

Online Appendix for

Trade Policy in a Model of Sequential Production

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S.1 Further derivation details for Lemma 3

In Appendix A.3, we argue that $\underline{\tau} > 1$. A sufficient condition for this is that $\partial \tilde{v}_1(1, 1)/\partial \delta > 0$ holds. Evaluating the derivative in Eq. (A.5) at $\tau = \delta = 1$, we obtain

$$\frac{\partial \tilde{v}_1(1, 1)}{\partial \delta} = \lambda_1 \left[f(\Lambda) - \frac{1}{\exp[\lambda_1] - 1} \right], \quad (\text{S.1})$$

with

$$f(\Lambda) \equiv \frac{(x-1)(x-1-\Lambda)}{2x-1-\Lambda} \left[1 - \frac{x(1-\Lambda)}{(2x-1-\Lambda)^2} \right] \quad (\text{S.2})$$

and x given as implicit function of Λ by Eq. (6). To proceed, we differentiate $f(\Lambda)$ and obtain

$$f'(\Lambda) = \frac{x(x-1)}{(2x-1-\Lambda)^5} \frac{x-1-\Lambda}{\Lambda^2} H(x, \Lambda), \quad (\text{S.3})$$

with

$$\begin{aligned} H(x, \Lambda) = & [2x(x-1-\Lambda) + \Lambda(1+\Lambda)] [(2x-1-\Lambda)^2 - x(1-\Lambda)] \left[\frac{\Lambda}{x-1-\Lambda} + \ln \left(\frac{x-1-\Lambda}{x-1} \right) \right] \\ & + (x-1)(x-1-\Lambda)(2x+1+\Lambda)(1-\Lambda) \left[\frac{\Lambda}{x-1-\Lambda} + \ln \left(\frac{x-1-\Lambda}{x-1} \right) \right] \\ & - 2[x(x-1) + (x-1-\Lambda)(1-\Lambda)] \frac{\Lambda^2(2x-1-\Lambda)}{x-1-\Lambda}. \end{aligned}$$

It follows that $H(x, \Lambda) >, =, < 0$ if and only if $\hat{H}(x, \Lambda) >, =, < 0$, with

$$\hat{H}(x, \Lambda) \equiv \ln \left(\frac{x-1-\Lambda}{x-1} \right) + \frac{\Lambda}{x-1-\Lambda} - 2 \frac{\Omega_1(x, \Lambda)}{\Omega_2(x, \Lambda)} \frac{\Lambda^2(2x-1-\Lambda)}{x-1-\Lambda}, \quad (\text{S.4})$$

$\Omega_1(x, \Lambda) \equiv x(x-1) + (x-1-\Lambda)(1-\Lambda) > 0$, and

$$\begin{aligned} \Omega_2(x, \Lambda) \equiv & [2x(x-1-\Lambda) + \Lambda(1+\Lambda)] [(2x-1-\Lambda)^2 - x(1-\Lambda)] \\ & + (x-1)(x-1-\Lambda)(2x+1+\Lambda)(1-\Lambda) > 0. \end{aligned}$$

To determine the sign of $\hat{H}(x, \Lambda)$, we differentiate it with respect to x . This gives

$$\begin{aligned} \hat{H}'_x(x, \Lambda) = \frac{\Lambda^2 \Omega_2(x, \Lambda)^{-2}}{(x-1)(x-1-\Lambda)^2} & \left\{ 2\Omega_1(x, \Lambda)\Omega_2(x, \Lambda)(1+\Lambda)(x-1) - \Omega_2(x, \Lambda)^2 \right. \\ & \left. - 2 \left[\frac{\partial \Omega_1(\cdot)}{\partial x} \Omega_2(x, \Lambda) - \frac{\partial \Omega_2(\cdot)}{\partial x} \Omega_1(x, \Lambda) \right] (2x-1-\Lambda)(x-1-\Lambda)(x-1) \right\}. \end{aligned}$$

Expanding this derivative, we compute

$$\hat{H}'_x(x, \Lambda) = \frac{\Lambda^2 \Omega_2(x, \Lambda)^{-2}}{(x-1)} \frac{2x-1-\Lambda}{x-1-\Lambda} h(x, \Lambda),$$

with

$$\begin{aligned} h(x, \Lambda) \equiv & 8x^5(3+\Lambda) - 4x^4(27+18\Lambda-5\Lambda^2) + x^3(183+207\Lambda-27\Lambda^2-43\Lambda^3) \\ & - x^2(1+\Lambda)(141+115\Lambda-77\Lambda^2-19\Lambda^3) + x(1+\Lambda)(45+94\Lambda-18\Lambda^2-42\Lambda^3+\Lambda^4) \\ & - 3 - 26\Lambda - 31\Lambda^2 + 8\Lambda^3 + 19\Lambda^4 + 2\Lambda^5 - \Lambda^6. \end{aligned}$$

To make progress in the determination of the sign of $h(x, \Lambda)$, we compute its higher derivatives as¹

$$\begin{aligned} h'_x(x, \Lambda) = & 40x^4(3+\Lambda) - 16x^3(27+18\Lambda-5\Lambda^2) + 3x^2(183+207\Lambda-27\Lambda^2-43\Lambda^3) \\ & - 2x(1+\Lambda)(141+115\Lambda-77\Lambda^2-19\Lambda^3) + (1+\Lambda)(45+94\Lambda-18\Lambda^2-42\Lambda^3+\Lambda^4) \end{aligned}$$

$$\begin{aligned} h''_x(x, \Lambda) = & 2 \left[80x^3(3+\Lambda) - 24x^2(27+18\Lambda-5\Lambda^2) + 3x(183+207\Lambda-27\Lambda^2-43\Lambda^3) \right. \\ & \left. - (1+\Lambda)(141+115\Lambda-77\Lambda^2-19\Lambda^3) \right] \end{aligned}$$

$$h'''_x(x, \Lambda) = 6 \left[80x^2(3+\Lambda) - 16x(27+18\Lambda-5\Lambda^2) + 183+207\Lambda-27\Lambda^2-43\Lambda^3 \right]$$

$$h^{iv}_x(x, \Lambda) = 96 \left[10x(3+\Lambda) - 27 - 18\Lambda + 5\Lambda^2 \right]$$

Noting $h^{iv}_x(x, \Lambda) > 0$ holds for all $x > 2.79$, it follows that $h'''_x(x, \Lambda) > h'''_x(2.79, \Lambda) > 5,075 + 157\Lambda + 1,177\Lambda^2 - 259\Lambda^3 > 0$. As a consequence, we have $h''_x(x, \Lambda) > h''_x(2.79, \Lambda) > 3,117 - 298\Lambda + 1,340\Lambda^2 - 528\Lambda^3 + 38\Lambda^4 > 0$, establishing $h'_x(x, \Lambda) > h'_x(2.79, \Lambda) > 1,420 - 287\Lambda + 970\Lambda^2 - 529\Lambda^3 + 65\Lambda^4 + \Lambda^5 > 0$. We can thus safely conclude that $h(x, \Lambda) > h(2.79, \Lambda) > 512 - 146\Lambda + 510\Lambda^2 - 346\Lambda^3 + 52\Lambda^4 + 4\Lambda^5 - \Lambda^6 > 0$ and thus $\hat{H}'_x(x, \Lambda) > 0$. Noting that $\lim_{x \rightarrow \infty} \hat{H}(x, \Lambda) = 0$, we finally conclude that $\hat{H}(x, \Lambda) < 0$ and thus $f'(\Lambda) < 0$. In view of Eq. (S.1), this establishes

$$\frac{\partial \tilde{v}_1(1, 1)}{\partial \delta} > \lambda_1 L \left[f(1) - \frac{1}{\exp[\lambda_1] - 1} \right]. \quad (\text{S.5})$$

¹Alternatively, one can make use of mathematical software packages to confirm that $h(x, \Lambda) > 0$ if $x > 2.79$ and $0 < \Lambda \leq 1$

Making use of Eq. (6), we further note that $\lim_{\Lambda \rightarrow 1} x = 2 \exp[\lambda_1] / \{\exp(\lambda_1) - 1\}$. Substituting into Eq. (S.2), we compute $f(1) = 1 / \{\exp[\lambda_1] - 1\}$. This implies that the right-hand side of Eq. (S.5) is equal to zero, which is sufficient for $\partial \tilde{v}_1(1, 1) / \partial \delta > 0$ to hold for all $\Lambda < 1$ and completes the proof.