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## 7 Supplementary Material: Summary tables

We present here the summary tables of the different tests performed with the UNet-based emulators and for the two steps of the emulator evaluation.

Table SM.T1: Summary of the results obtained in the 1st step of evaluation (UPRCM world). The scores are the one used to summarize the maps on figures 6,7, for the different versions of the emulator tested in the paper. The first line (RCM truth) is the RCM absolute values (except for the first section). On each sections, the three first lines give the mean and the range of the ensemble of emulators described in Section 4.4. For the ensemble minimums and maximums the name ( E1 to E30 ) of the emulator corresponding to the value is given in brackets. “Main Emulator” is the one used for the results section (3). “Cheap” emulator is the one including only the variables at 700hpa described in Section 4.3. “Emul-Hist” is the emulator trained only on the historical period (section 4.2). “NO\_smoothing” and “TAS.included” correspond to the emulator trained with no prior smoothing of the data, and the one including TAS among the inputs (Section 4.3).

|               | RMSE       |           | ACC        |            | Variance ratio |           | Wasserstein distance |           |
|---------------|------------|-----------|------------|------------|----------------|-----------|----------------------|-----------|
|               | SQ_05      | MEAN      | SQ_05      | MEAN       | SQ_05          | MEAN      | SQ_05                | MEAN      |
| MAX           | 0.70 (E6)  | 0.86 (E6) | 0.93 (E11) | 0.97 (E11) | 0.95 (E9)      | 0.98 (E9) | 0.14 (E6)            | 0.24 (E6) |
| MEAN          | 0.67       | 0.82      | 0.92       | 0.96       | 0.93           | 0.96      | 0.08                 | 0.14      |
| MIN           | 0.64 (E19) | 0.8 (E11) | 0.91 (E13) | 0.96 (E6)  | 0.89 (E6)      | 0.92 (E6) | 0.04 (E9)            | 0.11 (E8) |
| Main Emulator | 0.67       | 0.83      | 0.92       | 0.96       | 0.93           | 0.95      | 0.08                 | 0.13      |
| UPRCM         | 0.43       | 2.10      | 0.79       | 0.95       | 0.68           | 0.92      | 0.14                 | 1.82      |
| Cheap         | 0.92       | 1.11      | 0.87       | 0.93       | 0.94           | 0.98      | 0.08                 | 0.15      |
| Emul-Hist     | 1.16       | 1.35      | 0.90       | 0.95       | 0.88           | 0.93      | 0.74                 | 0.87      |
| No.Smoothing  | 0.62       | 0.76      | 0.93       | 0.97       | 0.95           | 0.96      | 0.07                 | 0.13      |
| TAS.included  | 0.36       | 0.55      | 0.96       | 0.98       | 0.96           | 0.98      | 0.05                 | 0.08      |
|               |            |           |            |            |                |           |                      | 0.15      |

  

| PRESENT PERIOD 2006-2025 |             |            |            |             |                          |           |             |             |                 |
|--------------------------|-------------|------------|------------|-------------|--------------------------|-----------|-------------|-------------|-----------------|
| RCM truth values         | CLIMATOLOGY |            |            |             | Number of days over 30°C |           |             |             | 99th Percentile |
|                          | SQ_05       | MEAN       | SQ_05      | MEAN        | SQ_05                    | MEAN      | SQ_05       | MEAN        | SQ_95           |
| MAX                      | -2.166      | 9.879      | 16.518     | 0.000       | 3.356                    | 23.500    | 12.138      | 23.817      | 28.580          |
| MEAN                     | -0.06 (E7)  | 0.06 (E16) | 0.31 (E22) | -5.27 (E22) | -0.57 (E22)              | 1.72 (E9) | -0.61 (E22) | -0.19 (E22) | 0.32 (E9)       |
| MIN                      | 1.00 (E1)   | -0.30 (E1) | 0.07 (E4)  | -12.22 (E6) | -1.34 (E6)               | 0.32      | -0.87       | -0.38       | 0.07            |
| Main Emulator            | 1.00        | -0.08      | 0.03       | -10.06      | -1.68                    | 0.06      | -0.96       | -0.45       | 0.06            |
| UPRCM                    | 0.81        | -2.88      | 8.02       | -22.15      | -2.80                    | 0.08      | -4.22       | -0.32       | 7.4             |
| Cheap                    | 1.00        | -0.16      | 0.03       | -7.77       | -1.59                    | 0.09      | -0.84       | -0.30       | 0.19            |
| Emul-Hist                | 1.00        | -0.39      | -0.28      | -16.93      | -2.79                    | 0.02      | -1.66       | -0.98       | -0.49           |
| No.Smoothing             | 1.00        | -0.14      | -0.01      | -8.59       | -1.32                    | 0.04      | -0.77       | -0.31       | 0.14            |
| TAS.included             | 1.00        | -0.12      | -0.03      | -6.24       | -0.98                    | 0.07      | -0.65       | -0.25       | 0.07            |

  

| FUTURE PERIOD 2080-2100 |             |             |             |              |                          |            |             |             |                 |
|-------------------------|-------------|-------------|-------------|--------------|--------------------------|------------|-------------|-------------|-----------------|
| RCM truth values        | CLIMATOLOGY |             |             |              | Number of days over 30°C |            |             |             | 99th Percentile |
|                         | SQ_05       | MEAN        | SQ_05       | MEAN         | SQ_05                    | MEAN       | SQ_05       | MEAN        | SQ_95           |
| MAX                     | -0.19       | 11.53       | 17.89       | 0            | 16.80                    | 112.29     | 14.04       | 25.7        | 30.55           |
| MEAN                    | -0.16 (E7)  | 0 (E16)     | 0.18 (E22)  | -30.67 (E11) | -5.75 (E11)              | 0.36 (E22) | -0.89 (E22) | -0.35 (E22) | 0.17 (E22)      |
| MIN                     | 1.00 (E1)   | -0.39 (E13) | -0.14 (E12) | -61.91 (E6)  | -10.63 (E6)              | 0 (E30)    | -1.15       | -0.57       | -0.06           |
| Main Emulator           | 1.00        | -0.16       | -0.04       | -52.41       | -9.23                    | 0.01       | -1.44 (E2)  | -0.83 (E6)  | -0.24 (E6)      |
| UPRCM                   | 0.80        | -2.87       | 7.79        | -107.14      | -15.35                   | 1.66       | -1.25       | -0.63       | -0.03           |
| Cheap                   | 1.00        | -0.37       | 0.09        | -46.35       | -9.08                    | 0.01       | -4.42       | -0.34       | 7.29            |
| Emul-Hist               | 1.00        | -1.46       | -1.27       | -100.01      | -16.02                   | 0          | -1.45       | -0.73       | -0.11           |
| No.Smoothing            | 1.00        | -0.25       | 0.06        | -45.37       | -8.17                    | 0          | -3.74       | -2.48       | -1.67           |
| TAS.included            | 1.00        | -0.14       | 0.04        | -26.54       | -5.15                    | 0.06       | -1.04       | -0.58       | -0.15           |
|                         |             |             |             |              |                          |            | -0.72       | -0.33       | 0.01            |

  

| CLIMATE CHANGE 2080-2100 vs 2006-2025 |                       |            |           |              |                          |            |             |             |                 |
|---------------------------------------|-----------------------|------------|-----------|--------------|--------------------------|------------|-------------|-------------|-----------------|
| RCM truth values                      | Annual Mean Delta TAS |            |           |              | Number of days over 30°C |            |             |             | 99th Percentile |
|                                       | SQ_05                 | MEAN       | SQ_05     | MEAN         | SQ_05                    | MEAN       | SQ_05       | MEAN        | SQ_95           |
| MAX                                   | 1.34                  | 1.65       | 2.02      | -0.08        | 13.45                    | 90.14      | 1.01        | 1.88        | 2.79            |
| MEAN                                  | -0.16 (E28)           | -0.06 (E9) | 0.05 (E1) | -28.17 (E11) | -4.89 (E11)              | 0.31 (E22) | -0.37 (E11) | -0.06 (E11) | 0.48 (E11)      |
| MIN                                   | 0.95                  | -0.19      | 0.01      | -39.67       | -6.7                     | 0.1        | -0.73       | -0.19       | 0.31            |
| Main Emulator                         | 0.96                  | -0.18      | -0.07     | -44.58       | -7.56                    | 0.1        | -0.84 (E16) | -0.28 (E10) | 0.22 (E16)      |
| UPRCM                                 | 0.82                  | -0.3       | 0.15      | -87.4        | -12.56                   | 1.54       | -0.77       | -0.18       | 0.38            |
| Cheap                                 | 0.94                  | -0.3       | 0.07      | -41.43       | -7.47                    | 0.11       | -0.81       | -0.03       | 0.64            |
| Emul-Hist                             | 0.87                  | -1.23      | -0.99     | -84.12       | -13.23                   | 0.07       | -1.03       | -0.43       | 0.21            |
| No.Smoothing                          | 0.96                  | -0.25      | -0.11     | -38.92       | -6.86                    | 0.08       | -0.84       | -0.27       | -0.92           |
| TAS.included                          | 0.98                  | -0.09      | -0.01     | -23.52       | -4.18                    | 0.16       | -0.51       | -0.08       | 0.36            |

Table SM.T2: Summary of the results obtained in the 2nd step of evaluation (GCM world). The scores are the one used to summarize the maps on figures 6,7, for the different versions of the emulator tested in the paper. The first line (RCM truth) is the RCM absolute values (except for the first section). On each sections, the three first lines give the mean and the range of the ensemble of emulators described in Section 4.4. For the ensemble minimums and maximums the name ( E1 to E30 ) of the emulator corresponding to the value is given in brackets. “Main Emulator” is the one used for the results section (3). “Cheap” emulator is the one including only the variables at 700hpa described in Section 4.3. “Emul-Hist” is the emulator trained only on the historical period (Section 4.2). “NO\_smoothing” and “TAS\_included” correspond to the emulator trained with no prior smoothing of the data, and the one including TAS among the inputs (Section 4.3).

|               | RMSE        |             |             | ACC        |            |            | Variance ratio |             |             | Wasserstein distance |             |             |
|---------------|-------------|-------------|-------------|------------|------------|------------|----------------|-------------|-------------|----------------------|-------------|-------------|
|               | SQ 05       | Mean        | SQ95        | SQ 05      | Mean       | SQ95       | SQ 05          | Mean        | SQ95        | SQ 05                | Mean        | SQ95        |
| MAX           | 1.44 (Em30) | 2.27 (Em7)  | 2.85 (Em22) | 0.70 (Em8) | 0.81 (Em2) | 0.96 (Em8) | 0.96 (Em8)     | 1.01 (Em22) | 1.09 (Em26) | 0.29 (Em2)           | 0.82 (Em7)  | 1.23 (Em27) |
| MEAN          | 1.40        | 2.24        | 2.80        | 0.69       | 0.81       | 0.93       | 0.93           | 0.99        | 1.06        | 0.21                 | 0.75        | 1.11        |
| MIN           | 1.38 (Em14) | 2.20 (Em12) | 2.76 (Em4)  | 0.69 (Em7) | 0.81 (Em1) | 0.90 (Em6) | 0.90 (Em6)     | 0.96 (Em6)  | 1.03 (Em20) | 0.13 (Em12)          | 0.66 (Em12) | 1.00 (Em30) |
| Main Emulator | 1.46        | 2.25        | 2.83        | 0.69       | 0.81       | 0.95       | 0.95           | 1.00        | 1.07        | 0.29                 | 0.78        | 1.12        |
| UPRCM         | 1.76        | 3.34        | 10.17       | 0.61       | 0.78       | 0.73       | 0.73           | 0.97        | 1.28        | 0.24                 | 2.17        | 9.73        |
| Cheap         | 1.48        | 2.25        | 2.83        | 0.69       | 0.80       | 0.93       | 0.93           | 0.98        | 1.03        | 0.46                 | 0.84        | 1.14        |
| Emul-Hist     | 1.54        | 2.16        | 2.75        | 0.68       | 0.8        | 0.89       | 0.89           | 0.95        | 1.02        | 0.16                 | 0.28        | 0.55        |
| No.Smoothing  | 1.50        | 2.29        | 2.96        | 0.59       | 0.80       | 0.83       | 0.83           | 0.94        | 1.07        | 0.18                 | 0.53        | 0.92        |
| TAS_included  | 1.81        | 2.48        | 4.13        | 0.65       | 0.79       | 0.83       | 0.83           | 1.03        | 1.21        | 0.16                 | 0.84        | 2.93        |

| PRESENT PERIOD 2006-2025 |            |             |             |             |             |                          |             |              |             |              |             |             |
|--------------------------|------------|-------------|-------------|-------------|-------------|--------------------------|-------------|--------------|-------------|--------------|-------------|-------------|
| CLIMATOLOGY              |            |             |             |             |             | 99th Percentile          |             |              |             |              |             |             |
| Number of days over 30°C |            |             |             |             |             | Number of days over 30°C |             |              |             |              |             |             |
| RCM truth values         | -2.17      | 9.90        | 16.52       | 0.00        | 3.36        | 23.50                    | 12.14       | 23.82        | 28.58       |              |             |             |
| Emul vs RCM truth        |            |             |             |             |             |                          |             |              |             |              |             |             |
| MAX                      | SpaCor     | SQ 05       | Mean        | SQ95        | SpaCor      | SQ 05                    | Mean        | SQ95         | SpaCor      | SQ 05        | Mean        | SQ95        |
| MEAN                     | 1.00 (Em3) | 0.41 (Em19) | 1.00 (Em7)  | 1.41 (Em27) | 0.96 (Em1)  | -1.35 (Em22)             | 5.10 (Em22) | 34.22 (Em22) | 1.00 (Em7)  | 0.3 (Em22)   | 0.96 (Em22) | 1.63 (Em5)  |
| MIN                      | 1.00 (Em1) | 0.27        | 0.93        | 1.31        | 0.95        | -2.1                     | 3.08        | 23.78        | 1.00        | -0.01        | 0.73        | 1.34        |
| Main Emulator            | 1.00 (Em1) | 0.13 (Em27) | 0.83 (Em12) | 1.20 (Em12) | 0.94 (Em21) | -3.10 (Em21)             | 1.64 (Em6)  | 13.31 (Em20) | 1.00 (Em27) | -0.35 (Em27) | 0.50 (Em20) | 1.05 (Em20) |
| UPRCM                    | 1.00       | 0.43        | 0.96        | 1.29        | 0.95        | -1.61                    | 3.23        | 24.06        | 1.00        | 0.26         | 0.83        | 1.43        |
| Cheap                    | 0.70       | -3.16       | 1.19        | 9.99        | 0.16        | -22.53                   | -1.49       | 9.95         | 0.58        | -4.63        | 0.77        | 9.38        |
| Emul-Hist                | 1.00       | 0.61        | 1.05        | 1.40        | 0.93        | -4.775                   | 0.33        | 17.63        | 1.00        | 0.032        | 0.61        | 1.28        |
| No.Smoothing             | 1.00       | 0.10        | 0.66        | 0.98        | 0.86        | -10.49                   | -2.05       | 0.12         | 1.00        | -0.98        | -0.17       | 0.46        |
| TAS_included             | 1.00       | -0.05       | 0.68        | 1.20        | 0.73        | -5.26                    | 1.95        | 19.75        | 0.98        | -0.97        | 0.52        | 2.39        |
|                          | 0.97       | -1.38       | 0.55        | 3.12        | 0.61        | -5.22                    | 3.86        | 33.57        | 0.95        | -1.77        | 0.60        | 2.88        |

| FUTURE PERIOD 2080-2100  |             |             |            |             |            |                          |              |              |             |              |              |             |
|--------------------------|-------------|-------------|------------|-------------|------------|--------------------------|--------------|--------------|-------------|--------------|--------------|-------------|
| CLIMATOLOGY              |             |             |            |             |            | 99th Percentile          |              |              |             |              |              |             |
| Number of days over 30°C |             |             |            |             |            | Number of days over 30°C |              |              |             |              |              |             |
| RCM truth values         | -0.19       | 11.53       | 17.89      | 0.00        | 16.80      | 112.29                   | 14.04        | 25.70        | 30.55       |              |              |             |
| Emul vs RCM truth        |             |             |            |             |            |                          |              |              |             |              |              |             |
| MAX                      | SpaCor      | SQ 05       | Mean       | SQ95        | SpaCor     | SQ 05                    | Mean         | SQ95         | SpaCor      | SQ 05        | Mean         | SQ95        |
| MEAN                     | 1.00 (Em22) | 0.17 (Em11) | 0.73 (Em7) | 1.18 (Em16) | 0.97 (Em8) | -13.17 (Em22)            | 0.86 (Em22)  | 32.27 (Em22) | 1.00 (Em4)  | -0.33 (Em22) | 0.41 (Em22)  | 1.25 (Em5)  |
| MIN                      | 1.00 (Em1)  | 0.01        | 0.65       | 1.07        | 0.96       | -26.27                   | -3.05        | 12.458       | 1.00        | -0.70        | 0.14         | 0.99        |
| Main Emulator            | 1.00        | 0.16        | 0.69       | 1.11        | 0.96       | -39.84 (Em20)            | -6.69 (Em20) | 0.42 (Em20)  | 0.99 (Em27) | -1.00 (Em2)  | -0.14 (Em20) | 0.74 (Em20) |
| UPRCM                    | 0.70        | -3.28       | 1.00       | 9.59        | 0.07       | -109.27                  | -12.11       | 21.34        | 0.56        | -5.19        | 0.30         | 9.04        |
| Cheap                    | 1.00        | 0.3         | 0.70       | 1.03        | 0.93       | -40.66                   | -8.31        | 0.18         | 1.00        | -1.14        | -0.29        | 0.69        |
| Emul-Hist                | 1.00        | -0.96       | -0.50      | -0.10       | 0.76       | -99.23                   | -15.87       | 0.03         | 0.99        | -3.26        | -1.93        | -0.77       |
| No.Smoothing             | 0.997       | -0.64       | 0.22       | 0.77        | 0.54       | -86.63                   | -9.42        | 23.84        | 0.98        | -2.15        | -0.35        | 1.59        |
| TAS_included             | 0.97        | -1.43       | 0.39       | 2.86        | 0.59       | -63.43                   | 2.64         | 87.53        | 1.00        | -2.13        | 0.17         | 2.33        |

| CLIMATE CHANGE 2080-2100 vs 2006-2025 |             |              |              |              |            |                          |              |             |             |              |              |             |
|---------------------------------------|-------------|--------------|--------------|--------------|------------|--------------------------|--------------|-------------|-------------|--------------|--------------|-------------|
| Annual Mean Delta TAS                 |             |              |              |              |            | 99th Percentile          |              |             |             |              |              |             |
| Number of days over 30°C              |             |              |              |              |            | Number of days over 30°C |              |             |             |              |              |             |
| RCM truth values                      | 1.33        | 1.65         | 2.02         | -0.08        | 13.45      | 90.14                    | 1.01         | 1.88        | 2.79        |              |              |             |
| Emul vs RCM truth                     |             |              |              |              |            |                          |              |             |             |              |              |             |
| MAX                                   | SpaCor      | SQ 05        | Mean         | SQ95         | SpaCor     | SQ 05                    | Mean         | SQ95        | SpaCor      | SQ 05        | Mean         | SQ95        |
| MEAN                                  | 0.91 (Em28) | -0.39 (Em28) | -0.26 (Em28) | -0.13 (Em11) | 0.94 (Em8) | -28.15 (Em22)            | -4.24 (Em22) | 9.98 (Em22) | 0.76 (Em12) | -1.06 (Em11) | -0.51 (Em11) | 0.22 (Em4)  |
| MIN                                   | 0.89        | -0.44        | -0.28        | -0.155       | 0.917      | -37.72                   | -6.13        | 3.56        | 0.666       | -1.19        | -0.59        | 0.14        |
| Main Emulator                         | 0.87 (Em22) | -0.47 (Em6)  | -0.31 (Em6)  | -0.18 (Em10) | 0.88 (Em6) | -49.90 (Em6)             | -8.33 (Em20) | 0.27 (Em20) | 0.47 (Em13) | -1.38 (Em13) | -0.66 (Em30) | 0.02 (Em12) |
| UPRCM                                 | 0.89        | -0.41        | -0.27        | -0.14        | 0.91       | -37.94                   | -5.92        | 3.58        | 0.63        | -1.26        | -0.59        | 0.15        |
| Cheap                                 | 0.56        | -0.54        | -0.19        | 0.07         | 0.02       | -88.65                   | -10.62       | 13.54       | 0.04        | -1.36        | -0.47        | 0.48        |
| Emul-Hist                             | 0.91        | -0.49        | -0.34        | -0.20        | 0.91       | -47.58                   | -8.64        | 0.18        | 0.66        | -1.60        | -0.91        | -0.24       |
| No.Smoothing                          | 0.73        | -1.44        | -1.16        | -0.99        | -0.59      | -94.60                   | -13.83       | 0.11        | 0.36        | -2.59        | -1.77        | -0.96       |
| TAS_included                          | 0.73        | -0.76        | -0.46        | -0.27        | 0.37       | -80.50                   | -11.37       | 9.01        | 0.60        | -1.56        | -0.87        | -0.19       |
|                                       | 0.68        | -0.44        | -0.17        | 0.10         | 0.56       | -59.36                   | -1.22        | 56.57       | 0.29        | -1.31        | -0.43        | 0.43        |