

Supplementary Material of “Ordinal Mixed-Effects Random Forests”

S1 Additional information and results on the simulation study

S1.1 Descriptive statistics of simulation datasets

In the simulation study presented in Section 4, we set ordinal responses with three possible categories and we design a two-level data structure with $I = 10$ groups and $n_i = 100$ observations each, for a total number of 1000 units. We report in table S1 the distribution of the number of observations observed in the 100 simulated ordinal responses, for each DGP, and in Table S2 the percentage of runs with zero observation per each category-group couple, for each DGP.

DGP	Response category	mean $\pm$ sd (min–max)	
		Train	Test
1	1	126.03 $\pm$ 23.42 (77–180)	31.05 $\pm$ 5.88 (19–45)
	2	230.04 $\pm$ 15.10 (194–263)	57.02 $\pm$ 3.74 (48–65)
	3	445.05 $\pm$ 34.44 (368–515)	110.81 $\pm$ 8.60 (91–128)
2	1	81.88 $\pm$ 12.75 (56–108)	20.00 $\pm$ 3.24 (13–27)

	2	123.80 ± 12.34 (99–149)	30.47 ± 3.08 (24–37)
	3	595.52 ± 22.57 (547–641)	148.33 ± 5.62 (136–160)
3	1	157.95 ± 47.23 (65–268)	39.00 ± 11.83 (16–66)
	2	197.12 ± 22.33 (148–252)	48.78 ± 5.57 (36–62)
	3	446.11 ± 58.97 (322–572)	111.04 ± 14.80 (80–143)
4	1	99.02 ± 26.70 (49–165)	24.22 ± 6.62 (12–41)
	2	116.72 ± 15.92 (79–156)	28.76 ± 4.00 (19–39)
	3	585.39 ± 39.03 (494–665)	145.89 ± 9.80 (123–166)
5	1	122.34 ± 14.51 (91–158)	30.05 ± 3.64 (22–39)
	2	237.51 ± 13.78 (204–281)	58.88 ± 3.51 (50–70)
	3	441.41 ± 21.66 (387–494)	109.81 ± 5.49 (96–123)
6	1	79.10 ± 9.37 (57–102)	19.30 ± 2.38 (14–25)
	2	126.29 ± 10.86 (98–159)	31.08 ± 2.73 (24–39)
	3	595.74 ± 15.72 (562–631)	148.49 ± 3.93 (140–157)
7	1	131.95 ± 22.85 (84–184)	32.48 ± 5.76 (20–45)
	2	228.75 ± 15.38 (190–270)	56.71 ± 3.86 (47–67)
	3	440.46 ± 32.06 (359–521)	109.65 ± 8.04 (89–130)
8	1	82.71 ± 13.84 (52–116)	20.16 ± 3.48 (13–28)
	2	125.73 ± 12.55 (97–154)	30.94 ± 3.13 (24–38)
	3	592.81 ± 21.92 (546–640)	147.65 ± 5.43 (136–159)
9	1	115.79 ± 9.13 (94–134)	28.46 ± 2.24 (23–33)
	2	239.12 ± 11.78 (212–272)	59.24 ± 2.92 (53–67)
	3	446.28 ± 12.72 (408–484)	111.11 ± 3.21 (102–121)
10	1	76.54 ± 7.36 (56–92)	18.69 ± 1.85 (14–22)
	2	126.20 ± 9.89 (97–148)	31.01 ± 2.51 (24–36)
	3	598.41 ± 11.06 (576–627)	149.15 ± 2.74 (143–156)

11	1	284.14 ± 17.52 (249–352)	70.60 ± 4.39 (62–88)
	2	94.76 ± 8.00 (74–112)	23.21 ± 2.02 (18–27)
	3	422.23 ± 19.47 (348–460)	105.06 ± 4.85 (87–115)
12	1	286.59 ± 28.45 (231–387)	71.10 ± 7.08 (57–96)
	2	92.86 ± 8.07 (78–121)	22.74 ± 2.05 (19–30)
	3	421.75 ± 30.80 (323–489)	104.96 ± 7.72 (80–122)

Table S1: Distribution of the number of observations observed in the simulated ordinal responses in the 12 DGPs presented in Section 4, across the 100 runs, for both train and test sets. Values are reported as mean $\pm$ standard deviation (min–max).

Table S2: Percentage of runs with zero observations per each category-group couple, for each DGP. Only non-zero percentages are reported; omitted categories have value 0. The symbol ‘–’ indicates that all three categories have value 0.

DGP	Group	Train	Test
1	Overall	–	–
	1	$C_1 = 1$	$C_1 = 7, C_2 = 1$
	2	$C_1 = 1$	$C_1 = 10$
	3	–	$C_1 = 10$
	4	$C_1 = 1$	$C_1 = 13, C_2 = 3$
	5	–	$C_1 = 9$
	6	$C_1 = 1$	$C_1 = 11$

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DGP Group Train			Test
	7	$C_1 = 1$	$C_1 = 11$
	8	–	$C_1 = 12$
	9	–	$C_1 = 17$
	10	$C_1 = 1$	$C_1 = 9$
2	Overall	–	–
	1	$C_1 = 1$	$C_1 = 18, C_2 = 1$
	2	$C_1 = 1$	$C_1 = 17, C_2 = 4$
	3	$C_1 = 1$	$C_1 = 15$
	4	$C_1 = 1$	$C_1 = 22, C_2 = 3$
	5	–	$C_1 = 19, C_2 = 6$
	6	–	$C_1 = 14, C_2 = 4$
	7	$C_1 = 2$	$C_1 = 13, C_2 = 5$
	8	–	$C_1 = 20, C_2 = 6$
	9	–	$C_1 = 15, C_2 = 2$
	10	$C_1 = 1$	$C_1 = 14, C_2 = 8$
3	Overall	–	–
	1	$C_1 = 11, C_2 = 1$	$C_1 = 17, C_2 = 5, C_3 = 1$
	2	$C_1 = 4$	$C_1 = 26, C_2 = 4$
	3	$C_1 = 10$	$C_1 = 22, C_2 = 4, C_3 = 1$
	4	$C_1 = 9$	$C_1 = 21, C_2 = 6$
	5	$C_1 = 9, C_2 = 1$	$C_1 = 25, C_2 = 6$
	6	$C_1 = 6$	$C_1 = 15, C_2 = 2, C_3 = 1$
	7	$C_1 = 4, C_2 = 1$	$C_1 = 22, C_2 = 6$
	8	$C_1 = 6, C_2 = 2$	$C_1 = 17, C_2 = 2$
	9	$C_1 = 8$	$C_1 = 18, C_2 = 6$
	10	$C_1 = 6, C_2 = 1$	$C_1 = 21, C_2 = 4$
4	Overall	–	–
	1	$C_1 = 7, C_2 = 1$	$C_1 = 27, C_2 = 6$
	2	$C_1 = 5, C_2 = 1$	$C_1 = 17, C_2 = 12$

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DGP Group Train			Test
	3	$C_1 = 3, C_2 = 2$	$C_1 = 28, C_2 = 12$
	4	$C_1 = 7$	$C_1 = 29, C_2 = 12$
	5	$C_1 = 5, C_2 = 1$	$C_1 = 26, C_2 = 13$
	6	$C_1 = 3$	$C_1 = 18, C_2 = 3$
	7	$C_1 = 7, C_2 = 1$	$C_1 = 23, C_2 = 10$
	8	$C_1 = 4, C_2 = 1$	$C_1 = 24, C_2 = 8$
	9	$C_1 = 4$	$C_1 = 23, C_2 = 8$
	10	$C_1 = 5$	$C_1 = 21, C_2 = 10$
5	Overall	–	–
	1	–	$C_1 = 4$
	2	–	$C_1 = 7, C_2 = 1$
	3	–	$C_1 = 3$
	4	–	$C_1 = 4$
	5	–	$C_1 = 5$
	6	–	$C_1 = 8$
	7	–	$C_1 = 7$
	8	$C_1 = 1$	$C_1 = 3$
	9	–	$C_1 = 6$
	10	–	$C_1 = 7$
6	Overall	–	–
	1	–	$C_1 = 14, C_2 = 4$
	2	–	$C_1 = 15, C_2 = 4$
	3	–	$C_1 = 15, C_2 = 2$
	4	–	$C_1 = 8, C_2 = 5$
	5	–	$C_1 = 14, C_2 = 6$
	6	–	$C_1 = 14, C_2 = 3$
	7	–	$C_1 = 15, C_2 = 3$
	8	–	$C_1 = 18, C_2 = 2$
	9	–	$C_1 = 14, C_2 = 4$

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DGP Group Train			Test
	10	–	$C_1 = 8, C_2 = 3$
7	Overall	–	–
	1	–	$C_1 = 4$
	2	$C_1 = 1$	$C_1 = 12, C_2 = 1$
	3	–	$C_1 = 7$
	4	–	$C_1 = 11$
	5	$C_1 = 1$	$C_1 = 9$
	6	–	$C_1 = 9$
	7	$C_1 = 1$	$C_1 = 11, C_2 = 2$
	8	–	$C_1 = 12, C_2 = 1$
	9	$C_1 = 1$	$C_1 = 6, C_2 = 1$
	10	–	$C_1 = 6, C_2 = 2$
8	Overall	–	–
	1	–	$C_1 = 13, C_2 = 6$
	2	$C_1 = 2$	$C_1 = 20, C_2 = 6$
	3	–	$C_1 = 20, C_2 = 4$
	4	–	$C_1 = 18, C_2 = 3$
	5	$C_1 = 1$	$C_1 = 11, C_2 = 2$
	6	$C_1 = 1$	$C_1 = 15, C_2 = 2$
	7	$C_1 = 1$	$C_1 = 15, C_2 = 5$
	8	$C_1 = 1$	$C_1 = 18, C_2 = 7$
	9	–	$C_1 = 9, C_2 = 7$
	10	$C_1 = 1$	$C_1 = 18, C_2 = 3$
9	Overall	–	–
	1	–	$C_1 = 2$
	2	–	$C_1 = 5$
	3	–	$C_1 = 4$
	4	–	$C_1 = 6$
	5	–	$C_1 = 1$

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DGP Group Train			Test
	6	–	$C_1 = 4$
	7	–	$C_1 = 6$
	8	–	$C_1 = 4$
	9	–	$C_1 = 4$
	10	–	$C_1 = 5$
10	Overall	–	–
	1	–	$C_1 = 11, C_2 = 2$
	2	–	$C_1 = 11, C_2 = 2$
	3	–	$C_1 = 15, C_2 = 6$
	4	–	$C_1 = 21, C_2 = 3$
	5	–	$C_1 = 17, C_2 = 3$
	6	–	$C_1 = 21, C_2 = 4$
	7	–	$C_1 = 7, C_2 = 3$
	8	–	$C_1 = 12, C_2 = 5$
	9	–	$C_1 = 7, C_2 = 5$
	10	–	$C_1 = 17, C_2 = 5$
11	Overall	–	–
	1	–	$C_2 = 6$
	2	–	$C_2 = 14$
	3	–	$C_2 = 10$
	4	–	$C_2 = 8$
	5	–	$C_1 = 1, C_2 = 5$
	6	–	$C_2 = 9$
	7	–	$C_2 = 6$
	8	–	$C_2 = 14$
	9	–	$C_2 = 9$
	10	–	$C_2 = 10$
12	Overall	–	–
	1	–	$C_1 = 1, C_2 = 9$

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DGP Group Train		Test
2	–	$C_2 = 11, C_3 = 1$
3	–	$C_2 = 8$
4	–	$C_1 = 1, C_2 = 9$
5	–	$C_2 = 6$
6	–	$C_2 = 10$
7	–	$C_2 = 7$
8	–	$C_2 = 9$
9	–	$C_2 = 8$
10	–	$C_2 = 8$

S1.2 Results of the simulation study

For each model and DGP of the simulation study presented in Section 4 of the main paper, the mean predictive performance over the 100 simulation runs, with the corresponding standard deviation in parentheses are reported in Table S3.

DGP	Model	Accuracy	Precision	Recall	RMSE	MAE	Cohen's k	W Cohen's k	Cardoso	Ballante
1	CLM	0.5622	0.4502	0.3784	0.8883	0.5563	0.0940	0.1556	0.5646	0.3024
		(0.0447)	(0.0827)	(0.0290)	(0.0640)	(0.0652)	(0.0510)	(0.0702)	(0.0484)	(0.0606)
	CLMM	0.5713	0.4709	0.4120	0.8421	0.5234	0.1535	0.2566	0.5503	0.2742
		(0.0429)	(0.0728)	(0.0359)	(0.0601)	(0.0591)	(0.0556)	(0.0838)	(0.0458)	(0.0376)
	ordforest	0.6384	0.5660	0.5071	0.7238	0.4165	0.3216	0.4669	0.4765	0.2101
		(0.0357)	(0.0607)	(0.0479)	(0.0496)	(0.0448)	(0.0637)	(0.0792)	(0.0412)	(0.0308)
	OMERF _{CLM}	0.6520	0.5740	0.5777	0.6843	0.3889	0.3954	0.5787	0.4683	0.2007
		(0.0324)	(0.0381)	(0.0381)	(0.0478)	(0.0407)	(0.0491)	(0.0566)	(0.0360)	(0.0255)
	OMERF	0.6564	0.5832	0.5735	0.6730	0.3807	0.3982	0.5764	0.4617	0.1983
		(0.0349)	(0.0419)	(0.0403)	(0.0460)	(0.0413)	(0.0513)	(0.0561)	(0.0376)	(0.0266)
2	CLM	0.7506	0.5621	0.4228	0.7185	0.3395	0.1670	0.2724	0.3816	0.2179
		(0.0301)	(0.1057)	(0.0394)	(0.0592)	(0.0463)	(0.0656)	(0.0985)	(0.0415)	(0.0722)
	CLMM	0.7512	0.5435	0.4377	0.7091	0.3345	0.1962	0.3132	0.3791	0.1806
		(0.0291)	(0.1046)	(0.0373)	(0.0572)	(0.0435)	(0.0594)	(0.0899)	(0.0395)	(0.0636)
	ordforest	0.8025	0.6484	0.5513	0.5524	0.2343	0.4145	0.5986	0.2911	0.0864
		(0.0260)	(0.0792)	(0.0491)	(0.0506)	(0.0334)	(0.0673)	(0.0777)	(0.0369)	(0.0206)
	OMERF _{CLM}	0.8038	0.6301	0.6337	0.5129	0.2193	0.5091	0.6991	0.2920	0.0960
		(0.0300)	(0.0546)	(0.0481)	(0.0489)	(0.0350)	(0.0651)	(0.0524)	(0.0403)	(0.0204)
	OMERF	0.8084	0.6376	0.6219	0.5044	0.2132	0.5067	0.6985	0.2842	0.0909
		(0.0272)	(0.0528)	(0.0483)	(0.0472)	(0.0323)	(0.0608)	(0.0530)	(0.0377)	(0.0193)
3	CLM	0.5665	0.4841	0.3698	0.9830	0.6155	0.0677	0.1119	0.5848	0.4839
		(0.0669)	(0.1047)	(0.0303)	(0.1160)	(0.1170)	(0.0528)	(0.0804)	(0.0753)	(0.1528)
	CLMM	0.6236	0.5278	0.4963	0.8104	0.4725	0.2893	0.4412	0.5082	0.2330
		(0.0560)	(0.0580)	(0.0505)	(0.0893)	(0.0818)	(0.0776)	(0.0971)	(0.0647)	(0.0503)

	ordforest	0.6077 (0.0486)	0.4986 (0.0654)	0.4536 (0.0519)	0.8591 (0.0927)	0.5104 (0.0802)	0.2250 (0.0756)	0.3442 (0.1153)	0.5327 (0.0586)	0.2543 (0.0508)
	OMERF _{CLM}	0.6715 (0.0377)	0.5913 (0.0411)	0.5935 (0.0409)	0.6824 (0.0477)	0.3750 (0.0439)	0.4238 (0.0572)	0.6184 (0.0623)	0.4515 (0.0409)	0.1820 (0.0274)
	OMERF	0.6740 (0.0413)	0.5971 (0.0420)	0.5876 (0.0453)	0.6752 (0.0514)	0.3702 (0.0475)	0.4219 (0.0624)	0.6163 (0.0646)	0.4472 (0.0449)	0.1792 (0.0296)
4	CLM	0.7407 (0.0464)	0.6022 (0.0910)	0.4027 (0.0364)	0.7709 (0.0922)	0.3737 (0.0769)	0.1441 (0.0708)	0.2215 (0.1063)	0.4063 (0.0654)	0.3059 (0.0803)
		0.7536 (0.0392)	0.5924 (0.0968)	0.4713 (0.0402)	0.7288 (0.0816)	0.3435 (0.0643)	0.2659 (0.0691)	0.3936 (0.0888)	0.3839 (0.0553)	0.2118 (0.0845)
	CLMM	0.7802 (0.0392)	0.6294 (0.1044)	0.4987 (0.0486)	0.6512 (0.0819)	0.2901 (0.0604)	0.3360 (0.0794)	0.4842 (0.1001)	0.3389 (0.0576)	0.1259 (0.0479)
		0.8013 (0.0323)	0.6314 (0.0481)	0.6265 (0.0440)	0.5345 (0.0577)	0.2288 (0.0405)	0.5114 (0.0568)	0.6982 (0.0576)	0.3007 (0.0447)	0.0933 (0.0204)
	ordforest	0.8013 (0.0323)	0.6314 (0.0481)	0.6265 (0.0440)	0.5345 (0.0577)	0.2288 (0.0405)	0.5114 (0.0568)	0.6982 (0.0576)	0.3007 (0.0447)	0.0933 (0.0204)
		0.8013 (0.0323)	0.6314 (0.0481)	0.6265 (0.0440)	0.5345 (0.0577)	0.2288 (0.0405)	0.5114 (0.0568)	0.6982 (0.0576)	0.3007 (0.0447)	0.0933 (0.0204)
	OMERF _{CLM}	0.8013 (0.0323)	0.6314 (0.0481)	0.6265 (0.0440)	0.5345 (0.0577)	0.2288 (0.0405)	0.5114 (0.0568)	0.6982 (0.0576)	0.3007 (0.0447)	0.0933 (0.0204)
		0.8013 (0.0323)	0.6314 (0.0481)	0.6265 (0.0440)	0.5345 (0.0577)	0.2288 (0.0405)	0.5114 (0.0568)	0.6982 (0.0576)	0.3007 (0.0447)	0.0933 (0.0204)
	OMERF	0.8063 (0.0331)	0.6438 (0.0535)	0.6141 (0.0417)	0.5334 (0.0609)	0.2252 (0.0417)	0.5057 (0.0572)	0.6895 (0.0593)	0.2927 (0.0464)	0.0880 (0.0216)
		0.8063 (0.0331)	0.6438 (0.0535)	0.6141 (0.0417)	0.5334 (0.0609)	0.2252 (0.0417)	0.5057 (0.0572)	0.6895 (0.0593)	0.2927 (0.0464)	0.0880 (0.0216)
	CLM	0.5575 (0.0301)	0.4612 (0.0922)	0.3826 (0.0292)	0.8663 (0.0430)	0.5458 (0.0380)	0.1035 (0.0519)	0.1810 (0.0733)	0.5628 (0.0288)	0.2884 (0.0355)
		0.5575 (0.0301)	0.4612 (0.0922)	0.3826 (0.0292)	0.8663 (0.0430)	0.5458 (0.0380)	0.1035 (0.0519)	0.1810 (0.0733)	0.5628 (0.0288)	0.2884 (0.0355)
5	CLMM	0.5568 (0.0326)	0.4636 (0.0767)	0.3921 (0.0313)	0.8564 (0.0424)	0.5405 (0.0397)	0.1188 (0.0550)	0.2055 (0.0757)	0.5629 (0.0314)	0.2852 (0.0311)
		0.5568 (0.0326)	0.4636 (0.0767)	0.3921 (0.0313)	0.8564 (0.0424)	0.5405 (0.0397)	0.1188 (0.0550)	0.2055 (0.0757)	0.5629 (0.0314)	0.2852 (0.0311)
	ordforest	0.6417 (0.0320)	0.5756 (0.0595)	0.5145 (0.0431)	0.7053 (0.0432)	0.4052 (0.0388)	0.3374 (0.0612)	0.4852 (0.0666)	0.4683 (0.0362)	0.2058 (0.0239)
		0.6417 (0.0320)	0.5756 (0.0595)	0.5145 (0.0431)	0.7053 (0.0432)	0.4052 (0.0388)	0.3374 (0.0612)	0.4852 (0.0666)	0.4683 (0.0362)	0.2058 (0.0239)
	OMERF _{CLM}	0.6273 (0.0359)	0.5469 (0.0442)	0.5516 (0.0430)	0.7196 (0.0464)	0.4218 (0.0435)	0.3549 (0.0592)	0.5326 (0.0616)	0.4960 (0.0383)	0.2220 (0.0274)
		0.6273 (0.0359)	0.5469 (0.0442)	0.5516 (0.0430)	0.7196 (0.0464)	0.4218 (0.0435)	0.3549 (0.0592)	0.5326 (0.0616)	0.4960 (0.0383)	0.2220 (0.0274)
	OMERF _{no ran slope}	0.6409 (0.0335)	0.5676 (0.0435)	0.5599 (0.0424)	0.6906 (0.0438)	0.3990 (0.0401)	0.3752 (0.0571)	0.5510 (0.0587)	0.4795 (0.0362)	0.2103 (0.0248)
		0.6409 (0.0335)	0.5676 (0.0435)	0.5599 (0.0424)	0.6906 (0.0438)	0.3990 (0.0401)	0.3752 (0.0571)	0.5510 (0.0587)	0.4795 (0.0362)	0.2103 (0.0248)
	OMERF	0.6306 (0.0337)	0.5530 (0.0403)	0.5455 (0.0395)	0.7032 (0.0439)	0.4117 (0.0404)	0.3561 (0.0548)	0.5348 (0.0592)	0.4897 (0.0366)	0.2187 (0.0258)
		0.6306 (0.0337)	0.5530 (0.0403)	0.5455 (0.0395)	0.7032 (0.0439)	0.4117 (0.0404)	0.3561 (0.0548)	0.5348 (0.0592)	0.4897 (0.0366)	0.2187 (0.0258)
6	CLM	0.7513 (0.0211)	0.5469 (0.1071)	0.4287 (0.0382)	0.7018 (0.0472)	0.3307 (0.0329)	0.1766 (0.0616)	0.2919 (0.0945)	0.3768 (0.0296)	0.1777 (0.0601)
		0.7513 (0.0211)	0.5469 (0.1071)	0.4287 (0.0382)	0.7018 (0.0472)	0.3307 (0.0329)	0.1766 (0.0616)	0.2919 (0.0945)	0.3768 (0.0296)	0.1777 (0.0601)
	CLMM	0.7509 (0.0219)	0.5418 (0.1025)	0.4305 (0.0408)	0.6990 (0.0464)	0.3297 (0.0322)	0.1810 (0.0658)	0.2984 (0.1017)	0.3765 (0.0292)	0.1741 (0.0568)
		0.7509 (0.0219)	0.5418 (0.1025)	0.4305 (0.0408)	0.6990 (0.0464)	0.3297 (0.0322)	0.1810 (0.0658)	0.2984 (0.1017)	0.3765 (0.0292)	0.1741 (0.0568)
	ordforest	0.8098 (0.0202)	0.6743 (0.0670)	0.5635 (0.0521)	0.5327 (0.0450)	0.2221 (0.0262)	0.4397 (0.0657)	0.6155 (0.0781)	0.2794 (0.0288)	0.0808 (0.0158)
		0.8098 (0.0202)	0.6743 (0.0670)	0.5635 (0.0521)	0.5327 (0.0450)	0.2221 (0.0262)	0.4397 (0.0657)	0.6155 (0.0781)	0.2794 (0.0288)	0.0808 (0.0158)
	OMERF _{CLM}	0.8016 (0.0264)	0.6237 (0.0522)	0.6238 (0.0554)	0.5185 (0.0462)	0.2226 (0.0322)	0.5013 (0.0649)	0.6829 (0.0623)	0.2970 (0.0369)	0.0997 (0.0197)
		0.8016 (0.0264)	0.6237 (0.0522)	0.6238 (0.0554)	0.5185 (0.0462)	0.2226 (0.0322)	0.5013 (0.0649)	0.6829 (0.0623)	0.2970 (0.0369)	0.0997 (0.0197)
	OMERF _{no ran slope}	0.8103 (0.0253)	0.6405 (0.0509)	0.6266 (0.0542)	0.5020 (0.0456)	0.2111 (0.0309)	0.5117 (0.0619)	0.6956 (0.0587)	0.2823 (0.0356)	0.0902 (0.0173)
		0.8103 (0.0253)	0.6405 (0.0509)	0.6266 (0.0542)	0.5020 (0.0456)	0.2111 (0.0309)	0.5117 (0.0619)	0.6956 (0.0587)	0.2823 (0.0356)	0.0902 (0.0173)
	OMERF	0.8097 (0.0248)	0.6384 (0.0508)	0.6233 (0.0544)	0.5051 (0.0449)	0.2126 (0.0301)	0.5091 (0.0644)	0.6914 (0.0614)	0.2840 (0.0345)	0.0911 (0.0175)
		0.8097 (0.0248)	0.6384 (0.0508)	0.6233 (0.0544)	0.5051 (0.0449)	0.2126 (0.0301)	0.5091 (0.0644)	0.6914 (0.0614)	0.2840 (0.0345)	0.0911 (0.0175)
7	CLM	0.5572 (0.0389)	0.4427 (0.0950)	0.3808 (0.0324)	0.9073 (0.0653)	0.5710 (0.0607)	0.0946 (0.0558)	0.1576 (0.0741)	0.5734 (0.0427)	0.3018 (0.0637)
		0.5572 (0.0389)	0.4427 (0.0950)	0.3808 (0.0324)	0.9073 (0.0653)	0.5710 (0.0607)	0.0946 (0.0558)	0.1576 (0.0741)	0.5734 (0.0427)	0.3018 (0.0637)
	CLMM	0.5733 (0.0376)	0.4830 (0.0729)	0.4248 (0.0404)	0.8484 (0.0600)	0.5256 (0.0545)	0.1739 (0.0641)	0.2792 (0.0841)	0.5522 (0.0415)	0.2762 (0.0426)
		0.5733 (0.0376)	0.4830 (0.0729)	0.4248 (0.0404)	0.8484 (0.0600)	0.5256 (0.0545)	0.1739 (0.0641)	0.2792 (0.0841)	0.5522 (0.0415)	0.2762 (0.0426)
	ordforest	0.6296 (0.0339)	0.5632 (0.0545)	0.5022 (0.0388)	0.7459 (0.0526)	0.4333 (0.0450)	0.3087 (0.0547)	0.4537 (0.0650)	0.4881 (0.0408)	0.2206 (0.0309)
		0.6296 (0.0339)	0.5632 (0.0545)	0.5022 (0.0388)	0.7459 (0.0526)	0.4333 (0.0450)	0.3087 (0.0547)	0.4537 (0.0650)	0.4881 (0.0408)	0.2206 (0.0309)
	OMERF _{CLM}	0.6263 (0.0398)	0.5450 (0.0413)	0.5493 (0.0407)	0.7331 (0.0598)	0.4294 (0.0526)	0.3516 (0.0567)	0.5331 (0.0639)	0.4987 (0.0448)	0.2234 (0.0343)
		0.6263 (0.0398)	0.5450 (0.0413)	0.5493 (0.0407)	0.7331 (0.0598)	0.4294 (0.0526)	0.3516 (0.0567)	0.5331 (0.0639)	0.4987 (0.0448)	0.2234 (0.0343)
	OMERF _{no ran slope}	0.6437 (0.0391)	0.5701 (0.0433)	0.5593 (0.0403)	0.6946 (0.0529)	0.3993 (0.0473)	0.3776 (0.0565)	0.5608 (0.0597)	0.4765 (0.0435)	0.2069 (0.0311)
		0.6437 (0.0391)	0.5701 (0.0433)	0.5593 (0.0403)	0.6946 (0.0529)	0.3993 (0.0473)	0.3776 (0.0565)	0.5608 (0.0597)	0.4765 (0.0435)	0.2069 (0.0311)
	OMERF	0.6358 (0.0389)	0.5586 (0.0424)	0.5500 (0.0410)	0.7150 (0.0596)	0.4144 (0.0516)	0.3620 (0.0568)	0.5372 (0.0648)	0.4873 (0.0447)	0.2167 (0.0347)
		0.6358 (0.0389)	0.5586 (0.0424)	0.5500 (0.0410)	0.7150 (0.0596)	0.4144 (0.0516)	0.3620 (0.0568)	0.5372 (0.0648)	0.4873 (0.0447)	0.2167 (0.0347)
8	CLM	0.7481 (0.0278)	0.5639 (0.1033)	0.4249 (0.0337)	0.7148 (0.0629)	0.3396 (0.0452)	0.1743 (0.0585)	0.2871 (0.0908)	0.3827 (0.0400)	0.2115 (0.0753)
		0.7481 (0.0278)	0.5639 (0.1033)	0.4249 (0.0337)	0.7148 (0.0629)	0.3396 (0.0452)	0.1743 (0.0585)	0.2871 (0.0908)	0.3827 (0.0400)	0.2115 (0.0753)
	CLMM	0.7504 (0.0278)	0.5489 (0.1033)	0.4398 (0.0337)	0.7018 (0.0629)	0.3317 (0.0452)	0.2044 (0.0585)	0.3281 (0.0908)	0.3778 (0.0400)	0.1788 (0.0753)
		0.7504 (0.0278)	0.5489 (0.1033)	0.4398 (0.0337)	0.7018 (0.0629)	0.3317 (0.0452)	0.2044 (0.0585)	0.3281 (0.0908)	0.3778 (0.0400)	0.1788 (0.0753)

		(0.0272)	(0.1025)	(0.0392)	(0.0578)	(0.0417)	(0.0658)	(0.0931)	(0.0381)	(0.0622)
	ordforest	0.8007	0.6544	0.5500	0.5601	0.2383	0.4175	0.5897	0.2954	0.0897
		(0.0237)	(0.0682)	(0.0427)	(0.0520)	(0.0327)	(0.0604)	(0.0738)	(0.0350)	(0.0161)
	OMERF _{CLM}	0.7939	0.6111	0.6133	0.5346	0.2338	0.4858	0.6746	0.3072	0.1035
		(0.0276)	(0.0508)	(0.0500)	(0.0565)	(0.0368)	(0.0587)	(0.0595)	(0.0394)	(0.0217)
	OMERF _{no ran slope}	0.8017	0.6277	0.6136	0.5200	0.2234	0.4950	0.6828	0.2951	0.0960
		(0.0273)	(0.0497)	(0.0474)	(0.0557)	(0.0358)	(0.0570)	(0.0597)	(0.0400)	(0.0203)
	OMERF	0.8010	0.6248	0.6108	0.5219	0.2246	0.4922	0.6800	0.2961	0.0962
		(0.0290)	(0.0553)	(0.0505)	(0.0573)	(0.0374)	(0.0612)	(0.0615)	(0.0420)	(0.0206)
9	CLM	0.5623	0.4739	0.3819	0.8525	0.5345	0.1056	0.1792	0.5556	0.2843
		(0.0257)	(0.0925)	(0.0259)	(0.0349)	(0.0307)	(0.0473)	(0.0639)	(0.0247)	(0.0393)
	CLMM	0.5621	0.4712	0.3815	0.8522	0.5344	0.1055	0.1790	0.5557	0.2840
		(0.0256)	(0.0916)	(0.0265)	(0.0342)	(0.0302)	(0.0492)	(0.0636)	(0.0239)	(0.0389)
	ordforest	0.6607	0.6067	0.5302	0.6757	0.3790	0.3661	0.5133	0.4444	0.1919
		(0.0291)	(0.0612)	(0.0404)	(0.0410)	(0.0351)	(0.0544)	(0.0619)	(0.0336)	(0.0237)
	OMERF _{CLM}	0.6497	0.5709	0.5763	0.6803	0.3884	0.3909	0.5741	0.4680	0.2035
		(0.0335)	(0.0423)	(0.0412)	(0.0420)	(0.0381)	(0.0538)	(0.0520)	(0.0356)	(0.0243)
	OMERF	0.6551	0.5813	0.5703	0.6652	0.3780	0.3946	0.5722	0.4605	0.1995
		(0.0310)	(0.0433)	(0.0385)	(0.0412)	(0.0362)	(0.0506)	(0.0530)	(0.0338)	(0.0226)
10	CLM	0.7552	0.5340	0.4367	0.6853	0.3204	0.1934	0.3187	0.3688	0.1646
		(0.0204)	(0.1068)	(0.0428)	(0.0446)	(0.0301)	(0.0674)	(0.1023)	(0.0284)	(0.0538)
	CLMM	0.7554	0.5345	0.4378	0.6842	0.3197	0.1962	0.3217	0.3683	0.1635
		(0.0207)	(0.1055)	(0.0431)	(0.0448)	(0.0305)	(0.0680)	(0.1016)	(0.0288)	(0.0544)
	ordforest	0.8153	0.6802	0.5757	0.5182	0.2132	0.4550	0.6328	0.2703	0.0773
		(0.0209)	(0.0591)	(0.0507)	(0.0430)	(0.0267)	(0.0696)	(0.0756)	(0.0296)	(0.0151)
	OMERF _{CLM}	0.8062	0.6329	0.6354	0.5112	0.2170	0.5096	0.6904	0.2896	0.0949
		(0.0251)	(0.0476)	(0.0500)	(0.0439)	(0.0294)	(0.0601)	(0.0544)	(0.0343)	(0.0181)
	OMERF	0.8127	0.6470	0.6329	0.5006	0.2090	0.5151	0.6952	0.2794	0.0875
		(0.0226)	(0.0448)	(0.0474)	(0.0422)	(0.0264)	(0.0552)	(0.0550)	(0.0314)	(0.0166)
11	CLM	0.8609	0.7553	0.7393	0.4253	0.1537	0.7573	0.8936	0.2144	0.0463
		(0.0221)	(0.0413)	(0.0362)	(0.0447)	(0.0261)	(0.0374)	(0.0233)	(0.0327)	(0.0122)
	CLMM	0.8716	0.7745	0.7633	0.3987	0.1393	0.7770	0.9059	0.1977	0.0417
		(0.0206)	(0.0372)	(0.0370)	(0.0453)	(0.0249)	(0.0350)	(0.0223)	(0.0317)	(0.0113)
	ordforest	0.8506	0.7286	0.6847	0.4774	0.1766	0.7317	0.8690	0.2312	0.0493
		(0.0218)	(0.0653)	(0.0354)	(0.0566)	(0.0310)	(0.0385)	(0.0312)	(0.0364)	(0.0127)
	OMERF _{CLM}	0.7860	0.6140	0.6136	0.6377	0.2796	0.6200	0.7679	0.3419	0.1147
		(0.0291)	(0.0537)	(0.0315)	(0.0642)	(0.0448)	(0.0477)	(0.0462)	(0.0437)	(0.0216)
	OMERF	0.8393	0.6988	0.6646	0.5188	0.1977	0.7135	0.8499	0.2510	0.0815
		(0.0198)	(0.0706)	(0.0249)	(0.0517)	(0.0295)	(0.0325)	(0.0297)	(0.0333)	(0.0144)
12	CLM	0.8367	0.7026	0.6685	0.5185	0.1999	0.7061	0.8453	0.2579	0.0580
		(0.0243)	(0.0792)	(0.0436)	(0.0651)	(0.0373)	(0.0428)	(0.0375)	(0.0396)	(0.0159)
	CLMM	0.8755	0.7767	0.7618	0.3926	0.1350	0.7821	0.9091	0.1920	0.0385
		(0.0205)	(0.0405)	(0.0389)	(0.0423)	(0.0237)	(0.0348)	(0.0197)	(0.0306)	(0.0096)
	ordforest	0.8277	0.6782	0.6430	0.5620	0.2215	0.6861	0.8205	0.2753	0.0678
		(0.0240)	(0.0877)	(0.0306)	(0.0639)	(0.0394)	(0.0415)	(0.0403)	(0.0411)	(0.0189)
	OMERF _{CLM}	0.7905	0.6135	0.6141	0.6356	0.2757	0.6254	0.7703	0.3365	0.1060
		(0.0274)	(0.0529)	(0.0274)	(0.0635)	(0.0425)	(0.0430)	(0.0433)	(0.0414)	(0.0212)
	OMERF	0.8220	0.6629	0.6336	0.5738	0.2296	0.6786	0.8163	0.2834	0.0891
		(0.0230)	(0.0952)	(0.0239)	(0.0606)	(0.0373)	(0.0378)	(0.0376)	(0.0388)	(0.0207)

Table S3: Mean and standard deviations of prediction performances, measured by nine indices, of the compared methods across 100 runs of the 12 DGPs listed in Table 1 in the main paper. Section S1.3 reports the average confusion matrices (averaged over the 100 runs) for each model and DGP.

S1.3 Additional results of the simulation study: confusion matrices and computation time

Table S4 reports the average confusion matrices (averaged over runs) for each model and DGP of the Simulation study presented in Section 4.

Table S5 complements the predictive results of Section 4 by reporting the number of iterations required by the OMERF-type algorithms and their computation time. Overall, the computational burden is strongly scenario-dependent. In the random-intercept DGPs 1–4, OMERF generally converges in a moderate number of iterations, especially when the random-intercept signal is stronger: for example, the average number of iterations decreases from 14.41 in DGP 1 to 5.12 in DGP 3, and from 30.33 in DGP 2 to 7.73 in DGP 4. This suggests that, when the group-level component is identifiable, the alternating estimation of the forests and mixed-effects parts stabilizes relatively quickly. By contrast, the most problematic cases are those in which the algorithm reaches, or gets close to, the maximum number of iterations, set to `itmax`=100. This occurs mainly in the random-slope DGPs 5–8 and in the no-random-effects DGPs 9–10. In the random-slope settings, the high number of iterations reflects the additional difficulty of estimating a more complex random-effects structure, especially when the random-slope signal is small relative to the fixed-effects component. In these same cases, the underlying CLMM step also frequently exhibits convergence difficulties, indicating that the issue is not only due to the outer OMERF iteration but also to the numerical instability of fitting cumulative-link mixed models with weakly identified or complex random effects. In DGPs 9 and 10, where no random effects are present by construction, OMERF and OMERF_{CLM} also reach almost 100 iterations on average. This behavior is expected and should be interpreted differently: since the true random-effects variance is zero, the estimated random effects are close to zero and the relative convergence criterion used by the algorithm

Table S4: Mean confusion matrices (averaged over the simulation runs) for each model and DGP. Standard deviations in parentheses. Rows T1, T2, T3 denote the true class; columns P1, P2, P3 the predicted class.

DGP	T	CLM			CLMM			ORDFOR			OMERF			OMERF_NO_RAN_SLOPE			OMERF_CLM		
		P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	T1	3.38	6.27	21.40	5.62	9.67	15.76	9.64	12.25	9.16	14.77	11.70	4.58	-	-	-	16.25	10.40	4.40
		(2.95)	(3.34)	(4.24)	(4.04)	(3.71)	(3.61)	(5.29)	(3.57)	(2.96)	(5.63)	(3.44)	(2.04)	-	-	-	(5.50)	(2.97)	(2.16)
	T2	2.45	7.43	47.14	3.56	11.52	41.94	6.40	20.19	30.43	11.34	25.00	20.68	-	-	-	13.53	23.15	20.34
		(2.23)	(4.41)	(3.29)	(2.66)	(4.64)	(3.99)	(3.55)	(5.69)	(4.53)	(4.07)	(4.87)	(3.69)	-	-	-	(3.78)	(4.52)	(3.46)
	T3	2.16	7.65	101.00	3.07	11.26	96.48	1.76	11.92	97.13	2.81	17.22	90.78	-	-	-	3.72	16.83	90.26
		(2.38)	(4.31)	(12.37)	(2.52)	(4.61)	(12.32)	(1.48)	(3.88)	(10.97)	(1.80)	(3.82)	(9.94)	-	-	-	(1.92)	(3.53)	(9.29)
2	T1	6.02	0.70	13.28	6.73	1.44	11.83	10.57	3.62	5.81	12.25	5.51	2.24	-	-	-	12.96	5.08	1.96
		(3.00)	(1.34)	(2.79)	(3.01)	(1.97)	(2.64)	(3.67)	(2.26)	(2.07)	(3.33)	(2.21)	(1.51)	-	-	-	(3.26)	(2.17)	(1.29)
	T2	2.11	0.45	27.91	2.32	0.99	27.16	4.04	5.09	21.34	6.50	9.98	13.99	-	-	-	7.40	10.30	12.77
		(1.42)	(1.01)	(2.97)	(1.48)	(1.39)	(2.88)	(2.17)	(2.76)	(2.65)	(2.41)	(3.35)	(3.11)	-	-	-	(2.70)	(3.44)	(3.18)
	T3	4.64	0.93	142.76	5.22	1.49	141.62	1.49	2.97	143.87	2.07	7.77	138.49	-	-	-	2.63	9.16	136.54
		(2.49)	(1.68)	(7.33)	(2.52)	(2.00)	(7.44)	(1.63)	(2.29)	(7.11)	(1.48)	(2.89)	(7.12)	-	-	-	(1.61)	(3.38)	(7.09)
3	T1	7.72	0.78	30.50	19.77	7.48	11.75	15.01	6.49	17.50	23.72	10.22	5.06	-	-	-	24.80	9.59	4.61
		(8.01)	(1.76)	(7.33)	(12.35)	(3.52)	(4.14)	(10.88)	(4.35)	(6.12)	(11.05)	(3.35)	(2.06)	-	-	-	(10.40)	(3.47)	(2.01)
	T2	4.63	0.67	43.48	8.94	8.62	31.22	8.74	7.32	32.72	11.26	17.18	20.34	-	-	-	12.56	16.75	19.47
		(5.03)	(1.58)	(4.92)	(5.61)	(5.15)	(5.22)	(5.89)	(5.14)	(4.75)	(4.43)	(4.97)	(4.10)	-	-	-	(4.56)	(4.93)	(3.80)
	T3	5.69	1.10	104.25	7.35	8.10	95.59	5.98	6.56	98.50	3.72	14.22	93.10	-	-	-	4.63	14.45	91.96
		(5.77)	(2.85)	(19.64)	(4.50)	(4.33)	(18.51)	(4.35)	(4.08)	(19.65)	(1.90)	(4.12)	(16.24)	-	-	-	(2.02)	(3.74)	(15.25)
4	T1	6.01	0.06	18.15	11.52	0.60	12.10	11.38	1.62	11.22	15.85	4.48	3.89	-	-	-	16.39	4.65	3.18
		(3.69)	(0.31)	(4.92)	(5.61)	(1.05)	(3.27)	(5.26)	(1.56)	(3.81)	(6.01)	(2.09)	(2.03)	-	-	-	(5.83)	(2.03)	(1.73)
	T2	2.47	0.07	26.22	4.06	0.47	24.23	4.33	2.14	22.26	6.28	7.75	14.73	-	-	-	7.24	8.54	12.98
		(2.21)	(0.36)	(3.77)	(2.39)	(0.93)	(3.56)	(2.61)	(2.08)	(3.10)	(2.68)	(2.89)	(2.70)	-	-	-	(2.72)	(3.23)	(2.89)
	T3	4.61	0.05	141.23	7.21	0.80	137.88	2.76	1.50	141.63	2.37	6.77	136.75	-	-	-	2.80	8.67	134.42
		(2.68)	(0.41)	(11.59)	(3.39)	(1.44)	(11.92)	(2.35)	(1.32)	(11.93)	(1.77)	(3.11)	(11.56)	-	-	-	(1.91)	(2.96)	(11.25)
5	T1	2.97	8.21	18.87	3.64	9.18	17.23	8.59	13.78	7.68	12.81	12.45	4.79	13.50	12.00	4.55	14.46	10.70	4.89
		(2.56)	(3.42)	(3.97)	(2.64)	(3.36)	(3.55)	(3.88)	(3.59)	(2.58)	(3.95)	(2.89)	(2.18)	(4.34)	(3.04)	(1.85)	(4.09)	(2.74)	(2.23)
	T2	1.93	9.70	47.25	2.29	11.42	45.17	5.57	23.44	29.87	11.82	24.54	22.52	11.47	25.67	21.74	13.88	22.48	22.52
		(1.68)	(4.89)	(3.81)	(1.82)	(4.96)	(4.09)	(2.78)	(6.05)	(3.92)	(3.62)	(3.35)	(5.23)	(3.60)	(3.59)	(3.51)	(5.08)	(3.59)	(3.60)
	T3	1.65	10.04	98.12	2.12	12.09	95.60	1.66	12.64	95.51	3.62	18.22	87.97	3.38	18.23	88.20	4.87	17.21	87.73
		(1.59)	(4.87)	(8.79)	(1.78)	(5.33)	(9.01)	(1.44)	(3.78)	(7.62)	(2.04)	(4.29)	(7.57)	(2.00)	(3.84)	(2.33)	(3.88)	(7.29)	(7.03)
6	T1	5.83	1.24	12.23	5.89	1.54	11.87	9.83	4.08	5.39	11.53	5.23	2.54	11.70	5.17	2.43	11.63	5.28	2.39
		(2.53)	(1.44)	(2.80)	(2.68)	(1.87)	(3.01)	(3.21)	(2.38)	(2.26)	(3.22)	(2.10)	(1.51)	(3.30)	(2.14)	(1.42)	(3.32)	(2.24)	(1.51)
	T2	2.06	0.93	28.09	2.04	1.09	27.95	3.79	6.79	20.50	6.66	10.74	13.68	6.55	10.78	13.75	7.42	11.00	12.66
		(1.55)	(1.26)	(2.34)	(1.44)	(1.34)	(2.31)	(2.09)	(3.15)	(2.55)	(2.29)	(3.49)	(2.78)	(2.28)	(3.59)	(2.82)	(2.31)	(3.16)	(2.50)
	T3	4.08	1.76	142.65	4.15	1.99	142.35	0.94	3.13	144.42	1.89	7.85	138.75	1.84	7.98	138.67	2.42	9.29	136.78
		(2.44)	(2.17)	(5.39)	(2.34)	(2.30)	(5.51)	(0.99)	(1.97)	(4.95)	(1.34)	(2.67)	(5.34)	(1.33)	(2.86)	(5.61)	(1.58)	(2.85)	(5.24)
7	T1	4.22	5.78	22.48	7.10	9.49	15.89	10.40	11.74	10.34	14.89	11.78	5.81	15.25	12.11	5.12	16.32	10.45	5.71
		(3.48)	(3.58)	(4.87)	(4.70)	(3.83)	(4.15)	(4.82)	(3.71)	(3.32)	(4.98)	(3.26)	(2.54)	(4.96)	(3.06)	(2.33)	(4.80)	(3.11)	(2.50)
	T2	3.08	6.64	46.99	4.71	12.00	40.00	6.51	18.50	31.70	11.62	21.61	23.48	11.62	22.69	22.40	14.09	19.30	23.32
		(2.83)	(4.30)	(3.97)	(3.52)	(5.48)	(4.64)	(3.69)	(5.78)	(4.22)	(3.07)	(4.67)	(3.69)	(3.44)	(4.97)	(3.71)	(3.08)	(4.06)	(3.60)
	T3	3.01	6.71	99.93	3.78	10.97	94.90	2.17	11.19	96.26	4.17	15.56	89.92	3.42	16.18	90.05	5.38	15.35	88.92
		(2.66)	(4.13)	(11.22)	(2.79)	(4.80)	(11.42)	(1.46)	(3.86)	(10.88)	(2.58)	(3.83)	(10.15)	(1.89)	(4.22)	(10.59)	(3.01)	(3.77)	(9.93)
8	T1	6.09	1.04	13.03	6.84	1.72	11.60	10.08	3.76	6.32	11.97	5.38	2.81	12.09	5.35	2.72	12.34	5.17	2.65
		(2.91)	(1.61)	(3.21)	(3.37)	(2.03)	(2.73)	(3.02)	(2.22)	(2.55)	(3.57)	(2.27)	(1.65)	(3.59)	(2.34)	(1.52)	(3.47)	(2.23)	(1.64)
	T2	2.27	0.64	28.03	2.53	1.21	27.20	4.18	5.84	20.92	7.00	9.80	14.14	6.96	9.93	14.05	7.97	9.79	13.18
		(1.66)	(1.34)	(3.23)	(1.80)	(1.57)	(3.05)	(2.60)	(3.19)	(2.84)	(2.50)	(2.79)	(2.74)	(2.39)	(3.46)	(2.54)	(2.54)	(3.58)	(2.82)
	T3	4.39	1.31	141.95	4.71	1.86	141.08	1.44	2.99	143.22	2.27	7.95	137.43	2.26	8.08	137.31	2.84	9.16	135.65
		(2.58)	(2.03)	(6.79)	(2.64)	(2.36)	(7.01)	(1.12)	(1.99)	(6.48)	(1.76)	(2.81)	(6.92)	(1.74)	(2.92)	(6.97)	(1.95)	(3.04)	(6.71)
9	T1	2.33	8.32	17.81	2.27	8.42	17.77	8.25	13.45	6.76	12.59	12.14	3.73	-	-	-	14.39	10.35	3.72
		(1.82)	(2.56)	(3.47)	(1.77)	(2.56)	(3.47)	(3.33)	(3.25)	(2.54)	(3.19)	(2.78)	(2.00)	-	-	-	(3.10)	(2.67)	(1.91)
	T2	1.47	10.40	47.37	1.46	10.49	47.29	4.70	25.01	29.53	10.77	27.03	21.44	-	-	-	13.64	24.42	21.18
		(1.68)	(3.67)	(3.85)	(1.61)	(3.88)	(3.94)	(2.62)	(5.20)	(4.01)	(3.95)	(5.22)	(3.63)	-	-	-	(4.27)	(4.71)	(3.69)
	T3	1.43	10.62	99.06	1.42	10.69	99.00	1.12	11.90	98.09	2.85	17.64	90.62	-	-	-	3.85	16.90	90.36
		(1.32)	(4.15)	(6.13)	(1.28)	(4.13)	(6.08)	(1.06)	(3.86)	(5.85)	(1.77)	(4.28)	(6.05)	-	-	-	(1.99)	(4.40)	(6.01)
10	T1	5.99	1.69	11.01	6.01	1.76	10.92	9.91	4.10	4.68	11.58	4.76	2.35	-	-	-	11.75	4.75	2.19
		(2.75)	(1.74)	(2.44)	(2.75)	(1.74)	(2.40)	(3.00)	(1.97)	(1.89)	(2.98)	(2.16)	(1.35)	-	-	-	(2.71)	(2.17)	(1.32)
	T2	2.04	1.21	27.76	2.05	1.29	27.67	3.61	7.16	20.24	6.47	10.94	13.60	-	-	-	7.25	11.14	12.62
		(1.38)	(1.35)	(2.44)	(1.39)	(1.42)	(2.42)	(1.76)	(2.86)	(3.14)	(2.22)	(3.09)	(2.94)	-	-	-	(2.26)	(3.24)	(3.04)
	T3	4.02	2.16	142.97	4.03	2.20	142.92	1.00	3.09	145.06	1.96	8.11	139.08	-	-	-	2.41	9.32	137.42
		(2.11)	(2.17)	(3.98)	(2.17)	(2.23)	(3.89)	(0.97)	(1.87)	(3.57)	(1.48)	(3.42)	(4.59)	-	-	-	(1.59)	(3.65)	(4.77)
11	T1	63.99	5.07	1.54	64.32	5.12	1.16	64.93	2.64	3.03	67.33	1.55	1.72	-	-	-	57.68	6.47	6.45
		(5.03)	(1.98)	(1.35)	(5.17)	(2.10)	(1.12)	(5.10)	(1.65)	(1.98)	(4.83)	(1.42)	(1.39)	-	-	-	(5.21)	(3.13)	(3.04)
	T2	6.95	8.79	7.47	6.21	10.28	6.72	8.76	4.38	10.07	13.49	2.82	6.90	-	-	-	14.06	2.56	

can become unstable, making convergence hard to declare even though the predictive performance remains comparable to that of the ordinal forests. The large standard deviations of the computation times in DGPs 2 and 5 further indicate that the mean running time is affected by a few unusually slow runs, likely corresponding to cases in which the clmm optimizer struggles or fails to converge. Finally, the comparison with the random-intercept-only version in DGPs 5–8 shows that simplifying the random-effects structure can substantially reduce the number of iterations and computation time, while often producing very similar predictive performance. Thus, Table S5 highlights a practical trade-off: the full OMERF specification is preferable when the random-effects structure is substantively important, but simpler random-effects specifications may be computationally attractive when the random-slope signal is weak and prediction is the main goal.

S2 Additional simulation study

To complement the scenario described in the simulation study in Section 4, we design a second simulation study that explores the effect of varying key factors such as the number and balance of response categories, as well as the number and size of groups. Specifically, we simulate the case of 4-categories ordinal responses and we design a two-level data structure with $I = 15$ groups with n_i observations each. To consider a scenario with varying group sizes, we sample n_i from a uniform distribution ($n_i \sim U(50, 100)$). For what concerns fixed- and random- effects, we follow the 12 DGPs presented in Section 4. We report in table S6 the distribution of the number of observations observed in the 100 simulated ordinal responses, for each DGP.

Results are reported in Table S7. OMERF and the ordinal forests are run with default inputs, as specified in Section 4.2. As expected, the simulation results closely

Table S5: Average (SD) number of iterations and computation time (in seconds) across simulation runs, by DGP and algorithm.

DGP	OMERF		OMERF _{no ran slope}		OMERF _{clm}	
	niter	time (s)	niter	time (s)	niter	time (s)
1	14.41 (21.16)	45.09 (7.21)	—	—	31.31 (34.06)	9.80 (10.95)
2	30.33 (32.97)	584.75 (5363.28)	—	—	51.70 (38.67)	16.31 (13.53)
3	5.12 (1.11)	45.69 (4.25)	—	—	7.23 (6.03)	2.38 (2.26)
4	7.73 (5.48)	32.16 (1.61)	—	—	18.08 (24.82)	3.99 (5.70)
5	88.49 (27.19)	760.50 (4869.14)	61.02 (39.35)	56.84 (16.82)	98.38 (11.45)	216.83 (47.49)
6	89.97 (25.13)	136.23 (48.97)	75.87 (35.60)	43.75 (14.96)	100.00 (0.00)	139.05 (40.43)
7	62.60 (38.68)	112.76 (54.41)	20.55 (26.92)	31.89 (5.16)	94.63 (18.52)	123.28 (28.91)
8	63.37 (39.17)	104.81 (50.63)	35.77 (34.69)	32.43 (6.93)	97.00 (13.55)	128.09 (21.28)
9	98.68 (8.27)	50.76 (2.61)	—	—	99.04 (9.60)	18.94 (2.09)
10	98.63 (10.03)	50.07 (6.78)	—	—	99.54 (3.88)	19.65 (2.90)
11	37.17 (37.07)	28.54 (5.71)	—	—	61.49 (38.76)	9.68 (6.17)
12	7.43 (8.13)	26.44 (3.24)	—	—	15.83 (22.98)	2.60 (4.06)

mirror the patterns observed in the main simulation study, with OMERF consistently emerging as the best-performing approach or showing performance comparable to that of the alternative models. The only exceptions are DGPs 11 and 12, where - consistent with the findings previously reported in the main study - the linearity of the predictor allows CLM and CLMM to outperform the other models. These results support the conclusion that, even when varying the number of categories in the ordinal response or the number and size of groups, the performance of OMERF remains consistent. Table S8 reports the average confusion matrices (averaged over runs) for each model and DGP of this simulation study.

DGP	Response category	mean $\pm$ sd (min–max)	
		Train	Test
1	1	113.81 $\pm$ 20.23 (76–187)	27.91 $\pm$ 4.99 (19–46)
	2	119.37 $\pm$ 15.02 (80–158)	29.29 $\pm$ 3.77 (19–39)
	3	280.15 $\pm$ 20.13 (243–328)	69.52 $\pm$ 5.05 (60–81)
	4	390.14 $\pm$ 37.63 (302–492)	97.00 $\pm$ 9.38 (75–122)
2	1	77.33 $\pm$ 12.41 (55–119)	18.82 $\pm$ 3.15 (13–29)
	2	62.37 $\pm$ 8.77 (45–85)	15.09 $\pm$ 2.20 (11–21)
	3	166.02 $\pm$ 14.55 (125–212)	41.00 $\pm$ 3.66 (31–52)
	4	597.65 $\pm$ 38.55 (523–684)	148.91 $\pm$ 9.65 (130–171)
3	1	151.88 $\pm$ 42.54 (74–301)	37.52 $\pm$ 10.68 (18–75)
	2	108.74 $\pm$ 17.39 (64–163)	26.70 $\pm$ 4.33 (15–40)
	3	235.36 $\pm$ 25.02 (179–303)	58.33 $\pm$ 6.20 (44–75)
	4	407.31 $\pm$ 59.83 (256–556)	101.35 $\pm$ 14.98 (63–139)
4	1	96.56 $\pm$ 24.42 (54–190)	23.68 $\pm$ 6.09 (13–47)
	2	62.40 $\pm$ 11.00 (38–92)	15.11 $\pm$ 2.77 (9–23)
	3	152.23 $\pm$ 17.80 (113–209)	37.54 $\pm$ 4.47 (28–52)

	4	592.15 ± 51.70 (465–716)	147.52 ± 12.92 (116–178)
5	1	108.31 ± 15.19 (79–141)	26.60 ± 3.81 (19–35)
	2	121.14 ± 11.80 (91–149)	29.87 ± 2.97 (22–37)
	3	290.15 ± 20.53 (240–357)	72.03 ± 5.13 (60–89)
	4	380.37 ± 27.12 (320–455)	94.59 ± 6.76 (80–113)
6	1	74.61 ± 10.01 (50–100)	18.07 ± 2.54 (12–24)
	2	61.32 ± 6.29 (48–76)	14.87 ± 1.61 (12–18)
	3	169.51 ± 14.90 (140–210)	41.93 ± 3.68 (35–52)
	4	594.56 ± 32.21 (533–680)	148.19 ± 8.11 (133–169)
7	1	119.12 ± 21.41 (71–179)	29.31 ± 5.39 (17–44)
	2	119.76 ± 13.60 (90–152)	29.40 ± 3.46 (22–38)
	3	277.44 ± 21.02 (228–348)	68.81 ± 5.20 (57–86)
	4	383.77 ± 34.83 (292–482)	95.45 ± 8.75 (72–120)
8	1	78.76 ± 12.89 (47–108)	19.13 ± 3.28 (11–26)
	2	61.49 ± 7.59 (42–80)	14.89 ± 1.94 (10–20)
	3	167.32 ± 15.89 (136–205)	41.32 ± 4.02 (34–51)
	4	592.51 ± 35.22 (516–692)	147.64 ± 8.84 (128–172)
9	1	102.47 ± 9.60 (74–124)	25.10 ± 2.39 (18–31)
	2	120.02 ± 11.63 (98–164)	29.46 ± 2.95 (24–40)
	3	297.56 ± 20.62 (252–357)	73.86 ± 5.12 (63–89)
	4	378.94 ± 22.40 (331–428)	94.27 ± 5.55 (82–106)
10	1	73.06 ± 8.52 (52–96)	17.72 ± 2.14 (12–23)
	2	61.18 ± 6.50 (48–80)	14.78 ± 1.65 (11–20)
	3	170.56 ± 14.51 (136–222)	42.18 ± 3.60 (34–55)
	4	594.16 ± 32.15 (532–678)	148.04 ± 8.05 (133–169)
11	1	302.34 ± 23.60 (240–376)	75.11 ± 5.92 (59–94)

	2	58.68 ± 6.95 (41–73)	14.16 ± 1.73 (10–18)
	3	109.92 ± 10.51 (84–135)	26.95 ± 2.66 (20–33)
	4	432.42 ± 27.37 (363–495)	107.61 ± 6.84 (90–123)
12	1	305.95 ± 30.50 (235–388)	75.97 ± 7.65 (58–97)
	2	57.73 ± 7.36 (44–73)	13.91 ± 1.87 (10–18)
	3	107.46 ± 10.19 (85–132)	26.39 ± 2.55 (21–32)
	4	432.20 ± 34.22 (360–517)	107.58 ± 8.62 (89–129)

Table S6: Distribution of the number of observations observed in the simulated ordinal responses in the 12 DGPs presented in Section S2, across the 100 runs, for both train and test sets. Values are reported as mean $\pm$ sd (min–max).

DGP	Model	Accuracy	Precision	Recall	RMSE	MAE	Cohen’s k	W Cohen’s k	Cardoso	Ballante
1	CLM	0.4485	0.4038	0.2968	1.2016	0.8113	0.0945	0.1968	0.6697	0.3887
		(0.0364)	(0.0753)	(0.0264)	(0.0710)	(0.0656)	(0.0447)	(0.0614)	(0.0325)	(0.0313)
	CLMM	0.4620	0.4236	0.3255	1.1411	0.7643	0.1394	0.2884	0.6541	0.3596
		(0.0394)	(0.0474)	(0.0281)	(0.0700)	(0.0670)	(0.0491)	(0.0699)	(0.0361)	(0.0326)
	ordforest	0.5579	0.4706	0.4233	0.9605	0.5909	0.3126	0.5117	0.5631	0.2141
		(0.0334)	(0.0720)	(0.0330)	(0.0634)	(0.0532)	(0.0448)	(0.0618)	(0.0347)	(0.0262)
	OMERF _{CLM}	0.5636	0.4594	0.4713	0.9311	0.5715	0.3526	0.6091	0.5637	0.2044
		(0.0361)	(0.0434)	(0.0347)	(0.0667)	(0.0574)	(0.0455)	(0.0480)	(0.0362)	(0.0249)
	OMERF	0.5749	0.5041	0.4659	0.9210	0.5592	0.3604	0.5961	0.5474	0.2217
		(0.0355)	(0.0605)	(0.0323)	(0.0635)	(0.0556)	(0.0447)	(0.0475)	(0.0356)	(0.0336)
2	CLM	0.6704	0.4721	0.3326	1.0219	0.5289	0.1553	0.3111	0.4842	0.2307
		(0.0299)	(0.0826)	(0.0311)	(0.0814)	(0.0615)	(0.0543)	(0.0870)	(0.0375)	(0.0322)
	CLMM	0.6690	0.4718	0.3434	0.9938	0.5161	0.1781	0.3578	0.4819	0.2208
		(0.0295)	(0.0616)	(0.0313)	(0.0695)	(0.0537)	(0.0521)	(0.0860)	(0.0346)	(0.0297)
	ordforest	0.7400	0.5666	0.4605	0.7502	0.3514	0.4140	0.6532	0.3743	0.1098
		(0.0247)	(0.0891)	(0.0350)	(0.0602)	(0.0395)	(0.0525)	(0.0561)	(0.0333)	(0.0295)
	OMERF _{CLM}	0.7386	0.5193	0.5115	0.7266	0.3447	0.4788	0.7158	0.3888	0.1101
		(0.0272)	(0.0676)	(0.0377)	(0.0619)	(0.0411)	(0.0454)	(0.0446)	(0.0348)	(0.0191)
	OMERF	0.7475	0.5787	0.5064	0.7173	0.3348	0.4878	0.7141	0.3741	0.1255
		(0.0282)	(0.0632)	(0.0367)	(0.0669)	(0.0449)	(0.0481)	(0.0484)	(0.0390)	(0.0268)
3	CLM	0.4609	0.3948	0.2834	1.4377	0.9600	0.0621	0.1278	0.6959	0.4452
		(0.0548)	(0.0995)	(0.0267)	(0.1540)	(0.1462)	(0.0460)	(0.0793)	(0.0559)	(0.0592)
	CLMM	0.5274	0.4778	0.3983	1.1238	0.7042	0.2578	0.4762	0.6109	0.3026
		(0.0511)	(0.0461)	(0.0350)	(0.1083)	(0.0981)	(0.0569)	(0.0849)	(0.0531)	(0.0429)
	ordforest	0.5233	0.4198	0.3727	1.1974	0.7546	0.2305	0.3943	0.6305	0.2855

		(0.0367)	(0.0797)	(0.0447)	(0.1293)	(0.1006)	(0.0634)	(0.1009)	(0.0461)	(0.0529)
	OMERF _{CLM}	0.5901	0.4814	0.4871	0.9314	0.5515	0.3855	0.6575	0.5447	0.1878
		(0.0379)	(0.0510)	(0.0349)	(0.0728)	(0.0611)	(0.0457)	(0.0457)	(0.0392)	(0.0277)
	OMERF	0.5974	0.5377	0.4841	0.9296	0.5466	0.3915	0.6527	0.5341	0.2069
		(0.0388)	(0.0615)	(0.0308)	(0.0786)	(0.0667)	(0.0444)	(0.0459)	(0.0429)	(0.0370)
4	CLM	0.6673	0.5298	0.3121	1.1352	0.5929	0.1226	0.2447	0.5091	0.2648
		(0.0412)	(0.0922)	(0.0283)	(0.1148)	(0.0955)	(0.0529)	(0.0936)	(0.0544)	(0.0465)
	CLMM	0.6778	0.4970	0.3686	1.0425	0.5352	0.2282	0.4255	0.4859	0.2192
		(0.0413)	(0.0867)	(0.0332)	(0.1110)	(0.0876)	(0.0606)	(0.0902)	(0.0532)	(0.0428)
	ordforest	0.7112	0.5269	0.3992	0.9270	0.4518	0.3140	0.5272	0.4382	0.1622
		(0.0342)	(0.0830)	(0.0403)	(0.1003)	(0.0696)	(0.0667)	(0.0908)	(0.0484)	(0.0421)
	OMERF _{CLM}	0.7367	0.5203	0.5047	0.7428	0.3524	0.4746	0.7285	0.3921	0.1081
		(0.0356)	(0.0714)	(0.0374)	(0.0792)	(0.0554)	(0.0509)	(0.0481)	(0.0473)	(0.0219)
	OMERF	0.7464	0.5881	0.4973	0.7443	0.3466	0.4809	0.7190	0.3794	0.1271
		(0.0330)	(0.0665)	(0.0334)	(0.0810)	(0.0549)	(0.0500)	(0.0522)	(0.0463)	(0.0278)
5	CLM	0.4486	0.4167	0.3049	1.1580	0.7836	0.1094	0.2198	0.6628	0.3767
		(0.0336)	(0.0818)	(0.0293)	(0.0590)	(0.0519)	(0.0549)	(0.0628)	(0.0277)	(0.0268)
	CLMM	0.4511	0.4184	0.3126	1.1461	0.7745	0.1217	0.2406	0.6611	0.3698
		(0.0341)	(0.0686)	(0.0285)	(0.0583)	(0.0535)	(0.0516)	(0.0603)	(0.0285)	(0.0268)
	ordforest	0.5709	0.4979	0.4395	0.9251	0.5618	0.3364	0.5310	0.5471	0.2041
		(0.0278)	(0.0783)	(0.0310)	(0.0537)	(0.0426)	(0.0399)	(0.0570)	(0.0291)	(0.0235)
	OMERF _{CLM}	0.5394	0.4391	0.4489	0.9896	0.6216	0.3183	0.5542	0.5913	0.2270
		(0.0308)	(0.0479)	(0.0333)	(0.0595)	(0.0517)	(0.0427)	(0.0493)	(0.0303)	(0.0226)
	OMERF _{no ran slope}	0.5666	0.5048	0.4584	0.9292	0.5696	0.3487	0.5764	0.5542	0.2324
		(0.0317)	(0.0540)	(0.0308)	(0.0595)	(0.0505)	(0.0434)	(0.0486)	(0.0325)	(0.0299)
	OMERF	0.5551	0.4891	0.4455	0.9647	0.5971	0.3316	0.5468	0.5704	0.2453
		(0.0327)	(0.0568)	(0.0331)	(0.0617)	(0.0536)	(0.0451)	(0.0506)	(0.0324)	(0.0333)
6	CLM	0.6657	0.4682	0.3348	0.9879	0.5154	0.1617	0.3386	0.4817	0.2259
		(0.0229)	(0.0673)	(0.0301)	(0.0729)	(0.0476)	(0.0538)	(0.0946)	(0.0291)	(0.0262)
	CLMM	0.6650	0.4700	0.3398	0.9818	0.5127	0.1702	0.3522	0.4819	0.2239
		(0.0237)	(0.0631)	(0.0317)	(0.0677)	(0.0453)	(0.0529)	(0.0927)	(0.0292)	(0.0250)
	ordforest	0.7446	0.5706	0.4709	0.7289	0.3386	0.4287	0.6674	0.3629	0.1078
		(0.0232)	(0.0911)	(0.0377)	(0.0631)	(0.0395)	(0.0540)	(0.0559)	(0.0336)	(0.0245)
	OMERF _{CLM}	0.7281	0.5013	0.4933	0.7488	0.3615	0.4562	0.6913	0.4016	0.1211
		(0.0279)	(0.0733)	(0.0432)	(0.0675)	(0.0445)	(0.0524)	(0.0525)	(0.0356)	(0.0231)
	OMERF _{no ran slope}	0.7403	0.5785	0.5025	0.7257	0.3425	0.4754	0.7031	0.3813	0.1345
		(0.0265)	(0.0598)	(0.0377)	(0.0617)	(0.0402)	(0.0521)	(0.0476)	(0.0351)	(0.0257)
	OMERF	0.7380	0.5665	0.4951	0.7316	0.3472	0.4698	0.6972	0.3850	0.1383
		(0.0278)	(0.0660)	(0.0394)	(0.0665)	(0.0430)	(0.0537)	(0.0508)	(0.0373)	(0.0262)
7	CLM	0.4463	0.4011	0.3012	1.2232	0.8269	0.1002	0.1987	0.6754	0.3947
		(0.0387)	(0.0814)	(0.0293)	(0.0814)	(0.0743)	(0.0509)	(0.0746)	(0.0354)	(0.0361)
	CLMM	0.4689	0.4269	0.3398	1.1518	0.7655	0.1618	0.3061	0.6531	0.3571
		(0.0343)	(0.0512)	(0.0349)	(0.0704)	(0.0605)	(0.0520)	(0.0800)	(0.0313)	(0.0304)
	ordforest	0.5506	0.4607	0.4212	0.9890	0.6113	0.3045	0.4981	0.5744	0.2218
		(0.0325)	(0.0724)	(0.0365)	(0.0622)	(0.0516)	(0.0507)	(0.0636)	(0.0322)	(0.0276)
	OMERF _{CLM}	0.5314	0.4230	0.4400	1.0195	0.6445	0.3064	0.5479	0.6018	0.2320
		(0.0349)	(0.0441)	(0.0336)	(0.0743)	(0.0661)	(0.0456)	(0.0579)	(0.0373)	(0.0264)
	OMERF _{no ran slope}	0.5587	0.4995	0.4564	0.9595	0.5918	0.3395	0.5767	0.5664	0.2417
		(0.0366)	(0.0520)	(0.0322)	(0.0722)	(0.0633)	(0.0482)	(0.0541)	(0.0369)	(0.0355)
	OMERF	0.5422	0.4745	0.4373	1.0014	0.6272	0.3142	0.5403	0.5874	0.2534
		(0.0371)	(0.0639)	(0.0325)	(0.0743)	(0.0656)	(0.0488)	(0.0586)	(0.0375)	(0.0370)
8	CLM	0.6633	0.4577	0.3286	1.0302	0.5388	0.1477	0.3123	0.4909	0.2359
		(0.0291)	(0.0806)	(0.0323)	(0.0843)	(0.0607)	(0.0528)	(0.0949)	(0.0372)	(0.0296)
	CLMM	0.6648	0.4727	0.3464	0.9956	0.5204	0.1814	0.3688	0.4856	0.2258
		(0.0294)	(0.0670)	(0.0354)	(0.0843)	(0.0596)	(0.0552)	(0.0946)	(0.0382)	(0.0309)

	ordforest	0.7344 (0.0253)	0.5592 (0.0957)	0.4531 (0.0410)	0.7745 (0.0673)	0.3651 (0.0444)	0.4039 (0.0578)	0.6321 (0.0700)	0.3847 (0.0358)	0.1197 (0.0281)
	OMERF _{CLM}	0.7230 (0.0263)	0.4949 (0.0674)	0.4873 (0.0411)	0.7748 (0.0612)	0.3763 (0.0414)	0.4480 (0.0493)	0.6779 (0.0507)	0.4127 (0.0332)	0.1242 (0.0197)
	OMERF _{no ran slope}	0.7348 (0.0277)	0.5608 (0.0684)	0.4918 (0.0356)	0.7457 (0.0634)	0.3554 (0.0438)	0.4648 (0.0487)	0.6935 (0.0468)	0.3917 (0.0363)	0.1354 (0.0266)
	OMERF	0.7321 (0.0246)	0.5620 (0.0699)	0.4873 (0.0348)	0.7515 (0.0607)	0.3596 (0.0404)	0.4586 (0.0455)	0.6885 (0.0487)	0.3949 (0.0335)	0.1384 (0.0272)
9	CLM	0.4501 (0.0288)	0.4211 (0.0767)	0.3046 (0.0243)	1.1155 (0.0468)	0.7579 (0.0400)	0.1123 (0.0426)	0.2321 (0.0607)	0.6563 (0.0230)	0.3630 (0.0242)
		0.4508 (0.0284)	0.4208 (0.0761)	0.3053 (0.0235)	1.1154 (0.0477)	0.7573 (0.0401)	0.1137 (0.0416)	0.2330 (0.0604)	0.6559 (0.0226)	0.3629 (0.0241)
	CLMM	0.4508 (0.0284)	0.4208 (0.0761)	0.3053 (0.0235)	1.1154 (0.0477)	0.7573 (0.0401)	0.1137 (0.0416)	0.2330 (0.0604)	0.6559 (0.0226)	0.3629 (0.0241)
		0.5851 (0.0301)	0.5114 (0.0791)	0.4519 (0.0342)	0.8815 (0.0510)	0.5289 (0.0428)	0.3577 (0.0471)	0.5557 (0.0563)	0.5276 (0.0298)	0.1908 (0.0200)
	OMERF _{CLM}	0.5594 (0.0310)	0.4540 (0.0464)	0.4686 (0.0331)	0.9401 (0.0621)	0.5795 (0.0509)	0.3476 (0.0452)	0.5860 (0.0500)	0.5670 (0.0324)	0.2087 (0.0208)
		0.5736 (0.0316)	0.5124 (0.0586)	0.4598 (0.0360)	0.9131 (0.0608)	0.5556 (0.0503)	0.3560 (0.0469)	0.5742 (0.0519)	0.5434 (0.0343)	0.2283 (0.0278)
	OMERF	0.5736 (0.0316)	0.5124 (0.0586)	0.4598 (0.0360)	0.9131 (0.0608)	0.5556 (0.0503)	0.3560 (0.0469)	0.5742 (0.0519)	0.5434 (0.0343)	0.2283 (0.0278)
	CLM	0.6626 (0.0219)	0.4619 (0.0596)	0.3390 (0.0349)	0.9775 (0.0599)	0.5125 (0.0395)	0.1681 (0.0521)	0.3547 (0.0887)	0.4831 (0.0258)	0.2225 (0.0247)
		0.6618 (0.0221)	0.4578 (0.0590)	0.3383 (0.0346)	0.9795 (0.0608)	0.5140 (0.0401)	0.1669 (0.0529)	0.3526 (0.0889)	0.4841 (0.0262)	0.2235 (0.0248)
	CLMM	0.6618 (0.0221)	0.4578 (0.0590)	0.3383 (0.0346)	0.9795 (0.0608)	0.5140 (0.0401)	0.1669 (0.0529)	0.3526 (0.0889)	0.4841 (0.0262)	0.2235 (0.0248)
		0.7546 (0.0216)	0.5898 (0.0830)	0.4908 (0.0372)	0.7059 (0.0557)	0.3219 (0.0336)	0.4574 (0.0494)	0.6865 (0.0559)	0.3511 (0.0292)	0.0988 (0.0227)
	OMERF _{CLM}	0.7405 (0.0254)	0.5236 (0.0750)	0.5148 (0.0411)	0.7224 (0.0610)	0.3418 (0.0394)	0.4825 (0.0478)	0.7136 (0.0444)	0.3853 (0.0336)	0.1144 (0.0220)
		0.7466 (0.0247)	0.5889 (0.0577)	0.5098 (0.0356)	0.7130 (0.0591)	0.3336 (0.0382)	0.4883 (0.0467)	0.7143 (0.0465)	0.3743 (0.0339)	0.1321 (0.0223)
	OMERF	0.7466 (0.0247)	0.5889 (0.0577)	0.5098 (0.0356)	0.7130 (0.0591)	0.3336 (0.0382)	0.4883 (0.0467)	0.7143 (0.0465)	0.3743 (0.0339)	0.1321 (0.0223)
10	CLM	0.6626 (0.0219)	0.4619 (0.0596)	0.3390 (0.0349)	0.9775 (0.0599)	0.5125 (0.0395)	0.1681 (0.0521)	0.3547 (0.0887)	0.4831 (0.0258)	0.2225 (0.0247)
		0.6618 (0.0221)	0.4578 (0.0590)	0.3383 (0.0346)	0.9795 (0.0608)	0.5140 (0.0401)	0.1669 (0.0529)	0.3526 (0.0889)	0.4841 (0.0262)	0.2235 (0.0248)
	CLMM	0.6618 (0.0221)	0.4578 (0.0590)	0.3383 (0.0346)	0.9795 (0.0608)	0.5140 (0.0401)	0.1669 (0.0529)	0.3526 (0.0889)	0.4841 (0.0262)	0.2235 (0.0248)
		0.7546 (0.0216)	0.5898 (0.0830)	0.4908 (0.0372)	0.7059 (0.0557)	0.3219 (0.0336)	0.4574 (0.0494)	0.6865 (0.0559)	0.3511 (0.0292)	0.0988 (0.0227)
	OMERF _{CLM}	0.7405 (0.0254)	0.5236 (0.0750)	0.5148 (0.0411)	0.7224 (0.0610)	0.3418 (0.0394)	0.4825 (0.0478)	0.7136 (0.0444)	0.3853 (0.0336)	0.1144 (0.0220)
		0.7466 (0.0247)	0.5889 (0.0577)	0.5098 (0.0356)	0.7130 (0.0591)	0.3336 (0.0382)	0.4883 (0.0467)	0.7143 (0.0465)	0.3743 (0.0339)	0.1321 (0.0223)
	OMERF	0.7466 (0.0247)	0.5889 (0.0577)	0.5098 (0.0356)	0.7130 (0.0591)	0.3336 (0.0382)	0.4883 (0.0467)	0.7143 (0.0465)	0.3743 (0.0339)	0.1321 (0.0223)
	CLM	0.8185 (0.0201)	0.7040 (0.0691)	0.5889 (0.0279)	0.5986 (0.0568)	0.2387 (0.0312)	0.7083 (0.0305)	0.9027 (0.0184)	0.2830 (0.0317)	0.0620 (0.0117)
		0.8295 (0.0224)	0.6760 (0.0834)	0.6109 (0.0322)	0.5531 (0.0568)	0.2149 (0.0320)	0.7274 (0.0338)	0.9164 (0.0170)	0.2616 (0.0351)	0.0514 (0.0116)
	CLMM	0.8295 (0.0224)	0.6760 (0.0834)	0.6109 (0.0322)	0.5531 (0.0568)	0.2149 (0.0320)	0.7274 (0.0338)	0.9164 (0.0170)	0.2616 (0.0351)	0.0514 (0.0116)
		0.8088 (0.0196)	0.6121 (0.0918)	0.5520 (0.0292)	0.6520 (0.0573)	0.2631 (0.0314)	0.6847 (0.0310)	0.8877 (0.0198)	0.2994 (0.0303)	0.0597 (0.0107)
	OMERF _{CLM}	0.7109 (0.0309)	0.5689 (0.0350)	0.4574 (0.0240)	1.0140 (0.0875)	0.5082 (0.0690)	0.5296 (0.0448)	0.7272 (0.0478)	0.4795 (0.0449)	0.1865 (0.0276)
		0.7829 (0.0225)	0.6565 (0.0532)	0.5124 (0.0222)	0.7910 (0.0693)	0.3404 (0.0451)	0.6423 (0.0334)	0.8411 (0.0278)	0.3546 (0.0370)	0.1239 (0.0181)
	OMERF	0.7829 (0.0225)	0.6565 (0.0532)	0.5124 (0.0222)	0.7910 (0.0693)	0.3404 (0.0451)	0.6423 (0.0334)	0.8411 (0.0278)	0.3546 (0.0370)	0.1239 (0.0181)
11	CLM	0.7918 (0.0224)	0.6763 (0.0482)	0.5312 (0.0323)	0.7329 (0.0868)	0.3081 (0.0471)	0.6556 (0.0378)	0.8575 (0.0330)	0.3381 (0.0378)	0.0810 (0.0158)
		0.8386 (0.0216)	0.7023 (0.0832)	0.6225 (0.0322)	0.5362 (0.0543)	0.2022 (0.0304)	0.7408 (0.0329)	0.9217 (0.0161)	0.2481 (0.0344)	0.0476 (0.0112)
	CLMM	0.8386 (0.0216)	0.7023 (0.0832)	0.6225 (0.0322)	0.5362 (0.0543)	0.2022 (0.0304)	0.7408 (0.0329)	0.9217 (0.0161)	0.2481 (0.0344)	0.0476 (0.0112)
		0.7827 (0.0202)	0.5897 (0.0956)	0.5053 (0.0235)	0.7977 (0.0803)	0.3402 (0.0449)	0.6341 (0.0330)	0.8345 (0.0332)	0.3574 (0.0353)	0.0872 (0.0211)
	OMERF _{CLM}	0.7275 (0.0317)	0.5877 (0.0376)	0.4715 (0.0256)	0.9702 (0.0841)	0.4692 (0.0666)	0.5545 (0.0464)	0.7519 (0.0433)	0.4557 (0.0455)	0.1662 (0.0283)
		0.7602 (0.0280)	0.6112 (0.0516)	0.4887 (0.0254)	0.8799 (0.0794)	0.3977 (0.0554)	0.6032 (0.0427)	0.8011 (0.0359)	0.4005 (0.0425)	0.1384 (0.0221)
	OMERF	0.7602 (0.0280)	0.6112 (0.0516)	0.4887 (0.0254)	0.8799 (0.0794)	0.3977 (0.0554)	0.6032 (0.0427)	0.8011 (0.0359)	0.4005 (0.0425)	0.1384 (0.0221)
	CLM	0.7918 (0.0224)	0.6763 (0.0482)	0.5312 (0.0323)	0.7329 (0.0868)	0.3081 (0.0471)	0.6556 (0.0378)	0.8575 (0.0330)	0.3381 (0.0378)	0.0810 (0.0158)
		0.8386 (0.0216)	0.7023 (0.0832)	0.6225 (0.0322)	0.5362 (0.0543)	0.2022 (0.0304)	0.7408 (0.0329)	0.9217 (0.0161)	0.2481 (0.0344)	0.0476 (0.0112)
	CLMM	0.8386 (0.0216)	0.7023 (0.0832)	0.6225 (0.0322)	0.5362 (0.0543)	0.2022 (0.0304)	0.7408 (0.0329)	0.9217 (0.0161)	0.2481 (0.0344)	0.0476 (0.0112)
		0.7827 (0.0202)	0.5897 (0.0956)	0.5053 (0.0235)	0.7977 (0.0803)	0.3402 (0.0449)	0.6341 (0.0330)	0.8345 (0.0332)	0.3574 (0.0353)	0.0872 (0.0211)
	OMERF _{CLM}	0.7275 (0.0317)	0.5877 (0.0376)	0.4715 (0.0256)	0.9702 (0.0841)	0.4692 (0.0666)	0.5545 (0.0464)	0.7519 (0.0433)	0.4557 (0.0455)	0.1662 (0.0283)
		0.7602 (0.0280)	0.6112 (0.0516)	0.4887 (0.0254)	0.8799 (0.0794)	0.3977 (0.0554)	0.6032 (0.0427)	0.8011 (0.0359)	0.4005 (0.0425)	0.1384 (0.0221)
	OMERF	0.7602 (0.0280)	0.6112 (0.0516)	0.4887 (0.0254)	0.8799 (0.0794)	0.3977 (0.0554)	0.6032 (0.0427)	0.8011 (0.0359)	0.4005 (0.0425)	0.1384 (0.0221)

Table S7: Mean and standard deviations of prediction performances, measured by nine indices, of the compared methods across 100 runs of the 12 DGPs presented in Section S2.

Table S8: Mean confusion matrices (averaged over the simulation runs) for each model and DGP. Standard deviations in parentheses. Rows T1–T4 denote the true class; columns P1–P4 the predicted class.

DGP	T	CLM				CLMM				ORDFOR				OMERF				OMERF_NO_RAN_SLOPE				OMERF_CLM			
		P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4
1	T1	3.27 (2.86)	0.00 (0.00)	13.25 (4.42)	11.39 (3.71)	5.80 (3.68)	0.01 (0.10)	14.75 (3.88)	7.35 (2.69)	9.74 (4.69)	1.65 (1.70)	13.18 (2.89)	3.34 (1.86)	14.38 (5.14)	1.15 (1.75)	11.13 (3.20)	1.25 (1.11)	-	-	-	-	14.94 (4.63)	3.68 (2.13)	7.91 (3.01)	1.38 (1.20)
	T2	1.61 (1.58)	0.00 (0.00)	11.19 (4.29)	16.49 (3.71)	2.78 (2.18)	0.02 (0.20)	13.97 (3.80)	12.52 (3.67)	4.93 (2.64)	1.32 (1.30)	16.63 (3.40)	6.41 (2.38)	9.32 (3.31)	1.19 (1.96)	15.37 (3.16)	3.41 (1.67)	-	-	-	-	10.08 (3.29)	3.54 (2.44)	11.99 (2.90)	3.68 (1.68)
	T3	2.05 (1.99)	0.00 (0.00)	21.03 (8.20)	46.44 (6.68)	3.30 (2.35)	0.00 (0.00)	25.84 (7.20)	40.38 (5.76)	4.79 (2.95)	1.46 (1.40)	35.93 (7.51)	27.34 (5.54)	9.40 (3.68)	1.96 (2.52)	38.62 (6.24)	19.86 (3.95)	-	-	-	-	10.41 (3.68)	6.09 (3.48)	32.07 (5.31)	20.95 (3.95)
	T4	1.80 (1.68)	0.00 (0.00)	19.13 (6.40)	76.07 (13.86)	2.71 (2.11)	0.02 (0.20)	22.58 (5.92)	71.69 (12.85)	0.91 (1.01)	0.39 (0.65)	17.87 (4.17)	77.83 (11.09)	1.54 (1.36)	0.48 (1.10)	20.56 (4.22)	74.42 (10.05)	-	-	-	-	1.94 (1.53)	1.54 (1.47)	18.00 (3.86)	75.52 (9.80)
2	T1	6.15 (2.61)	0.00 (0.00)	2.83 (2.36)	9.84 (3.16)	6.85 (2.83)	0.00 (0.00)	4.10 (2.36)	7.87 (3.00)	10.83 (3.27)	0.56 (0.81)	4.61 (2.09)	2.82 (1.59)	12.14 (3.48)	0.42 (1.11)	5.47 (2.18)	0.79 (0.83)	-	-	-	-	11.91 (3.37)	1.75 (1.57)	4.42 (2.03)	0.74 (0.85)
	T2	1.39 (1.40)	0.00 (0.00)	1.49 (1.34)	12.21 (2.46)	1.50 (1.34)	0.00 (0.00)	2.25 (1.69)	11.34 (2.52)	3.30 (1.82)	0.48 (0.90)	5.94 (2.40)	5.37 (2.19)	5.63 (2.16)	0.39 (0.94)	6.89 (2.25)	2.18 (1.37)	-	-	-	-	5.61 (1.95)	1.60 (1.36)	5.93 (2.09)	1.95 (1.30)
	T3	2.00 (1.48)	0.00 (0.00)	2.49 (2.52)	36.51 (3.53)	2.26 (1.72)	0.00 (0.00)	3.20 (3.79)	35.54 (3.79)	3.19 (2.21)	0.41 (0.79)	11.88 (4.28)	25.52 (4.07)	6.27 (2.73)	0.48 (1.13)	18.40 (4.04)	15.85 (3.53)	-	-	-	-	6.90 (2.87)	2.37 (1.98)	16.83 (3.58)	14.90 (3.59)
	T4	3.97 (2.50)	0.00 (0.00)	3.52 (2.83)	141.42 (10.93)	4.03 (2.19)	0.00 (0.00)	5.20 (10.76)	139.68 (20.72)	0.78 (0.89)	0.12 (0.36)	5.57 (2.78)	142.41 (10.41)	1.40 (1.20)	0.13 (0.39)	10.97 (3.57)	136.41 (10.88)	-	-	-	-	1.55 (1.39)	0.80 (0.86)	11.58 (3.28)	134.98 (10.28)
3	T1	8.52 (8.94)	0.00 (0.00)	2.95 (4.27)	26.05 (7.16)	20.27 (10.40)	0.00 (0.00)	11.52 (3.61)	5.73 (2.63)	15.71 (10.73)	0.51 (0.47)	10.30 (4.77)	11.00 (5.74)	24.86 (9.51)	0.80 (1.46)	10.13 (3.27)	1.73 (1.41)	-	-	-	-	24.89 (9.51)	2.72 (2.08)	8.08 (3.21)	1.83 (1.37)
	T2	3.67 (3.92)	0.00 (0.00)	2.20 (3.38)	20.83 (3.90)	7.06 (4.02)	0.00 (0.00)	10.42 (3.72)	9.22 (3.09)	6.85 (4.70)	0.33 (0.68)	8.33 (4.52)	11.19 (3.89)	10.18 (3.78)	0.92 (1.39)	11.81 (3.22)	3.79 (1.84)	-	-	-	-	10.36 (3.56)	2.71 (2.08)	9.86 (3.23)	3.77 (1.93)
	T3	5.83 (5.70)	0.00 (0.00)	3.38 (5.29)	49.12 (6.91)	7.18 (3.73)	0.01 (0.00)	18.01 (5.66)	33.13 (5.98)	9.08 (5.93)	0.40 (0.74)	16.53 (8.91)	32.32 (7.11)	9.87 (4.00)	0.98 (1.57)	27.24 (7.01)	20.24 (4.47)	-	-	-	-	9.84 (3.57)	4.27 (2.93)	23.99 (6.06)	20.23 (4.32)
	T4	6.25 (6.08)	0.00 (0.00)	3.81 (5.51)	91.29 (20.28)	6.26 (4.11)	0.00 (0.00)	15.31 (5.21)	79.78 (18.20)	4.63 (3.47)	0.28 (0.64)	11.80 (5.26)	84.64 (18.17)	2.29 (1.44)	0.38 (0.87)	17.89 (4.93)	80.79 (15.65)	-	-	-	-	2.43 (1.50)	1.49 (1.58)	16.84 (4.65)	80.59 (15.54)
4	T1	6.92 (4.01)	0.00 (0.00)	0.37 (1.01)	16.39 (4.34)	12.10 (5.49)	0.00 (0.00)	2.62 (2.64)	8.96 (3.29)	12.20 (5.34)	0.16 (0.47)	3.07 (2.27)	8.25 (3.27)	16.22 (5.79)	0.22 (0.60)	5.64 (2.30)	1.60 (1.23)	-	-	-	-	16.30 (5.63)	1.70 (1.52)	4.24 (2.07)	1.44 (1.09)
	T2	1.70 (1.29)	0.00 (0.00)	0.16 (0.56)	13.25 (2.67)	2.81 (1.92)	0.00 (0.00)	1.30 (1.46)	11.00 (2.81)	3.63 (2.07)	0.12 (0.36)	2.72 (2.15)	8.64 (2.60)	5.57 (2.74)	0.20 (0.79)	6.12 (2.43)	3.22 (1.82)	-	-	-	-	5.88 (2.57)	1.15 (1.10)	5.20 (2.08)	2.88 (1.61)
	T3	2.48 (1.94)	0.00 (0.00)	0.20 (0.64)	34.86 (3.84)	3.27 (2.04)	0.00 (0.00)	1.81 (1.93)	32.46 (4.15)	2.59 (2.59)	0.17 (0.37)	5.05 (3.51)	28.16 (3.33)	5.58 (2.71)	0.32 (0.89)	14.51 (4.33)	17.13 (3.38)	-	-	-	-	5.89 (2.59)	2.19 (2.06)	13.66 (3.69)	15.80 (3.46)
	T4	4.69 (2.63)	0.00 (0.00)	0.59 (1.38)	142.24 (13.96)	6.45 (3.27)	0.00 (0.00)	3.25 (2.88)	137.82 (14.20)	1.96 (1.72)	0.12 (0.36)	3.57 (2.69)	141.87 (14.24)	1.49 (1.31)	0.16 (0.51)	9.70 (3.75)	136.17 (13.62)	-	-	-	-	1.62 (1.25)	0.79 (1.15)	11.29 (4.14)	133.82 (13.85)
5	T1	3.11 (2.32)	0.00 (0.00)	14.43 (3.50)	9.06 (3.03)	3.75 (2.44)	0.00 (0.00)	15.09 (2.32)	7.76 (2.71)	9.23 (3.40)	1.66 (1.56)	12.82 (3.51)	2.89 (1.76)	12.07 (3.65)	0.91 (1.51)	12.09 (3.06)	1.54 (1.18)	-	-	-	-	12.74 (3.70)	0.99 (1.66)	11.56 (3.08)	1.31 (1.12)
	T2	1.51 (1.49)	0.00 (0.00)	14.26 (3.57)	14.10 (3.82)	1.92 (1.61)	0.00 (0.00)	15.32 (3.56)	12.63 (3.68)	4.75 (2.38)	1.59 (1.58)	17.86 (3.18)	5.67 (2.52)	8.74 (2.75)	1.11 (2.18)	16.43 (2.95)	3.59 (1.71)	-	-	-	-	9.09 (2.86)	1.12 (1.99)	16.33 (2.90)	3.33 (1.73)
	T3	1.78 (1.55)	0.00 (0.00)	26.83 (8.72)	43.42 (7.10)	2.27 (1.91)	0.00 (0.00)	29.07 (8.15)	40.69 (6.45)	3.80 (2.17)	1.70 (1.55)	40.91 (6.18)	25.62 (4.53)	10.54 (2.49)	1.49 (2.43)	39.68 (6.15)	20.32 (3.56)	-	-	-	-	9.58 (3.41)	1.42 (2.54)	41.32 (6.57)	19.71 (3.62)
	T4	1.70 (1.60)	0.00 (0.00)	22.77 (7.15)	70.12 (11.39)	2.41 (1.92)	0.00 (0.00)	24.41 (6.53)	67.77 (10.57)	0.62 (0.80)	0.31 (0.54)	18.05 (4.21)	75.61 (8.21)	2.13 (1.55)	0.36 (0.84)	21.10 (4.38)	71.00 (7.44)	-	-	-	-	1.47 (1.34)	0.36 (0.72)	21.52 (4.06)	71.24 (7.48)
6	T1	5.95 (2.48)	0.00 (0.00)	4.20 (2.38)	7.92 (3.05)	6.25 (2.63)	0.00 (0.00)	4.46 (2.32)	7.36 (2.83)	10.59 (2.59)	0.58 (1.07)	4.46 (1.98)	2.44 (1.51)	11.33 (2.92)	0.18 (0.61)	5.63 (1.98)	0.93 (0.96)	-	-	-	-	11.66 (2.90)	0.24 (0.67)	5.25 (1.96)	0.92 (1.02)
	T2	1.35 (1.17)	0.00 (0.00)	2.20 (1.83)	11.32 (2.44)	1.43 (1.33)	0.00 (0.00)	2.41 (3.59)	11.03 (2.41)	3.47 (1.99)	0.42 (0.82)	6.35 (2.56)	4.63 (2.03)	5.73 (1.99)	0.10 (0.41)	7.05 (2.12)	1.99 (1.48)	-	-	-	-	5.68 (1.97)	0.18 (0.50)	7.09 (2.21)	1.92 (1.48)
	T3	1.79 (1.42)	0.00 (0.00)	3.17 (2.49)	36.97 (3.34)	1.79 (1.47)	0.00 (0.00)	3.59 (3.25)	36.55 (3.25)	2.64 (1.65)	0.36 (0.63)	13.43 (4.51)	25.50 (3.68)	6.39 (2.71)	0.22 (0.71)	18.75 (4.63)	16.56 (3.40)	-	-	-	-	6.26 (2.62)	0.16 (0.42)	19.06 (4.49)	16.45 (3.40)
	T4	3.64 (1.42)	0.00 (0.00)	5.21 (1.39)	139.34 (3.34)	3.83 (1.47)	0.00 (0.00)	5.89 (3.25)	138.47 (3.25)	0.93 (1.65)	0.06 (0.63)	5.56 (4.51)	141.64 (3.68)	1.53 (2.71)	0.07 (0.71)	12.14 (4.63)	134.45 (3.40)	-	-	-	-	1.57 (2.62)	0.04 (0.42)	12.34 (4.49)	133.42 (3.40)

DGP	T	CLM				CLMM				ORDFOR				OMERF				OMERF_NO_RAN_SLOPE				OMERF_CLM			
		P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4
7		(2.20)	(0.00)	(3.03)	(8.40)	(2.11)	(0.00)	(3.16)	(8.33)	(0.98)	(0.24)	(2.47)	(8.48)	(1.37)	(0.29)	(3.72)	(8.41)	(1.22)	(0.20)	(3.69)	(8.20)	(1.27)	(0.91)	(3.74)	(8.36)
	T1	4.07	0.00	12.97	12.27	7.47	0.00	14.81	7.03	10.55	1.37	13.30	4.09	14.29	1.00	11.87	2.15	15.53	0.83	11.22	1.73	14.99	2.96	9.05	2.31
	T2	(3.42)	(0.00)	(4.00)	(4.20)	(4.94)	(0.00)	(3.89)	(3.01)	(4.90)	(1.24)	(3.46)	(2.16)	(4.93)	(1.65)	(3.18)	(1.50)	(4.88)	(1.54)	(3.54)	(1.18)	(4.51)	(1.79)	(3.15)	(1.48)
	T3	1.74	0.00	11.03	16.63	3.19	0.00	14.34	11.87	5.08	1.12	15.98	7.22	9.41	0.74	14.42	4.83	9.63	0.74	14.80	4.23	10.16	2.48	11.58	5.18
	T4	(1.72)	(0.00)	(3.66)	(4.25)	(2.39)	(0.00)	(3.36)	(3.47)	(2.65)	(1.39)	(3.73)	(2.88)	(3.41)	(1.38)	(3.50)	(2.06)	(3.34)	(1.49)	(3.33)	(1.98)	(3.22)	(1.86)	(3.05)	(2.16)
8		(2.34)	(0.00)	(7.88)	(6.65)	(2.46)	(0.00)	(7.58)	(5.90)	(2.45)	(1.35)	(7.26)	(5.54)	(4.09)	(2.01)	(5.84)	(4.09)	(3.81)	(1.87)	(6.73)	(4.25)	(4.03)	(2.55)	(5.39)	(4.24)
	T1	2.17	0.00	18.79	74.49	3.80	0.00	21.74	69.91	1.10	0.42	17.58	76.35	2.59	0.44	21.17	71.25	1.95	0.28	21.45	71.77	3.11	1.90	17.81	72.63
	T2	(1.89)	(0.00)	(6.50)	(13.17)	(2.50)	(0.00)	(5.93)	(11.90)	(1.13)	(0.70)	(3.97)	(9.91)	(1.54)	(0.83)	(4.64)	(9.54)	(1.25)	(0.67)	(4.39)	(9.42)	(1.71)	(1.50)	(4.39)	(8.69)
	T3	6.33	0.00	3.08	9.72	7.37	0.00	4.18	7.58	10.62	0.52	4.54	3.45	11.83	0.27	5.88	1.15	11.95	0.31	5.72	1.15	11.28	1.37	5.14	1.34
	T4	(3.00)	(0.00)	(2.04)	(2.94)	(3.41)	(0.00)	(2.38)	(2.67)	(3.58)	(0.85)	(2.24)	(1.81)	(3.38)	(0.76)	(2.20)	(0.98)	(3.40)	(0.88)	(2.22)	(0.99)	(3.44)	(1.39)	(2.36)	(1.11)
9		(1.51)	(0.00)	1.53	11.85	1.56	0.00	2.41	10.92	3.39	0.43	5.69	5.38	5.58	0.25	6.89	2.17	5.60	0.28	6.81	2.20	5.77	1.27	5.66	2.19
	T1	(1.31)	(0.00)	(1.48)	(2.26)	(1.37)	(0.00)	(1.50)	(2.21)	(1.79)	(0.79)	(2.41)	(2.24)	(2.07)	(0.69)	(2.15)	(1.36)	(2.15)	(0.81)	(2.17)	(1.29)	(2.17)	(1.35)	(1.98)	(1.40)
	T2	2.21	0.00	1.99	37.12	2.25	0.00	3.29	35.78	3.28	0.38	11.93	25.73	6.07	0.30	17.24	17.11	6.63	0.40	17.58	16.71	7.35	1.92	15.94	16.11
	T3	(1.50)	(0.00)	(2.01)	(3.43)	(1.55)	(0.00)	(2.22)	(3.64)	(1.87)	(0.75)	(4.40)	(3.53)	(2.69)	(0.80)	(4.06)	(3.50)	(2.70)	(0.83)	(4.30)	(3.35)	(2.63)	(1.78)	(4.05)	(3.49)
	T4	4.26	0.00	3.80	139.58	4.41	0.00	5.68	137.55	1.98	0.08	5.81	140.77	1.67	0.08	11.93	133.92	1.57	0.10	11.92	134.05	1.89	0.98	12.04	132.73
10		(2.62)	(0.00)	(3.02)	(9.83)	(2.57)	(0.00)	(3.26)	(9.58)	(1.06)	(0.27)	(2.79)	(9.50)	(1.36)	(0.31)	(3.70)	(9.59)	(1.39)	(0.39)	(3.53)	(9.63)	(1.50)	(1.15)	(3.79)	(9.16)
	T1	2.44	0.00	15.55	7.11	2.47	0.01	15.53	7.09	8.68	2.09	12.48	1.85	11.64	0.93	11.50	1.03	11.52	2.85	8.12	1.21	(3.03)	(2.06)	(2.36)	(1.05)
	T2	(2.03)	(0.00)	(2.95)	(2.70)	(2.01)	(0.00)	(2.89)	(2.70)	(3.07)	(1.81)	(2.66)	(1.38)	(3.43)	(1.83)	(3.02)	(0.94)	(2.62)	(1.16)	(1.83)	(0.94)	9.75	3.28	13.34	3.09
	T3	1.07	0.01	15.33	13.05	1.09	0.01	15.34	13.02	3.95	1.97	19.24	4.30	8.28	0.88	17.65	2.65	5.60	0.28	6.81	2.20	(2.17)	(1.35)	(1.98)	(1.40)
	T4	(1.16)	(0.10)	(3.65)	(3.22)	(1.22)	(0.10)	(3.73)	(3.26)	(2.08)	(1.83)	(3.12)	(1.77)	(2.36)	(2.07)	(3.02)	(1.58)	(2.63)	(2.67)	(2.85)	(1.85)	(1.46)	5.80	36.27	20.33
11		(1.52)	(0.00)	30.17	42.17	1.52	0.00	30.30	42.04	3.31	1.66	45.09	23.80	9.14	1.15	44.79	18.78	11.46	5.80	36.27	20.33	7.35	1.92	15.94	16.11
	T1	(1.38)	(0.00)	(5.97)	(4.99)	(1.37)	(0.00)	(5.96)	(4.90)	(1.73)	(1.49)	(6.72)	(4.87)	(3.46)	(2.35)	(6.97)	(4.18)	(2.70)	(0.83)	(4.30)	(3.35)	(2.63)	(1.78)	(4.05)	(3.49)
	T2	0.99	0.00	25.67	67.61	1.04	0.00	25.63	67.60	0.68	0.27	18.81	74.51	1.55	0.30	22.01	70.41	1.57	0.10	11.92	134.05	1.89	0.98	12.04	132.73
	T3	(1.06)	(0.00)	(5.60)	(8.44)	(1.10)	(0.00)	(5.43)	(8.24)	(0.84)	(0.57)	(4.17)	(5.99)	(1.33)	(0.75)	(4.62)	(6.02)	(1.60)	(1.28)	(4.46)	(5.80)	(1.52)	1.05	4.33	0.82
	T4	6.11	0.00	4.70	6.91	6.08	0.00	4.70	6.94	10.91	0.52	4.07	2.22	11.69	0.20	4.98	0.85	11.52	1.05	4.33	0.82	(2.62)	(1.16)	(1.83)	(0.94)
12		(2.63)	(0.00)	2.45	(2.67)	(2.58)	(0.00)	(2.32)	(2.65)	(2.60)	(0.76)	(2.02)	(1.69)	(2.79)	(0.67)	(1.96)	(0.99)	(2.62)	(1.16)	(1.83)	(0.94)	5.91	1.12	5.69	2.06
	T1	1.46	0.00	2.35	10.87	1.44	0.00	2.49	10.85	3.39	0.56	6.51	4.32	6.04	0.16	6.42	2.16	5.91	1.12	5.69	2.06	(2.11)	(1.39)	(2.13)	(1.29)
	T2	(1.31)	(0.00)	(1.71)	(2.16)	(1.28)	(0.00)	(1.72)	(2.16)	(1.89)	(0.86)	(2.05)	(1.89)	(1.95)	(0.58)	(2.01)	(1.25)	(2.11)	(1.39)	(2.13)	(1.29)	7.09	1.89	18.37	14.83
	T3	1.92	0.00	3.61	36.65	1.95	0.00	3.61	36.62	2.64	0.52	15.21	23.81	6.08	0.28	19.57	15.65	7.09	1.89	18.37	14.83	(2.73)	(1.82)	(4.10)	(3.45)
	T4	(1.32)	(0.00)	(2.37)	(3.49)	(1.27)	(0.00)	(2.46)	(3.50)	(1.73)	(0.85)	(4.25)	(3.76)	(2.44)	(0.89)	(4.05)	(3.45)	(2.73)	(1.82)	(4.10)	(3.45)	1.32	0.56	12.29	133.87
13		3.84	0.00	6.41	137.79	3.90	0.00	6.49	137.65	0.73	0.08	5.84	141.39	1.14	0.05	12.01	134.84	1.32	0.56	12.29	133.87	(1.13)	(0.77)	(3.68)	(7.72)
	T1	(2.04)	(0.00)	(3.53)	(8.00)	(2.01)	(0.00)	(3.59)	(8.05)	(0.76)	(0.27)	(2.71)	(8.23)	(1.13)	(0.26)	(3.32)	(8.02)	(1.13)	(0.77)	(3.68)	(7.72)	56.47	0.00	13.55	5.09
	T2	69.59	0.39	4.68	0.45	70.09	1.10	3.64	0.28	70.21	0.77	2.71	1.42	71.93	0.00	2.67	0.51	66.47	0.00	13.55	5.09	(6.77)	(0.00)	(3.87)	(2.57)
	T3	(5.94)	(1.03)	(2.41)	(0.74)	(5.93)	(1.42)	(2.05)	(0.55)	(5.92)	(0.98)	(1.90)	(1.16)	(6.49)	(0.00)	(2.23)	(0.66)	(6.77)	(0.00)	(3.87)	(2.57)	11.53	0.00	1.51	1.12
	T4	6.71	0.27	6.00	1.18	6.37	0.83	6.07	0.89	7.60	0.63	3.55	2.38	11.78	0.00	1.29	1.09	11.53	0.00	1.51	1.12	(2.27)	(0.00)	(1.27)	(1.04)
14		(1.97)	(0.79)	(1.94)	(1.08)	(1.99)	(1.16)	(1.65)	(0.94)	(2.26)	(0.82)	(1.92)	(1.52)	(2.20)	(0.00)	(1.12)	(1.06)	(2.27)	(0.00)	(1.27)	(1.04)	14.41	0.00	4.52	8.02
	T1	4.91	0.19	13.06	8.79	4.12	0.81	14.17	7.85	5.99	0.88	7.52	12.56	14.08	0.00	4.81	8.06	14.41	0.00	4.52	8.02	(2.26)	(0.00)	(1.44)	(1.34)
	T2	(2.23)	(0.56)	(3.71)	(2.53)	(2.07)	(1.05)	(3.99)	(2.71)	(2.51)	(0.95)	(3.16)	(3.18)	(3.39)	(0.00)	(2.33)	(2.64)	(3.45)	(0.00)	(2.23)	(2.49)	(2.26)	(0.00)	4.76	8.84
	T3	0.52	0.06	6.73	100.30	0.33	0.09	6.61	100.58	0.97	0.24	3.69	102.71	4.38	0.00	4.73	98.50	4.90	0.00	4.55	98.16	12.79	0.00	4.76	8.84
	T4	(0.78)	(0.28)	(2.60)	(7.46)	(0.57)	(0.29)	(2.76)	(7.48)	(0.97)	(0.47)	(2.06)	(7.13)	(2.02)	(0.00)	(2.24)	(7.18)	(2.15)	(0.00)	(2.43)	(7.33)	(3.15)	(0.00)	(2.12)	(2.71)
15		(69.82)	(0.00)	4.20	1.95	71.28	0.99	3.32	0.38	69.57	0.38	2.43	3.59	68.74	0.00	5.23	2.00	61.49	0.00	10.32	4.16	(7.78)	(0.00)	(3.80)	(2.09)
	T1	(7.91)	(0.00)	(2.18)	(1.71)	(7.64)	(1.53)	(1.96)	(0.62)	(8.22)	(0.79)	(1.77)	(2.35)	(8.57)	(0.00)	(2.91)	(1.81)	(7.78)	(0.00)	(3.80)	(2.09)	9.92	0.00	2.21	1.78
	T2	7.56	0.00	3.43	2.92	6.48	1.10	5.62	0.71	7.69	0.20	1.97	4.05	10.42	0.00	1.71	1.78	9.92	0.00	2.21	1.78	(2.26)	(0.00)	(1.44)	(1.34)
	T3	(2.27)	(0.00)	(2.13)	(1.90)	(1.73)	(1.53)	(1.87)	(0.81)	(2.08)	(0.51)	(1.53)	(1.85)	(2.19)	(0.00)	(1.30)	(1.36)	(2.26)	(0.00)	(1.44)	(1.34)	(2.26)	(0.00)	4.76	8.84
	T4	7.00	0.00	7.44	11.95	3.51	0.88	14.34	7.66	8.07	0.38	4.05	13.89	13.33	0.00	3.87	9.19	12.79	0.00	4.76	8.84	(3.15)	(0.00)	(2.12)	(2.71)
16		(2.41)	(0.00)	(3.65)	(2.86)	(1.80)	(1.39)	(3.31)	(2.28)	(2.44)	(0.74)	(2.41)	(2.99)	(3.34)	(0.00)	(2.34)	(2.79)	(3.15)	(0.00)	(2.12)	(2.71)	5.42	0.00	5.56	96.60
	T1	2.17	0.00	5.43	99.98	0.37	0.10	6.11	101.00	2.74	0.29	3.14	101.41	5.50	0.00	4.51	97.57	5.42	0.00	5.56	96.60	(2.31)	(0.00)	(2.33)	(9.23)
	T2	(1.58)	(0.00)	(2.59)	(8.91)	(0.63)	(0.33)	(2.47)	(8.94)	(1.82)	(0.56)	(2.19)	(9.02)	(2.47)	(0.00)	(2.41)	(9.14)	(2.31)	(0.00)	(2.33)	(9.23)				
	T3	69.82	0.00	4.20	1.95	71.28	0.99	3.32	0.38	69.57	0.38	2.43	3.59	68.74	0.00	5.23	2.00	61.49	0.00	10.32	4.16	(7.78)	(0.00)	(3.80)	(2.09)
	T4	(7.91)	(0.00)	(2.18)	(1.71)	(7.64)	(1.53)	(1.96)	(0.62)	(8.22)	(0.79)	(1.77)	(2.35)	(8.57)	(0.00)	(2.91)	(1.81)	(7.78)	(0.00)	(3.80)	(2.09)	9.92	0.00	2.21	1.78

S3 Additional information on the case studies data

S3.1 Case study 1

Table S9 reports a detailed description of the variables used in case study 1, along with corresponding descriptive statistics. All indicators variables are built by PISA by combining multiple responses to questionnaires in numerical indicators.

Variable name	Description	Type	Distribution
mate3	Output variable, mathematics test score	ordered factor	1:48% 2:44% 3:8%
gender	Student gender (0 = male, 1 = female)	factor	1:53% 0:47%
immig	Student immigration status (0 = native Italian; 1 = 1 st -generation immigrant; 2 = 2 nd -generation immigrant)	factor	0:89% 1:8% 2:3%
grade	School grade attended (10 = regular student; 9 = late enrolled student; 11 = early enrolled student)	factor	9:10% 10:85% 11:5%
video_games	Indicator of frequency of use at home of video or online games	numeric	3.27 ± 1.59 [1.00–6.00]
internet_quality	Indicator of quality of access to ICT at school	numeric	-0.18 ± 0.83 [-2.80–2.89]
internet_availability	Indicator of ICT availability outside of school	numeric	5.70 ± 0.91 [0.00–6.00]
SCHRISK	Indicator of perceived school safety risks	numeric	0.01 ± 0.82 [-0.46–3.05]
BULLIED	Indicator of being bullied	numeric	-0.47 ± 0.87 [-1.23–4.69]

Continued on next page

Variable name	Variable description	Variable type	Distribution
BELONG	Indicator of sense of belonging to school	numeric	0.01 ± 0.89 [-3.26–2.78]
COOPAGR	Indicator of cooperation (agreement)	numeric	0.12 ± 1.00 [-5.24–6.13]
TEACHSUP	Indicator of perceived teachers support	numeric	-0.20 ± 1.11 [-2.91–1.56]
FAMSUP	Indicator of perceived family support	numeric	-0.02 ± 0.93 [-3.01–1.96]
PERSEVAGR	Indicator of perseverance (agreement)	numeric	0.07 ± 0.98 [-5.91–4.89]
ASSERAGR	Indicator of assertiveness (agreement)	numeric	-0.03 ± 1.01 [-8.23–7.23]
EMPATAGR	Indicator of empathy (agreement)	numeric	0.01 ± 0.99 [-6.46–4.69]
EMOCOAGR	Indicator of emotional control (agreement)	numeric	-0.09 ± 0.98 [-5.17–5.58]
STRESAGR	Indicator of stress resistance (agreement)	numeric	-0.18 ± 1.00 [-5.26–5.49]
CURIOAGR	Indicator of curiosity (agreement)	numeric	0.09 ± 0.96 [-4.95–4.18]
study_time	Total time for all homework in all subjects per week	numeric	3.50 ± 1.51 [1.00–6.00]
HISCED	Highest level of education of parents	numeric	7.03 ± 2.14 [1.00–10.00]
ESCS	socio-economic family index	numeric	-0.01 ± 0.87 [-3.23–2.78]

Table S9: Student-level variables extracted from the OECD-PISA database. Numerical distributions are reported in terms of mean $\pm$ sd [min–max].

S3.2 Case study 2

Table S10 reports the selected ESS MD variables used for case study 2. For a complete description of the variables, please refer to the ESS codebook.

Variable	Type	Description	Distribution ¹
stflife	Ordinal	How satisfied with life as a whole	0: 25 (0.6%), 1: 10 (0.2%) 2: 70 (1.6%), 3: 142 (3.2%) 4: 232 (5.3%), 5: 328 (7.5%) 6: 336 (7.7%), 7: 798 (18%) 8: 1,264 (29%), 9: 770 (18%) 10: 411 (9.4%)
group	Factor	Country	28 levels
agea	Integer	Age	47.76 (16.63) 47.00 [35.00, 61.00]
gndr	Binary	Gender	1 (Male): 1937 (44%) 2 (Female): 2449 (56%)
health	Integer	Subjective general health	2.08 (0.84) 2.00 [1.00, 3.00]
netustm	Numeric	Internet use, how much time on typical day, in minutes	246.15 (193.94) 180.00 [105.00, 360.00]
hinctnta	Numeric	Household's total net income	6.15 (2.65) 6.00 [4.00, 8.00]
eisced	Integer	Level of education on a 1-7 scale	4.65 (1.70) 4.00 [3.00, 6.00]
mainact	Factor	Main activity in the last 7 days	1: Paid work 2,353 (54%) 2: Education 337 (7.7%) 3: Unemployed, looking for job 151 (3.4%) 4: Unemployed, not looking for job 108 (2.5%) 5: Permanently sick or disabled 149 (3.4%) 6: Retired 656 (15%) 7: Community or military service 3 (<0.1%) 8: Housework, looking after children 582 (13%) 9: Other 47 (1.1%)
ppltrst	Integer	Trust in people	5.50 (2.39) 6.00 [4.00, 7.00]
pplfair	Integer	Perceived fairness of people	5.92 (2.16) 6.00 [5.00, 8.00]
pplhlp	Integer	Perceived helpfulness of people	5.31 (2.15) 5.00 [4.00, 7.00]
trstplt	Integer	Trust in politicians	4.15 (2.30) 4.00 [2.00, 6.00]
trstlgl	Integer	Trust in the legal system	5.73 (2.50) 6.00 [4.00, 8.00]
rlgdgr	Integer	How religious are you	4.39 (3.03) 5.00 [2.00, 7.00]

¹n (%); Mean (SD); Median [Q1, Q3]

Table S10: Subject-level variables extracted from the ESS MD. A complete description of the survey can be found in the ESS codebook.