

Need-blind College Admissions, Financial Aid and Student Loan Programs

Online Appendix

We consider the model described in Section 3, but without assuming independence between the distributions of v_i and y_i . Let G denote the joint cumulative distribution function on $V \times Y$, and g the corresponding density function. We say that students' ability and need are *negatively correlated* if

$$\frac{\int_x^1 v g(v, y) dv}{\int_x^1 g(v, y) dv} \text{ weakly decreases in } y > 0 \text{ for every } x \in (0, 1), \quad (1)$$

that is, the mean ability of students above any given cutoff $x \in (0, 1)$ weakly decreases in their need level.

The characterization of equilibria in the three financial aid regimes considered in Section 4, and their comparison in terms of needy student enrollment, can be extended to certain environments where students' ability and need are negatively correlated. We provide a sufficient condition for this to be the case below. To do so, we outline how the arguments in Section 4 can be extended to accommodate negative correlation in each of the three regimes. We retain the assumption that the colleges' budgets are insufficient to implement the ideal outcome that all students above ability v^* enroll and fill both colleges' capacities exactly.

A.1 Fully need-blind regime

Recall that each college c selects a single admission cutoff $x_c \in [0, 1]$ and a fixed aid probability $\beta_c(v, y) \equiv b_c \in (0, 1)$ in this regime. Let $\{(x_c^r, b_c^r)\}_{c=A,B}$ denote an equilibrium in this regime where the scholarship budgets satisfy $M_A > M_B$.

We first show that $x_A^r > x_B^r$. Suppose to the contrary that $x_B^r \geq x_A^r$. If $b_A^r < 1$, A could increase both x_A^r and b_A^r slightly to fill capacity, which would improve the average ability of incoming students, a contradiction. If $b_A^r = 1$ then a strictly larger measure of students would enroll in college A than in B , violating capacity constraint for at least one college, unless $x_A^r = x_B^r$ and $b_A^r = b_B^r = 1$. But, this latter case is impossible because it would imply $x_A^r = x_B^r = v^*$ and thus, contradict the assumption that the ideal outcome cannot be implemented.

This establishes that $x_A^r > x_B^r$. It then follows that $b_A^r > b_B^r$, from binding capacity and budget constraints for both colleges written below:

$$\begin{aligned}
& [G(1, 0) - G(x_A^r, 0)] + [G(1, \bar{y}) - G(1, 0) - G(x_A^r, \bar{y}) + G(x_A^r, 0)][b_A^r + (1 - b_B^r)b_A^r] = \frac{1}{2} \\
& [G(1, 0) + G(x_A^r, 0) - 2G(x_B^r, 0)] + [G(1, \bar{y}) - G(1, 0) - G(x_A^r, \bar{y}) + G(x_A^r, 0)][b_B^r + (1 - b_A^r)b_B^r] \\
& \quad + 2[G(x_A^r, \bar{y}) - G(x_A^r, 0) - G(x_B^r, \bar{y}) + G(x_B^r, 0)]b_B^r = \frac{1}{2} \\
& \int_{\substack{x_A^d \leq v \leq 1 \\ 0 < y}} y[b_A^r + (1 - b_B^r)b_A^r] dG = M_A \\
& \int_{\substack{x_A^d \leq v \leq 1 \\ 0 < y}} y[b_B^r + (1 - b_A^r)b_B^r] dG + \int_{\substack{x_B^d \leq v \leq x_A^d \\ 0 < y}} yb_B^r dG = M_B
\end{aligned}$$

where the first two equations are binding capacity constraints for colleges A and B , respectively, and the third and fourth are binding budget constraints for A and B , respectively. In the limit case where $M_A = M_B$, $x_A^r = x_B^r$ and $b_A^r = b_B^r$.

A.2 Need-dependent financial aid regime (NBd)

The proof of Lemma 3 relies on the argument that the colleges allocate scholarships most cost-effectively by funding from the least needy students (above the cutoff) until its budget is exhausted, because the average ability of students above the cutoff is the same regardless of their need level. Indeed, funding from the least needy students is most cost-effective if

(*) the average ability of students above the cutoff weakly decreases in their need level y .

With negative correlation between ability and need, (*) holds for the college that sets a higher ability cutoff, which is the one with a higher budget, A . Thus, the proof of Lemma 3 goes through if (*) also holds for the college B that sets a lower cutoff. For college B , the average ability of students with $v_i \geq x_B^d$ in S_B and those with $v_i \in (x_B^d, x_A^d)$ in S_A is

$$\frac{\int_{x_B^d}^1 vg(v, y)dv + \int_{x_B^d}^{x_A^d} vg(v, y)dv}{\int_{x_B^d}^1 g(v, y)dv + \int_{x_B^d}^{x_A^d} g(v, y)dv} \tag{2}$$

for $y \in (0, n_A^d]$. Therefore, Lemma 3 holds under the following condition.

Condition NC: (2) weakly decreases in y for every $x_B^d < x_A^d$.

As an example, $g(v, y) = (1 - v)^{1+y}$ exhibits negative correlation and satisfies Condition NC. Note that Condition NC is a sufficient condition, so Lemma 3 may hold more broadly beyond Condition NC.

A.3 Ability-dependent financial aid regime (NBa)

The proofs for both Lemmas 4 and 5 are valid for general G , i.e., do not rely on the independence between the distribution of v and that of y .

A.4 Comparison of the three regimes

We have verified above that the equilibrium characterization in Section 4 for the three financial aid regimes extend to the environments where Condition NC holds. We now outline that the comparison of three regimes in terms of needy student enrollment also extends, in particular, that it is largest when aid allocation can be based on students' need but not on their ability.

We first compare the outcome of NBf, denoted by $\{(x_c^r, b_c^r)\}_{c=A,B}$, and that of NBd, denoted by $\{(x_c^d, n_c^d)\}_{c=A,B}$, for the case that $M_A > M_B$. Recall that $x_A^r > x_B^r$ and $b_A^r > b_B^r$, and that $x_A^d > x_B^d$ and $n_A^d > n_B^d$ (cf. Figure 1(b) and Figure 2(b)).

We need to show that $x_B^d > x_B^r$. Suppose to the contrary that $x_B^r \geq x_B^d$. If $x_B^r \geq x_A^d$, as the outcome moves from $\{(x_c^d, n_c^d)\}_{c=A,B}$ to $\{(x_c^r, b_c^r)\}_{c=A,B}$, it is clear that a larger measure of total needy students enroll and their average need level increases because those with $v_i < x_B^r$ and $y_i \leq n_A^r$ are replaced by those with $v_i \geq x_B^r$ and $y_i > n_A^r$. This means that the total scholarship spend is strictly larger in $\{(x_c^d, n_c^d)\}_{c=A,B}$, which is a contradiction because total budgets are exhausted in both regimes. The same conclusion holds by an analogous reasoning in the case $x_A^d > x_B^r \geq x_B^d$ as well, establishing that $x_B^d > x_B^r$.

It remains to show that the equilibrium ability cutoff in NBd is higher than that in NBa. We note that the corresponding proof of Proposition 1 is valid for general G , i.e., does not rely on the independence between the distribution of v and that of y . Hence, the same result holds for the current case.