different periods**

P values			
Median	1Q ahead	2Q ahead	4Q ahead
Panel A: SPF consumption forecasts - I(1) test			
<i>Period: 1981Q4-2019Q4</i>			
Dickey-Fuller	0.908	0.902	0.912
<i>Period: 1999Q1-2015Q2</i>			
Dickey-Fuller	0.113	0.127	0.117
<i>Period: 2001Q4-2018Q4</i>			
Dickey-Fuller	0.516	0.509	0.509
Panel B: SPF consumption forecasts - I(2) test			
<i>Period: 1981Q4-2019Q4</i>			
Dickey-Fuller	0.00	0.00	0.00
<i>Period: 1999Q1-2015Q2</i>			
Dickey-Fuller	0.00	0.00	0.00
<i>Period: 2001Q4-2018Q4</i>			
Dickey-Fuller	0.00	0.00	0.00
Evidence A.4: Mean consumption and dividend forecasts are I(1) but not I(2).			

Table A.7: **Integration properties of median (and mean) forecasts of dividend**

P values		
1-year ahead	Median forecasts	Mean forecasts
Panel A: I(1) test		
<i>IBES dividend forecasts (2003Q1-2015Q3)</i>		
Dickey-Fuller	0.950	0.949
Panel B: I(2) test		
<i>IBES dividend forecasts (2003Q1-2015Q3)</i>		
Dickey-Fuller	0.00	0.00
Evidence A.5: Median (and mean) aggregate dividend forecasts are I(1) but not I(2).		

F No cointegration of median and mean forecasts

F.1 No cointegration with deflating with inflation expectation from Michigan Surveys of Consumers

Table A.8: No cointegration between stock price forecasts and fundamental forecasts, deflated with inflation expectation from Michigan Surveys of Consumers

Stock price forecasts and consumption forecasts		
	Livingston & SPF	Shiller & SPF
PP (Z_t test)	-2.732	-2.537
10% critical value	-2.890	-2.599
DF-GLS	0.621	-1.971
10% critical value	-1.859	-1.995
KPSS	0.257	0.183
5% critical value	0.146	0.146
Engle-Granger	-1.735	-3.398
5% critical value	-4.334	-4.415
Gregory-Hansen	-4.59	-4.76
5% critical value	-5.50	-5.50

F.2 No cointegration with imposing theoretical restrictions

Table A.9: Cointegration tests between mean stock price forecasts and SPF consumption forecasts with the cointegrating vector $(1, -1)$

Panel A: <i>Livingston stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	4Q ahead	4Q P & 1Q C
PP (Z_t test)	<i>n.a</i>	-2.184	-2.042	-2.032
10% critical value	<i>n.a</i>	-2.586	-2.586	-2.586
DF-GLS	<i>n.a</i>	-0.085	0.186	0.161
10% critical value	<i>n.a</i>	-1.839	-1.815	-1.815
KPSS	<i>n.a</i>	0.387	0.184	0.183
5% critical value	<i>n.a</i>	0.146	0.146	0.146
Panel B: <i>Shiller stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	4Q ahead	4Q P & 1Q C
PP (Z_t test)	-2.050	-2.121	-2.157	-2.153
10% critical value	-2.594	-2.594	-2.594	-2.594
DF-GLS	-1.785	-1.794	-1.803	-1.815
10% critical value	-1.895	-1.895	-1.895	-1.895
KPSS	0.421	0.425	0.428	0.429
5% critical value	0.146	0.146	0.146	0.146
Panel C: <i>CFO stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	3Q ahead	4Q ahead
PP (Z_t test)	-2.050	-2.121	-2.157	-2.153
10% critical value	-2.594	-2.594	-2.594	-2.594
DF-GLS	-1.785	-1.794	-1.803	-1.815
10% critical value	-1.895	-1.895	-1.895	-1.895
KPSS	0.421	0.425	0.428	0.429
5% critical value	0.146	0.146	0.146	0.146

Evidence A.6: Mean stock price forecasts and consumption forecasts are not cointegrated with the vector $(1, -1)$.

F.3 No cointegration without imposing theoretical restrictions

This section tests cointegration between forecasts of stock prices and aggregate consumption (or dividend) without imposing the theory-implied cointegrating vector $(1, -1)$, using the Engle–Granger (EG) test. Again, the last column of Panel A and B are the testing results of using 4-quarter-ahead stock price forecasts and 1-quarter-ahead consumption forecasts.

The test results uniformly suggest that we cannot reject the null hypothesis that stock price forecasts are not cointegrated with consumption (or dividend) forecasts at the 10% significance level and over the same or different forecasting horizons, as all test statistics are greater than the corresponding 10% critical values.

Median forecasts (stock price and consumption). Table A.10 reports the results of testing the cointegration between median stock price forecasts and median consumption forecasts over the same or different horizons.³² The test results uniformly suggest that we cannot reject the null hypothesis that stock price forecasts are not cointegrated with consumption forecasts at the 10% significance level and over the same or different forecasting horizons, as all test statistics are greater than the corresponding 10% critical values.

Table A.10: **No cointegration test between median stock price forecasts and consumption forecasts (without imposing any vector)**

Engle–Granger test				
Panel A: Livingston stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	<i>n.a</i>	-1.695	-1.662	-1.686
10% critical value	<i>n.a</i>	-4.014	-4.014	-4.014
Panel B: Shiller stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	-3.317	-3.337	-3.144	-3.233
10% critical value	-4.038	-4.038	-4.038	-4.038
Panel C: CFO stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	-2.654	-2.793	-2.950	-3.087
10% critical value	-4.031	-4.031	-4.031	-4.031

Evidence A.7: Median stock price forecasts are not cointegrated with consumption forecasts without imposing any cointegrating vector.

Mean forecasts (stock price and consumption). Similarly, Table A.11 reports the

³²We report the Engle–Granger test results without incorporating the trend component. Our test results are robust when the trend is included. The Engle–Granger test’s critical values have been calculated by MacKinnon (1996).

results of testing the cointegration between mean stock price forecasts and mean consumption forecasts over the same or different horizons. We do not find cointegration between mean stock price forecasts and consumption forecasts.

Table A.11: **Cointegration test for mean SPF forecasts without imposing the cointegrating vector $(1, -1)$**

Engle–Granger test				
Panel A: <i>Livingston stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	<i>n.a</i>	-1.977	-1.994	-1.997
10% critical value	<i>n.a</i>	-4.000	-4.000	-4.000
Panel B: <i>Shiller stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	-3.334	-3.283	-3.102	-3.140
10% critical value	-4.038	-4.038	-4.038	-4.038
Panel C: <i>CFO stock price forecasts and SPF consumption forecasts</i>				
Mean	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	-3.334	-3.283	-3.102	-3.140
10% critical value	-4.031	-4.031	-4.031	-4.031

Evidence A.8: Mean stock price forecasts are not cointegrated with consumption forecasts without imposing any cointegrating vector.

Forecasts of stock price and dividends. Table A.12 reports the results for testing the cointegration between stock price forecasts and dividend forecasts. Test results also suggest that stock price forecasts are not cointegrated with dividends forecasts.

Table A.12: No cointegration between stock price forecasts and dividend forecasts (without imposing any vector)

Engle–Granger test		
	Median forecasts	Mean forecasts
Engle-Granger test	-1.038	-1.051
10% critical value	-3.130	-3.130

Evidence A.9: Median (and mean) stock price forecasts and dividend forecasts are not cointegrated without imposing any cointegrating vector.

G No cointegration properties of individual-level forecasts

G.1 No cointegration between stock price forecasts and aggregate output forecasts made over different horizons

This section tests cointegration between individual-level forecasts of output and stock prices over different forecasting horizons and with imposing the theory-implied $(1, -1)$ cointegrating vector. Figure A.1 (or A.2) reports the test results produced by the PP, DF-GLS and KPSS tests, using 2-quarter (or 4-quarter) ahead stock price forecasts and 4-quarter (or 2-quarter) ahead output forecasts for 11 forecasters. Similar to the results in Section 4.2, the test statistics of PP and DF-GLS tests for all forecasters stay above the critical values in all cases. For KPSS tests, the test statistics for all forecasters are above the corresponding critical values (except for the forecasters with the ID number 171 and 378). These results suggest that for all forecasters, there is no cointegration between stock price forecasts and output forecasts made over different horizons.

Figure A.1: Cointegration tests between individual-level 2Q-ahead stock price forecasts and 4Q-ahead output forecasts

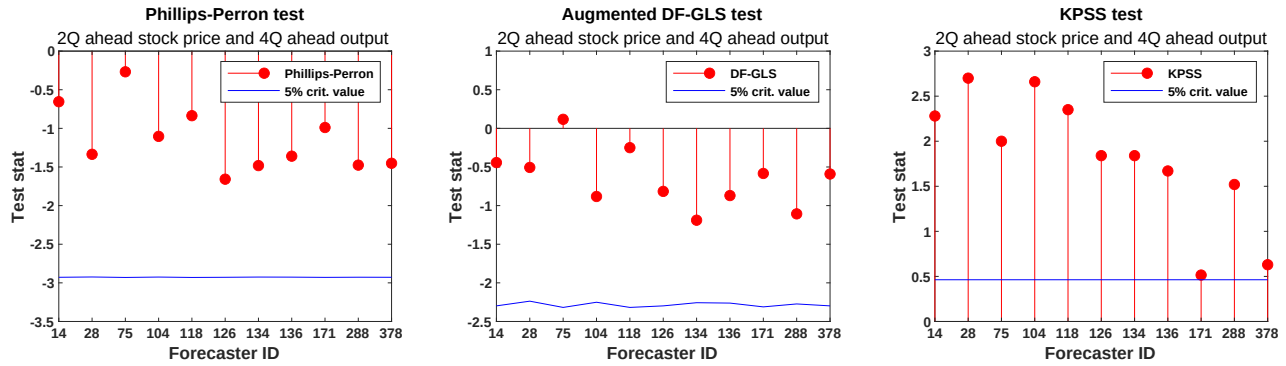
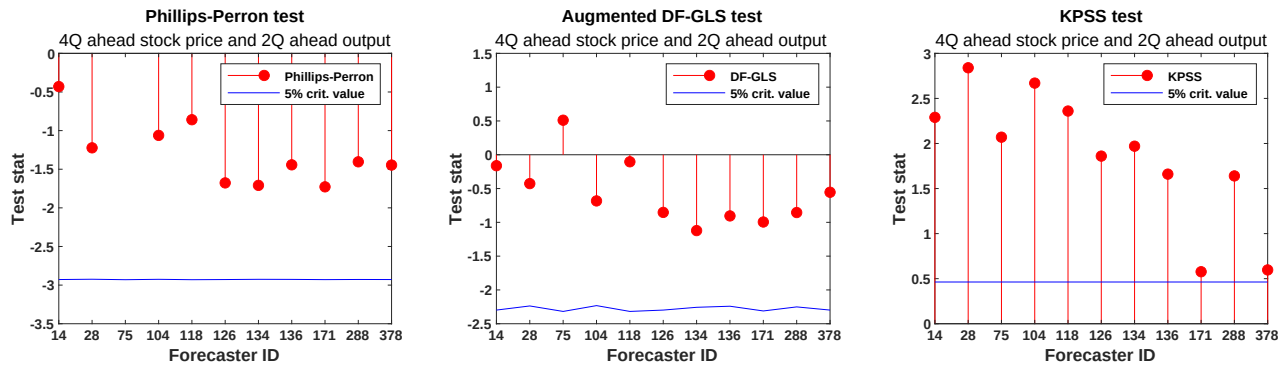


Figure A.2: Cointegration tests between individual-level 4Q-ahead stock price forecasts and 2Q-ahead output forecasts



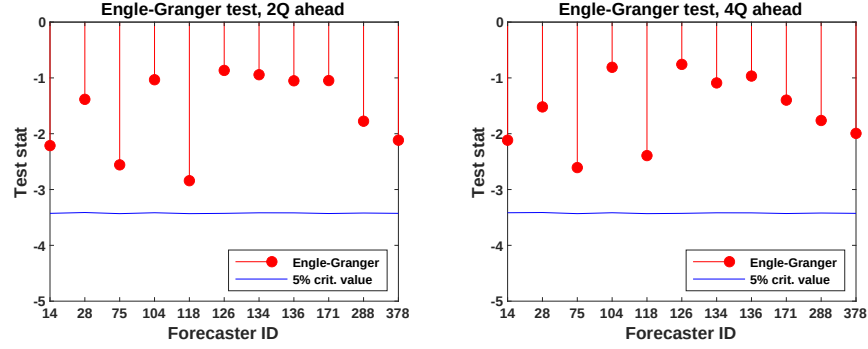
Evidence A.10: Individual stock price forecasts and output forecasts over different horizons are not cointegrated with the vector $(1, -1)$.

G.2 Individual-level cointegration tests without imposing theoretical restrictions

This section tests cointegration between individual-level stock price forecasts and output forecasts without imposing any cointegrating vector. Figure A.3 reports the testing results produced by EG test, using 2-quarter (or 4-quarter) ahead stock price forecasts and output forecasts for 11 forecasters made over the same forecasting horizon. Figure A.4 reports the same results using forecasts made over different forecasting horizons. The test statistics of EG tests stay above the critical values in all cases. This suggests no cointegration between stock price forecasts and output forecasts for any cointegrating vector.

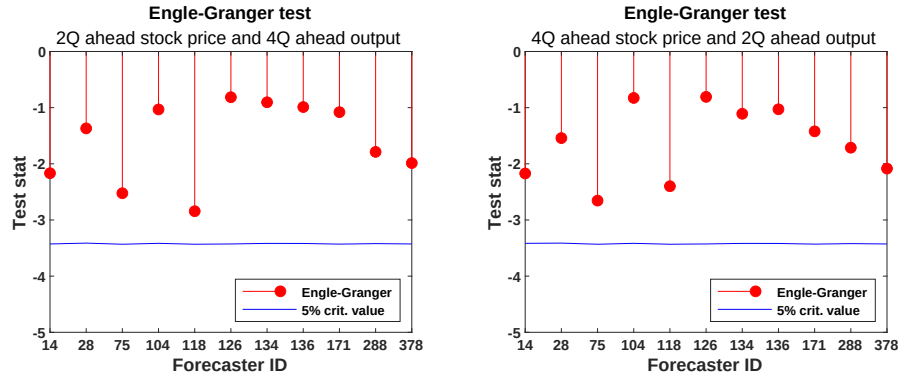
The forecasters in Livingston survey are experts from various sectors. According to their IDs in Figure 1, these sectors include academic institutions, commercial banking, investment banking and nonfinancing business. Overall, our results imply that all professional forecasters, regardless of their sectors, do not utilize the cointegration relation between stock prices and output implied in standard asset pricing models to forecast.

Figure A.3: Cointegration tests between individual-level forecasts of output and stock prices, made over the same horizon



Evidence A.11: Individual stock price forecasts and output forecasts over the same horizon are not cointegrated without imposing any cointegrating vector.

Figure A.4: Cointegration tests between individual-level forecasts of stock prices and output, made over different horizons



Evidence A.12: Individual stock price forecasts and output forecasts over different horizons are not cointegrated without imposing any cointegrating vector.

H Additional tables and figures for several testing issues

This section provides additional tables and figures for Section 8.

H.1 Structural breaks

H.1.1 Recursive Cusum tests for forecasts of stock price and consumption

Table A.13: Test for structural break in median forecasts of stock price-consumption ratios

Cusum test (OLS residual)				
Panel A: <i>Livingston stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	4Q ahead	4Q Y &1Q C
Test stats.	<i>n.a</i>	0.713	0.7643	0.7609
5% critical value	<i>n.a</i>	1.358	1.358	1.358
Panel B: <i>Shiller stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	4Q ahead	4Q Y &1Q C
Test stats.	1.037	1.044	0.978	0.988
5% critical value	1.358	1.358	1.358	1.358
Panel C: <i>CFO stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	1.165	1.155	1.147	1.139
5% critical value	1.358	1.358	1.358	1.358

Evidence A.13-1: No structural break is detected in median forecasts of stock price-consumption ratio.

Table A.14: Test for structural break in mean forecasts of stock price-consumption ratios

Test for structure break - Cusum test (OLS residual)				
Panel A: <i>Livingston stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	<i>n.a</i>	0.580	0.706	0.698
5% critical value	<i>n.a</i>	1.358	1.358	1.358
Panel B: <i>Shiller stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P &1Q C
Test stats.	1.059	1.019	0.991	1.001
5% critical value	1.358	1.358	1.358	1.358
Panel C: <i>CFO stock price forecasts and SPF consumption forecasts</i>				
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	1.278	1.276	1.274	1.271
5% critical value	1.358	1.358	1.358	1.358

Evidence A.13-2: No structural break is detected in mean forecasts of stock price-consumption ratios.

H.1.2 Gregory-Hansen tests for forecasts of stock price and consumption

Table A.15: **Gregory-Hansen cointegration tests between median stock price forecasts and consumption forecasts**

Gregory-Hansen cointegration test (assuming structural break)				
Panel A: Livingston stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P & 1Q C
Test stats.	<i>n.a</i>	-5.08	-5.20	-5.23
5% critical value	<i>n.a</i>	-5.50	-5.50	-5.50
Panel B: Shiller stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q P & 1Q C
Test stats.	-3.52	-3.94	-3.64	-3.52
5% critical value	-5.50	-5.50	-5.50	-5.50
Panel C: CFO stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	-5.47	-5.39	-5.24	-5.34
5% critical value	-5.50	-5.50	-5.50	-5.50

Evidence A.14-1: Median stock price forecasts are not cointegrated with consumption forecasts assuming the existence of a structural break.

Table A.16: **Gregory-Hansen cointegration tests between mean stock price forecasts and SPF consumption forecasts**

Gregory-Hansen cointegration test (assuming structural break)				
Panel A: Livingston stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q Y & 1Q C
Test stats.	<i>n.a</i>	-5.21	-5.26	-5.13
5% critical value	<i>n.a</i>	-5.50	-5.50	-5.50
Panel B: Shiller stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	4Q ahead	4Q Y & 1Q I
Test stats.	-3.51	-3.50	-3.56	-3.53
5% critical value	-5.50	-5.50	-5.50	-5.50
Panel C: CFO stock price forecasts and SPF consumption forecasts				
Median	1Q ahead	2Q ahead	3Q ahead	4Q ahead
Test stats.	-5.16	-5.14	-5.09	-5.05
5% critical value	-5.50	-5.50	-5.50	-5.50

Evidence A.14-2: Mean stock price forecasts are not cointegrated with consumption forecasts assuming the existence of a structural break.

H.1.3 Tests for stock price and dividend forecasts

Table A.17: **Test for structural break in forecasts of stock price-dividend ratio**
Addressing issues related to structural break

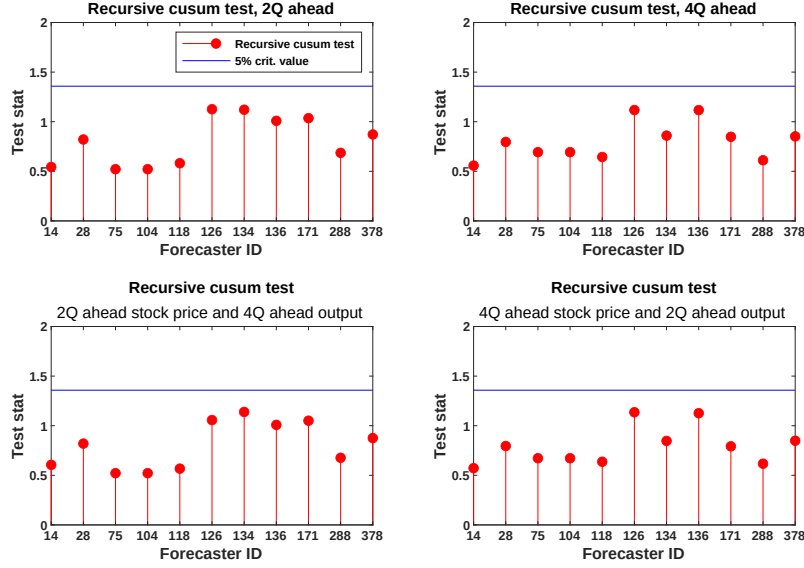
Panel A: Cusum test (OLS residual) for structural break		
	Median forecasts	Mean forecasts
Test stats.	0.557	0.539
5% critical value	1.224	1.224
Panel B: Gregory-Hansen cointegration test		
	1Q ahead	2Q ahead
Test stats.	-5.21	-5.24
5% critical value	-5.50	-5.50

Evidence A.15-1: No structural break is detected in median (or mean) forecasts of stock price-dividend ratio.

Evidence A.15-2: Median (or mean) forecasts of stock price are not cointegrated with forecasts of dividend assuming the existence of a structural break.

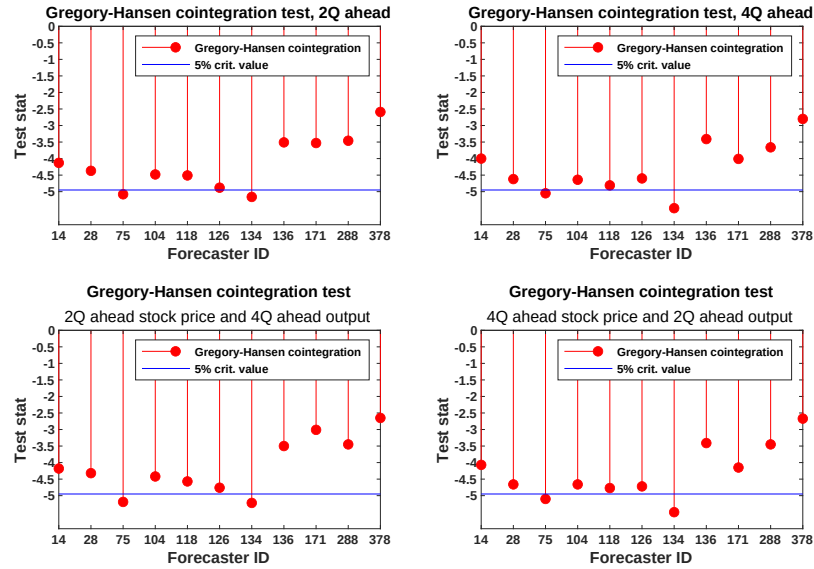
H.1.4 Tests for individual-level forecasts

Figure A.5: Illustration of individual-level Recursive Cusum tests for structural break



Evidence A.16-1: No structure break is detected in individual forecasts of stock price-output ratios.

Figure A.6: Illustration of individual-level Gregory-Hansen cointegration tests

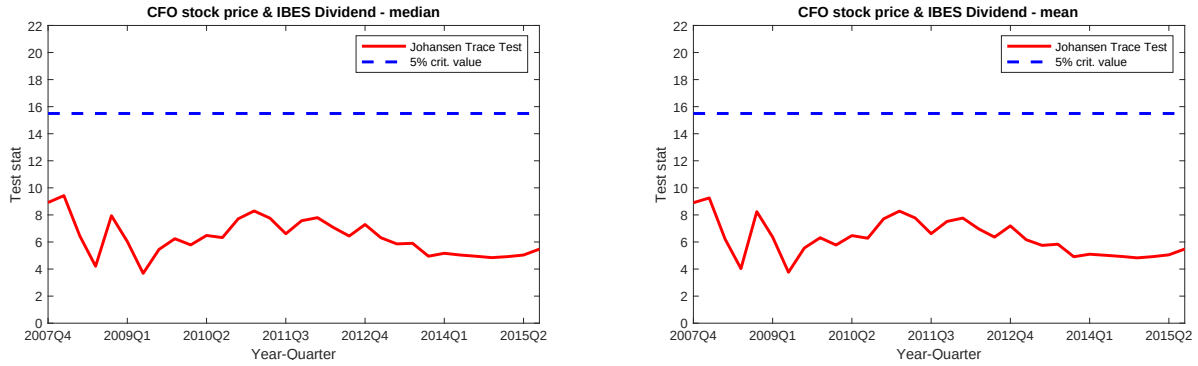


Evidence A.16-2: For almost all forecasters, individual-level forecasts of stock prices are not cointegrated with forecasts of output assuming the existence of structure break.

H.2 Sample size

H.2.1 Recursive Johansen Trace testing for stock price forecast and dividend forecast

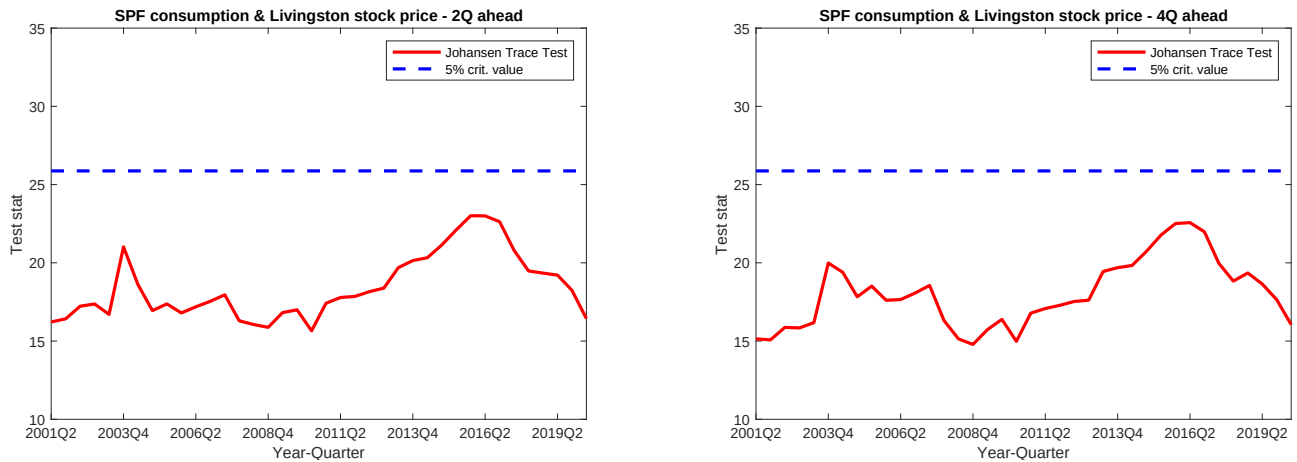
Figure A.7: Recursive Johansen trace test (rank = 0) for median and mean forecasts of stock prices (CFO) and dividend (IBES)



Evidence A.17-1: Forecasts of CFO stock prices are not cointegrated with IBES forecasts of dividends.

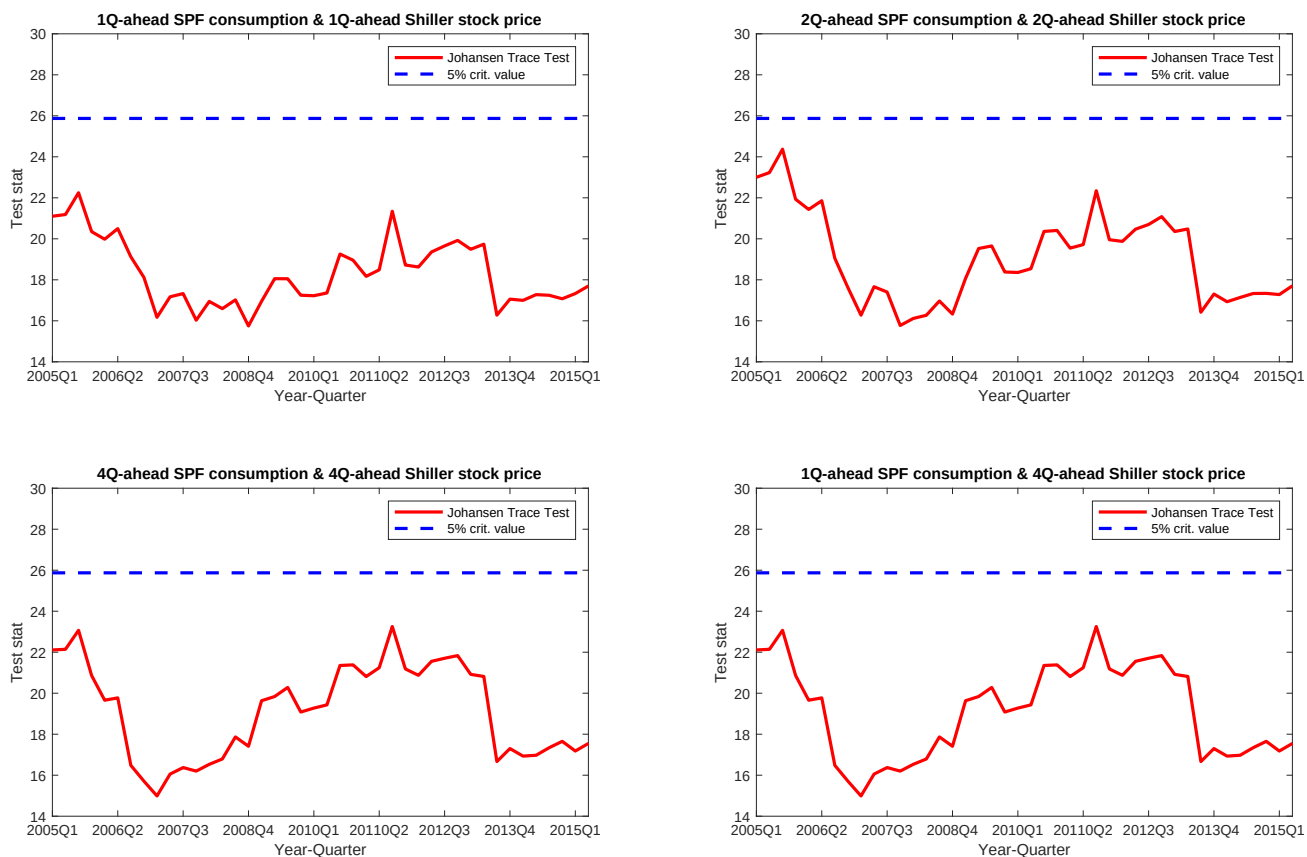
H.2.2 Recursive Johansen Trace testing for median forecasts

Figure A.8: Recursive Johansen trace test (rank = 0) for median forecasts of stock prices (Livingston) and consumption (SPF)



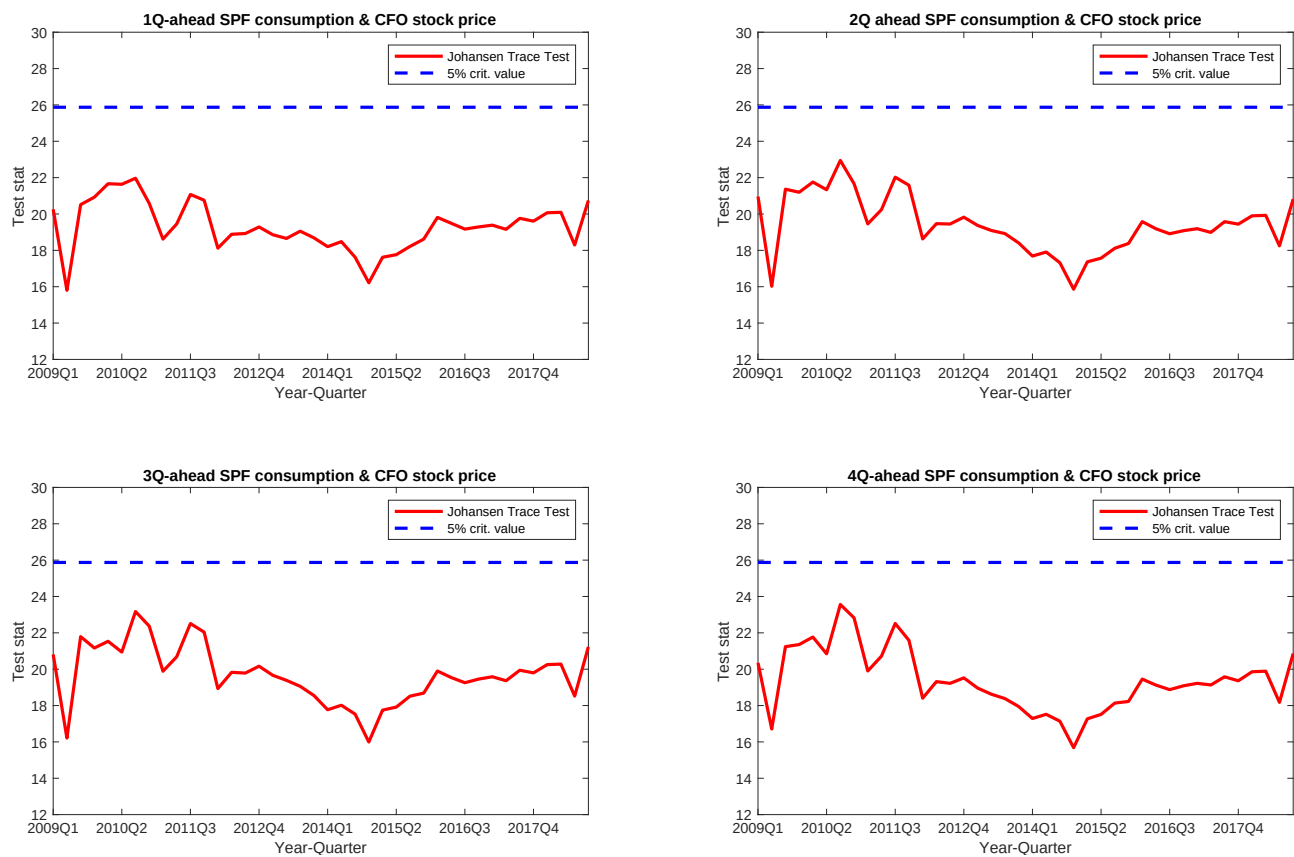
Evidence A.17-2: Median forecasts of Livingston stock prices are not cointegrated with median SPF consumption forecasts.

Figure A.9: Recursive Johansen trace test (rank = 0) for median forecasts of stock price (Shiller) and consumption (SPF)



Evidence A.17-3: Median forecasts of Shiller stock prices are not cointegrated with median SPF consumption forecasts.

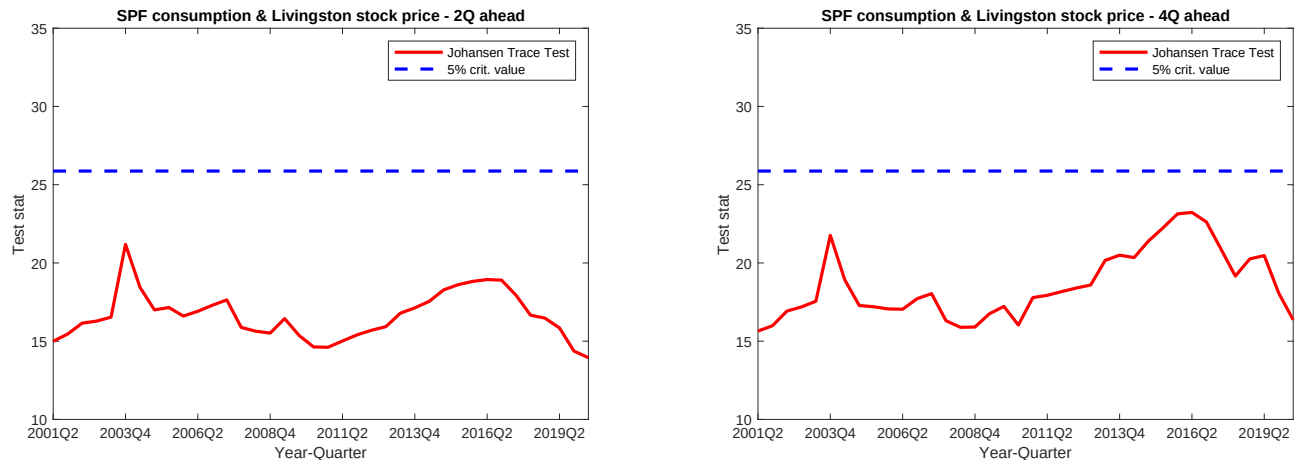
Figure A.10: Recursive Johansen trace test (rank = 0) for median forecasts of stock prices (CFO) and consumption (SPF)



Evidence A.17-4: Median forecasts of CFO stock prices are not cointegrated with median SPF consumption forecasts.

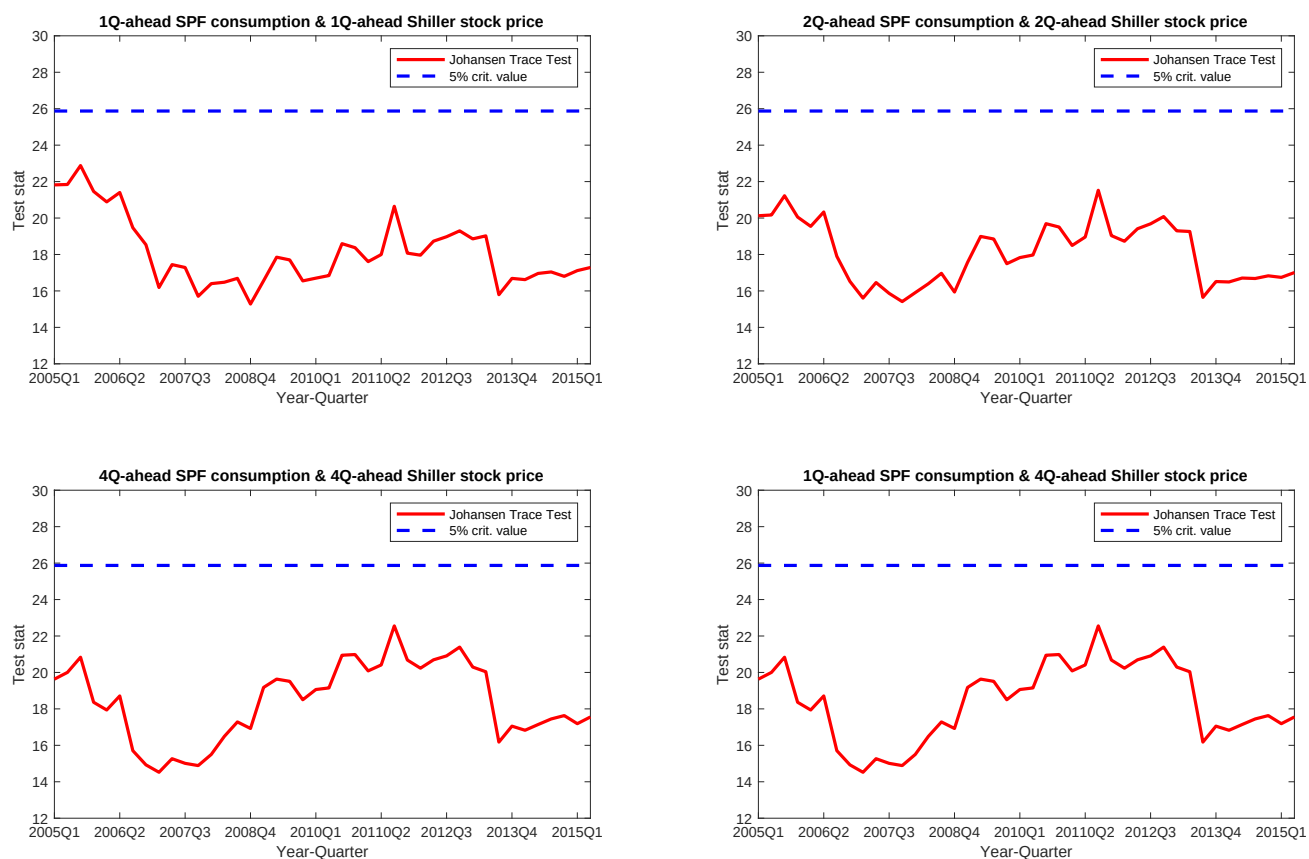
H.2.3 Recursive Johansen Trace testing for mean forecasts

Figure A.11: Recursive Johansen trace test ($\text{rank} = 0$) for mean Livingston stock price forecasts and SPF consumption forecasts



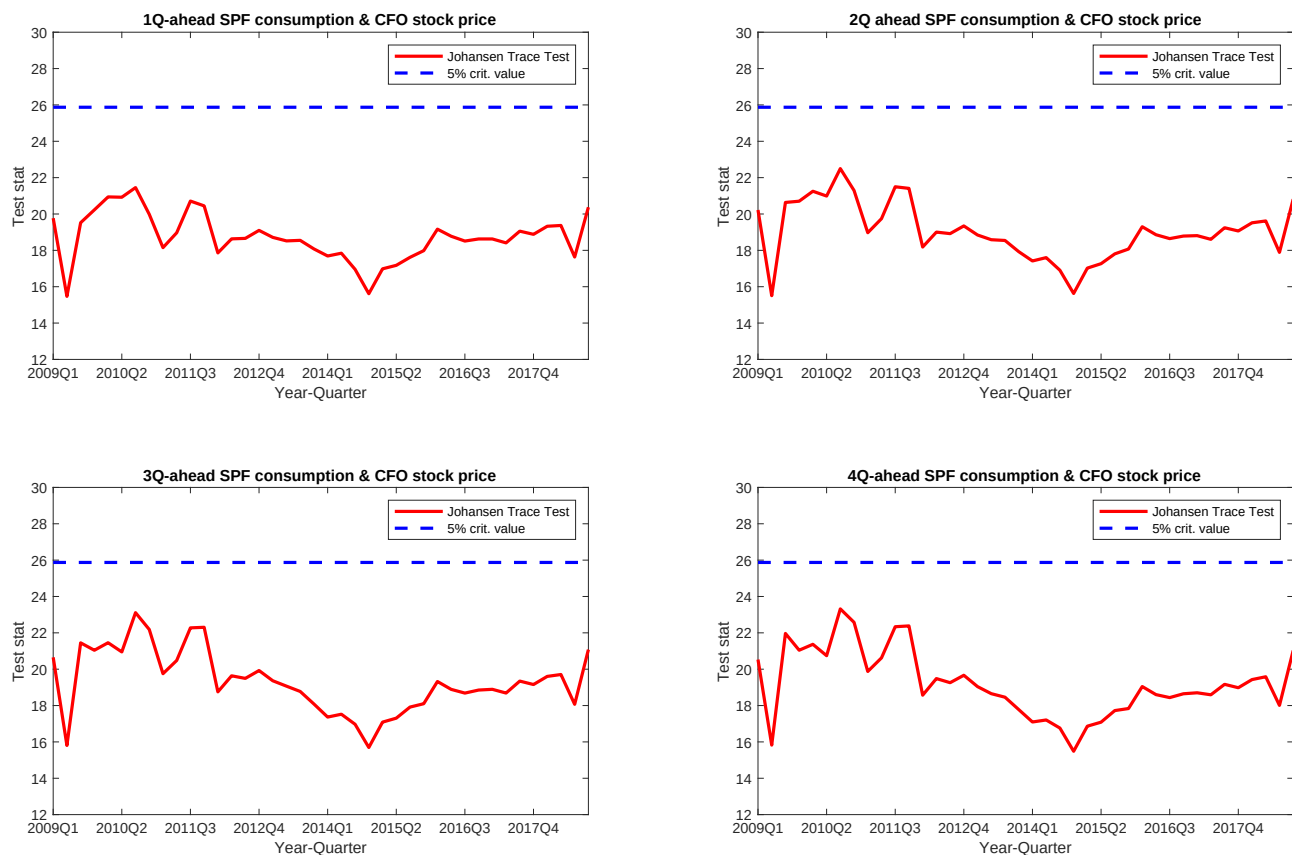
Evidence A.17-5: Mean forecasts of Livingston stock prices are not cointegrated with mean SPF consumption forecasts.

Figure A.12: Recursive Johansen trace test (rank = 0) for mean Shiller stock price forecasts and SPF consumption forecasts



Evidence A.17-6: Mean Shiller forecasts of stock prices are not cointegrated with mean SPF consumption forecasts.

Figure A.13: Recursive Johansen trace test (rank = 0) for mean CFO stock price forecasts and SPF consumption forecasts



Evidence A.17-7: Mean CFO forecasts of stock prices are not cointegrated with mean SPF consumption forecasts.

H.2.4 Recursive Johansen Trace testing for individual forecasts

Figure A.14: Recursive Johansen Trace test for individual-level forecasts (ID:126)

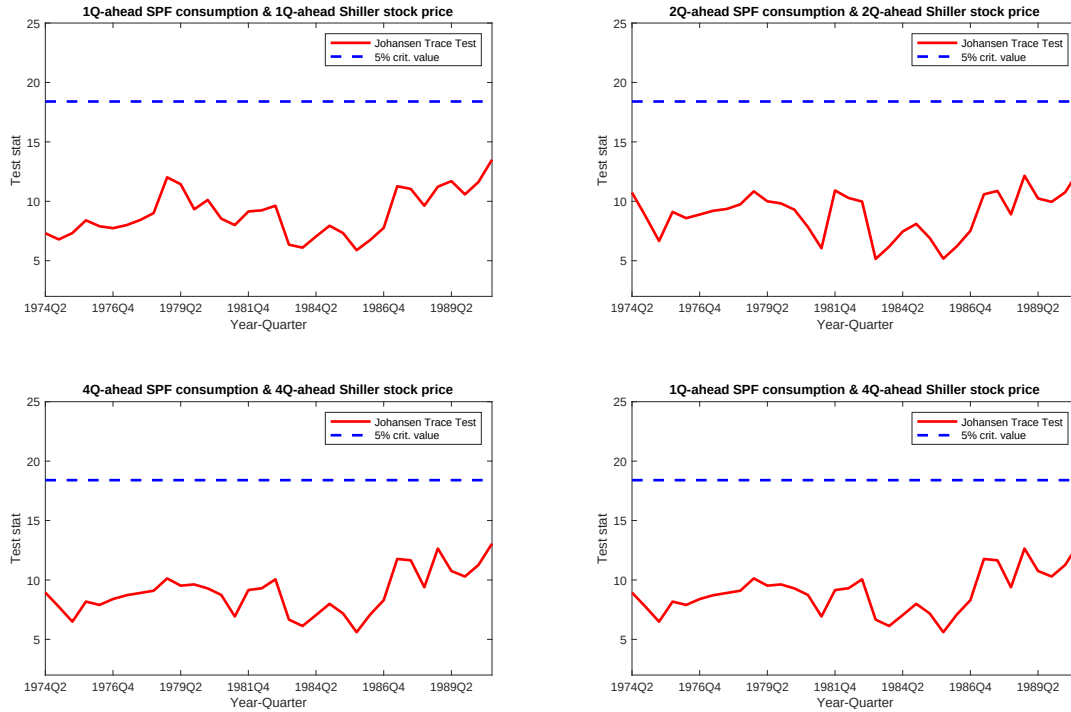


Figure A.15: Recursive Johansen Trace test for individual-level forecasts (ID:136)

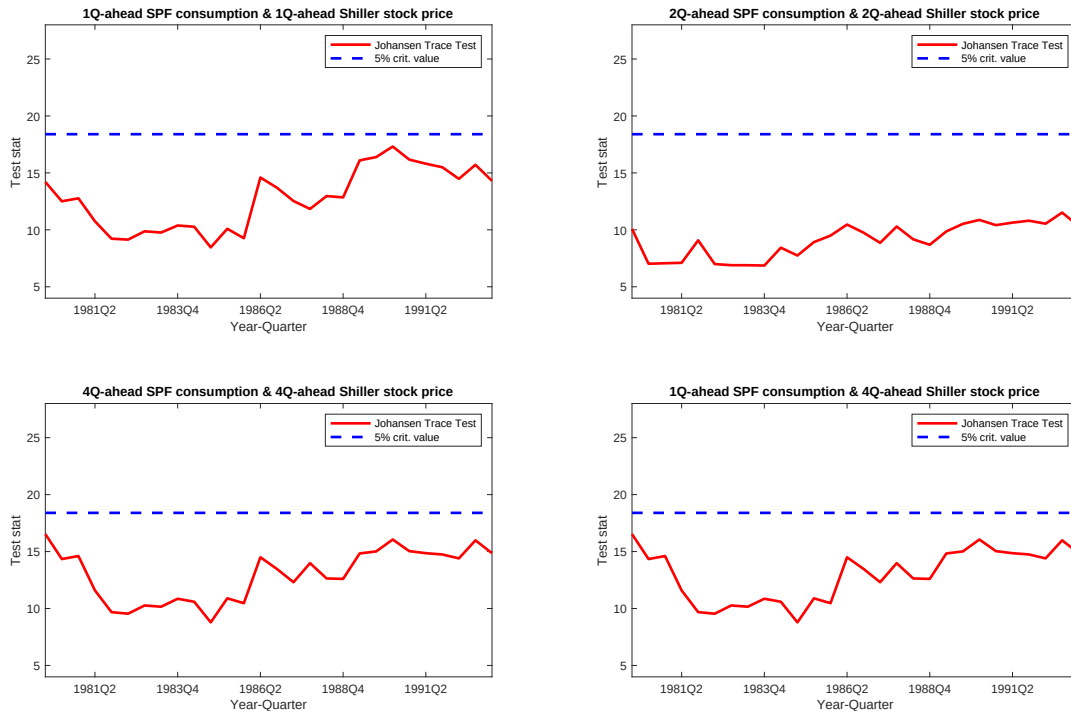


Figure A.16: Recursive Johansen Trace test for individual-level forecasts (ID:288)

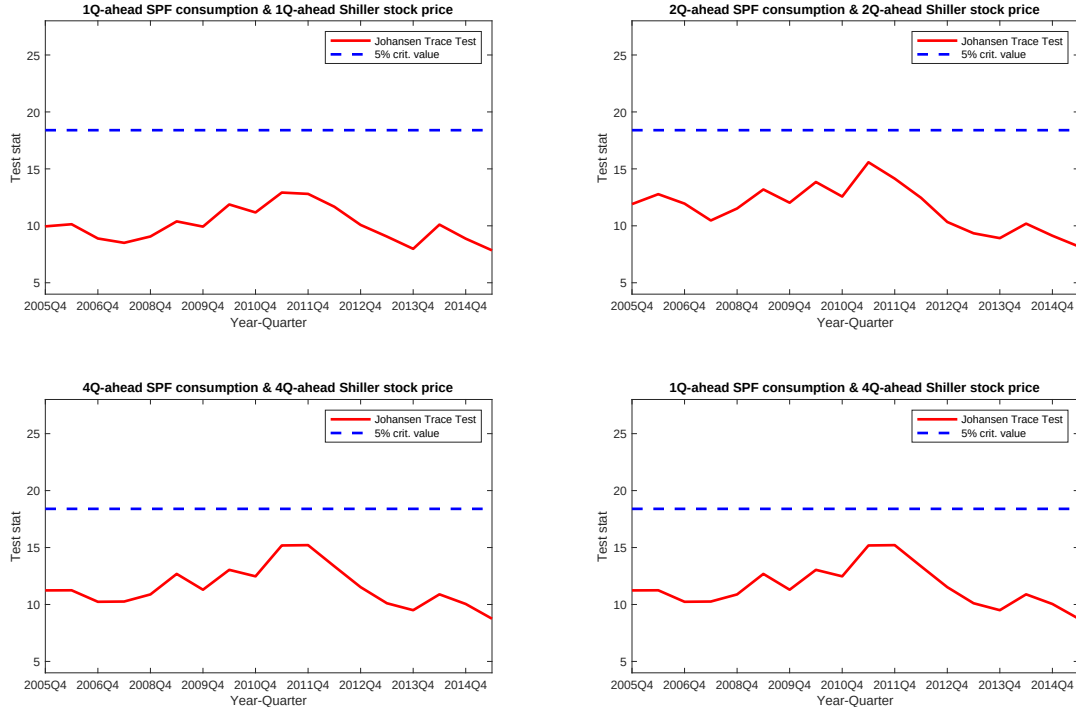
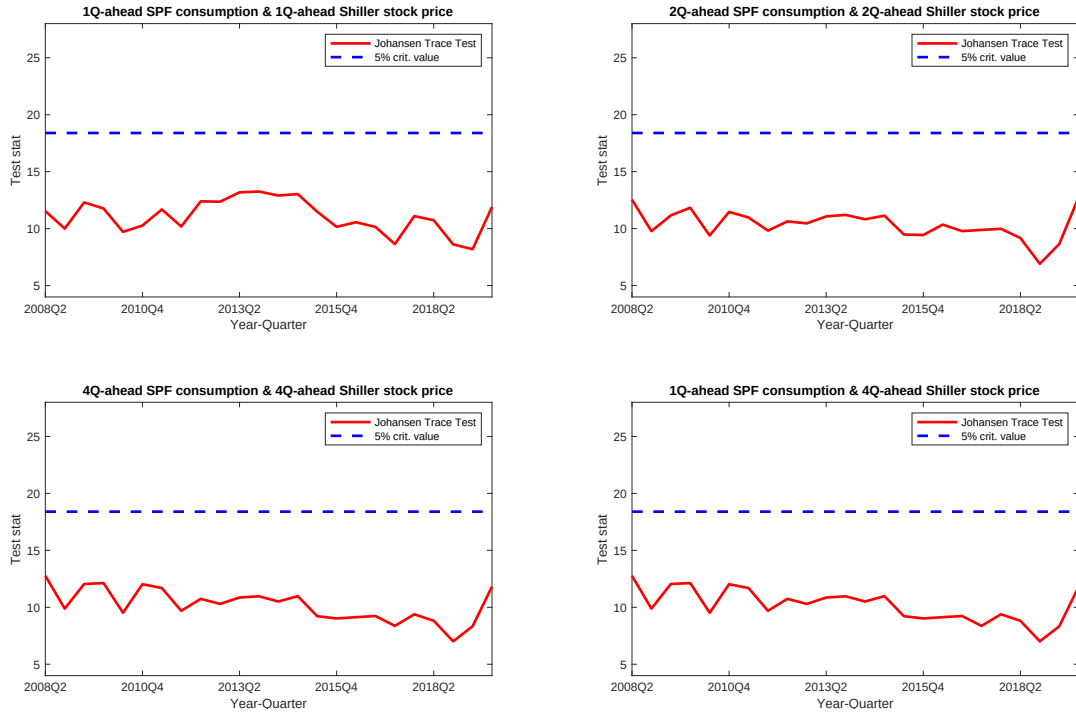


Figure A.17: Recursive Johansen Trace test for individual-level forecasts (ID:378)



H.2.5 Cointegration between forecasts made on different dates

Table A.18: No cointegration between $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_2} \log C_{i_2+j_2} : i_1 \neq i_2$

Panel A: $i_1 = 57H1, 57H2, \dots, 19H2; i_2 = 84Q1, 84Q2, \dots, 15Q2$				
I(1) test	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	-0.961	-0.858	-0.963	-0.856
10% critical value	-2.578	-2.578	-2.578	-2.578
DF-GLS	-0.714	-0.602	-0.708	-0.607
10% critical value	-1.781	-1.781	-1.781	-1.781
KPSS test	2.25	2.32	2.25	2.33
1% critical value	0.739	0.739	0.739	0.739
Panel B: $i_1 = 57H1, 57H2, \dots, 19H2; i_2 = 87Q4, 88Q1, \dots, 19Q1$				
I(1) test	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	-0.872	-0.773	-0.870	-0.775
10% critical value	-2.578	-2.578	-2.578	-2.578
DF-GLS	-0.715	-0.613	-0.718	-0.610
10% critical value	-1.781	-1.781	-1.781	-1.781
KPSS test	2.17	2.22	2.16	2.23
1% critical value	0.739	0.739	0.739	0.739
Panel C: $i_1 = 59H1, 59H2, \dots, 19H2; i_2 = 88Q4, 89Q1, \dots, 19Q1$				
	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t stat.)	-0.825	-0.732	-0.825	-0.733
10% critical value	-2.579	-2.579	-2.579	-2.579
DF-GLS	-0.842	-0.755	-0.840	-0.757
10% critical value	-1.786	-1.786	-1.786	-1.786
KPSS test	2.17	2.23	2.16	2.24
1% critical value	0.739	0.739	0.739	0.739

Note: cointegration tests utilize 126 pairs of observations in panel A and B and 122 pairs of observations in panel C.

Evidence A.18: Median stock price forecasts and consumption forecasts made at different dates are not cointegrated with the vector (1,-1).

Table A.19: No cointegration between median and mean forecasts of P and C

<i>KPSS test</i>	2Q ahead	4Q ahead
Panel A: median Livingston forecasts		
Test statistic	0.500	0.240
5% critical value	0.146	0.146
5% critical value (corrected)	0.185	0.185
Panel B: mean Livingston forecasts		
Test statistic	0.512	0.242
5% critical value	0.146	0.146
5% critical value (corrected)	0.185	0.185

Evidence A.19: Median/mean forecasts of stock price are not cointegrated with forecasts of consumption from the Monte Carlo study.

H.3 Multiple testing problem

Table A.20: Individual-level cointegration testing adjusted for multiple testing problem

PP tests	Using FDR sharpened q-values	
<i>Forecasts of stock prices and output</i>	No. of no cointegration	Proportion of no cointegration
2Q ahead	11	100%
4Q ahead	11	100%
2Q ahead & 4Q ahead	11	100%
4Q ahead & 2Q ahead	11	100%

Evidence A.20: Individual-level forecasts of stock price are not cointegrated with output forecasts, after adjusted for the multiple testing problem.

Table A.21: **Test results of Pesaran panel cross-sectional dependence test**

Number of panels: 11 in each test				
H_0 : forecasts are cross-sectionally independent.				
H_1 : forecasts are cross-sectionally dependent.				
	2Q ahead	4Q ahead	2Q & 4Q ahead	4Q & 2Q ahead
Forecasts of stock price-output ratio				
P-values	0.000	0.000	0.000	0.000
Average correlation coeff.	0.84	0.91	0.83	0.86

Table A.22: **P-values of Pesaran panel cointegration test with the cointegrating vector $(1, -1)$**

CADF panel unit root test Number of panels: 11 in each test				
H_0 : All panels are non-stationary;				
H_1 : At least one panel is stationary				
	1Q ahead	2Q ahead	3Q ahead	4Q ahead
P-values	0.510	0.288	0.412	0.367

Evidence A.21: The CADF panel cointegration tests suggest that individual forecasters' forecasts of stock prices are not cointegrated with forecasts of output.

I The model

This section develops a Lucas-type asset pricing model with adaptive learning and sentiment shocks, which simultaneously replicates the survey evidence and a set of equity pricing facts, such as the observed risk premium, high volatility of stock returns and high persistence and volatility of the price-dividend ratio.

I.1 Model setup

Consider an economy with a unit mass of infinitely lived investors, endowed with one unit of a stock that can be traded competitively in a stock market in each period. Each unit of stock pays dividend D_t , consisting of a perishable consumption goods. Besides, each investor receives extra wage income of endowment W_t of non-storable consumption goods at the beginning of

each period. The wage income follows

$$\log\left(1 + \frac{W_t}{D_t}\right) = (1 - p)\log(1 + \rho) + p\log\left(1 + \frac{W_{t-1}}{D_{t-1}}\right) + \log\epsilon_t^w, \quad (\text{A.10})$$

for $t = 0, 1, 2, \dots$. $\log\epsilon_t^w \sim iiN(-\frac{\sigma_w^2}{2}, \sigma_w^2)$ and $1 + \rho$ is the mean consumption-dividend ratio, with quarterly persistence of p . Similarly, $E(\epsilon_t^w) = 1$. The aggregate supply of consumption goods in this economy satisfies the feasibility constraint

$$C_t = D_t + W_t \quad (\text{A.11})$$

in each period. Dividends evolve according to

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \quad (\text{A.12})$$

for $t = 0, 1, 2, \dots$, where $\log\epsilon_t^d \sim iiN(-\frac{\sigma_d^2}{2}, \sigma_d^2)$. This indicates $E(\epsilon_t^d) = 1$, $\sigma_{\frac{D}{D}}^2 = a^2(e^{\sigma_d^2} - 1)$ and the expected dividend growth $E_{\frac{D}{D}} = a - 1$.

There is a continuum of identical investors with measure one, $i \in [0, 1]$. The representative investor has an identical constant-relative-risk-aversion (CRRA) expected utility function

$$\tilde{E}_t \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma}, \quad (\text{A.13})$$

where $\tilde{E}_t$ stands for the representative investor's subjective expectation, $\gamma \in (0, \infty)$ the coefficient of relative risk aversion and C_t^i the consumption at period t . She makes contingent plans for consumption C_t^i , bond holding B_t^i , and stock shares holding S_t^i , subject to the budget constraint

$$C_t^i + P_t S_t^i + B_t^i \leq (P_t + D_t) S_{t-1}^i + (1 + r_{t-1}) B_{t-1}^i + W_t, \quad (\text{A.14})$$

where P_t is stock price, r_{t-1} is the real interest rate on risk-free bonds issued in period $t - 1$ and W_t is the exogenous income. The initial endowments of stock shares and bonds are given: $S_{-1}^i = 1$ and $B_{-1}^i = 0$.

To avoid Ponzi schemes and ensure the existence of a maximum, the following finite lower

and upper bounds for S_t^i and B_t^i are assumed to always hold

$$S^L \leq S_t^i \leq S^U, \quad (\text{A.15})$$

and

$$B^L \leq B_t^i \leq B^U, \quad (\text{A.16})$$

where $S^L < 1 < S^U$ and $B^L < 0 < B^U$.

The representative investor is assumed to have perfect knowledge about the process of aggregate consumption supply (A.11), dividends (A.12) and wage (A.10), and can derive the correct consumption process. The fundamental shocks are jointly realized from a bi-variate normal distribution:

$$\begin{pmatrix} \log \epsilon_t^d \\ \log \epsilon_t^w \end{pmatrix} \sim iin \left[\begin{pmatrix} -\frac{\sigma_d^2}{2} \\ -\frac{\sigma_w^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & \rho_{d,w} \sigma_d \sigma_w \\ \rho_{w,d} \sigma_w \sigma_d & \sigma_w^2 \end{pmatrix} \right]. \quad (\text{A.17})$$

The representative investor faces the problem of finding a sequence of demand functions $\{C_t^i, S_t^i, B_t^i\}_{t=0}^\infty$ to maximize her subjective expected utility function (A.13), subject to the budget constraint (A.14) non-ponzi schemes constraints (A.15) and (A.16). Given a concave objective function (A.13) and a convex feasible set, optimal decisions of the representative investor can be characterized by two first-order conditions:

$$P_t = \delta \tilde{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} [P_{t+1} + D_{t+1}], \quad (\text{A.18})$$

$$1 + r_t = \frac{1}{\delta} \tilde{E}_t \left(\frac{C_{t+1}}{C_t} \right)^\gamma. \quad (\text{A.19})$$

Both conditions are standard, except that the expectations may not necessarily be rational; this will be discussed in detail below.³³

³³When deriving this equation, we also follow AMN (p.43 - 44) by assuming $\frac{C_{t,t+j}^i}{C_t^i} \approx \frac{C_{t,t+j}}{C_t}$.

I.2 Adding judgement in forecasting

Two features of the model are crucial for replicating the survey evidence. First, we follow the literature on asset pricing with adaptive learning and assume investors have imperfect knowledge about stock markets so that they do not know other investors' preferences and stock price beliefs, as in Carceles-Poveda and Giannitsarou (2008), Lansing (2010, 2019), and AMB. They cannot correctly deduce the equilibrium law of motion for asset prices. Instead, they form subjective price beliefs and learn from market prices.

We note that the usual belief specifications in asset pricing models with adaptive learning may be insufficient to replicate the survey evidence. Stock pricing models with adaptive learning usually work with detrended variables and incorporate learning about (detrended) stock prices; see Lansing (2010) and Carceles-Poveda and Giannitsarou (2008). This implies that agents know the evolution of the trend in the growth rate of stock prices and consumption exactly, and hence, stock price forecasts are cointegrated with consumption forecasts, as in FIRE models.³⁴ Some models allow for learning about the growth rates of stock prices. For instance, AMB develop a model of stock pricing, which quantitatively replicates some survey evidence and many asset-pricing moments and endogenously generates boom-bust asset price dynamics. In this model, stock price forecasts and consumption forecasts are cointegrated, because realized stock prices and consumption are cointegrated, and agents' beliefs about the long-run growth rate of stock prices will mean-revert to the long-run growth rate of consumption, despite agents' lack of knowledge of both features. Note that it is not desirable to reconcile the survey evidence by adding a non-stationary sentiment component to the consumption process in the AMB model, because it will yield price consumption ratios with unbounded volatility.

Second, we deviate from the literature on asset pricing models with adaptive learning in terms of modelling investors' stock price forecasts. Motivated by the discussion in Section 9.2, we assume that investors make forecasts about stock prices based on an econometric model, as seen in AMN and AMB, and add judgement to their forecasts.

³⁴This can be proved following Section 7 of Kuang and Mitra (2016).

I.3 Subjective forecasts of stock prices

Following Svensson (2003, 2005), Bullard et al. (2008), and Milani (2011), we incorporate judgement into investors' forecasts in modelling their expectation formation. Denote by $\tilde{E}_t \log P_{t+j}$ the investor's forecast of (log) stock prices made at period t over horizon j , $j > 0$. The investor's stock price forecasts consist of two components and, mathematically, follow

$$\tilde{E}_t \log P_{t+j} = \hat{E}_t \log P_{t+j} + \tilde{E}_t \log \zeta_{t+j}, \quad (\text{A.20})$$

$$\log \zeta_{t+j} = \log \zeta_{t+j-1} + \log v_{t+j}. \quad (\text{A.21})$$

The first component, $\hat{E}_t \log P_{t+j}$, is generated by adaptive learning from past stock prices as seen in AMN and explained below. The second component, $\tilde{E}_t \log \zeta_{t+j}$, is the judgement component (also called “sentiment”) and is new relative to AMN; we use “judgement” and “sentiment” interchangeably hereinafter in the paper. When $\tilde{E}_t \log \zeta_{t+j}$ is positive (or negative), the investor is more optimistic (or pessimistic) than the stock price forecasts produced by her econometric model. $\log v_{t+j}$ is an *i.i.d* innovation following a normal distribution, i.e., $\log v_{t+j} \sim iiN(-\frac{\sigma_v^2}{2}, \sigma_v^2)$.

Two assumptions are made regarding the sentiment (judgement) variable. First, $\log \zeta_t$ is assumed to be non-stationary and contain a unit root. Second, $\log \zeta_t$ is independent of the aggregate consumption process. Both assumptions are in the spirit of those made in Svensson (2003, 2005), Bullard et al. (2008), and Milani (2011). In those papers, the judgement variable is not intrinsically related to economic fundamentals at all. Moreover, the judgemental variable is stationary and an AR(1) process (given those papers work with detrended variables), while it is an (1) process in our model because we work with a model with non-stationary variables. Both assumptions are needed to reproduce the survey evidence that stock price forecasts are not cointegrated with consumption forecasts, as is shown later. Appendix H.5 shows that the quantitative results of the model are robust to a more generalized sentiment process.

The first component of stock price forecasts, $\hat{E}_t \log P_{t+j}$, is generated as in AMN. Specifically, the investor has the following forecasting model for risk-adjusted stock price growth

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}}{P_t} = b_t + e_t, \quad (\text{A.22})$$

$$b_t = b_{t-1} + \mu_t, \quad (\text{A.23})$$

where e_t and μ_t are *i.i.n* regression errors. Shocks are independent of each other and all fundamental shocks. The investor is uncertain about b_t and needs to learn about it using historical stock price data. She then makes forecasts using the estimated model. We define the investor's belief about b_t from learning as

$$\beta_t = \hat{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}}{P_t}\right]. \quad (\text{A.24})$$

As in AMN, the following constant-gain learning algorithm is from a Bayesian perspective

$$\beta_t = \beta_{t-1} + g\left(\left(\frac{C_{t-1}}{C_{t-2}}\right)^{-\gamma} \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right). \quad (\text{A.25})$$

There exists an upper bound $U \in (a^{1-\gamma}\rho_{\epsilon,t}, \frac{1}{\delta U^\zeta})$, such that:

$$0 < \beta_{t-1} + g < U, \quad (\text{A.26})$$

for any t , where U^ζ is the upper bound for sentiment ζ_t . Property (A.26) guarantees the existence of positive equilibrium PD ratios.

I.4 Model mechanism

I.4.1 Analytical solution and no cointegration between P and C forecasts

Substituting equation (A.20) and (A.21) into equation (A.18) yields

$$P_t = \delta \zeta_t \hat{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} P_{t+1}\right] + \delta \tilde{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} D_{t+1}\right], \quad (\text{A.27})$$

where we assume that agents believe that non-adjusted stock price growth is uncorrelated with fundamental growth. Using equation (A.24) and defining the forecast of risk-adjusted dividends

growth as

$$\beta_{d,t} = \tilde{E}_t[(\frac{C_{t+1}}{C_t})^{-\gamma} \frac{D_{t+1}}{D_t}], \quad (\text{A.28})$$

equation (A.27) implies that the equilibrium PD ratio is

$$\frac{P_t}{D_t} = \frac{\delta \beta_{d,t}}{1 - \delta \beta_t \zeta_t}, \quad (\text{A.29})$$

where $\beta_{d,t} = a^{1-\gamma} \rho_{\epsilon,t}$.³⁵ If $\zeta_t = 1$ for all t , our learning model with sentiment collapses to a version of AMN.³⁶ Moreover, following AMN and to ensure feasible and positive PD ratios, a projection facility is constructed to ensure expected utility to remain finite.

The following proposition facilitates understanding why stock price forecasts are not cointegrated with consumption forecasts in the model, as is in the survey data.

Proposition 4. . *Given the investor's price beliefs (A.20) - (A.23), we have (1) stock price forecasts $\tilde{E}_i \log P_{i+j}$ and consumption forecasts $\tilde{E}_k \log C_{k+l}$ are not cointegrated for any integer $i, j, k, l > 0$.*

Proof. Given (A.20) - (A.23), stock price forecasts are $\tilde{E}_i \log P_{i+j} = \hat{E}_t \log P_{t+j} + \tilde{E}_i \log \zeta_{i+j} = \gamma j * \log a + \log P_i + \bar{S}(i, j) + \log \zeta_i$, where $\bar{S}(i, j)$ is a stationary term and irrelevant to the proof and hence omitted.³⁷ The consumption forecasts are $\tilde{E}_k \log C_{k+l} = \tilde{E}_k \log D_{k+l} + \tilde{E}_k \log(1 + \frac{W_{k+l}}{D_{k+l}}) = \log D_k + l * \log a + \tilde{E}_k \log(1 + \frac{W_{k+l}}{D_{k+l}})$. Therefore, we have

$$\tilde{E}_i \log P_{i+j} - \tilde{E}_k \log C_{k+l} = (\log P_i - \log D_k) + \log \zeta_i + (\gamma j - l) \log a + \bar{S}(i, j) - \tilde{E}_k \log(1 + \frac{W_{k+l}}{D_{k+l}}). \quad (\text{A.30})$$

Given $\log \zeta_i$ is an I(1) process and not cointegrated with consumption forecasts, we have $(\tilde{E}_i \log P_{i+j} - \tilde{E}_k \log C_{k+l})$ is I(1). $\square$

Equation (A.30) suggests that even if price-to-dividend ratios are stationary, stock price forecasts are not cointegrated with consumption forecasts. Introducing the sentiment component directly to the investor's stock price forecasts, which is unrelated to consumption forecasts,

³⁵ $\rho_{\epsilon,t} = [(1+\rho) \frac{D_{t-1}}{C_{t-1}}]^{-(1-p)\gamma} E_t[(\epsilon_{t+1}^w)^{-\gamma} (\epsilon_{t+1}^d)^{1-\gamma}] = [(1+\rho) \frac{D_{t-1}}{C_{t-1}}]^{-(1-p)\gamma} e^{\gamma(1+\gamma) \frac{\sigma_w^2}{2}} e^{-\gamma(1-\gamma) \frac{\sigma_d^2}{2}} e^{-\gamma(1-\gamma) \rho_{w,d} \sigma_w \sigma_d}$.

³⁶ Also, if $\zeta_t = 1$ holds for all periods and the investor's subjective expectations uniformly equal the RE level, that is $\beta_t = \beta_{d,t}^{RE} = a^{1-\gamma} \rho_{\epsilon,t}$, the equilibrium PD ratio reduces to the PD ratio under RE, $PD^{RE} = \frac{\delta a^{1-\gamma} \rho_{\epsilon,t}}{1 - \delta a^{1-\gamma} \rho_{\epsilon,t}}$.

³⁷ The long-run growth rate of stock price growth is $\gamma j \log a$ as in AMN.

is the key to breaking the tight cointegration link between stock price forecasts and consumption forecasts and reconciling the survey evidence.³⁸ Note that the stationarity of stock price-to-consumption (or price-to-dividend) ratios arises mainly from the mean-reversion property of the model, as explained in the later sections.

I.4.2 Phase diagram

Consider the belief updating equation (A.25). When we shut down shocks by imposing an assumption that all stochastic shocks only fluctuate weakly around their unconditional mean values, jumps in β_t from updating are sufficiently small. Based on the forecast-updating scheme and using equilibrium PD ratio equation, we obtain the stock price growth as

$$\frac{P_t}{P_{t-1}} = a \frac{\rho_{\epsilon,t}}{\rho_{\epsilon,t-1}} \left(1 + \frac{\delta(\beta_t \zeta_t - \beta_{t-1} \zeta_{t-1})}{1 - \delta \beta_t \zeta_t} \right) \epsilon_t^d. \quad (\text{A.31})$$

and

$$\left(\frac{C_t}{C_{t-1}} \right)^{-\gamma} = a^{-\gamma} [(1 + \rho) \frac{D_{t-1}}{C_{t-1}}]^{(1-p)(-\gamma)} (\epsilon_t^d)^{-\gamma} (\epsilon_t^w)^{-\gamma}. \quad (\text{A.32})$$

After leading the forecast-updating equation one period ahead and combining it with equilibrium price ratio (A.31) and consumption growth (A.32), a second-order difference equation can be derived:

$$\Delta \beta_{t+1} = h_{t+1} \left[a^{1-\gamma} [(1 + \rho) \frac{D_{t-1}}{C_{t-1}}]^{(1-p)(-\gamma)} (\epsilon_t^w)^{-\gamma} (\epsilon_t^d)^{1-\gamma} \left(1 + \frac{\delta(\beta_t \zeta_t - \beta_{t-1} \zeta_{t-1})}{1 - \delta \beta_t \zeta_t} \right) \frac{\rho_{\epsilon,t}}{\rho_{\epsilon,t-1}} - \beta_t; \beta_t \right] \quad (\text{A.33})$$

Given initial conditions for aggregate states (W_0, D_0, P_{-1}) , the initial forecast state β_0 and the initial sentiment state ζ_0 , equation (A.33) describes the equilibrium dynamics of the forecast of price growth through learning. Further, given a sufficiently high persistence of consumption-dividend ratio $p^{postwar} \in \{0.95, 0.99\}$, we have a second order difference equation:

³⁸Note if investors learn about stock price growth instead of risk-adjusted stock price growth, the no cointegration between stock price forecasts and consumption forecasts will still hold.

$$\Delta\beta_{t+1} \approx h_{t+1}[a^{1-\gamma}(\epsilon_t^w)^{-\gamma}(\epsilon_t^d)^{1-\gamma}(1 + \frac{\delta(\beta_t\zeta_t - \beta_{t-1}\zeta_{t-1})}{1 - \delta\beta_t\zeta_t}) - \beta_t; \beta_t]. \quad (\text{A.34})$$

If we shut down shocks by imposing an assumption that all stochastic shocks only fluctuate weakly around their unconditional mean values (fundamental shocks $(\epsilon_t^w)^{-\gamma}(\epsilon_t^d)^{1-\gamma}$ fluctuate around $\rho_\epsilon = E[(\epsilon_t^w)^{-\gamma}(\epsilon_t^d)^{1-\gamma}]$ and the sentiment shock v_t changes relatively small from $\rho_v = E(v_t) = 1$, i.e. $\zeta_t = \zeta_{t-1}$), jumps in β across periods are sufficiently small.³⁹ Therefore, equation (A.34) approximates to a second-order difference equation

$$\Delta\beta_{t+1} \approx h_{t+1}[a^{1-\gamma}\rho_\epsilon(1 + \frac{\delta\zeta_t\Delta\beta_t}{1 - \delta\beta_t\zeta_t}) - \beta_t; \beta_t]. \quad (\text{A.35})$$

Thus, we have an approximate second-order difference updating system:

$$\Delta\beta_{t+1} \approx g[a^{1-\gamma}\rho_\epsilon(1 + \frac{\delta\zeta_t\Delta\beta_t}{1 - \delta\beta_t\zeta_t}) - \beta_t]. \quad (\text{A.36})$$

Figure A.18 illustrates the evolution of the vector (β_{t-1}, β_t) from this belief updating system. Consider an initial, low sentiment ζ^L . Four feasible areas are divided by the the 45° line $(\beta_{t-1} = \beta_t)$ and the curve “ $\beta_t = \beta_{t+1}$ ” and are denoted by *I*, *II*, *III*, and *IV*. Arrows indicate the direction of evolution in each area. Suppose the evolution initiates from area *I*. Area *I*, where $\beta_{t-1} > \beta_t$ and β_t keeps decreasing, shows downward momentum in the price change until β_t enters area *II*. Once in area *II*, β_t starts to increase and in the next period, beliefs will move towards area *III*. In area *III*, $\beta_{t-1} < \beta_t$ and β_t keeps increasing, which shows upward momentum. The upward and rightward movement in area *III* alters beliefs across time until β_t enters area *IV*. There, β_t starts to decrease, so that it goes back to area *I* and thereby displays mean reversion. Therefore, our model preserves the same mechanism as AMN and AMB that β_t is oscillating around its RE level, generating momentum and mean-reversion in stock prices. The latter contributes to generate stationary price-to-dividend (or price-to-consumption) ratios.

Now consider an increase in sentiment from ζ^L to ζ^H (a positive sentiment shock). The new,

³⁹This is also justified by our estimation results, as the sentiment shock only has around 0.001 standard deviation, implying a small change in sentiment across periods.

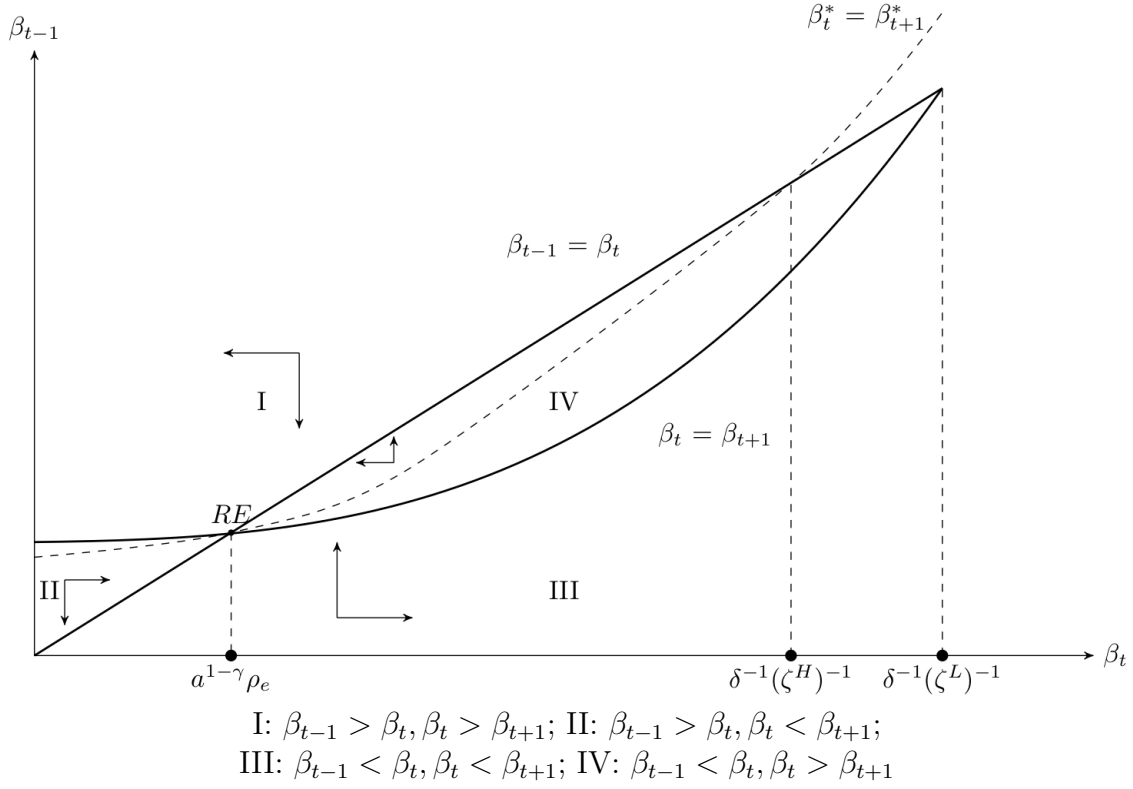


Figure A.18: **Phase diagram with a sentiment shift**

higher level of sentiment changes the slope of the curve “ $\beta_t = \beta_{t+1}$ ”: it creates a new dashed curve labeled as “ $\beta_t^* = \beta_{t+1}^*$ ”. As a result, the right-hand-side point of intersection between the 45° line ($\beta_{t-1} = \beta_t$) and the curve $\beta_t^* = \beta_{t+1}^*$ slides downwards along the 45° line. Its projection on the X-axis moves leftward from $\delta^{-1}(\zeta^L)^{-1}$ to $\delta^{-1}(\zeta^H)^{-1}$, while the left-hand-side point of intersection remains at the point “RE”. After this shift, area *III* expands by taking over some space from the old area *IV* (when sentiment is ζ^L), meaning that upward momentum is more likely to happen (and thus the PD ratio more likely to increase) after a positive sentiment shock. Similarly, area *I* expands by taking over some space from the old area *II*, meaning downward momentum is also more likely. Conversely, when the level of sentiment (judgement) is lower, both upward and downward momentum are less likely to happen.

I.5 Quantitative results

We calibrate some model parameters to the US data and estimate other (free) model parameters by the Simulated Method of Moments (SMM). This section evaluates the model’s ability to explain the survey stock price evidence and other US stock market facts. Test results suggest

that our estimated model robustly matches facts about stock markets (even equity premium) and forecasts of stock prices. Moreover, the estimated model passes the formal overall goodness of fit test based on asymptotic distribution.

I.5.1 Calibration

Following AMB, we set the mean US postwar consumption-to-dividend ratio $1 + \rho$ to 23 and its persistence p to 0.95. Moreover, as in AMB, the relation between consumption supply growth and dividends growth is calibrated as follows: the standard deviation of consumption supply growth is $1/7$ of the standard deviation of dividends growth, i.e. $\sigma_t^d > \sigma_t^c = \frac{1}{7}\sigma_t^d$, and their correlation is 0.2, i.e. $\rho_{d,c} = 0.2$.⁴⁰ Following Hall (1988), the relative risk aversion coefficient γ is calibrated to 5.

I.5.2 Estimating and matching

The SMM approach simulates the model to estimate free parameters (which are not fixed in the model and free to change), attempting to match selected sample statistics. Similar to the Generalized Method of Moments (GMM), it has an objective function:

$$[\hat{S} - \tilde{S}(\theta)]' \hat{W}^{-1} [\hat{S} - \tilde{S}(\theta)], \quad (\text{A.37})$$

where $\hat{S}$ is a vector of selected sample statistics, $\tilde{S}(\theta)$ is a vector of simulated statistics given a set of free parameters θ , and $\hat{W}$ is the estimated weight matrix. In line with AMB, we choose the variance-covariance matrix (the efficient matrix) of sample data as the weight.

After calibration, there remain five free parameters in the model: the discount factor δ , the constant gain parameter g , the mean and standard deviation of dividends growth (a and $\sigma_{\frac{\Delta D}{D}}$), and the standard deviation of sentiment shocks σ_ζ . From simulations, SMM finds a set of free parameters $\hat{\theta}$,

$$\hat{\theta} = (\tilde{\delta}, \tilde{g}, \tilde{a}, \tilde{\sigma}_{\frac{\Delta D}{D}}, \tilde{\sigma}_\zeta), \quad (\text{A.38})$$

⁴⁰In the specification of parametric relations between dividend growth and wage income growth, $\sigma_w^2 = [\frac{1}{49} - 1 - 2(\frac{0.2}{7} - 1)]\sigma_d^2$ and $\sigma_{w,d} = (\frac{0.2}{7} - 1)\sigma_d^2$ in the bi-variate distribution (A.17); see AMB for details.

such that

$$\hat{\theta} = \arg \min_{\theta} [\hat{S} - \tilde{S}(\theta)]' \hat{W}^{-1} [\hat{S} - \tilde{S}(\theta)], \quad (\text{A.39})$$

where $\hat{S}$ is a subset of data statistics for the US stock markets, and $\tilde{S}(\theta)$ is a vector of corresponding statistics implied by the model, given θ . Therefore, the SMM estimate $\hat{\theta}$ selects model parameters such that the model moments $\tilde{S}(\hat{\theta})$ fit the observed $\hat{S}$ as closely as possible.

In terms of statistical inference, individual and overall tests are performed. The individual test examines if the difference between each pair of observed statistic and simulated statistic is statistically significant. The overall test, based on asymptotic distribution, examines the joint goodness of fit between the simulated statistics and observed facts under the null that the estimated model is correct; see AMN for details.

If we choose $\hat{\Sigma}$ (the efficient matrix) as the estimated weight matrix, we encounter a nearly singular $\hat{\Sigma}^{-1}$, and as a result, only some statistics are targeted and included in $\hat{S}$. In particular, for realized moments, we exclude the coefficient β_{PD} of excess return predictability and for forecast moments, we target the coefficient ϕ_2 from the 2-quarter-ahead rationality test. Despite this, our model still explains almost all of the sample statistics individually, including dropped ones.

I.5.3 Estimation results

We incorporate some survey forecast moments in the estimating and matching process along with realized stock market moments.⁴¹ Table A.23 reports sample statistics and the corresponding statistics in the estimated model. All data moments are taken from AMN.

First, our model preserves the good properties of the models developed by AMN and AMB by quantitatively replicating a set of stock market moments. In particular, the estimated model successfully reproduces both the mean stock return Er^s and mean bond return Er^b , generating an equity premium of nearly 1.7% per quarter (around 4/5 of the actual mean equity

⁴¹We match US data moments by directly targeting the moments: mean stock return, mean bond return, mean price-dividend (PD) ratio, mean dividends growth, standard deviation of stock return, standard deviation of PD ratio, standard deviation of dividends growth, autocorrelation of PD ratio, coefficients of 5-year excess return predictability and coefficient of 2-quarter-ahead rationality test regression.

premium), while the AMN and AMB models replicate around half of the equity premium when the coefficient of relative risk aversion γ equals 5. To see this, the stock return from period 0 to T can be expressed as:

$$\prod_{t=0}^T \frac{P_t + D_t}{P_{t-1}} = \frac{PD_T + 1}{PD_0} \cdot \prod_{t=1}^T \frac{D_t}{D_{t-1}} \cdot \prod_{t=1}^{T-1} \left(1 + \frac{1}{PD_t}\right).$$

Since our estimation produces a lower average PD ratio (relative to AMN but insignificantly different from the data, according to the t-test), the estimated model generates a higher average stock return from the last term. The model experiences some difficulty in reproducing the coefficient of 5-year-ahead excess return predictability, β_{PD}^5 , which is not targeted in the estimation. This difference does not seem substantial. Except for β_{PD}^5 , all the other individual t-statistics are below 2.

Table A.23: Estimation outcomes with risk aversion $\gamma = 5$ and efficient matrix

	US data		Estimated model (efficient)	
	data moment	std. dev.	model moment	
	$\hat{S}_N$	$\hat{\sigma}_N$	$\hat{S}_N$	t-stat.
Mean stock return Er^s	2.25	0.59	1.93	0.55
Mean bond return Er^b	0.15	0.09	0.26	-1.22
Mean PD ratio EPD	123.91	6.94	116.81	1.02
Mean dividends growth $E\frac{\Delta D}{D}$	0.41	0.27	0.01	1.49
Std. dev. stock return σ_{r^s}	11.44	1.79	12.32	-0.49
Std. dev. PD ratio σ_{PD}	62.43	8.01	67.93	-0.69
Std. dev. dividends growth $\sigma\frac{\Delta D}{D}$	2.88	0.32	2.77	0.34
Autocorr. PD ratio $\rho_{PD,-1}$	0.97	0.03	0.96	0.23
Excess return predictability: 5-year-ahead				
Coefficient of excess return β_{PD}^5	-0.0041	0.0007	-0.0079	5.36
R^2 of excess return R_{PD}^2	0.2101	0.0580	0.2080	0.04
Rationality test: 2-quarter-ahead				
Coefficient of RE test ϕ_2	0.0558	0.0725	0.0445	0.16
Constant of RE test η_2	0.9786	0.0761	0.9702	0.11
4-quarter-ahead				
Coefficient of RE test ϕ_4	0.0619	0.0720	-0.0723	1.87
Constant of RE test η_4	1.0093	0.0800	1.1114	-1.27
Estimated parameters and test statistics				
Discount factor $\tilde{\delta}$			0.9926	
Filter gain $\tilde{g}$			0.0024	
Sentiment shock $\tilde{\sigma}_\zeta$			0.0010	
Test stat. $\tilde{Wald}$			14.29	
p-value of $\tilde{Wald}$			2.7%	

Note: this table reports US statistics, associated standard deviation, estimated statistics, parameter estimates and test statistics (individual and overall) from the estimated models with efficient weight matrix.

Notably, for stock price forecasts, although only the coefficient ϕ_2 from 2-quarter-ahead rationality test is targeted, all forecast statistics (ϕ_2 , η_2 , ϕ_4 and η_4) are successfully matched. In particular, 2-quarter-ahead rationality test statistics are replicated with accuracy, as both t statistics of ϕ_2 and η_2 are well below 0.2. Despite a negative ϕ_4 , the individual t-test suggests that the simulated statistic $\tilde{\phi}_4$ is still close enough to $\hat{\phi}_4$ from the US sample. Moreover, the

estimated model comfortably passes the overall goodness-of-fit Wald test at the 1% level. It is a noteworthy improvement (compared to the AMN model with $\gamma = 5$). Overall, the model replicates the survey evidence that RE is rejected by rationality tests and a standard set of asset pricing facts. Relative to AMN and AMB, our model requires a smaller gain parameter ($\tilde{g} = 0.0024$) due to the introduction of exogenous sentiment shocks, consistent with the analysis in Section [I.4.2](#).

I.6 Reproducing survey evidence and quantitative role for sentiment shocks

I.6.1 No cointegration between stock price forecasts and consumption forecasts

This section shows that our learning model is able to simultaneously produce: (1) forecasts of stock prices are not cointegrated with consumption forecasts made on the same or different dates, with or without imposing the cointegrating vector $(1, -1)$; (2) unit root tests reject the null hypothesis that realized stock price-to-consumption ratios are non-stationary (as in the data); (3) forecasts of stock prices (or consumption) over different horizons are cointegrated with cointegrating vector $(1, -1)$. This section studies large sample properties of the model, while the testing results for a shorter sample size (as the actual data) are presented later in robustness analysis. The estimated model in Table [A.23](#) is simulated at a quarterly frequency for $5000 + 40000$ periods. The first 5000 periods are discarded to reduce the influence of initial beliefs and the final 40000 periods are used to perform the tests.

Table [A.24](#) suggests that we cannot reject the null hypothesis that stock prices forecasts are not cointegrated with consumption forecasts with cointegrating vector $(1, -1)$ using PP and DF-GLS at the 10% significance level. Moreover, using KPSS tests, we can reject the null hypothesis that stock price forecasts are cointegrated with consumption forecasts with cointegrated vector $(1, -1)$ at the 1% level. Without imposing any cointegration restriction, we cannot reject that stock prices are not cointegrated with consumption forecasts at a 10% significance level using the Engle-Granger test, robust to all combinations of forecasting horizons for the two variables (Panel C).

Table A.24: **No cointegration between stock price forecasts and consumption forecasts in the model**

Panel A: forecasts made on the same period: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
Semi-annual	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	-0.581	-0.767	-1.114	-0.676	-1.197
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	-0.635	-0.955	-1.612	-0.792	-1.567
10% critical value	-1.622	-1.622	-1.622	-1.622	-1.622
KPSS	38.200	37.677	36.405	37.912	36.075
1% critical value	0.739	0.739	0.739	0.739	0.739
Panel B: forecasts made at different periods: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
Case 1: $i_1 = 2, 4, 6, 8, \dots, 40000$; $i_2 = 117, 118, 119, 120, \dots, 20116$					
	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	0.946	0.711	0.291	0.705	0.295
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	-0.576	0.239	0.299	0.458	0.305
10% critical value	-1.622	-1.624	-1.624	-1.625	-1.624
KPSS	31.335	31.344	31.359	31.348	31.355
1% critical value	0.739	0.739	0.739	0.739	0.739
Case 2: $i_1 = 2, 4, 6, 8, \dots, 40000$; $i_2 = 130, 131, 132, 133, \dots, 20129$					
PP (Z_t statistics)	0.926	0.694	0.277	0.686	0.282
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	0.733	0.420	0.299	0.416	0.304
10% critical value	-1.625	-1.625	-1.624	-1.625	-1.624
KPSS	31.410	31.419	31.433	31.423	31.429
1% critical value	0.739	0.739	0.739	0.739	0.739
Panel C: test results without imposing restriction: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
Engle-Granger	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
Test statistics	-1.193	-1.175	-1.177	-1.182	-1.173
10% crit. value	-2.460	-2.460	-2.460	-2.460	-2.460

Specifically, Panel A reports the test results when the forecasts of stock prices and consumption are made on the same date. The first three columns are the results when the two forecasts are made over the same horizons (1-quarter-, 2-quarter-, and 4-quarter-ahead) and the last two columns are when the two forecasts are made at different horizons. Semi-annual frequency is used as in our evidence using the Livingston Survey. Panel B reports the results when the two forecasts ($E_{i_1} \log P_{i_1+j_1}$ and $E_{i_2} \log C_{i_2+j_2}$) are made on different dates. Two cases are demonstrated as examples, corresponding to the Livingston Survey forecasts, except with

a longer sample. In case 1, we choose $i_1 = 2, 4, 6, 8, \dots, 40000$ and $i_2 = 117, 118, 119, 120, \dots, 20116$. In case 2, we have $i_1 = 2, 4, 6, 8, \dots, 40000$ and $i_2 = 130, 131, 132, 133, \dots, 20129$. All tests suggest the same conclusion—forecasts made on different dates are not cointegrated with the cointegrating vector $(1, -1)$. Similar results are obtained with shorter simulated samples of 122 semi-annual periods; see Section I.6.2.

The literature generally finds that realized price to consumption (or dividend) ratios are stationary; see Golez and Koudijs (2018). Table A.25 shows that the DF-GLS test rejects the hypothesis that realized stock price-to-consumption (PC) ratios are non-stationary (column 2), consistent with the US data (column 1).

Table A.25: **Stationarity of realized (log) PC ratios**

Stationarity test on realized <i>log</i> price-to-consumption ratio		
	US data	Model
DF-GLS	-1.700	-4.015
10% critical value	-1.619	-1.624

Note: Column 1 uses quarterly US data on real personal consumption expenditure (non-durable goods and services) and real S&P 500 stock prices from the sample period 1959Q1-2017Q2.

The results are robust to semi-annual frequency.

Table A.26: **P-values from cointegration tests among stock price (or consumption) forecasts**

Stationarity test among $E_t \log(X_{t+i}) - E_t \log(X_{t+j})$			
<i>X</i> is stock price or consumption			
<i>i</i> & <i>j</i>	1Q & 2Q	1Q & 4Q	2Q & 4Q
PP test			
Stock price (quarterly)	0.001	0.001	0.001
Stock price (semi-annual)	0.001	0.001	0.001
Consumption (quarterly)	0.001	0.001	0.001
Consumption (semi-annual)	0.001	0.001	0.001

Finally, Table A.26 suggests that the simulated stock price (or consumption) forecasts over different horizons are cointegrated with the cointegrating vector $(1, -1)$, for both quarterly and semi-annual frequencies and different pairs of forecast horizons $(i, j) = (1, 2)$, $(i, j) = (1, 4)$ and $(i, j) = (2, 4)$. These results are consistent with the Table 2.

I.6.2 Reproducing survey evidence using shorter samples

We test the cointegration relation between stock price forecasts and consumption forecasts with a shorter simulated sample of 61 years at semi-annual frequency, which is the same as the sample size in Panel A of Table 2. The model is simulated for $5000 + 244$ quarters and 1000 repetitions. The first 5000 periods are used to generate the stationary distribution of beliefs, and 122 even quarters from the final 244 quarters are used to conduct tests.

Table A.27: **No cointegration between stock price forecasts and consumption forecasts with short simulated samples: the frequency of rejecting the null hypothesis**

Sample length: 122 periods; repetition: 1000 times; $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP test , null: no cointegration					
Frequency of rej.	4%	5%	5%	5%	4%
DF-GLS test , null: no cointegration					
Frequency of rej.	5%	5%	6%	6%	6%
KPSS test , null: cointegration					
Frequency of rej.	96%	96%	97%	96%	96%

Table A.27 reports the frequency of rejecting the null hypothesis for three tests (PP, DF-GLS and KPSS). In 4% – 5% of the repetitions, the PP test rejects the null of no cointegration between stock price forecasts and consumption forecasts, which are made over the same or different horizons with the $(1, -1)$ cointegrating vector; the corresponding frequency of rejection is 5% – 6% for DF-GLS tests. In 96% – 97% of the repetitions, KPSS tests reject the null of cointegration between stock price forecasts and consumption forecasts. Therefore, in shorter samples, it is very likely that standard tests will conclude that the simulated stock price forecasts and consumption forecasts are not cointegrated. Note that we also find that DF-GLS tests reject the null of no cointegration between realized stock price and consumption at a 5% significance level in 80% of the repetitions.

I.6.3 Specifying a generalized sentiment process

We examine the robustness of the model results when sentiment is assumed to follow a more generalized process

$$\log \zeta_{t+j} = \log \zeta_{t+j-1} + k \Delta \log \zeta_{t+j-1} + \log v_{t+j}, \quad (\text{A.40})$$

where k is the auto-correlation coefficient of the changes in sentiment.

Table A.28: **Estimation outcomes for low, median and high sentiment growth persistence, with risk aversion $\gamma = 5$**

	Data moment	Estimated Model A		Estimated Model B		Estimated Model C	
		$(k = 0.1, \text{ efficient})$		$(k = 0.5, \text{ diagonal})$		$(k = 0.9, \text{ diagonal})$	
	$\hat{S}_N$	$\tilde{S}_N$	t-stat.	$\tilde{S}_N$	t-stat.	$\tilde{S}_N$	t-stat.
Er^s	2.25	1.94	0.53	1.50	1.26	1.63	1.05
Er^b	0.15	0.25	-1.15	0.38	-1.99	0.40	-1.61
E_{PD}	123.91	114.02	1.36	117.86	0.81	113.35	1.42
$E_{\frac{\Delta D}{D}}$	0.41	0.00	1.50	0.23	0.63	0.20	0.71
σ_{r^s}	11.44	12.10	-0.37	7.91	1.97	8.88	1.42
σ_{PD}	62.43	68.26	-0.73	65.38	-0.37	65.47	-0.38
$\sigma_{\frac{\Delta D}{D}}$	2.88	2.75	-0.38	3.51	-1.89	3.16	-0.82
$\rho_{PD,-1}$	0.97	0.96	0.14	0.98	0.35	0.96	0.02
Excess return predictability: 5-year-ahead							
β_{PD}^5	-0.0041	-0.0080	5.55	-0.0055	1.94	-0.0056	2.00
R_{PD}^2	0.2101	0.2045	0.10	0.3214	-1.92	0.3152	-1.81
Rationality test: 2-quarter-ahead							
ϕ_2	0.0558	0.0470	0.12	0.3172	-3.61	0.3614	-4.22
η_2	0.9786	0.9662	0.16	0.6907	3.79	0.6480	4.35
4-quarter-ahead							
ϕ_4	0.0619	-0.0563	1.65	0.0315	0.42	0.0432	0.26
η_4	1.0093	1.0904	-1.02	1.0047	0.06	1.0037	0.07

Table A.28 reports estimation results with low ($k = 0.1$, estimated Model A), median ($k = 0.5$, estimated Model B) and high ($k = 0.9$, estimated Model C) sentiment growth persistence. The estimated outcomes are generally similar to the estimated model in Table A.23, reproducing a standard set of asset pricing moments individually (according to the t-test) and robust to different choices of k . Specifically, the estimated Model A with low sentiment growth persistence reproduces the survey findings and almost all other individual moments (equity premium included), except for the 5-year-ahead excess return predictability coefficient, β_{PD}^5 . With higher sentiment growth persistence coefficients, estimated Model B and C improve in explaining β_{PD}^5 (t statistic no greater than 2) and generating ϕ_4 and η_4 from the 4-quarter-ahead rationality test that are closer to data moments. However, they fail to reproduce the

coefficients in the 2-quarter-ahead rationality test, ϕ_2 and η_2 .

Table A.29: **No cointegration between stock price forecasts and consumption forecasts in the model with a generalized sentiment process**

Panel A: forecasts made on the same period: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
Semi-annual	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	-0.535	-0.714	-1.046	-0.627	-1.125
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	-0.602	-0.918	-1.544	-0.759	-1.525
10% critical value	-1.622	-1.622	-1.622	-1.622	-1.622
KPSS	38.285	37.781	36.551	38.009	36.231
1% critical value	0.739	0.739	0.739	0.739	0.739
Panel B: forecasts made at different periods: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_1}$					
Case 1: $i_1 = 2, 4, 6, 8, \dots, 40000$; $i_2 = 117, 118, 119, 120, \dots, 20116$					
	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
PP (Z_t statistics)	1.175	0.929	0.494	0.922	0.498
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	0.847	0.706	0.531	0.702	0.537
10% critical value	-1.625	-1.625	-1.624	-1.625	-1.624
KPSS	33.760	33.769	33.782	33.772	33.778
1% critical value	0.739	0.739	0.739	0.739	0.739
Case 2: $i_1 = 2, 4, 6, 8, \dots, 40000$; $i_2 = 130, 131, 132, 133, \dots, 20129$					
PP (Z_t statistics)	1.153	0.910	0.479	0.902	0.484
10% critical value	-1.617	-1.617	-1.617	-1.617	-1.617
DF-GLS	0.832	0.661	0.527	0.656	0.532
10% critical value	-1.625	-1.625	-1.624	-1.625	-1.624
KPSS	33.827	33.835	33.848	33.838	33.844
1% critical value	0.739	0.739	0.739	0.739	0.739
Panel C: test results without imposing restriction: $E_{i_1} \log P_{i_1+j_1}$ & $E_{i_1} \log C_{i_1+j_2}$					
Engle-Granger	$j_1 = j_2 = 1$	$j_1 = j_2 = 2$	$j_1 = j_2 = 4$	$j_1 = 2, j_2 = 4$	$j_1 = 4, j_2 = 2$
Test statistics	-1.989	-1.945	-1.878	-1.972	-1.932
10% critical value	-2.460	-2.460	-2.460	-2.460	-2.460

We simulate Model A under the generalized sentiment process (A.40) for 40000 periods (quarters) and test if stock price forecasts are cointegrated, with consumption forecasts with or without cointegrating vector $(1, -1)$, similar to Section I.6.⁴² Table A.29 reports the testing results and suggests that the findings in Table A.24 is robust to the generalized sentiment process. In all panels, we find that stock price forecasts are not cointegrated with consumption

⁴²Model B and C yield the same results, which are omitted due to space limitations.

forecasts.

Finally, we test if there is cointegration among simulated stock price (or consumption) forecasts over different horizons, with the cointegrating vector $(1, -1)$ from Model A. The test result table is identical to Table [A.26](#) and hence omitted. Again, forecasts of stock price (or consumption) over two different horizons $((i, j) = (1, 2), \text{ or } (i, j) = (1, 4), \text{ or } (i, j) = (2, 4))$ are cointegrated with vector $(1, -1)$.