

Towards operating windows for pendant drop methods: Tensiometry and rheometry of elastic interfaces Supplemental Information

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1 Theory

To describe a pendant drop, we assume that there exists an interfacial stress at the liquid-liquid or liquid-gas interface, which can be written as [1]:

$$\boldsymbol{\sigma}_s = \sigma_{\alpha\beta}(\Gamma, T)\mathbf{I}_s + \boldsymbol{\tau}, \quad (1)$$

where $\boldsymbol{\sigma}_s$ is the surface stress, $\sigma_{\alpha\beta}(\Gamma, T)$ is the isotropic surface tension, which can be dependent on excess concentration Γ and temperature T and $\mathbf{I}_s$ is the surface unit tensor defined by $\mathbf{I}_s = \mathbf{I} - \mathbf{n}\mathbf{n}$, where $\mathbf{I}$ is the unit tensor and $\mathbf{n}$ is the unit normal vector to the interface.

Interfaces which are described by Eq. (1) are referred to as *complex*. In the case of a constant, isotropic surface tension, we refer to the interface as being *simple*, and Eq. (1) reduces to

$$\boldsymbol{\sigma}_s = \sigma_{\alpha\beta}\mathbf{I}_s. \quad (2)$$

1.1 Differential geometry of axisymmetric pendant drop interfaces

In this section, we give a short introduction on the description of the pendant drop problem using differential geometry. For a full overview of differential geometry, we refer to reader to, e.g., [2, 3]. The shape of a pendant drop can be described in an cylindrical coordinate system, with z the axial coordinate, r the radial coordinate, and ϕ the circumferential coordinate (see Fig. 1). Additionally, we use the curve coordinate s , which describes the length along the interface in the r - z plane, ranging from $s = 0$ at the apex to $s = L$ at the neck of the drop. The drop, which is assumed to have an axisymmetric shape, is attached to a needle that fixes the radius of the drop neck (denoted by R_n).

We denote the unit vectors in axial direction, radial direction and circumferential direction by $\mathbf{e}_z$, $\mathbf{e}_r$ and $\mathbf{e}_\phi$, respectively. Moreover, we will denote the unit vector that is tangent to the interface and in the r - z plane, and pointing in positive s direction, by $\mathbf{e}_s$. Note, that $\mathbf{e}_s$ is perpendicular to $\mathbf{e}_\phi$, but not necessarily perpendicular to $\mathbf{e}_z$ and $\mathbf{e}_r$. The non-zero derivatives of the unit vectors with respect to the coordinates are

$$\frac{\partial \mathbf{e}_r}{\partial \phi} = \mathbf{e}_\phi, \quad \frac{\partial \mathbf{e}_\phi}{\partial \phi} = -\mathbf{e}_r. \quad (3)$$

We wish to describe the position of any point on the interface, $\mathbf{x}_s$, using a pair of local surface coordinates [4]. In this work we assume that there exists a unique mapping from ϕ and z to $\mathbf{x}_s$. Note, that this choice precludes shapes that have multiple values for a given z coordinate, e.g. “folded” shapes. By assuming axisymmetry, the mapping can be fully defined by the function $r = r_s(z)$, where r_s is a function that maps the z coordinate to a point on the interface. Thus, we select the cylindrical (ϕ, z) coordinates as the local surface coordinates, such that

$$\mathbf{x}_s(\phi, z) = r_s(z)\mathbf{e}_r + z\mathbf{e}_z, \quad (4)$$

where we note that the ϕ -dependence in the right hand side is in $\mathbf{e}_r$ and we will drop the explicit dependence of r_s on z for readability.

A pair of tangential vectors on the surface, called the natural basis vectors $\mathbf{t}_{(1)}$ and $\mathbf{t}_{(2)}$, can be derived by analyzing how the surface positions, $\mathbf{x}_s$, vary along each of the surface coordinate directions:

$$\mathbf{t}_{(1)} = \frac{\partial \mathbf{x}_s}{\partial \phi} = r_s \mathbf{e}_\phi, \quad \mathbf{t}_{(2)} = \frac{\partial \mathbf{x}_s}{\partial z} = \frac{dr_s}{dz} \mathbf{e}_r + \mathbf{e}_z, \quad (5)$$

where $\mathbf{t}_{(1)}$ and $\mathbf{t}_{(2)}$ are tangent to the ϕ (circumferential) and s (arc/meridional) coordinates, respectively, as shown in Fig. 1. Note that $\mathbf{t}_{(1)}$ and $\mathbf{t}_{(2)}$ are mutually orthogonal vectors of

non-unit length, such that $\mathbf{t}_{(1)} \cdot \mathbf{t}_{(2)} = 0$. If $\mathbf{v}$ is a vector that is tangential to the interface, it can be written in term of the natural basis vectors as $\mathbf{v} = \sum_{i=1}^2 v^{(i)} \mathbf{t}_{(i)}$, where $v^{(i)}$ are the *contravariant* components of $\mathbf{v}$.

Since the natural basis vectors are orthogonal, the metric tensor is diagonal and given by

$$g_{ij} = \mathbf{t}_{(i)} \cdot \mathbf{t}_{(j)} = \begin{bmatrix} r_s^2 & 0 \\ 0 & 1 + \left(\frac{dr_s}{dz}\right)^2 \end{bmatrix}. \quad (6)$$

And thus for the metric tensor inverse we find

$$g^{ij} = [g_{ij}]^{-1} = \begin{bmatrix} r_s^{-2} & 0 \\ 0 & \left[1 + \left(\frac{dr_s}{dz}\right)^2\right]^{-1} \end{bmatrix}. \quad (7)$$

To form a complete basis in three dimensions, we require a third vector which is mutually orthogonal to $\mathbf{t}_{(2)}$ and $\mathbf{t}_{(1)}$ [4]. We identify this third direction as the unit vector to the interface, which is defined as:

$$\mathbf{n} = \frac{\mathbf{t}_{(2)} \times \mathbf{t}_{(1)}}{\mathcal{G}} = \frac{r_s}{\mathcal{G}} \left(\mathbf{e}_r - \frac{dr_s}{dz} \mathbf{e}_z \right), \quad (8)$$

where $\mathcal{G}$ is given by

$$\mathcal{G} = |\mathbf{t}_{(2)} \times \mathbf{t}_{(1)}| = r_s \left[1 + \left(\frac{dr_s}{dz} \right)^2 \right]^{1/2}. \quad (9)$$

Note that $\mathbf{n}$ is invariant to the choice of curvilinear coordinates.

It is convenient to give explicit definitions of the tangential unit vectors:

$$\mathbf{e}_\phi = \frac{\mathbf{t}_{(1)}}{|\mathbf{t}_{(1)}|}, \quad \mathbf{e}_s = \frac{\mathbf{t}_{(2)}}{|\mathbf{t}_{(2)}|} = \frac{r_s}{\mathcal{G}} \left[\frac{dr_s}{dz} \mathbf{e}_r + \mathbf{e}_z \right]. \quad (10)$$

Moreover, we can find an expression for $\mathbf{e}_r$ in terms of $\mathbf{e}_s$ and $\mathbf{n}$ by combining Eqs. (10) and (8):

$$\mathbf{e}_r = \frac{r_s}{\mathcal{G}} \mathbf{n} + \frac{dr_s}{dz} \frac{r_s}{\mathcal{G}} \mathbf{e}_s. \quad (11)$$

Finally, the inverse metric tensor can be used to find the dual basis vectors:

$$\mathbf{t}^{(1)} = g^{11} \mathbf{t}_{(1)} = \frac{\mathbf{e}_\phi}{r_s}, \quad \mathbf{t}^{(2)} = g^{22} \mathbf{t}_{(2)} = \left(\frac{r_s}{\mathcal{G}} \right)^2 \left(\frac{dr_s}{dz} \mathbf{e}_r + \mathbf{e}_z \right). \quad (12)$$

If $\mathbf{v}$ is a vector that is tangential to the interface, it can be written in term of the dual basis vectors as $\mathbf{v} = \sum_{i=1}^2 v_{(i)} \mathbf{t}^{(i)}$, where $v_{(i)}$ are the *covariant* components of $\mathbf{v}$.

To describe the stress at pendant drop interfaces, we will need to apply surface differentiation operations. We define the surface gradient operator ∇_s , as [5]

$$\nabla_s = \mathbf{I}_s \cdot \nabla = \mathbf{t}^{(1)} \frac{\partial}{\partial \phi} + \mathbf{t}^{(2)} \frac{\partial}{\partial z}, \quad (13)$$

where ∇ is the three-dimensional gradient operator. Thus, the gradient of a vector $\mathbf{f}$ is defined as:

$$\nabla_s \mathbf{f} = \mathbf{t}^{(1)} \frac{\partial \mathbf{f}}{\partial \phi} + \mathbf{t}^{(2)} \frac{\partial \mathbf{f}}{\partial z}. \quad (14)$$

Similarly, the divergence of a tensor $\mathbf{A}$ is given by:

$$\nabla_s \cdot \mathbf{A} = \mathbf{t}^{(1)} \cdot \frac{\partial \mathbf{A}}{\partial \phi} + \mathbf{t}^{(2)} \cdot \frac{\partial \mathbf{A}}{\partial z}. \quad (15)$$

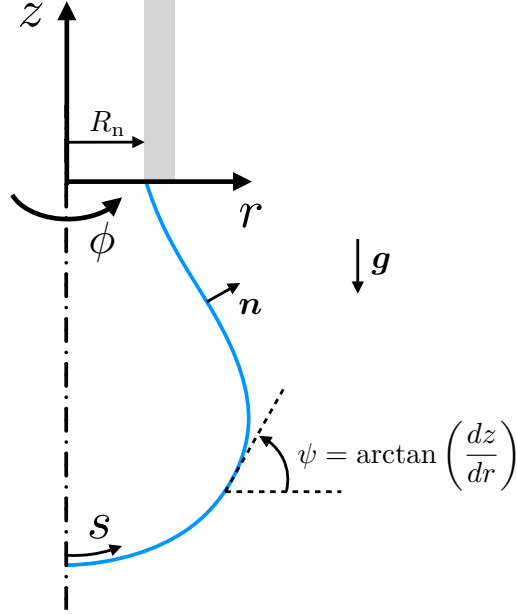


Figure 1: Geometric relations for a pendant drop.

1.2 Stress of an elastic interface

The stress at any viscoelastic fluid-fluid interface is given by [6]:

$$(\sigma_1 - \sigma_2) \cdot \mathbf{n} = \nabla_s \cdot \sigma_s, \quad (16)$$

where σ_1 and σ_2 are the Cauchy stress tensors for bulk phases 1 (inner phase) and 2 (outer phase), where $\mathbf{n}$ points towards 2. Pendant drop methods assume quasi-static conditions, such that the bulk flow fields can be neglected and the bulk stresses only contain a hydrostatic pressure term, reducing Eq. (16) to:

$$(\Delta P_0 - \Delta \rho g z) \mathbf{n} = -\nabla_s \cdot \sigma_s, \quad (17)$$

where ΔP_0 is the (positive) Laplace pressure jump at vertical location $z = 0$ ($\Delta P_0 = P_1(z = 0) - P_2(z = 0)$) and $\Delta \rho g z$ is the hydrostatic contribution, being g the magnitude of the gravitational acceleration vector, $\mathbf{g}$, and $\Delta \rho$ the density difference between phases 1 and 2, $\Delta \rho = \rho_1 - \rho_2$. Note that $\mathbf{g} \cdot z \mathbf{e}_z = -gz$ in Fig. 1, since $\mathbf{g}$ and $\mathbf{e}_z$ point in opposite directions.

It is convenient to diagonalize σ_s along its principal directions, such that σ_s contains only two unknown components. The principal directions at a given surface point $\mathbf{x}_s$ are the tangential $\mathbf{e}_s$ and $\mathbf{e}_\phi$ directions, such that σ_s can be expressed as:

$$\sigma_s = \sigma_s \mathbf{e}_s \mathbf{e}_s + \sigma_\phi \mathbf{e}_\phi \mathbf{e}_\phi. \quad (18)$$

It is straightforward to confirm that the principal axes are given by $\mathbf{e}_s$ and $\mathbf{e}_\phi$, since $\mathbf{e}_s$ is contained within the drop's symmetry plane and $\mathbf{e}_s \cdot \mathbf{e}_\phi = 0$.

Having diagonalized σ_s , we can now find an expression for its surface divergence, $\nabla_s \cdot \sigma_s$, by using Eq. (15) together with the expressions given in Section 1.1:

$$\nabla_s \cdot \sigma_s = -\frac{\sigma_\phi}{\mathcal{G}} \frac{dr_s}{dz} \mathbf{e}_s + \frac{\sigma_s}{\mathcal{G}} \frac{dr_s}{dz} \mathbf{e}_s + \frac{\partial \sigma_s}{\partial z} \frac{r_s}{\mathcal{G}} \mathbf{e}_s - \frac{\sigma_\phi}{\mathcal{G}} \mathbf{n} + \sigma_s \left(\frac{r_s}{\mathcal{G}} \right)^3 \frac{d^2 r_s}{dz^2} \mathbf{n}. \quad (19)$$

Having obtained an expression for $\nabla_s \cdot \boldsymbol{\sigma}_s$, we can now return to the stress balance equation. Eq. (17) is a vector equation; thus, to obtain its components along the normal and tangential directions, we take dot products with $\mathbf{n}$, $\mathbf{e}_s$, and $\mathbf{e}_\phi$, yielding:

$$\mathbf{n} : \quad (\Delta P_0 - \Delta \rho g z) = \frac{1}{\mathcal{G}} \sigma_\phi - \left(\frac{r_s}{\mathcal{G}} \right)^3 \frac{d^2 r_s}{dz^2} \sigma_s = \kappa_\phi \sigma_\phi + \kappa_s \sigma_s, \quad (20)$$

$$\mathbf{e}_s : \quad \frac{\partial \sigma_s}{\partial z} = \frac{\sigma_\phi - \sigma_s}{r_s} \frac{dr_s}{dz}, \quad (21)$$

$$\mathbf{e}_\phi : \quad 0 = 0, \quad (22)$$

where κ_s and κ_ϕ are the principal curvatures. Note that the dot products with $\mathbf{n}$ and $\mathbf{e}_s$ give the normal and tangential stress balance equations, while the dot product with $\mathbf{e}_\phi$ returns a trivial result. Moreover, note that in Eq. (21) the derivatives with respect to z on both sides can be converted to derivatives with respect to s by using the fact that $s = s(z)$ and applying the chain rule.

The expressions for the curvatures can alternatively be obtained by computing the curvature tensor, $\nabla_s \mathbf{n}$, which gives

$$\nabla_s \mathbf{n} = \frac{1}{\mathcal{G}} \mathbf{e}_\phi \mathbf{e}_\phi - \left(\frac{r_s}{\mathcal{G}} \right)^3 \frac{d^2 r_s}{dz^2} \mathbf{e}_s \mathbf{e}_s = \kappa_\phi \mathbf{e}_\phi \mathbf{e}_\phi + \kappa_s \mathbf{e}_s \mathbf{e}_s. \quad (23)$$

Note that $\nabla_s \mathbf{n}$ is a diagonal matrix, which makes κ_ϕ and κ_s the principal curvatures defined along the principal directions, $\mathbf{e}_\phi$ and the $\mathbf{e}_s$, respectively. The expressions for the curvatures are simplified considerably when described in terms of s and ψ , where ψ is the tangent angle formed between $\mathbf{e}_r$ and $\mathbf{e}_s$, as described in the following Sections.

1.3 Young-Laplace equation

The normal and tangential stress balance equations (Eqs. (20) and (21)) are applied to simple interfaces, whose stress is given by Eq. (2). The tangential stress balance returns a trivial result, whereas the normal stress balance yields the familiar Young-Laplace equation:

$$\Delta P_0 - \Delta \rho g z = (\kappa_\phi + \kappa_s) \sigma_{\alpha\beta}. \quad (24)$$

To simplify the computation of the curvature, the interface is described using the tangent angle ψ and curve coordinate s , as shown in Fig. 1 [7]. Note that s ranges from $s = 0$ at the apex to $s = L$ at the neck of the drop. In with follows, we use the notation r instead of r_s from the previous section, with the understanding that r now refers to the curve that describes the shape of the interface. The two coordinate systems are related through:

$$\frac{dr}{ds} = \cos(\psi), \quad \text{and} \quad \frac{dz}{ds} = \sin(\psi), \quad (25)$$

and the curvatures can be expressed in terms of their derivatives with respect to s , and are written as [7]

$$\kappa_\phi = \frac{1}{r} \frac{dz}{ds} = \frac{\sin(\psi)}{r}, \quad \text{and} \quad \kappa_s = \frac{d\psi}{ds}. \quad (26)$$

Substitution of Eq. (26) into Eq. (24) yields a first-order ordinary differential equation (with respect to s), which, together with the geometrical relations in Eq. (25), and proper boundary conditions, can be solved for r , ψ and z . The boundary conditions relevant for the pendant drop problem are [8]

$$r_{s=0} = 0, \quad \psi_{s=0} = 0, \quad z_{s=L} = 0. \quad (27)$$

The solution of Young-Laplace equation can be interpreted as starting at the axisymmetry axis ($r = 0$) and integrating the differential equations along s to find a shape. However, an “end-point” is not specified in this formulation, thus the integration can be continued until some arbitrary length. A more convenient way of solving the Young-Laplace equation is by imposing the volume of the drop.

1.3.1 Imposing droplet volume

The reference pressure ΔP_0 in Eq. (24) should be interpreted as an input parameter that, together with $\sigma_{\alpha\beta}$ and R_n completely describe the problem. Alternatively, we can use the volume of the droplet as an input parameter, in which case ΔP_0 becomes a Lagrange multiplier. To impose the volume, the following equation is added as a constraint [7]:

$$V = \int_0^L \pi r^2 \frac{dz}{ds} ds = \int_0^L \pi r^2 \sin(\psi) ds, \quad (28)$$

where V denotes the imposed volume. Note, that the domain length L is unknown with this approach, which will be tackled in the section on the numerical method.

1.4 Generalized Young-Laplace equation

For complex interfaces, the interfacial stress tensor is not necessarily isotropic, nor constant along the interface. For this case, a generalized version of the Young-Laplace equation can be obtained from separate normal and tangential stress balances from Eqs. (20) and (21), yielding: [7]

$$\Delta P_0 - \Delta \rho g z = (\kappa_\phi \sigma_\phi + \kappa_s \sigma_s), \quad (29)$$

$$r \frac{d\sigma_s}{ds} = \cos(\psi) (\sigma_\phi - \sigma_s). \quad (30)$$

Note that Eq. (30) is equivalent to the one provided in the main manuscript,

$$\sigma_\phi = \frac{d(\sigma_s r)}{dr}. \quad (31)$$

For a complex interface, a constitutive relation is needed to describe the interfacial stress σ_s . Here we assume a constant $\sigma_{\alpha\beta}$ and use the Neo-Hookean model proposed by Pepicelli et al. for the extra surface stress τ [9]. The principal stresses are then given by

$$\sigma_s = \sigma_{\alpha\beta} + \frac{K}{J} \ln J + \frac{G}{2} \left(\frac{1}{\lambda_\phi^2} - \frac{1}{\lambda_s^2} \right), \quad (32)$$

$$\sigma_\phi = \sigma_{\alpha\beta} + \frac{K}{J} \ln J - \frac{G}{2} \left(\frac{1}{\lambda_\phi^2} - \frac{1}{\lambda_s^2} \right), \quad (33)$$

where K and G are the interfacial bulk- and shear modulus, respectively, $J = \lambda_s \lambda_\phi$ is the relative change in area, and λ_s and λ_ϕ are the stretches in tangential- and circumferential direction, respectively.

The stretches λ_s and λ_ϕ are defined as

$$\lambda_\phi = \frac{r d\phi}{r^* d\phi} = \frac{r}{r^*}, \quad \text{and} \quad \lambda_s = \frac{ds}{ds^*}, \quad (34)$$

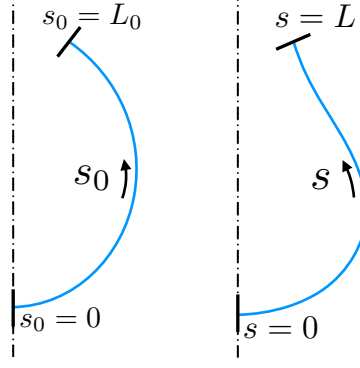


Figure 2: Curve coordinates for the simple problem: left is the “guessed” domain with curve coordinates s_0 and right is the actual domain with curve coordinate s . Note that when solving for the simple interface, L_0 is known (guessed) whereas L is unknown.

where the asterisk (*) is used to define the reference state for the elastic problem.

In addition to the boundary conditions used for the Young-Laplace equation (see Section 1.3), additional boundary conditions are required:

$$\left. \frac{d\sigma_s}{ds} \right|_{s=0} = 0, \quad \text{and} \quad \lambda_\phi|_{s=0} = \lambda_s|_{s=0} \quad (35)$$

where the first one is added to ensure a smooth solution at the apex, and the second one is added to avoid a singularity at the apex [7]. In contrast with [7], we also add the boundary condition

$$\sigma_\phi|_{s=0} = \sigma_s|_{s=0}, \quad (36)$$

which in our experience avoids a strong oscillation in the stress around $r = 0$.

2 Numerical method

In this section, the numerical procedure for solving the Young-Laplace equation for simple interfaces, as well as the generalized Young-Laplace equation for elastic interfaces is presented.

2.1 Simple interfaces

As mentioned in Section 1.3.1, the length of the 1D domain is unknown. This is tackled by introducing a “numerical” grid of length L_0 , where L_0 is a guess of the curve length (see Fig. 2). A linear mapping between the numerical grid and the actual grid is assumed: $s = s_0/C$ where $C = L_0/L$. For an arbitrary field on the interface we can write $f(s(s_0))$, and thus using the chain rule:

$$\frac{df}{ds_0} = \frac{df}{ds} \frac{ds}{ds_0} = \frac{1}{C} \frac{df}{ds} \rightarrow \frac{df}{ds} = C f', \quad (37)$$

where f' is the derivative on the numerical grid, i.e., $f' = df/ds_0$. Note that this adds C as an unknown to the system of equations. The equation that must be added to close the system is a boundary condition that imposed the needle radius:

$$r_{s=L} = R_n, \quad (38)$$

where R_n is the imposed radius of the needle.

The derivatives and integrals will be approximated using a spectral method [10]. Without going into detail, the 1D function will be represented at N nodal points at specific locations on the 1D domain (i.e., the Chebyshev points). The derivatives are then approximated by¹

$$\frac{df}{ds} = CDf, \quad (39)$$

where $df/ds \in \mathbb{R}^{N \times 1}$ are the nodal values of df/ds , $f \in \mathbb{R}^{N \times 1}$ are the nodal values of f and $D \in \mathbb{R}^{N \times N}$ is a differentiation matrix, defined on the numerical grid (i.e., it differentiates with respect to s_0). Integration is performed using

$$\int_0^L f ds \approx w^T f / C, \quad (40)$$

where $w \in \mathbb{R}^{N \times 1}$ is a discrete integration operator defined on the numerical grid (i.e., it integrates over s_0),

For simple interfaces, the equations to be solved are the Young-Laplace equation (Eq. (24)), the two geometrical relations in Eq. (25), the imposed volume Eq. (28) and the boundary condition for the needle radius (Eq. (38)). The unknowns of the problem are r , z and ψ , and we have the two additional unknowns C and ΔP_0 . After discretization using Eq. (39) and Eq. (40), the complete solution vector will be of length $3 \times N + 2$. The resulting non-linear system is solved using a Newton-Raphson iteration (a symbolic calculation of the Jacobian matrix is present in the `Tensiomet2` code).

2.2 Elastic interfaces

Elastic interfaces are solved by first solving the reference state assuming a simple interface (i.e., $\tau = \mathbf{0}$ in Eq. (1)). The deformed drops are all computed with respect to this reference state. We use an asterisk (*) to denote the reference state (e.g., s as it was defined for the simple interface problem becomes s^* when computing the elastic problem), see Fig. 3. Note, that this fixes the constant C (with $s^* = s_0/C$) when computing the elastic interface. However, since the domain changes length again for the elastic problem, an additional coordinate transformation is required.

Using the definition $\lambda_s = ds/ds^*$ (Eq. (34)), the derivatives on the current (elastic) domain become [7]:

$$\frac{df}{ds} = \frac{ds^*}{ds} \frac{df}{ds^*} = \frac{1}{\lambda_s} \frac{df}{ds^*} = \frac{1}{\lambda_s} C \frac{df}{ds_0} = \frac{C}{\lambda_s} f', \quad (41)$$

where f' again denotes the derivative on the numerical grid (i.e., the grid that was “guessed” for the simple interface). Due to the linearity of matrix-vector-multiplication, we can compute this by $(CDf) \odot \lambda_s^{-1}$, where $\odot$ is used to indicate component-wise multiplication and $\lambda_s^{-1} \in \mathbb{R}^{N \times 1}$ is the column vector contain the nodal values of λ_s^{-1} . Note, that integration also changes according to

$$\int_0^L f ds = \int_0^{L^*} f \lambda_s ds^* \approx (w \odot \lambda_s)^T f / C. \quad (42)$$

Finally, the new curve coordinate s can be computed according to

$$s = \int_0^{s^*} \lambda_s ds^* \approx W \lambda_s / C, \quad (43)$$

¹We will assume that the meaning of expressions involving row/column vectors and matrices is clear from the context, and they are therefore denoted using regular symbols (preferably lower case for row/column vectors, and upper case for matrices).

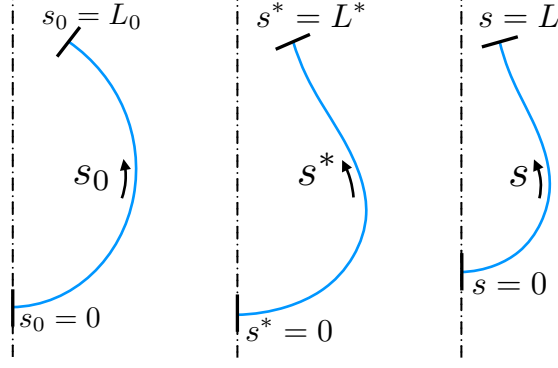


Figure 3: Curve coordinates for the elastic problem: left is the “guessed” domain with curve coordinates s_0 , middle is the solution of the simple interface with curve coordinate s^* and right is the current domain with curve coordinate s . Note that when solving for the elastic interface, both L_0 and L are known (and thus $C = L_0/L$ is also known).

where $W \in \mathbb{R}^{N \times N}$ is the lower triangular matrix generated using w (each row of W is equal to w^T , but with a lower triangular “mask” imposed). The result of $W\lambda_s/C$ is a column vector that contains the nodal values of s .

The equations solved for the elastic problem are the generalized Young-Laplace equations (Eqs. (29) and (30)), the constitutive relations (Eqs. (32) and (33)), the two geometrical relations in Eq. (25) (similar to the Young-Laplace equation for simple interfaces, but with the derivative replaced by Eq. (41)) and the integral for the volume Eq. (28)). The final equation is the expression for λ_ϕ in Eq. (34) (note that the expression for λ_s is not needed since the stretch is determined through the constitutive relations). If we discretize the unknowns $r, z, \psi, \sigma_s, \sigma_\phi, \lambda_s, \lambda_\phi$ using N unknowns, and we have one additional unknown ΔP_0 (note that C is not included as compared to the Young-Laplace equation for simple interfaces), the resulting system will be of size $7 \times N + 1$. This non-linear system is solved using a Newton-Raphson iteration (a symbolic calculation of the Jacobian matrix is present in the **Tensiomet2** code).

3 Results

We present selected results for parameter combinations of Wo , Ar , $\sigma_{\alpha\beta}$, and $\Delta A/A_0$ not shown in the main manuscript.

3.1 Forward problem

3.1.1 Drop profiles

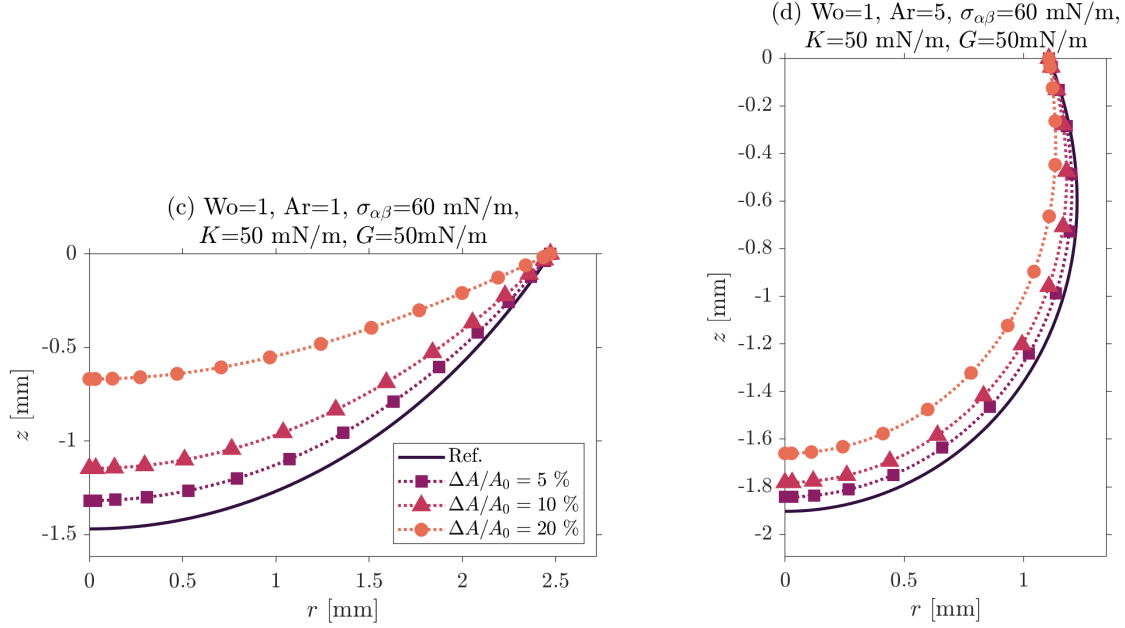


Figure 4: Reference and strained half-drop geometries for: (a) $Wo = 0.1$, $Ar = 1$; and (b) $Wo = 0.1$, $Ar = 5$. Profiles correspond to interfaces with $\sigma_{\alpha\beta} = 60$ mN/m, $K = 50$ mN/m, and $G = 50$ mN/m. The axisymmetric drop contour – cast in dimensional cylindrical coordinates to facilitate comparison with real experimental systems – is shown in black. The dotted lines represent the strained configurations at different total area strain magnitudes corresponding to $\Delta A/A_0 = 5$ (squares), 10 (triangles), and 20 (circles) %.

3.1.2 Strain and stress profiles for sample geometries.

Strain and stress profiles for different drop geometries, following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$, and the isotropic surface tension, $\sigma_{\alpha\beta}$. In each plot, the dimensionless arc length $\hat{s} = 0$ corresponds to the drop apex and $\hat{s} = 1$ is the capillary tip. Images of the unstrained configurations are displayed in the insets.

Table 1: Summary of parameters shown in Figs. 5 – 9. Results for $\Delta A/A_0 = 10\%$ and $\sigma_{\alpha\beta} = 60$ mN/m are shown in the main manuscript.

Figure number	$\Delta A/A_0$ [%]	$\sigma_{\alpha\beta}$ [mN/m]
Fig. 5	5	20
Fig. 6	10	20
Fig. 7	20	20
Fig. 8	5	60
Fig. 9	20	60

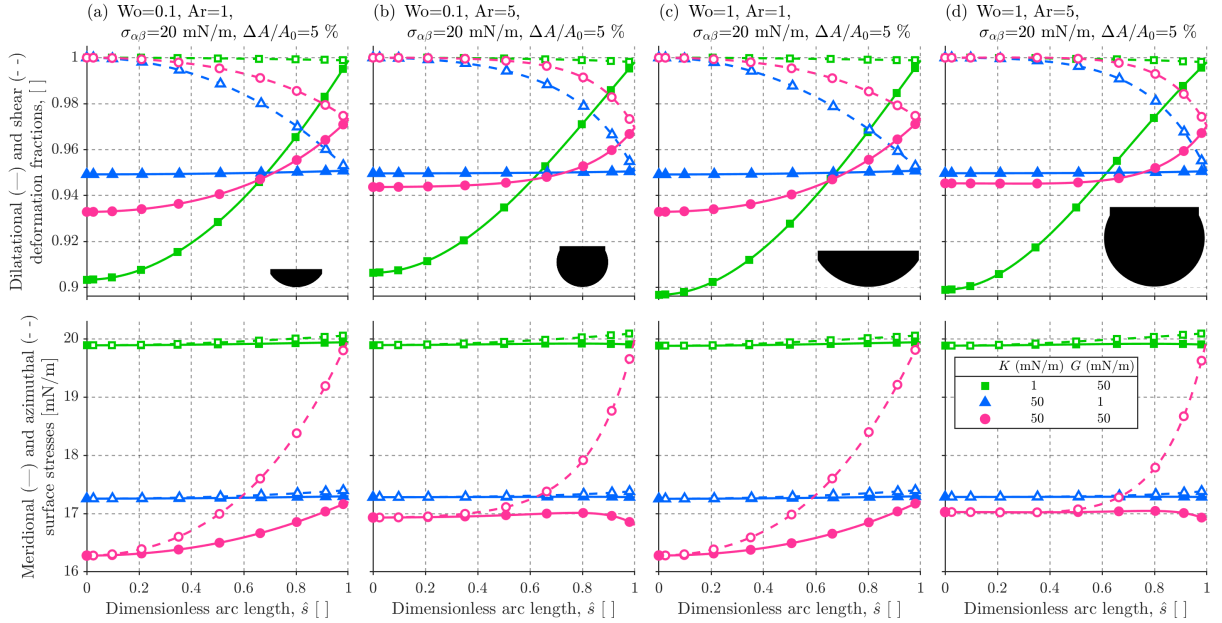


Figure 5: Strain (top) and stress (bottom) profiles for $\Delta A/A_0 = 5\%$ and $\sigma_{\alpha\beta} = 20$ mN/m. **Top:** Dilatational ($\lambda_s\lambda_\phi$, solid lines) and shear (λ_s/λ_ϕ , dotted lines) deformation fractions, illustrating the presence of mixed and/or inhomogeneous deformations in all cases. **Bottom:** Meridional (σ_s , solid lines) and azimuthal (σ_ϕ , dotted lines) stresses.

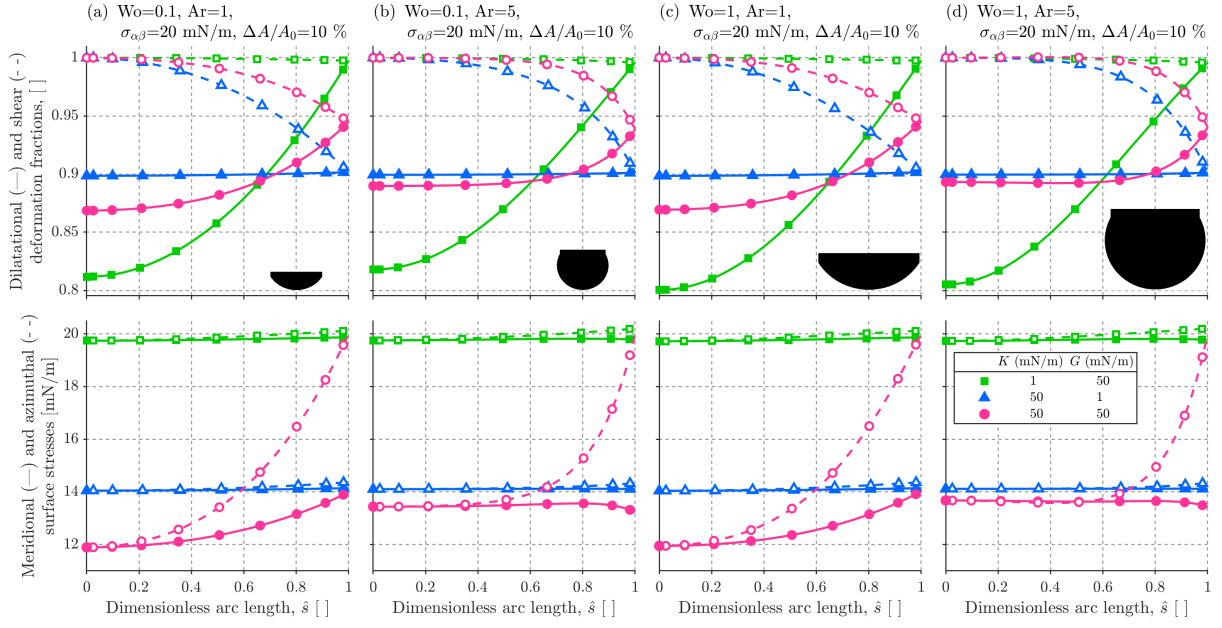


Figure 6: Strain (top) and stress (bottom) profiles for $\Delta A/A_0 = 10\%$ and $\sigma_{\alpha\beta} = 20$ mN/m. **Top:** Dilatational ($\lambda_s\lambda_\phi$, solid lines) and shear (λ_s/λ_ϕ , dotted lines) deformation fractions, illustrating the presence of mixed and/or inhomogeneous deformations in all cases. **Bottom:** Meridional (σ_s , solid lines) and azimuthal (σ_ϕ , dotted lines) stresses.

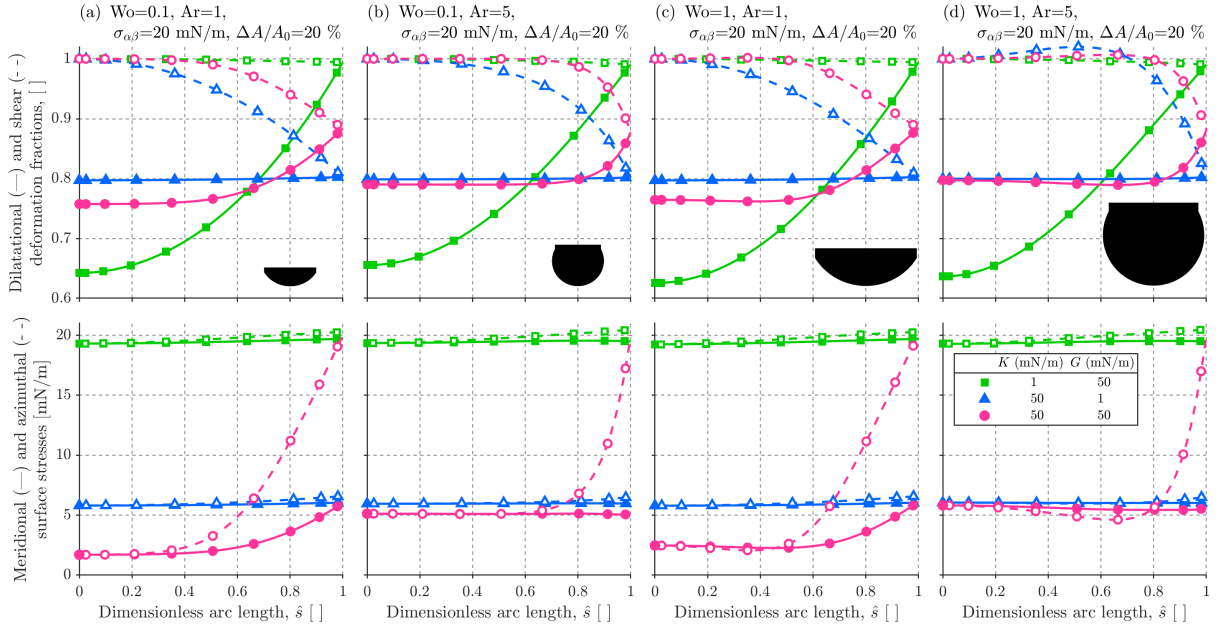


Figure 7: Strain (top) and stress (bottom) profiles for $\Delta A/A_0 = 20\%$ and $\sigma_{\alpha\beta} = 20$ mN/m. **Top:** Dilatational ($\lambda_s\lambda_\phi$, solid lines) and shear (λ_s/λ_ϕ , dotted lines) deformation fractions, illustrating the presence of mixed and/or inhomogeneous deformations in all cases. **Bottom:** Meridional (σ_s , solid lines) and azimuthal (σ_ϕ , dotted lines) stresses.

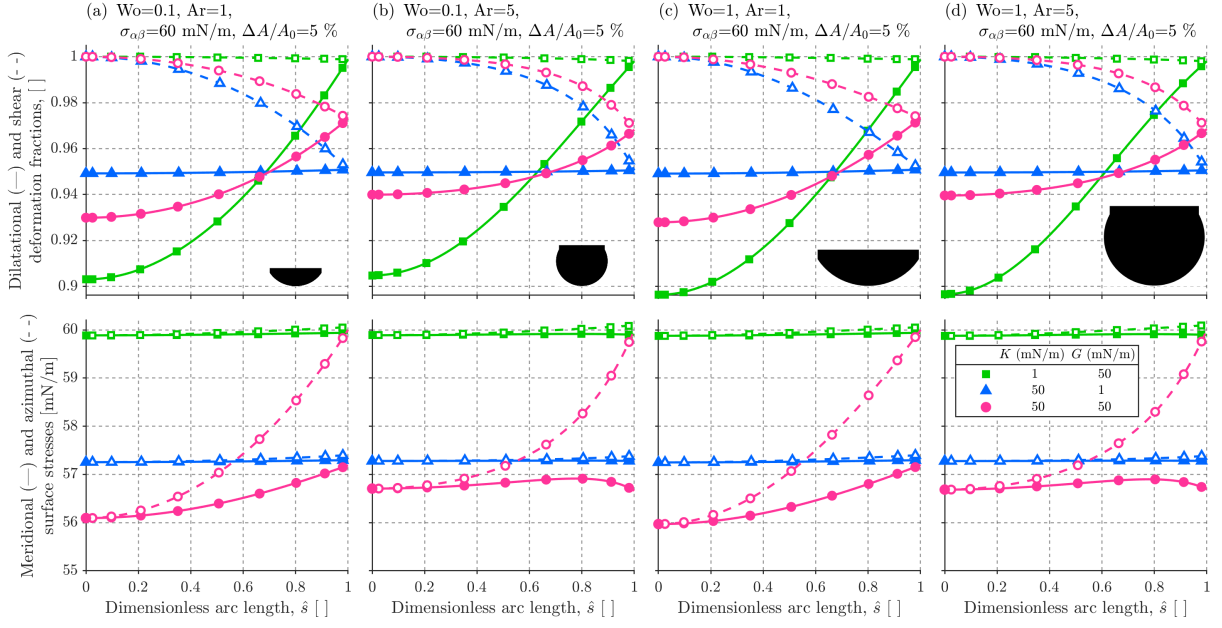


Figure 8: Strain (top) and stress (bottom) profiles for $\Delta A/A_0 = 5\%$ and $\sigma_{\alpha\beta} = 60$ mN/m. **Top:** Dilatational ($\lambda_s\lambda_\phi$, solid lines) and shear (λ_s/λ_ϕ , dotted lines) deformation fractions, illustrating the presence of mixed and/or inhomogeneous deformations in all cases. **Bottom:** Meridional (σ_s , solid lines) and azimuthal (σ_ϕ , dotted lines) stresses.

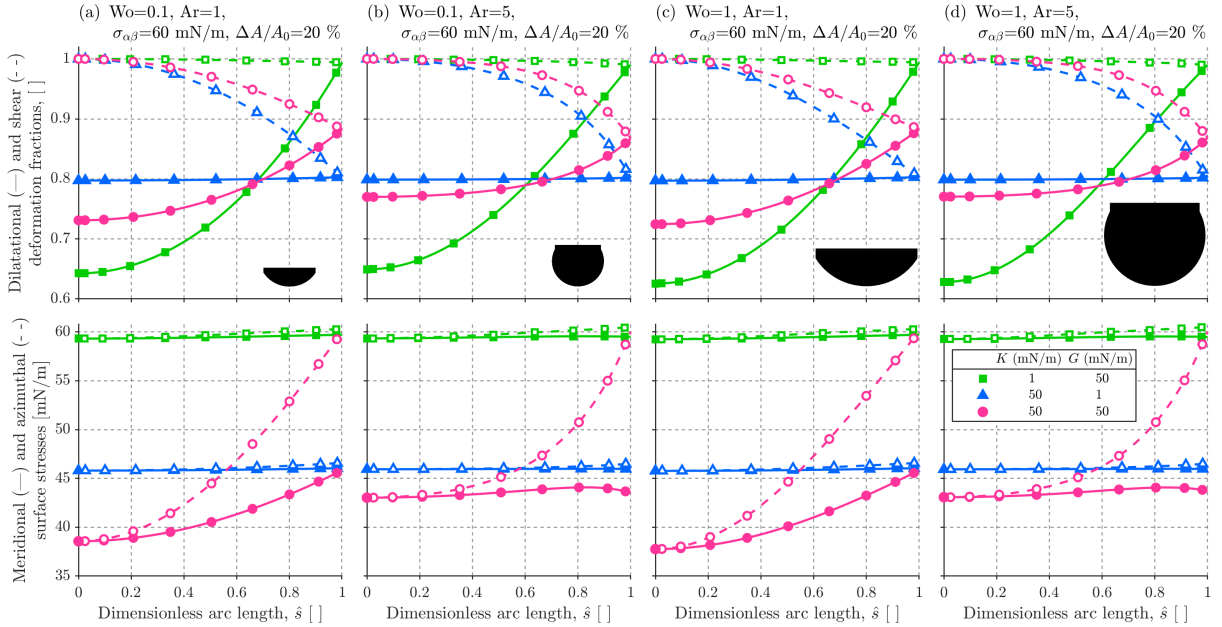


Figure 9: Strain (top) and stress (bottom) profiles for $\Delta A/A_0 = 20\%$ and $\sigma_{\alpha\beta} = 60$ mN/m. **Top:** Dilatational ($\lambda_s\lambda_\phi$, solid lines) and shear (λ_s/λ_ϕ , dotted lines) deformation fractions, illustrating the presence of mixed and/or inhomogeneous deformations in all cases. **Bottom:** Meridional (σ_s , solid lines) and azimuthal (σ_ϕ , dotted lines) stresses.

3.1.3 Dilatational Anisotropy Parameter (DAP).

Dilatational Anisotropy Parameter (DAP), following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$. Results are shown for low $\sigma_{\alpha\beta}$ ($\sigma_{\alpha\beta} = 20$ mN/m) in the top, and for high $\sigma_{\alpha\beta}$ ($\sigma_{\alpha\beta} = 60$ mN/m) in the bottom.

Table 2: Summary of parameters shown in Figs. 10 – 11. Results for $\Delta A/A_0 = 10\%$ are shown in the main manuscript.

Figure number	$\Delta A/A_0$ [%]
Fig. 10	5
Fig. 11	20

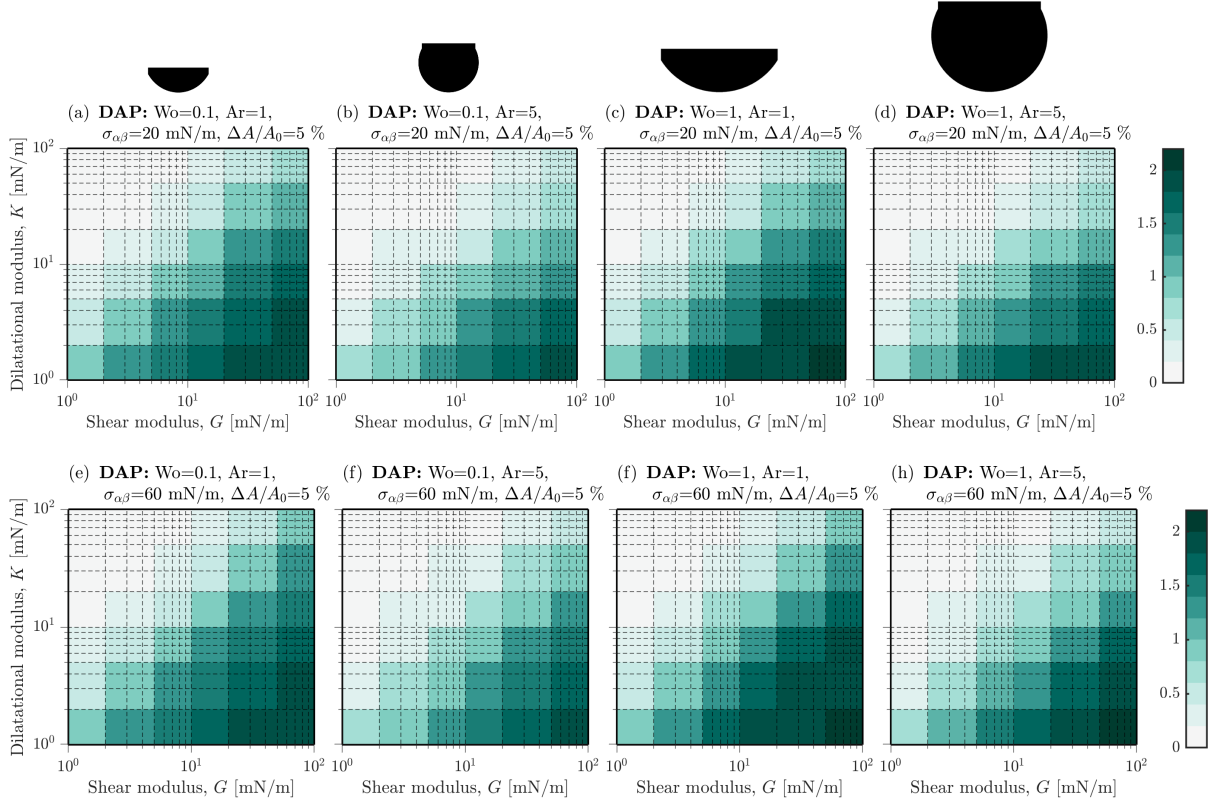


Figure 10: Dilatational anisotropy parameter (DAP) for strained drops at $\Delta A/A_0 = 5\%$, with $\sigma_{\alpha\beta} = 20$ mN/m (top, a–d) and $\sigma_{\alpha\beta} = 60$ mN/m (bottom, e–h). Results are shown for: (a,e) $Wo = 0.1$, $Ar = 1$; (b,f) $Wo = 0.1$, $Ar = 5$; (c,g) $Wo = 1$, $Ar = 1$; (d,h) $Wo = 1$, $Ar = 5$. A high K/G ratio yields a homogeneous dilatational deformation (i.e., low DAP). The magnitudes of $\sigma_{\alpha\beta}$, Wo , and Ar have a secondary effect.

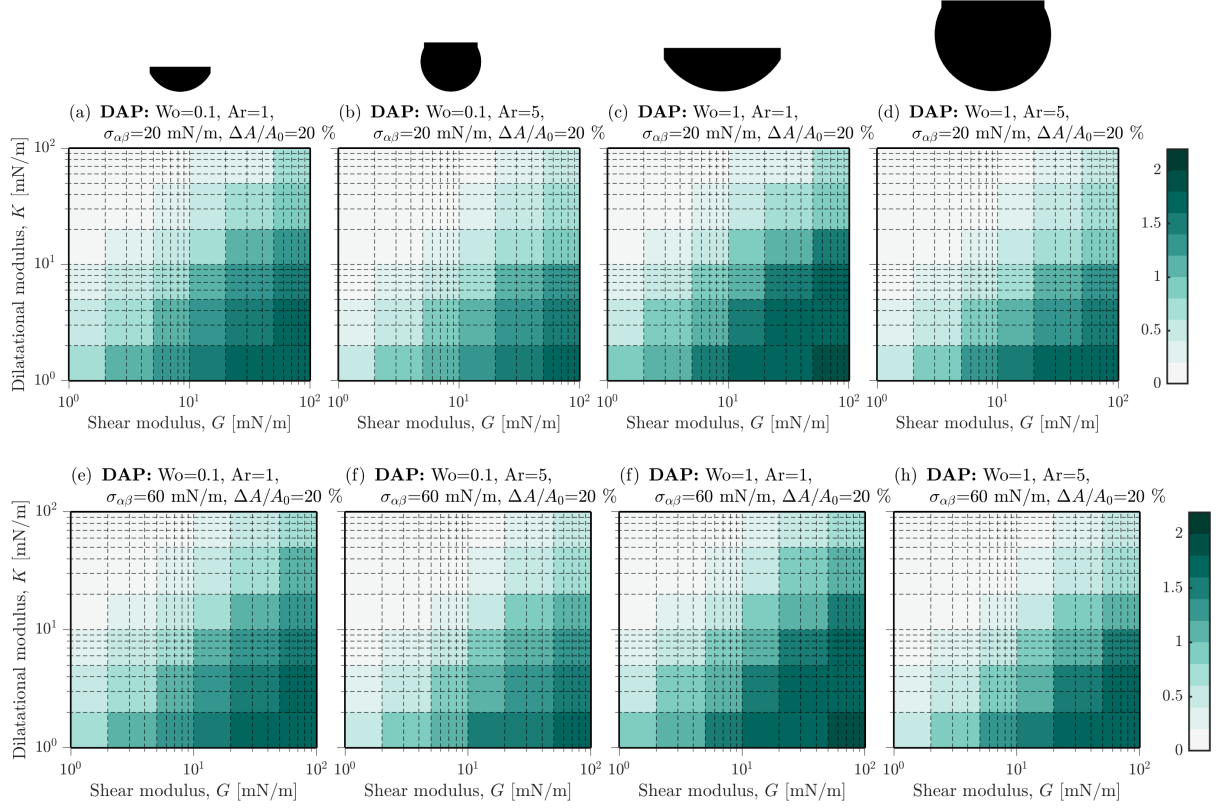


Figure 11: Dilatational anisotropy parameter (DAP) for strained drops at $\Delta A/A_0 = 5$ %, with $\sigma_{\alpha\beta} = 20$ mN/m (top, a–d) and $\sigma_{\alpha\beta} = 60$ mN/m (bottom, e–h). Results are shown for: (a,e) $Wo = 0.1$, $Ar = 1$; (b,f) $Wo = 0.1$, $Ar = 5$; (c,g) $Wo = 1$, $Ar = 1$; (d,h) $Wo = 1$, $Ar = 5$. A high K/G ratio yields a homogeneous dilatational deformation (i.e., low DAP). The magnitudes of $\sigma_{\alpha\beta}$, Wo , and Ar have a secondary effect.

3.1.4 Stress Anisotropy Parameter (SAP).

Stress Anisotropy Parameter (SAP), following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$. Results are shown for low $\sigma_{\alpha\beta}$ ($\sigma_{\alpha\beta} = 20$ mN/m) in the top, and for high $\sigma_{\alpha\beta}$ ($\sigma_{\alpha\beta} = 60$ mN/m) in the bottom.

Table 3: Summary of parameters shown in Figs. 12 – 13. Results for $\Delta A/A_0 = 10\%$ are shown in the main manuscript.

Figure number	$\Delta A/A_0$ [%]
Fig. 12	5
Fig. 13	20

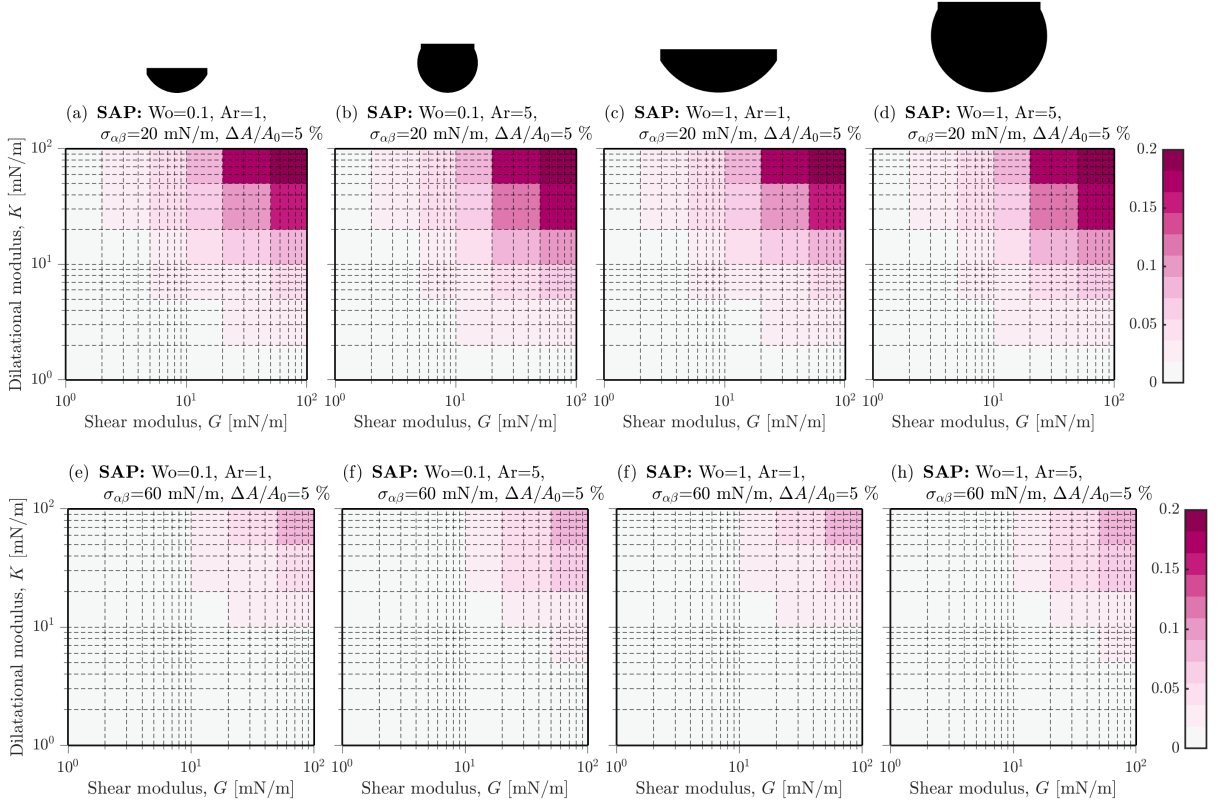


Figure 12: Stress anisotropy parameter (SAP) for strained drops at $\Delta A/A_0 = 5\%$, with $\sigma_{\alpha\beta} = 20$ mN/m (top, a–d) and $\sigma_{\alpha\beta} = 60$ mN/m (bottom, e–h). Results are shown for: (a,e) $Wo = 0.1$, $Ar = 1$; (b,f) $Wo = 0.1$, $Ar = 5$; (c,g) $Wo = 1$, $Ar = 1$; (d,h) $Wo = 1$, $Ar = 5$. High $\sigma_{\alpha\beta}/K$ and $\sigma_{\alpha\beta}/G$ ratios give a more homogeneous stress distribution along the drop interface (i.e., low SAP), while the magnitudes of Wo and Ar have a secondary effect.

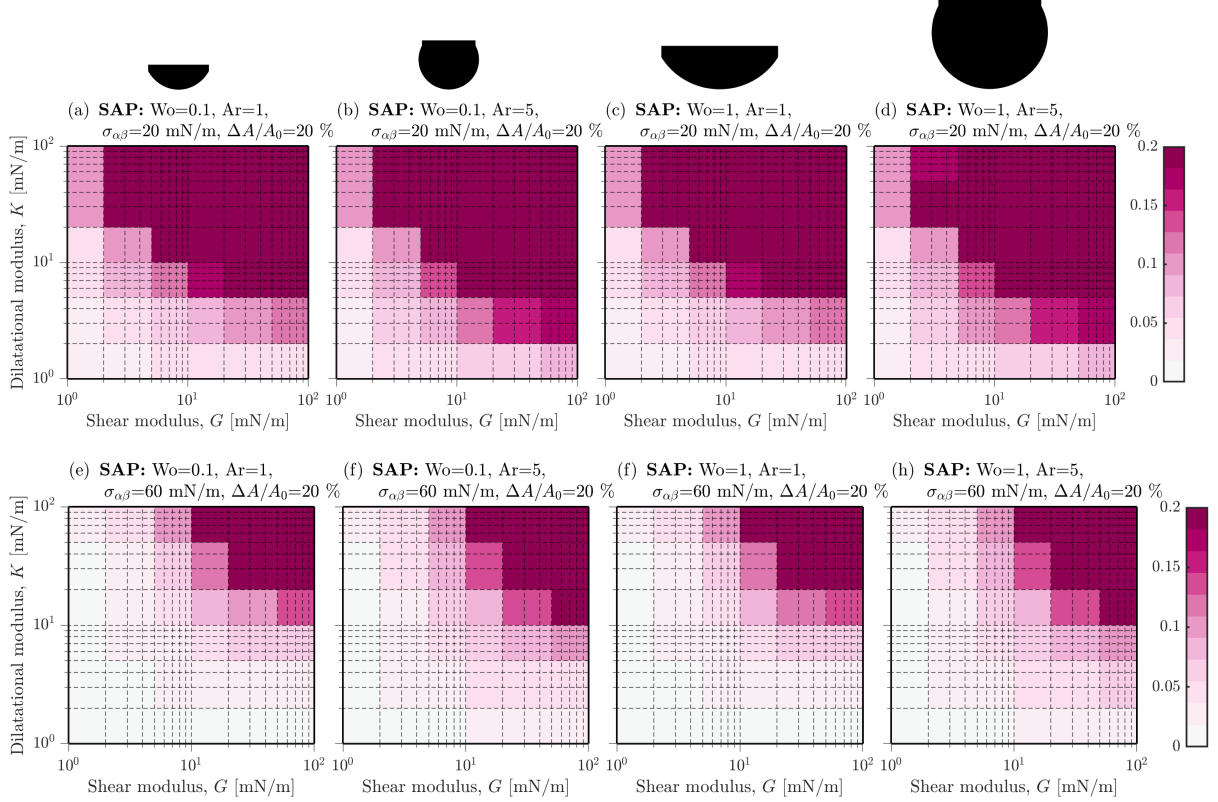


Figure 13: Stress anisotropy parameter (SAP) for strained drops at $\Delta A/A_0 = 20\%$, with $\sigma_{\alpha\beta} = 20$ mN/m (top, a-d) and $\sigma_{\alpha\beta} = 60$ mN/m (bottom, e-h). Results are shown for: (a,e) $Wo = 0.1$, $Ar = 1$; (b,f) $Wo = 0.1$, $Ar = 5$; (c,g) $Wo = 1$, $Ar = 1$; (d,h) $Wo = 1$, $Ar = 5$. High $\sigma_{\alpha\beta}/K$ and $\sigma_{\alpha\beta}/G$ ratios give a more homogeneous stress distribution along the drop interface (i.e., low SAP), while the magnitudes of Wo and Ar have a secondary effect.

3.2 Inverse problem

3.2.1 Apical strain errors, λ_{err}

Apical strain errors, λ_{err} , following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$, and the isotropic surface tension, $\sigma_{\alpha\beta}$. For each figure, results are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100$ mN/m and $1 \leq G \leq 100$ mN/m.

Table 4: Summary of parameters shown in Figs. 14 – 18. Results for $\Delta A/A_0 = 10\%$ and $\sigma_{\alpha\beta} = 60$ mN/m are shown in the main manuscript.

Figure number	$\Delta A/A_0$ [%]	$\sigma_{\alpha\beta}$ [mN/m]
Fig. 14	5	20
Fig. 15	10	20
Fig. 16	20	20
Fig. 17	5	60
Fig. 18	20	60

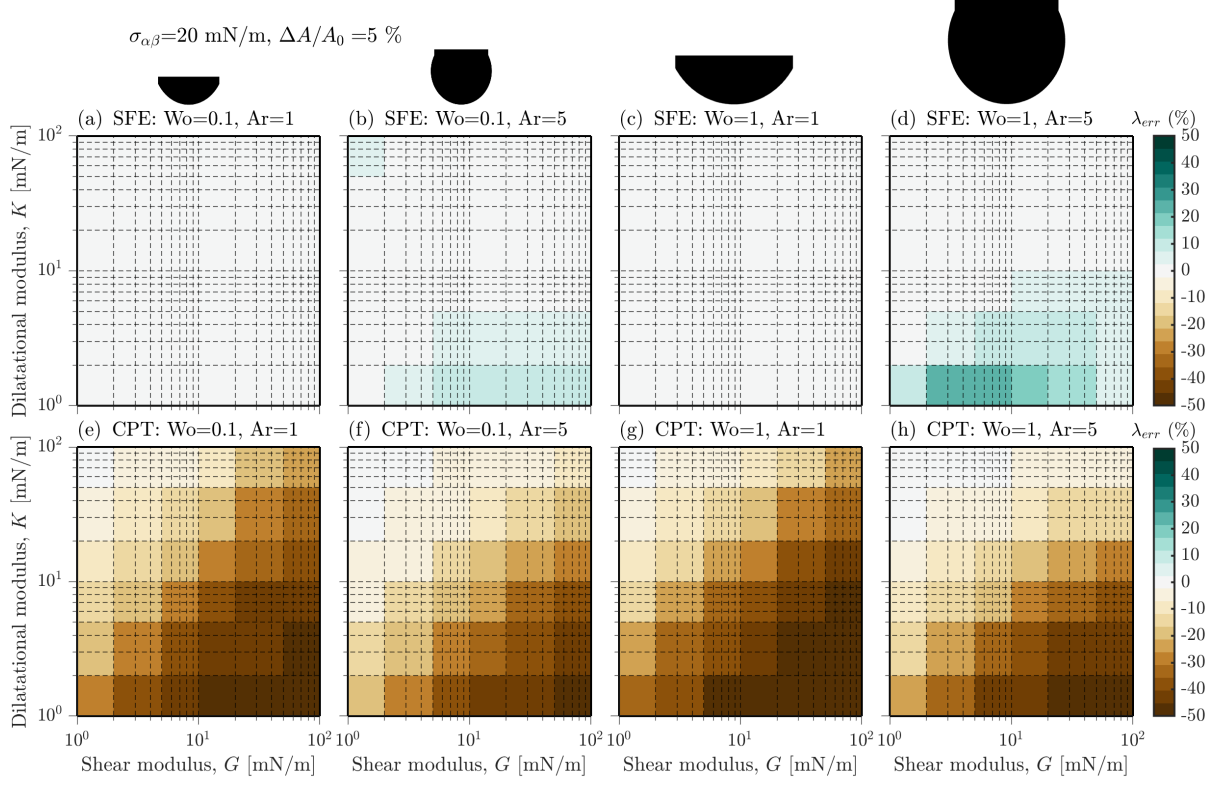


Figure 14: Apical strain errors from the solution of the Inverse Problem, λ_{err} , corresponding to $\Delta A/A_0 = 5 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1, Ar = 1$; (b, f) $Wo = 0.1, Ar = 5$; (c, g) $Wo = 1, Ar = 1$; and (d, h) $Wo = 1, Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Large errors in the apical strain predictions are incurred for CPT, with magnitudes proportional to G/K , arising from the anisotropy in the dilatational strain.

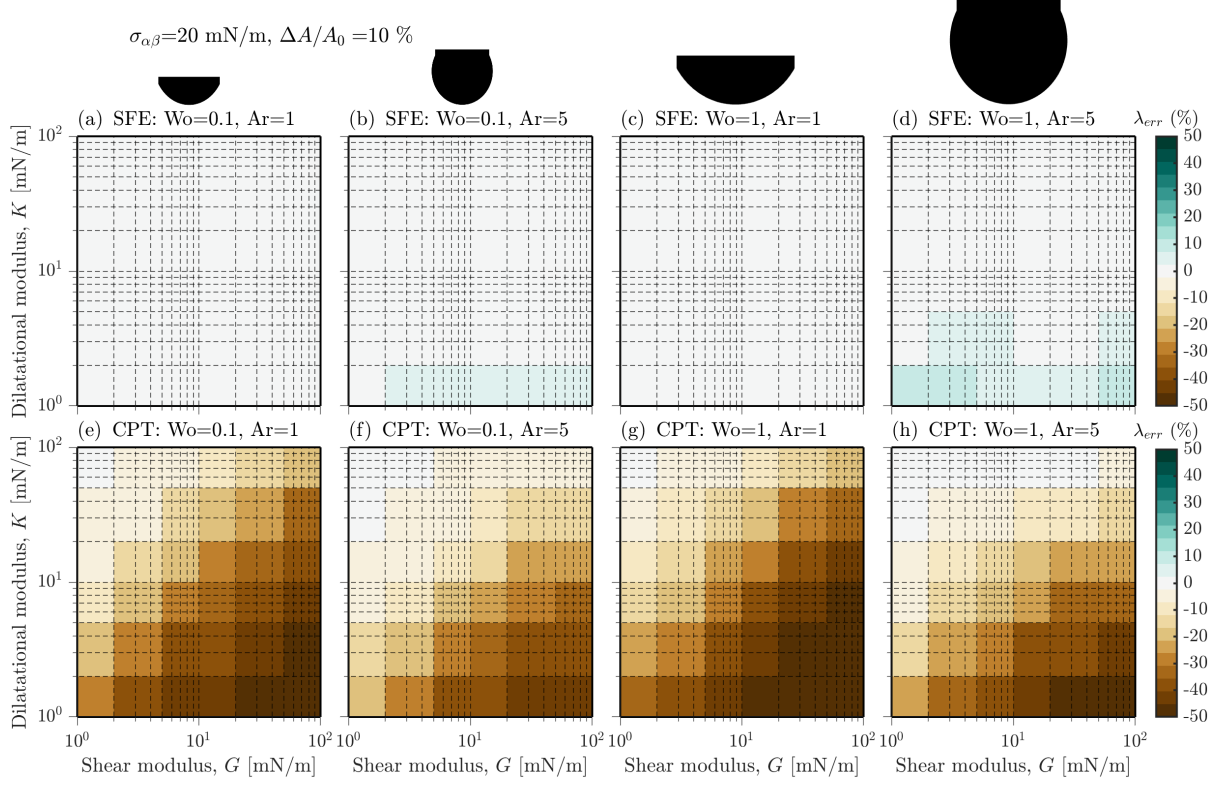


Figure 15: Apical strain errors from the solution of the Inverse Problem, λ_{err} , corresponding to $\Delta A/A_0 = 10 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Large errors in the apical strain predictions are incurred for CPT, with magnitudes proportional to G/K , arising from the anisotropy in the dilatational strain.

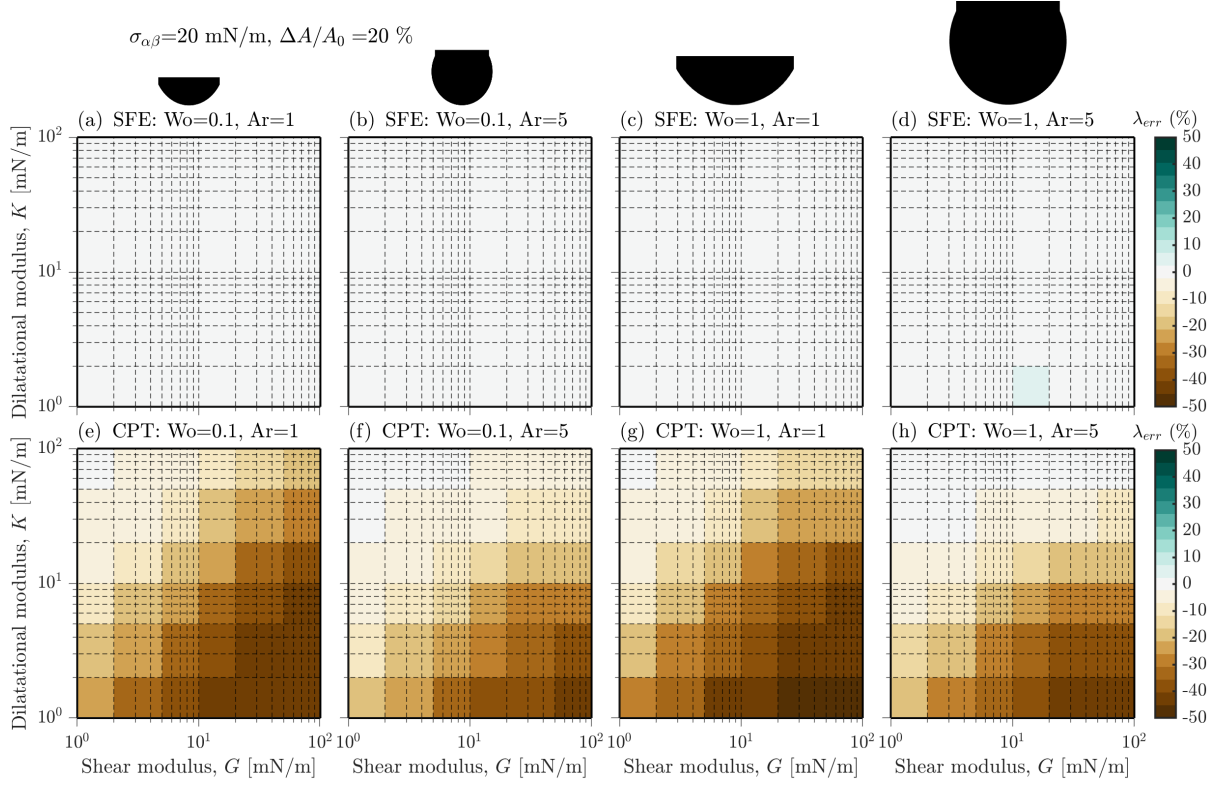


Figure 16: Apical strain errors from the solution of the Inverse Problem, λ_{err} , corresponding to $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Large errors in the apical strain predictions are incurred for CPT, with magnitudes proportional to G/K , arising from the anisotropy in the dilatational strain.

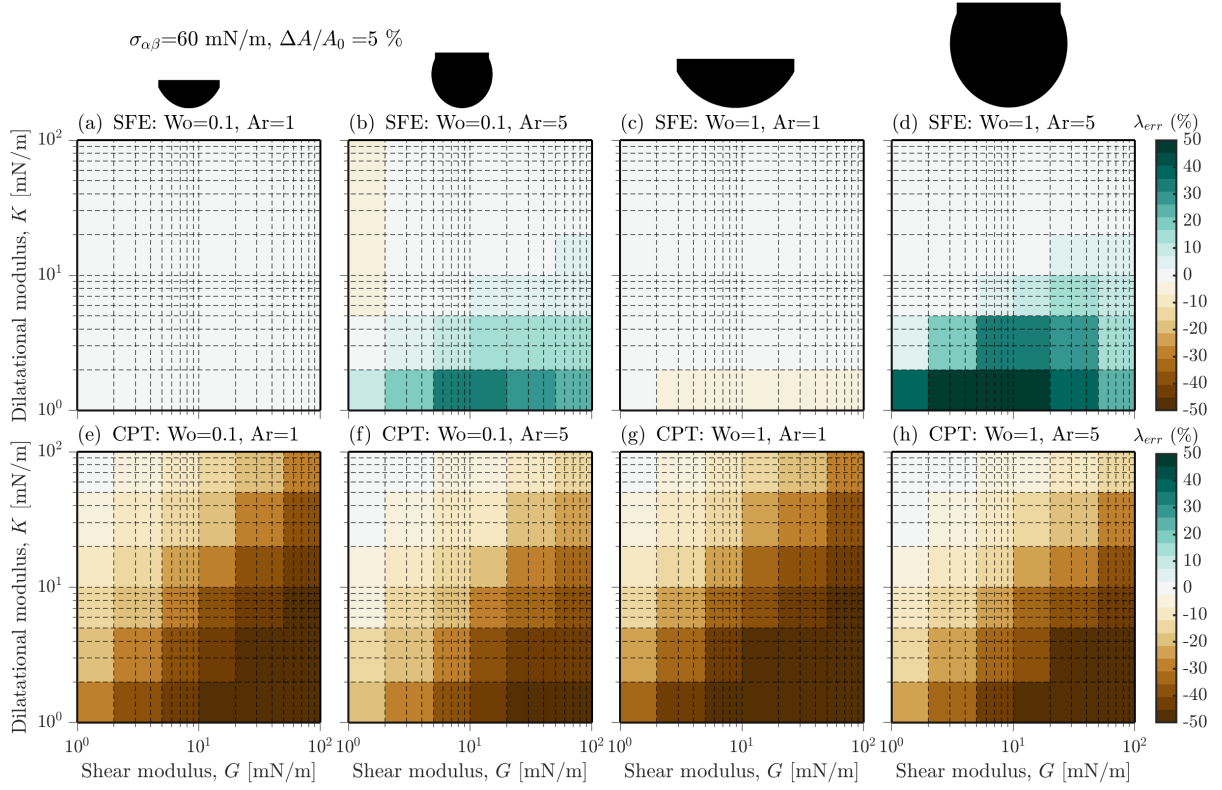


Figure 17: Apical strain errors from the solution of the Inverse Problem, λ_{err} , corresponding to $\Delta A/A_0 = 5 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Large errors in the apical strain predictions are incurred for CPT, with magnitudes proportional to G/K , arising from the anisotropy in the dilatational strain.

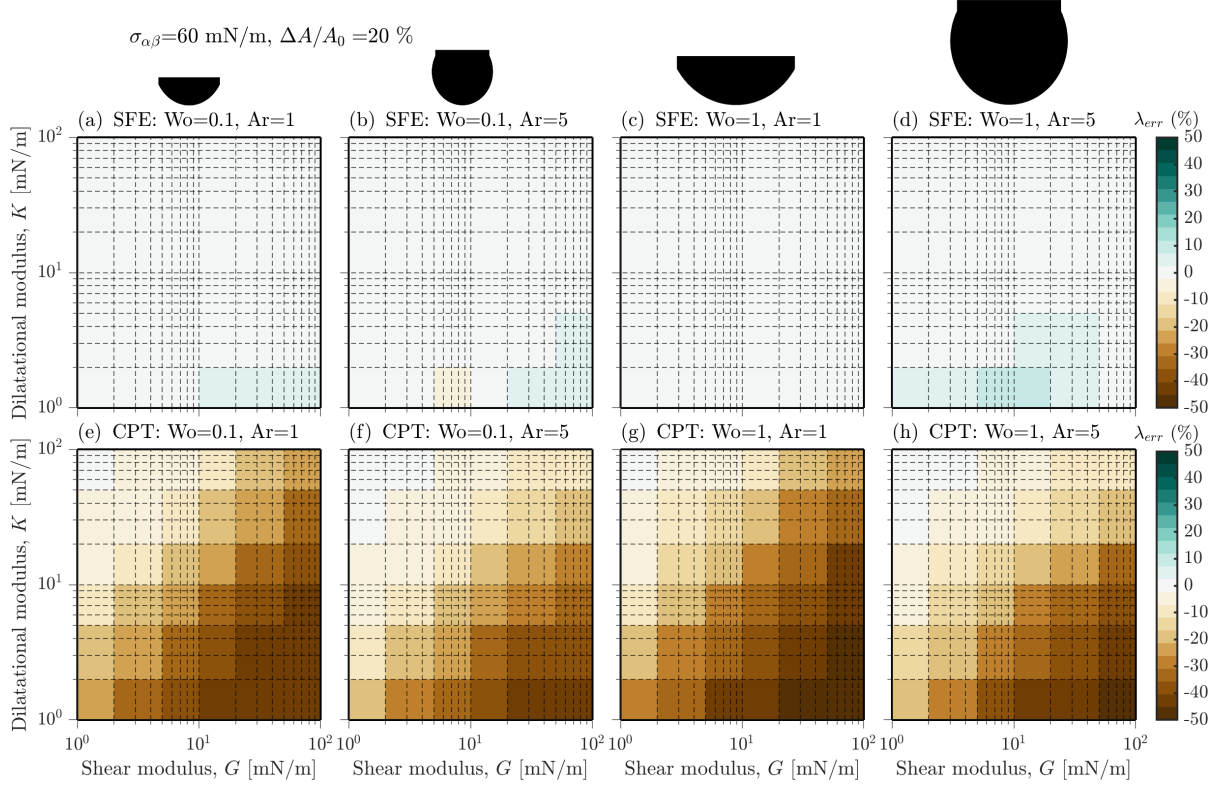


Figure 18: Apical strain errors from the solution of the Inverse Problem, λ_{err} , corresponding to $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Large errors in the apical strain predictions are incurred for CPT, with magnitudes proportional to G/K , arising from the anisotropy in the dilatational strain.

3.2.2 Apical stress errors, σ_{err}

Apical stress errors, σ_{err} , following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$, and the isotropic surface tension, $\sigma_{\alpha\beta}$. For each figure, results are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100$ mN/m and $1 \leq G \leq 100$ mN/m.

Table 5: Summary of parameters shown in Figs. 19 – 24.

Figure number	$\Delta A/A_0$ [%]	$\sigma_{\alpha\beta}$ [mN/m]
Fig. 19	5	20
Fig. 20	10	20
Fig. 21	20	20
Fig. 22	5	60
Fig. 23	10	60
Fig. 24	20	60

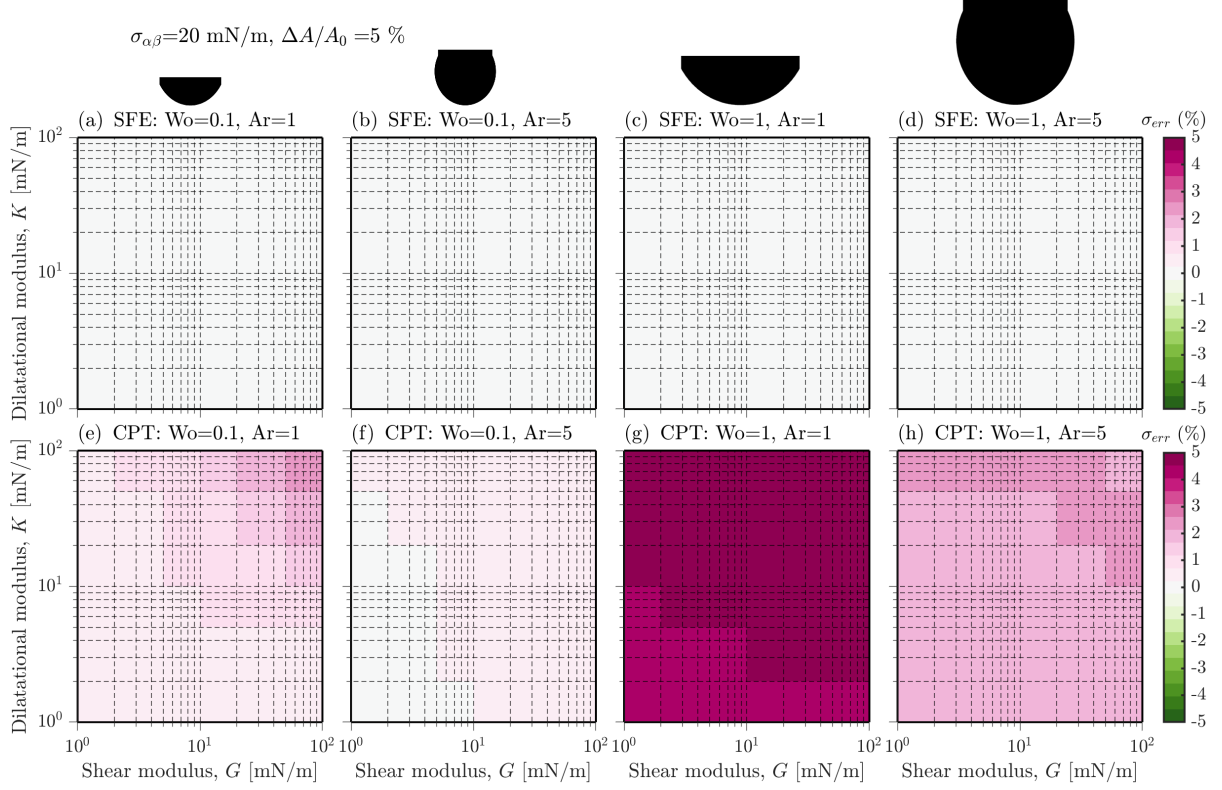


Figure 19: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 5 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

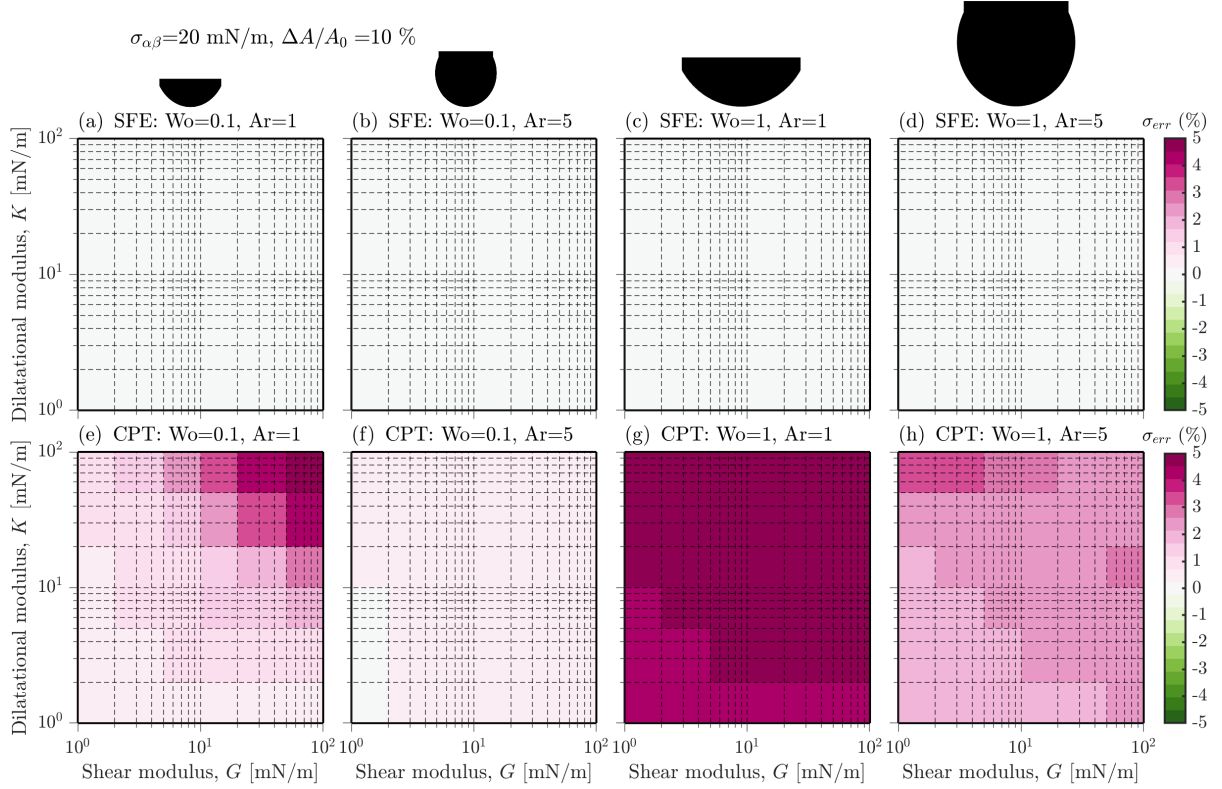


Figure 20: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 10 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

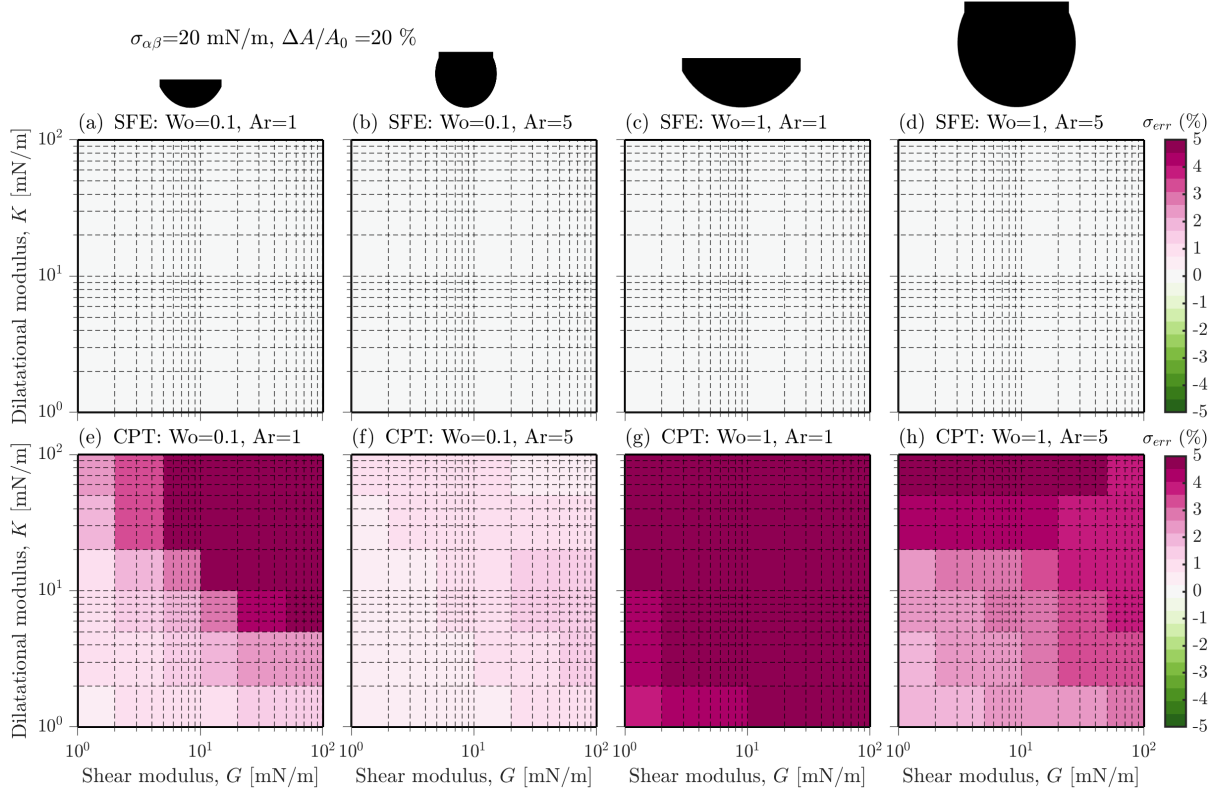


Figure 21: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

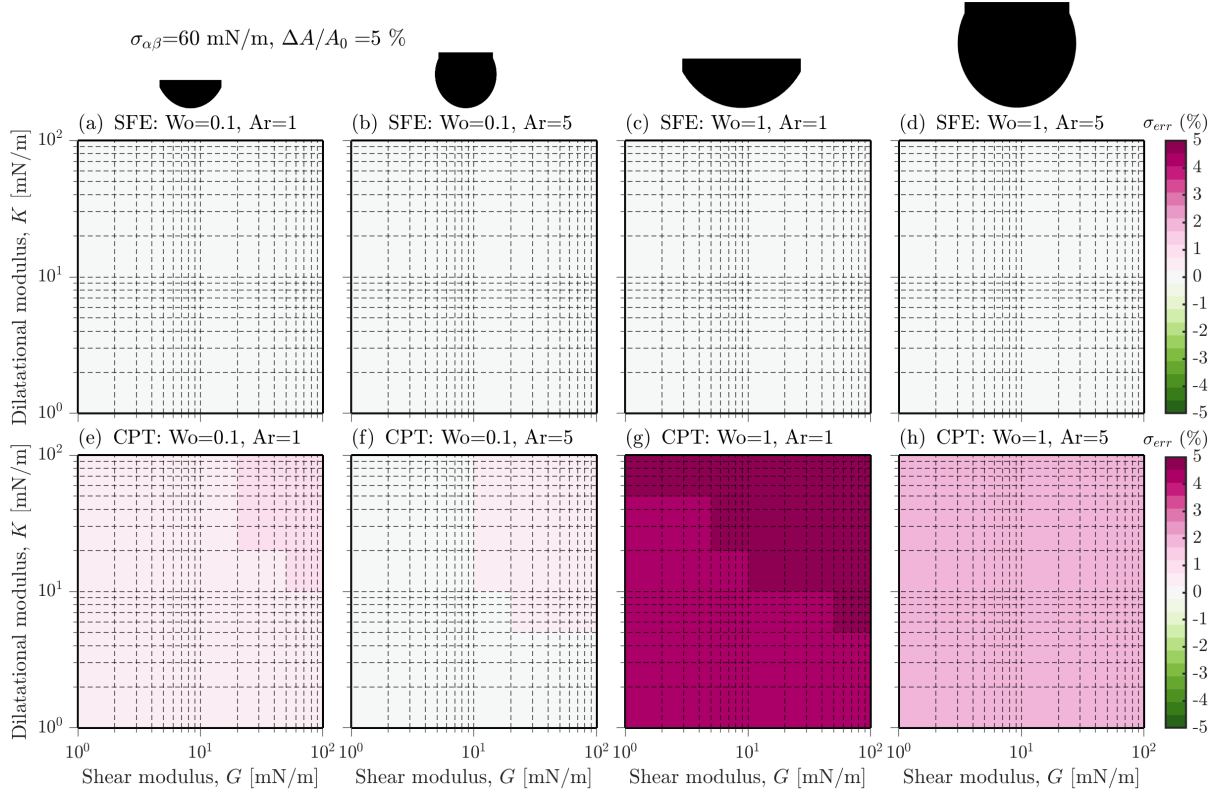


Figure 22: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 5 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

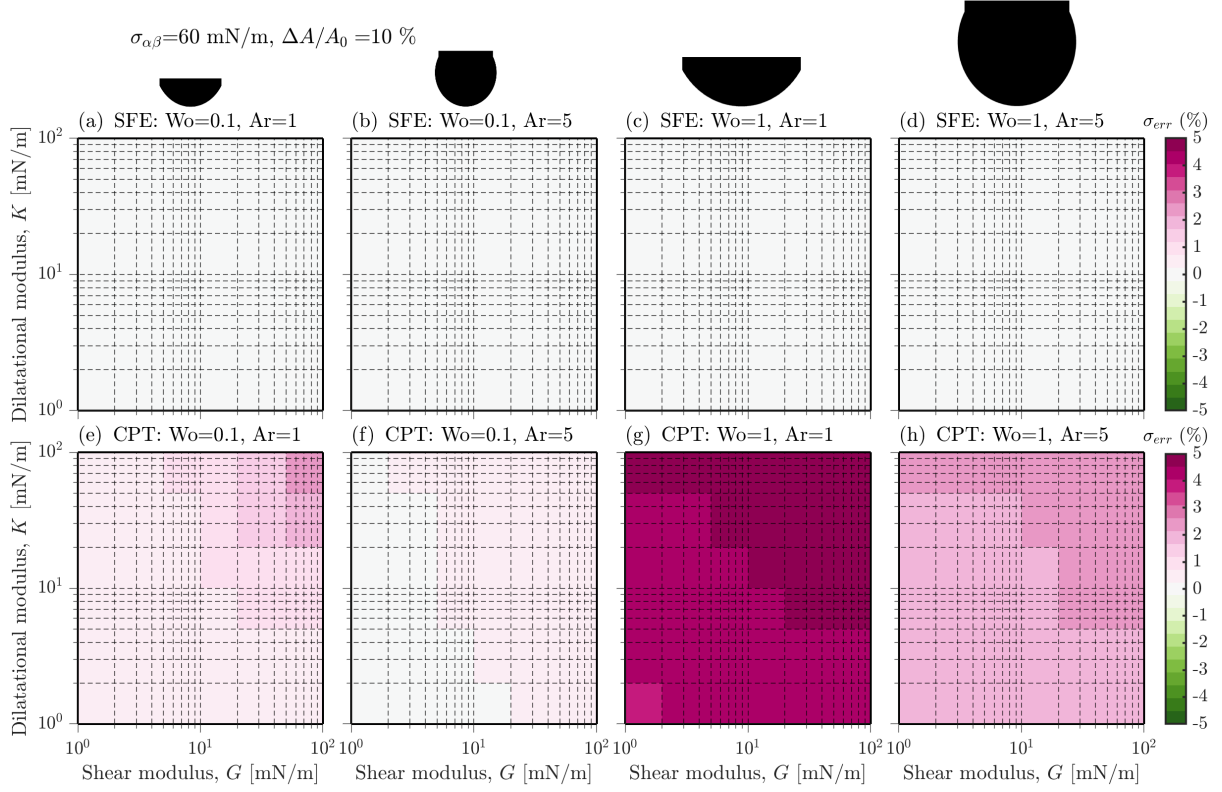


Figure 23: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 10 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

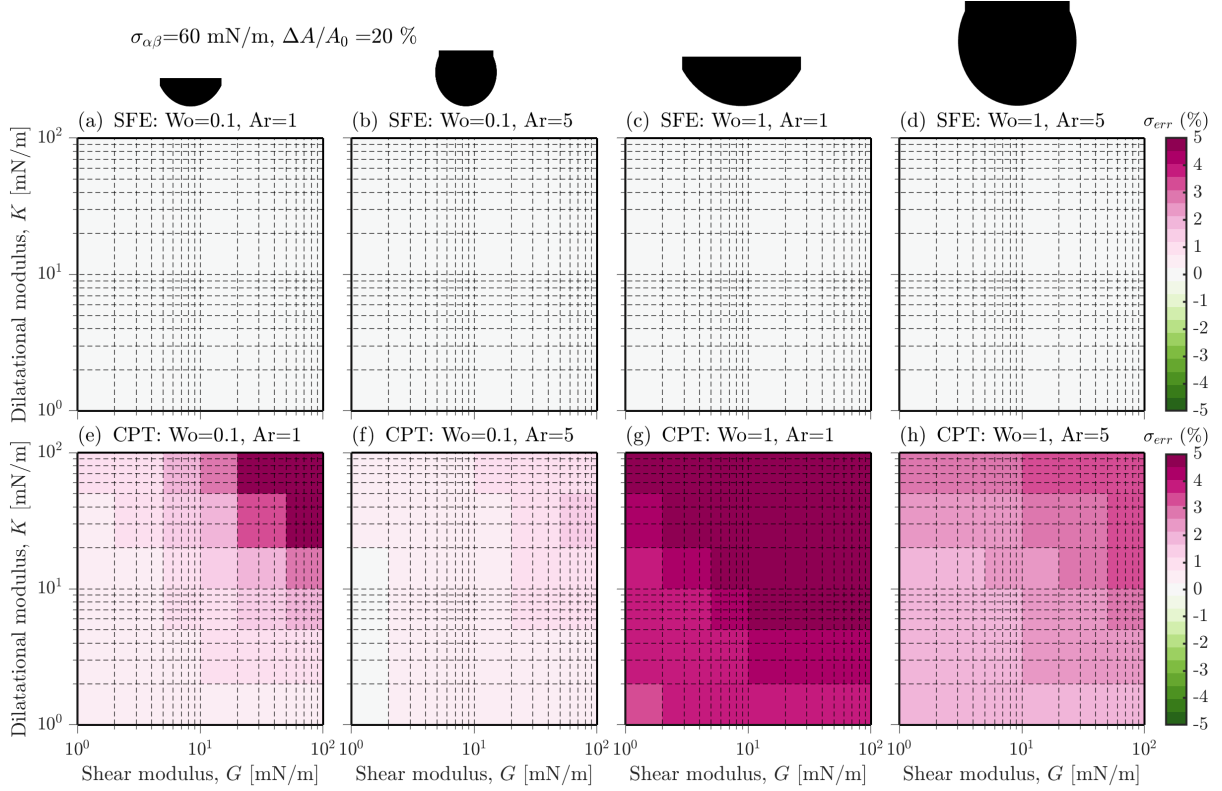


Figure 24: Apical stress errors from the solution of the Inverse Problem, σ_{err} , corresponding to $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo – Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1$, $Ar = 1$; (b, f) $Wo = 0.1$, $Ar = 5$; (c, g) $Wo = 1$, $Ar = 1$; and (d, h) $Wo = 1$, $Ar = 5$. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). The prediction of the apical strain is accurate for all methods, showing less than 5% error.

3.2.3 Dilatational modulus errors, K_{err}

Dilatational modulus errors, K_{err} , following a step-strain compression with different magnitudes for the total strain, $\Delta A/A_0$, and the isotropic surface tension, $\sigma_{\alpha\beta}$. For each figure, results are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100$ mN/m and $1 \leq G \leq 100$ mN/m.

Table 6: Summary of parameters shown in Figs. 25 – 29. Results for $\Delta A/A_0 = 10\%$ and $\sigma_{\alpha\beta} = 60$ mN/m are shown in the main manuscript.

Figure number	$\Delta A/A_0$ [%]	$\sigma_{\alpha\beta}$ [mN/m]
Fig. 25	5	20
Fig. 26	10	20
Fig. 27	20	20
Fig. 28	5	60
Fig. 29	20	60

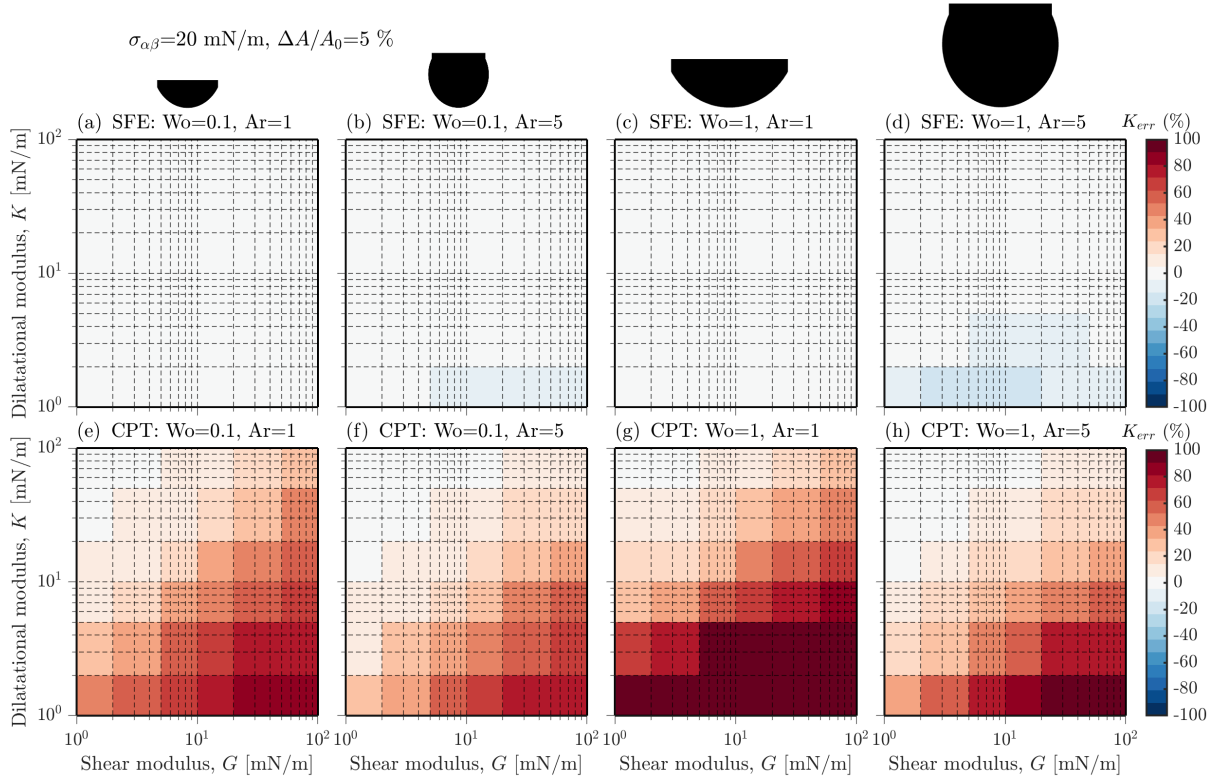


Figure 25: Elastic dilatational modulus errors from the solution of the Inverse Problem, K_{err} , are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100 \text{ mN/m}$ and $1 \leq G \leq 100 \text{ mN/m}$. Results are shown for $\Delta A/A_0 = 5 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo - Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1, Ar = 1$; (b, f) $Wo = 0.1, Ar = 5$; (c, g) $Wo = 1, Ar = 1$; and (d, h) $Wo = 1, Ar = 5$. Two different analytical routines are examined: SFE in the top row (a-d) and CPT in the bottom row (e-h). SFE incurs less than 5 % error for all cases, whereas CPT can show errors of over 100 % in the calculation of K , due to the assumption of a homogeneous strain field.

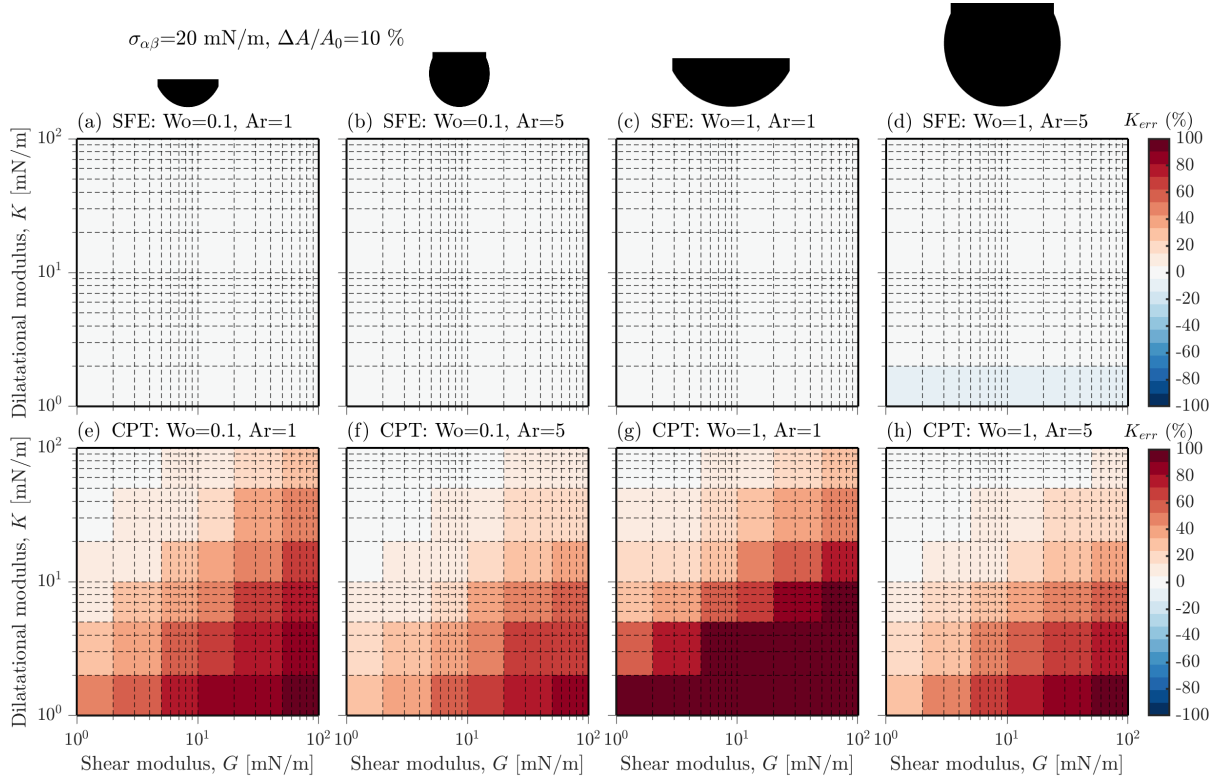


Figure 26: Elastic dilatational modulus errors from the solution of the Inverse Problem, K_{err} , are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100 \text{ mN/m}$ and $1 \leq G \leq 100 \text{ mN/m}$. Results are shown for $\Delta A/A_0 = 10 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo - Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1, Ar = 1$; (b, f) $Wo = 0.1, Ar = 5$; (c, g) $Wo = 1, Ar = 1$; and (d, h) $Wo = 1, Ar = 5$. Two different analytical routines are examined: SFE in the top row (a-d) and CPT in the bottom row (e-h). SFE incurs less than 5 % error for all cases, whereas CPT can show errors of over 100 % in the calculation of K , due to the assumption of a homogeneous strain field.

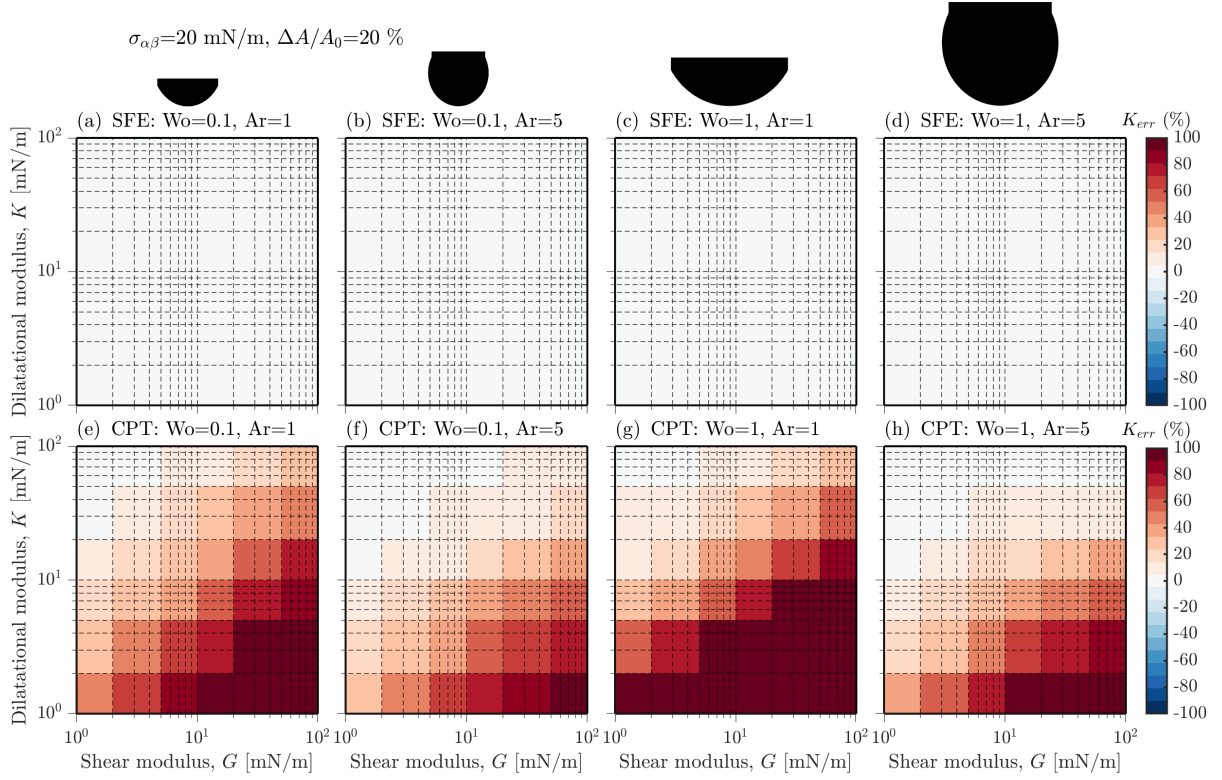
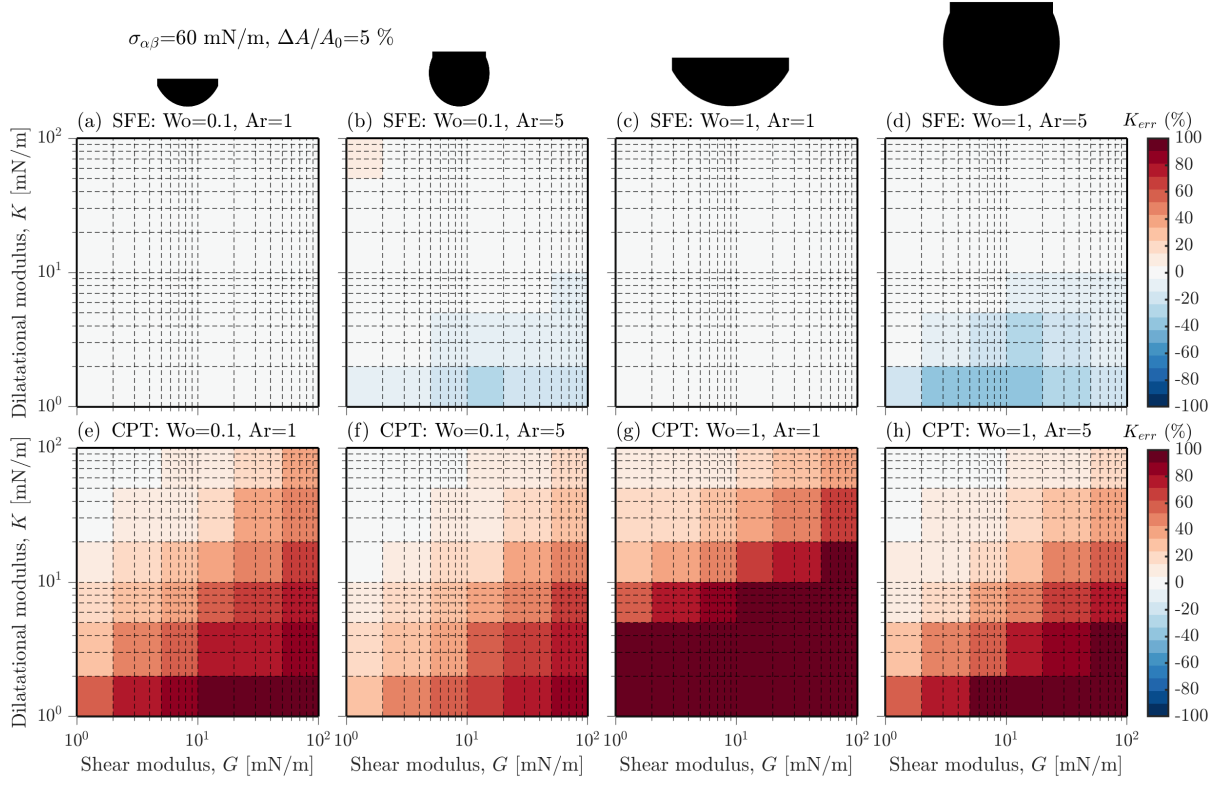


Figure 27: Elastic dilatational modulus errors from the solution of the Inverse Problem, K_{err} , are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100 \text{ mN/m}$ and $1 \leq G \leq 100 \text{ mN/m}$. Results are shown for $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 20 \text{ mN/m}$. Four different Wo - Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1, Ar = 1$; (b, f) $Wo = 0.1, Ar = 5$; (c, g) $Wo = 1, Ar = 1$; and (d, h) $Wo = 1, Ar = 5$. Two different analytical routines are examined: SFE in the top row (a-d) and CPT in the bottom row (e-h). SFE incurs less than 5 % error for all cases, whereas CPT can show errors of over 100 % in the calculation of K , due to the assumption of a homogeneous strain field.



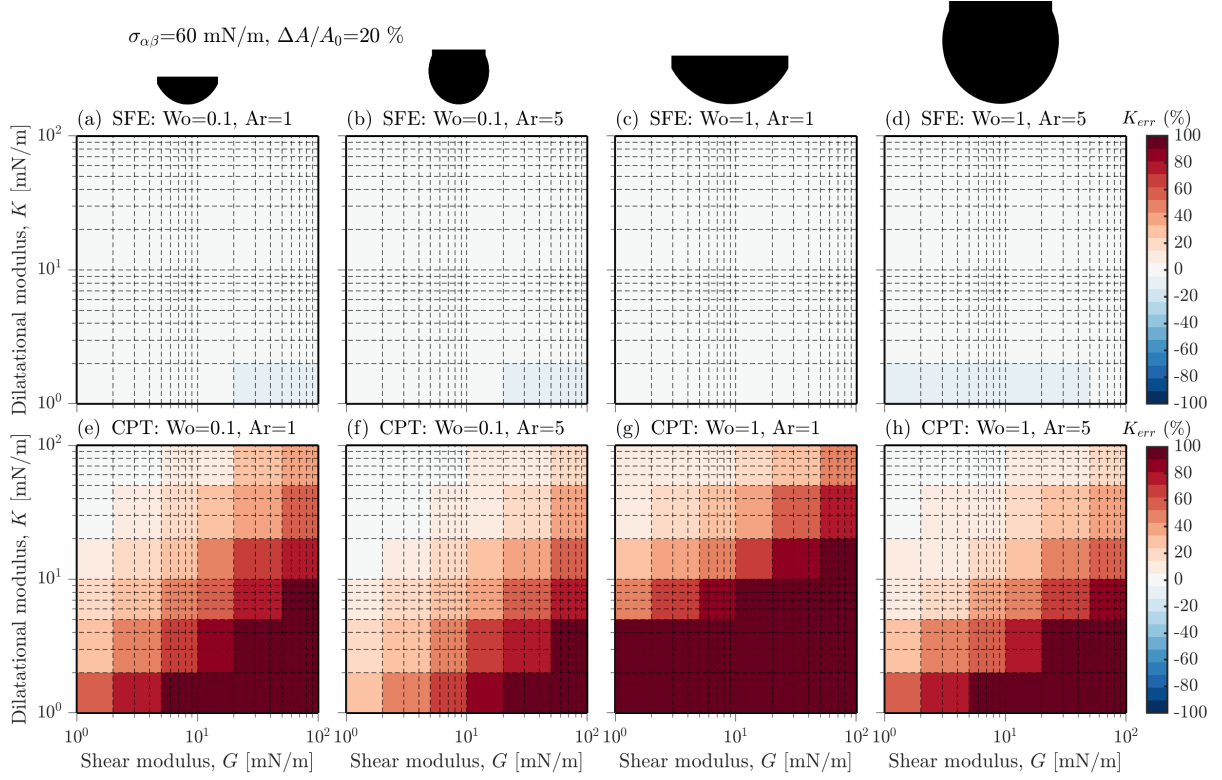


Figure 29: Elastic dilatational modulus errors from the solution of the Inverse Problem, K_{err} , are shown for different geometries (columns) and analytical routines (rows), for mechanical moduli values spanning $1 \leq K \leq 100 \text{ mN/m}$ and $1 \leq G \leq 100 \text{ mN/m}$. Results are shown for $\Delta A/A_0 = 20 \%$ and $\sigma_{\alpha\beta} = 60 \text{ mN/m}$. Four different Wo - Ar combinations are explored, with corresponding geometry schematics at the top of each column: (a, e) $Wo = 0.1, Ar = 1$; (b, f) $Wo = 0.1, Ar = 5$; (c, g) $Wo = 1, Ar = 1$; and (d, h) $Wo = 1, Ar = 5$. Two different analytical routines are examined: SFE in the top row (a-d) and CPT in the bottom row (e-h). SFE incurs less than 5 % error for all cases, whereas CPT can show errors of over 100 % in the calculation of K , due to the assumption of a homogeneous strain field.

3.3 “Noisy interfaces”: Inverse problem

Errors and normalized standard deviations for the apical strain, λ_{err} , and stress, σ_{err} , are shown for parameter values corresponding to $\Delta A/A_0 = 10\%$ and $\sigma_{\alpha\beta} = 60$ mN/m. Errors are computed using drop interface coordinates subjected to ~ 1 pixel random Gaussian noise, which simulates experimental contour detection errors. For each K – G combination, the errors are computed as the average from 10 independent noisy interface profiles and the normalized standard deviations are computed as outlined in the main manuscript. Results for the modulus error, K_{err} , are shown in the main manuscript.

3.3.1 Apical strain errors, λ_{err}

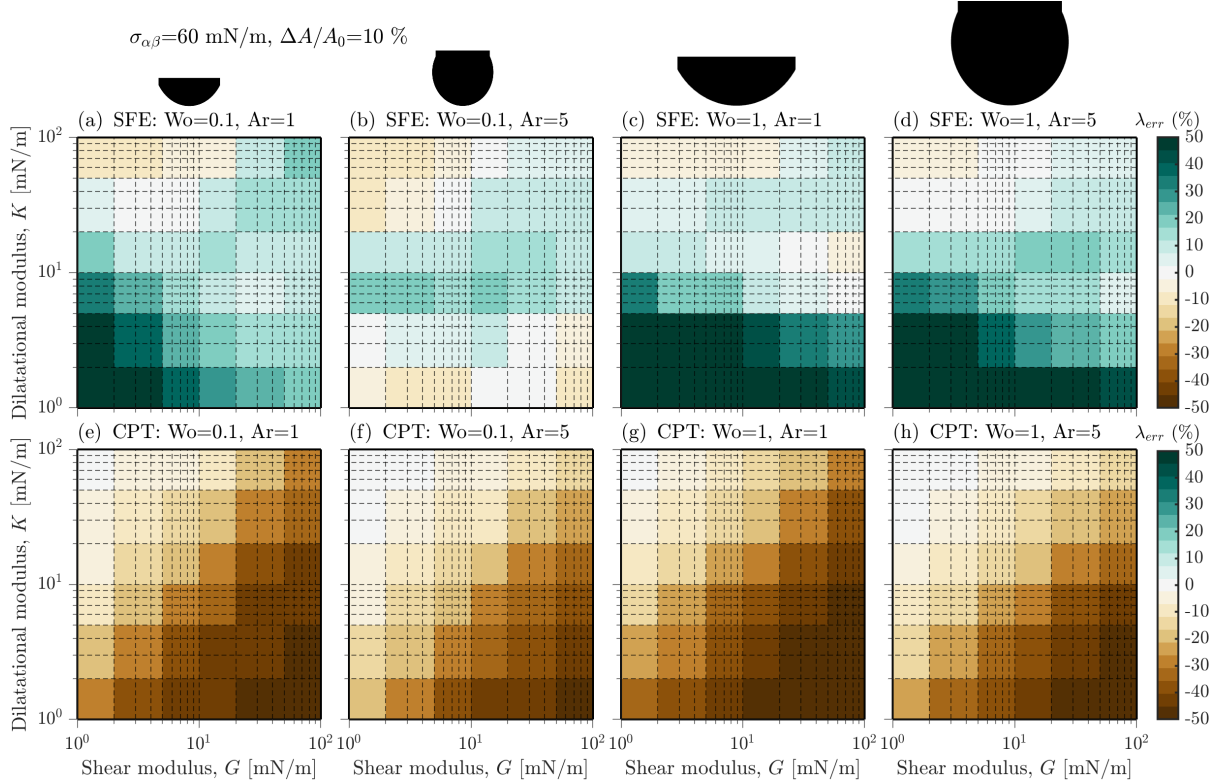


Figure 30: Apical strain errors, λ_{err} , from the solution of the Inverse Problem, are computed using drop interface coordinates subjected to ~ 1 pixel random Gaussian noise. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Whereas CPT is fairly insensitive to noise, substantial errors in K arise when SFE is applied to noisy interface profiles, particularly for low- Ar configurations.

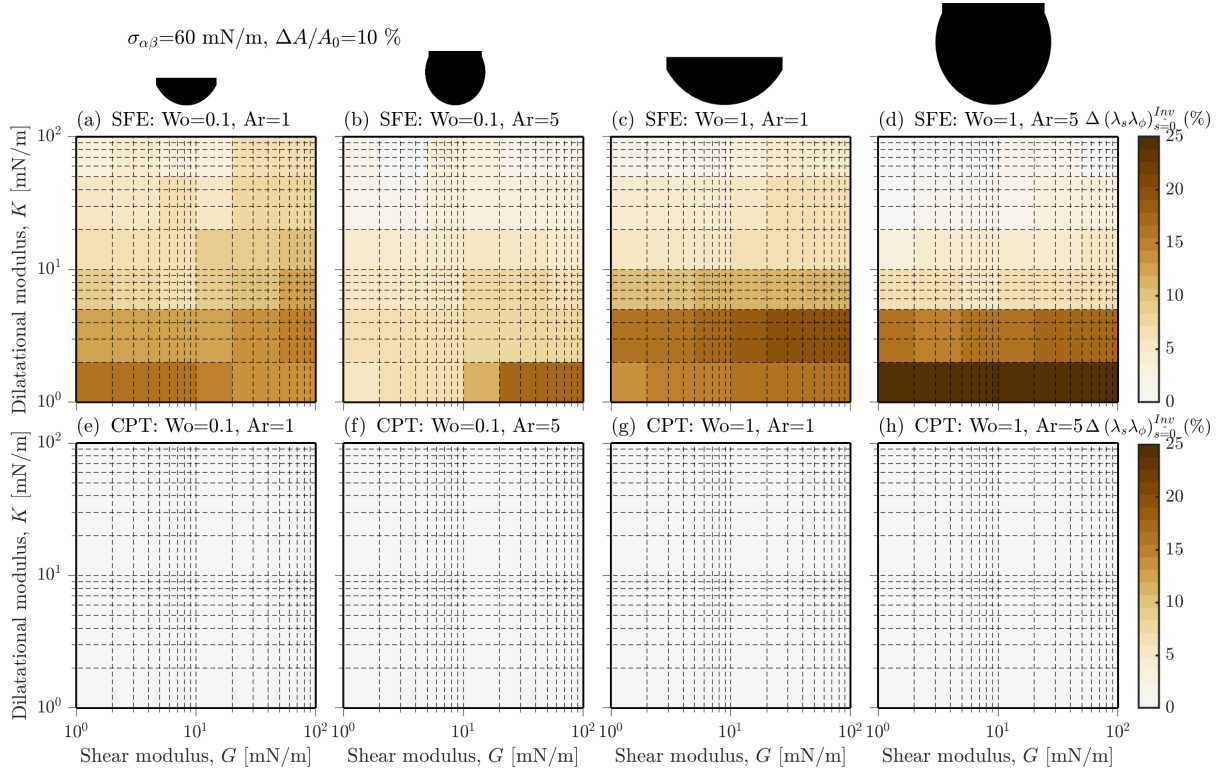


Figure 31: Normalized standard deviation, $\Delta\lambda_{\text{err}}$, arising from the calculation of λ_{err} from 10 independent noisy interface profiles. A large dispersion in the computed apical strain values is present in SFE, particularly at low Ar .

3.3.2 Apical stress errors, σ_{err}

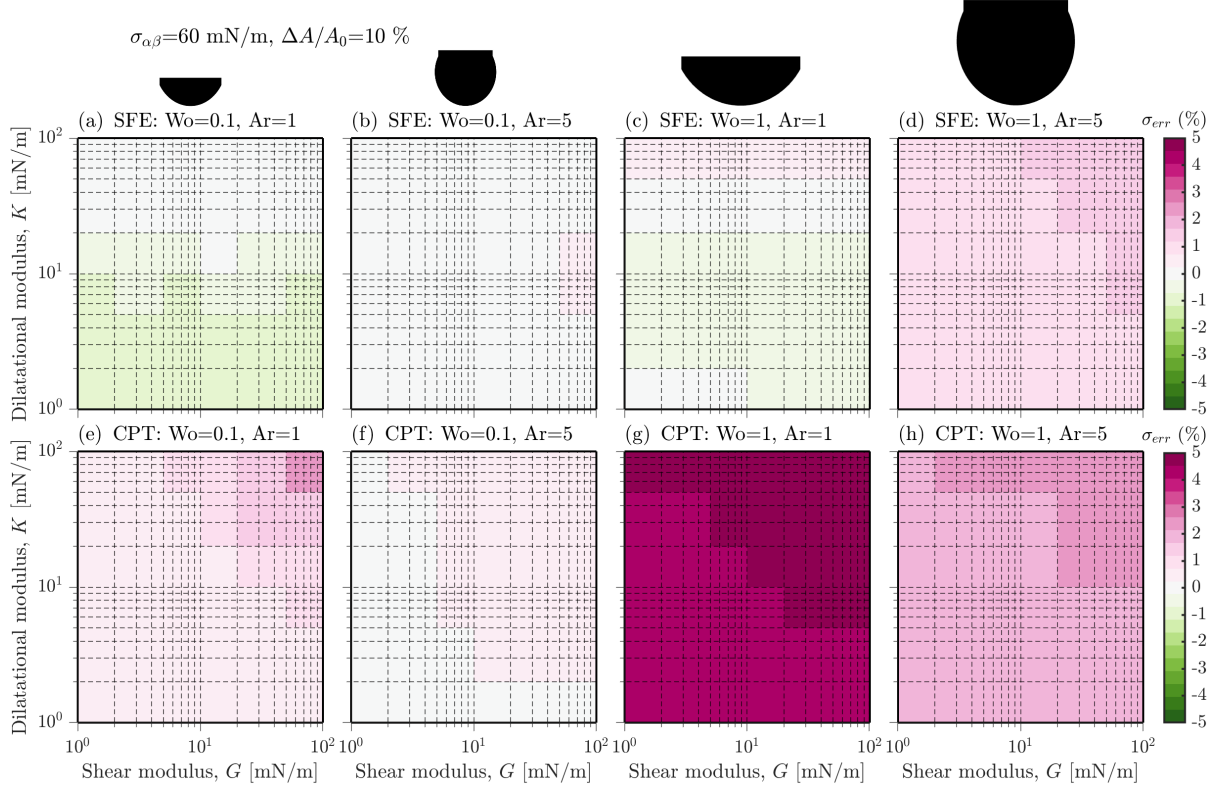


Figure 32: Apical stress errors, σ_{err} , from the solution of the Inverse Problem, are computed using drop interface coordinates subjected to ~ 1 pixel random Gaussian noise. Two different analytical routines are examined: SFE in the top row (a–d) and CPT in the bottom row (e–h). Both CPT and SFE remain accurate in the calculation of the apical stress, despite the errors introduced by the random Gaussian noise.

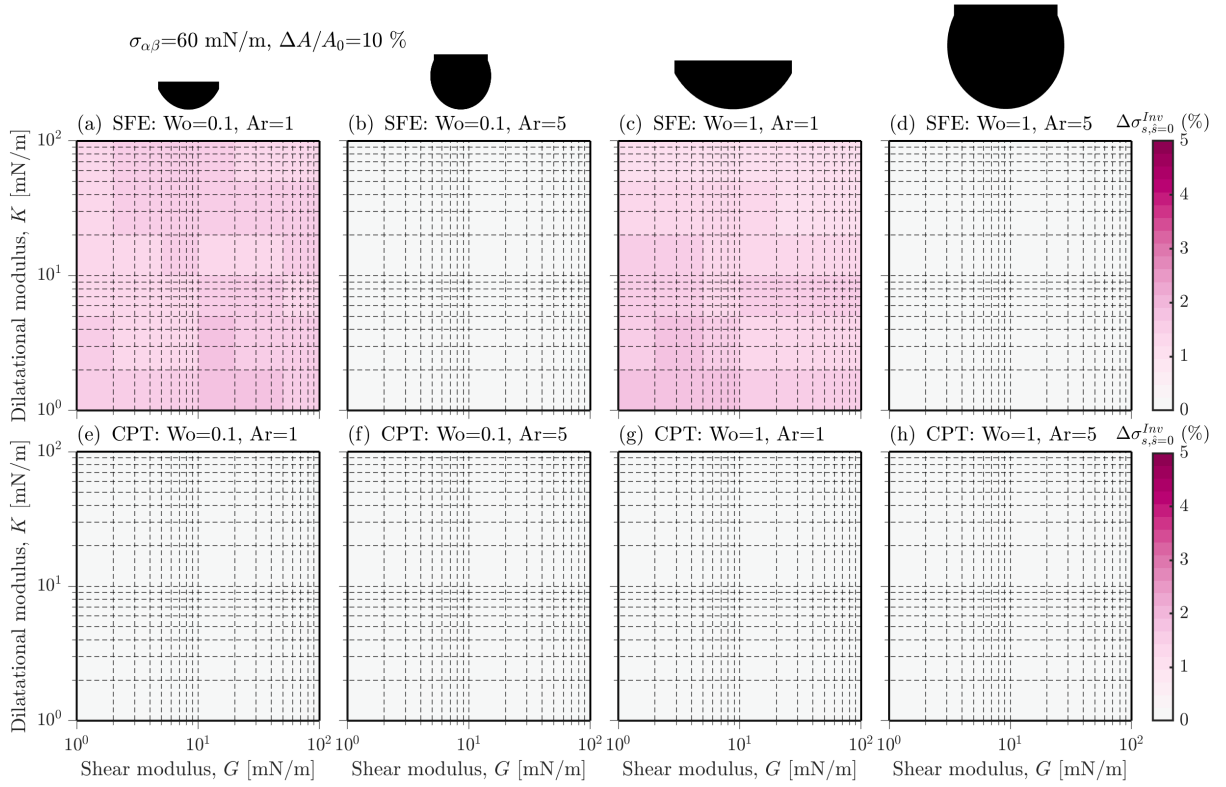


Figure 33: Normalized standard deviation, $\Delta\sigma_{err}$, arising from the calculation of σ_{err} from 10 independent noisy interface profiles. A large dispersion in the computed apical stress values is present in SFE, particularly at low Ar .

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