Supplementary Material for "The Role of Feedback in Morphological Computation with Compliant Bodies"

This document provides additional information on the experiments presented in the paper.

Definition of the used simulation parameters

Section 5 of the paper describes the construction process and the implementation of the simulation of the mass-spring networks. As pointed out the various construction parameters were randomly drawn from defined ranges. Table 1 shows the used parameters with a corresponding short description. Throughout this document will use the same names to describe the parameters of the individual tasks.

Parameters	**Description**
N	Number of mass points[1]
$width \times height$	Defines the two-dimensional area in which we randomly placed the N mass points
I	Number of inputs (compare to Figure 4 of the paper)
O	Number of outputs (compare to Figure 4 of the paper) - equals also to the number of feedbacks in our experiments
$k_{lim} = \begin{bmatrix} k_{1.min} & k_{1,max} \\ k_{3,min} & k_{3,max} \end{bmatrix}$	Defines the ranges from which the spring property parameters $k_1$and k_3 were drawn from.
$d_{lim} = \begin{bmatrix} d_{1.min} & d_{1,max} \\ d_{3,min} & d_{3,max} \end{bmatrix}$	Defines the ranges from which the damping parameters d_1 and $d_3$were drawn from.
c_{in}	Defines the percentage of all N mass points, which were randomly chosen to be input nodes.
c_{fb}	Defines the percentages of all N mass points, which received feedback. In the cases of multiple feedbacks, every single loop connected to c_{fb}% of all masses. As a consequence, there were nodes, which received multiple feedback loops.
$\mathbf{w}_{in,lim} = \begin{bmatrix} w_{in,min} & w_{in,max} \end{bmatrix}$	Defines the range from the *input* weights w_{in} have been randomly drawn (at the initialization).
$\mathbf{w}_{fb,lim} = \begin{bmatrix} w_{fb,min} & w_{in,max} \end{bmatrix}$	Defines the range from the *feedback* weights w_{fb} have been randomly drawn (at the initialization).
ν	amplitude of the noise applied during learning (see Figure 4 in Section 5.3)

Table 1: Summary and descriptions of all used parameters for the construction of the mass-spring networks.

Section 6.1: Three limit cycles

Table 2 summarizes the values used to construct the individual networks used for the experiments in Section 6.1. One can see that most of the parameters were set to the same values, only the the size of the network (i.e., N) was different. Note that the ranges (of all presented experiments) were found by trial and error. However, the choice for the range for the spring constants is not sensitive at all and no tedious search for good ranges was needed. For example, it is quite easy to find networks with random spring constants drawn from within [0.01 1000]. However, one can increase the probability to find good performing networks, if one reduces the range. It is reasonable to assume that biological systems will have optimized their ranges through an evolutionary process.

Variable Name	**Van der Pol (specialized)**	**Quadratic (specialized)**	**Lissajous (specialized)**	**General network**
N	10	10	20	25
$width \times height$	100×100	100×100	100×100	100×100
I	no input	no input	no input	no input
O	2	2	2	2
$\begin{bmatrix} k_{1.min} & k_{1,max} \\ k_{3,min} & k_{3,max} \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$
$\begin{bmatrix} d_{1.min} & d_{1,max} \\ d_{3,min} & d_{3,max} \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 10 \\ 100 & 200 \end{bmatrix}$
c_{in}	no input	no input	no input	no input
c_{fb}	20%	20%	20%	20%
$\begin{bmatrix} w_{in,min} & w_{in,max} \end{bmatrix}$	no input	no input	no input	no input
$\begin{bmatrix} w_{fb,min} & w_{in,max} \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$
ν	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$

Table 2: Values used to construct and simulate the mass-spring networks of Section 6.1, i.e., generating nonlinear limit cycles.

Section 6.2: Generation of different walking patterns using the same mass-spring network

Table 3 lists the parameter used for the construction of the network of Figure 8 of the paper, which has been employed to produce different walking patterns.

Variable Name	**Values**
N	50
$width$, $height$	10×10
I	no input
O	4
$\begin{bmatrix} k_{1.min} & k_{1,max} \\ k_{3,min} & k_{3,max} \end{bmatrix}$	$\begin{bmatrix} 10 & 200 \\ 100 & 200 \end{bmatrix}$
$\begin{bmatrix} d_{1.min} & d_{1,max} \\ d_{3,min} & d_{3,max} \end{bmatrix}$	$\begin{bmatrix} 0.1 & 2 \\ 100 & 200 \end{bmatrix}$
c_{in}	no input
c_{fb}	20%
$\begin{bmatrix} w_{in,min} & w_{in,max} \end{bmatrix}$	no input
$\begin{bmatrix} w_{fb,min} & w_{in,max} \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$
ν	1×10^{-3}

Table 3: list of used parameters for the experiment of Section 6.2, i.e., generating different walking patterns using one single network (see Figure 8 of the paper).

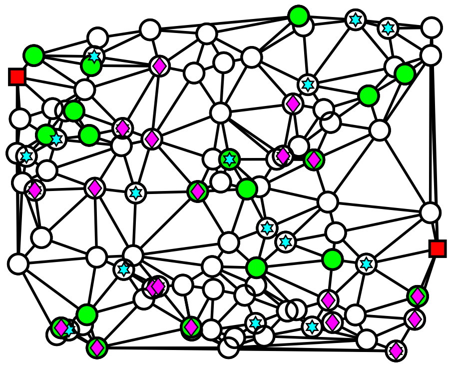

Figure 1: Network used for the experiments of Section 6.3, i.e., generation of input dependent limit cycles. The network consisted of $N = 100$ mass points and $L = 283$ springs. The different colors/shapes correspond to different the feedback the nodes get.

Section 6.3: Generation of different limit cycles depending on the input

Table 4 lists the parameter values used for the experiment described in Section 6.3 of the paper. Figure 1 shows the actual network used for the experiments.

Variable Name	**Values**
N	100
$width$, $height$	100×100
I	1
O	2
$k_{lim} = \begin{bmatrix} k_{1.min} & k_{1,max} \\ k_{3,min} & k_{3,max} \end{bmatrix}$	$\begin{bmatrix} 1 & 10 \\ 100 & 200 \end{bmatrix}$
$d_{lim} = \begin{bmatrix} d_{1.min} & d_{1,max} \\ d_{3,min} & d_{3,max} \end{bmatrix}$	$\begin{bmatrix} 1 & 10 \\ 100 & 200 \end{bmatrix}$
c_{in}	20%
c_{fb}	20%
$\mathbf{w}_{in,lim} = \begin{bmatrix} w_{in,min} & w_{in,max} \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$
$\mathbf{w}_{fb,lim} = \begin{bmatrix} w_{fb,min} & w_{in,max} \end{bmatrix}$	$\begin{bmatrix} -10 & 10 \end{bmatrix}$
ν	1×10^{-3}

Table 4: Parameters used for the construction and simulation of the network used for the experiments of Section 6.3. Figure 1 shows the actual network used for the experiments.