Supplementary materials to
General multiple tests for functional data
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This supplement contains all proofs, auxiliary theorems, and the extension of our methodology to multivariate functional data in Sections S1–S3. In addition, all results of the numerical experiments from Sections 6 and 7 of the main paper are given in Sections S4 and S5. Furthermore, results for additional simulation studies to investigate the effect of the heteroscedasticity level, sample sizes, and number of design time points can be found in Sections S6–S8. We also give some details on the competitors for the simulation studies in Section S9. Section S10 illustrates the limiting distribution of the test statistic and the parametric bootstrap counterpart. Finally, the computational time and speedup of the tests is analyzed in Section S11.

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S1. Multivariate Functional Data Setup

Instead of the univariate functional data setup as in Section 2, we may consider a more general setup that can handle multivariate functional data as well similarly to the setup in [12].

In the following, we consider the Hilbert space $\mathcal{L}_2^p(\mathcal{T})$, $p \in \mathbb{N}$, of all p-dimensional squared integrable functions over $\mathcal{T}$ with inner product

$$\langle .,. \rangle : \mathcal{L}_2^p(\mathcal{T}) \times \mathcal{L}_2^p(\mathcal{T}) \to \mathbb{R}, \quad \langle \mathbf{g}, \mathbf{h} \rangle := \int_{\mathcal{T}} \mathbf{g}^\top(t)\mathbf{h}(t)\,\mathrm{d}t.$$

Suppose we have $k \in \mathbb{N}$ independent multivariate functional samples

$$\mathbf{x}_{i1}, ..., \mathbf{x}_{in_i} \sim \mathrm{SP}_p(\boldsymbol{\eta}_i, \boldsymbol{\Gamma}_i) \text{ i.i.d.} \quad \text{for all } i \in \{1, ..., k\} \tag{S1}$$

with sample sizes $n_i \in \mathbb{N}, n_i \geq 2, i \in \{1, ..., k\}$, which take values in $\mathcal{L}_2^p(\mathcal{T})$. Here, $\boldsymbol{\eta}_i : \mathcal{T} \to \mathbb{R}^p$ denotes the mean function and $\boldsymbol{\Gamma}_i : \mathcal{T}^2 \to \mathbb{R}^{p\times p}$ denotes the covariance function of the ith sample for all $i \in \{1, ..., k\}$, which are usually unknown. The total sample size is denoted by $n := \sum_{i=1}^k n_i$. In addition to the univariate heteroscedastic contrast testing problem, the functional data may be multidimensional in this setup, i.e. $p \geq 1$. Let $\mathbf{H} \in \mathbb{R}^{r\times pk}$ denote a known matrix. Furthermore, let $\boldsymbol{\eta} := (\boldsymbol{\eta}_1^\top, ..., \boldsymbol{\eta}_k^\top)^\top : \mathcal{T} \to \mathbb{R}^{pk}$ be the vector of all mean functions and $\mathbf{c} : \mathcal{T} \to \mathbb{R}^r$ be a fixed measurable function, e.g. $\mathbf{c}(t) = \mathbf{0}_{pk}$ for all $t \in \mathcal{T}$. In the following, we consider the null and alternative hypothesis

$$\mathcal{H}_0 : \mathbf{H}\boldsymbol{\eta}(t) = \mathbf{c}(t) \text{ for all } t \in \mathcal{T} \quad \text{vs.} \quad \mathcal{H}_1 : \mathbf{H}\boldsymbol{\eta}(t) \neq \mathbf{c}(t) \text{ for some } t \in \mathcal{T}. \tag{S2}$$

For constructing suitable tests for the general testing problem (S2), the following assumptions are required.

Assumption 1. *Consider the k independent samples (S1). Let $\mathbf{v}_{ij} = (\mathbf{v}_{ij,v})_{v\in\{1,...,p\}} := \mathbf{x}_{ij} - \boldsymbol{\eta}_i$, $i \in \{1, ..., k\}$, $j \in \{1, ..., n_i\}$, denote the subject-effect functions. Additionally, assume that the following conditions (M1)–(M6) are fulfilled for all $i \in \{1, ..., k\}$.*

(M1) The functional data $\mathbf{x}_{i1}, ..., \mathbf{x}_{in_i} \sim SP_p(\boldsymbol{\eta}_i, \boldsymbol{\Gamma}_i)$ are i.i.d. and take values in $\mathcal{L}_2^p(\mathcal{T})$.

(M2) The mean functions satisfy $\boldsymbol{\eta}_i \in \mathcal{L}_2^p(\mathcal{T})$.

(M3) The function $\mathcal{T} \ni t \mapsto \boldsymbol{\Gamma}_i(t,t)$ is continuous and $\boldsymbol{\Gamma}_i(t,t)$ is positive definite for all $t \in \mathcal{T}$.

(M4) There exists a $\tau_i > 0$ such that $\frac{n_i}{n} \to \tau_i$ as $n \to \infty$.

(M5) It holds $\mathrm{E}\left[\left(\int_{\mathcal{T}} \mathbf{v}_{i1}^\top(t)\mathbf{v}_{i1}(t)\,\mathrm{d}t\right)^2\right] < \infty$.

(M6) For all $j \in \{1, 2\}$, it holds

$$\mathrm{E}_{\text{out}}\left[\sup_{t,s\in\mathcal{T},\,|t-s|<\delta} |||\mathbf{v}_{i1}(t)\mathbf{v}_{ij}^\top(t) - \mathbf{v}_{i1}(s)\mathbf{v}_{ij}^\top(s)|||\right] \to 0 \quad \textit{as } \delta \to 0,$$

where here and throughout $|||.|||$ denotes the spectral norm.

These assumptions are equivalent to the assumptions (A1)–(A6) in the main paper for the univariate case $p = 1$ and, thus, are natural extensions for the multivariate case. In the following remark, we discuss their practical meaning.

Remark S1. *(1) Assumptions (M1)-(M4) ensure that the mean function estimators are asymptotically Gaussian and (M1), (M3) and (M6) guarantee the uniform consistency of the covariance estimators. Additionally, (M5) is needed for the parametric bootstrap consistency, c.f. Section S3.*

(2) We have $\int_{\mathcal{T}} \boldsymbol{\Gamma}_{i,q_1q_1}(t,t)\,\mathrm{d}t < \infty$ and $\int_{\mathcal{T}^2} (\boldsymbol{\Gamma}_{i,q_1q_2}(t,s))^2\,\mathrm{d}(t,s) < \infty$ for all $i \in \{1, ..., k\}$, $q_1, q_2 \in \{1, ..., p\}$ under (M3). Here and throughout, $\mathbf{A}_{q_1q_2}$ denotes the entry (q_1, q_2) of the matrix $\mathbf{A}$.

(3) By applying the Cauchy-Schwarz inequality, it can be shown that (M6) is satisfied if

$$\mathrm{E}_{\text{out}}\left[\sup_{t\in\mathcal{T}} ||\mathbf{v}_{i1}(t)||\right] < \infty, \qquad \mathrm{E}_{\text{out}}\left[\sup_{t,s\in\mathcal{T},\,|t-s|<\delta} ||\mathbf{v}_{i1}(t) - \mathbf{v}_{i1}(s)||\right] \to 0$$

$$\textit{and} \quad \mathrm{E}_{\text{out}}\left[\sup_{t,s\in\mathcal{T},\,|t-s|<\delta} |||\mathbf{v}_{i1}(t)\mathbf{v}_{i1}'(t) - \mathbf{v}_{i1}(s)\mathbf{v}_{i1}'(s)|||\right] \to 0 \quad \textit{as } \delta \to 0$$

hold for all $i \in \{1, ..., k\}$. This is particularly satisfied if $\mathrm{E}_{\mathrm{out}}\left[\sup_{t\in\mathcal{T}}||\mathbf{v}_{i1}(t)||\right] < \infty$ *holds and* $\mathbf{v}_{i1}$ *takes values in an equicontinuous set of functions for all $i \in \{1, ..., k\}$.*

(4) The supremum in assumption (M6) and, therefore, in Remark S1 (3) can be replaced by the essential supremum. That is because we only need $\mathcal{L}_\infty(\mathcal{T})$-convergence of the covariance function estimators in the proofs, see Section S3 for details.

(5) Assumption (M6) implies that $(\mathbf{v}_{i1}(t)\mathbf{v}_{i1}'(t))_{t\in\mathcal{T}}$ is sample continuous with respect to the spectral norm for all $i \in \{1, ..., k\}$. Since the supremum of a continuous function is measurable, the outer expectation in (M6) can be replaced by the usual expectation. Due to the compactness of $\mathcal{T}$, $\mathbf{v}_{i1}$ is bounded almost surely for all $i \in \{1, ..., k\}$. Even the stronger result

$$\mathrm{E}\left[\sup_{t\in\mathcal{T}}||\mathbf{v}_{i1}(t)||_2^2\right] < \infty$$

follows from (M1) and (M6) for all $i \in \{1, ..., k\}$.

We define an unbiased estimator for the mean function of the ith group at t by

$$\widehat{\boldsymbol{\eta}}_i(t) := \frac{1}{n_i}\sum_{j=1}^{n_i}\mathbf{x}_{ij}(t)$$

and for the covariance function of the ith group at (t, s) by

$$\widehat{\boldsymbol{\Gamma}}_i(t,s) := \frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(\mathbf{x}_{ij}(t) - \widehat{\boldsymbol{\eta}}_i(t)\right)\left(\mathbf{x}_{ij}(s) - \widehat{\boldsymbol{\eta}}_i(s)\right)^\top \tag{S3}$$

for all $t, s \in \mathcal{T}, i \in \{1, ..., k\}$. Let

$$\widehat{\boldsymbol{\Lambda}}(t,s) := \bigoplus_{i=1}^{k}\left(\frac{n}{n_i}\widehat{\boldsymbol{\Gamma}}_i(t,s)\right)$$

be an estimator for

$$\boldsymbol{\Lambda}(t,s) := \bigoplus_{i=1}^{k}\left(\frac{1}{\tau_i}\boldsymbol{\Gamma}_i(t,s)\right) \in \mathbb{R}^{pk\times pk}$$

for all $t, s \in \mathcal{T}$, where $\bigoplus_{i=1}^k \mathbf{A}_i$ denotes the direct sum of the matrices $\mathbf{A}_1, ..., \mathbf{A}_k$. Then, the *point-wise Hotelling's T^2-test statistic* regarding the multivariate functional data setup is defined by

$$\mathrm{PH}_{n,\mathbf{H},\mathbf{c}}(t) := n(\mathbf{H}\widehat{\boldsymbol{\eta}}(t) - \mathbf{c}(t))^\top(\mathbf{H}\widehat{\boldsymbol{\Lambda}}(t,t)\mathbf{H}^\top)^+(\mathbf{H}\widehat{\boldsymbol{\eta}}(t) - \mathbf{c}(t)) \tag{S4}$$

for all $t \in \mathcal{T}$, where $\widehat{\boldsymbol{\eta}} := (\widehat{\boldsymbol{\eta}}_1^\top, ..., \widehat{\boldsymbol{\eta}}_k^\top)^\top$ denotes the vector of all mean function estimators. This leads to the *globalizing pointwise Hotelling's T^2-test* (GPH) *statistic* for the multivariate functional data setup, that is,

$$T_n(\mathbf{H},\mathbf{c}) := \int_{\mathcal{T}}\mathrm{PH}_{n,\mathbf{H},\mathbf{c}}(t)\,\mathrm{d}t.$$

The limiting distribution of the GPH statistic for the multivariate functional data setup under the null hypothesis is given by the following theorem.

Theorem S1. *Let $\mathbf{y} \sim GP_r(\mathbf{0}_r, \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top)$. Under (M1)-(M4), (M6) and the null hypothesis in (S2), we have*

$$T_n(\mathbf{H},\mathbf{c}) \xrightarrow{d} \int_{\mathcal{T}}\mathbf{y}^\top(t)(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{y}(t)\,\mathrm{d}t \tag{S5}$$

as $n \to \infty$.

We approximate the limiting distributions again via the parametric bootstrap approach. Hence, $\boldsymbol{\Gamma}_1, ..., \boldsymbol{\Gamma}_k$ are substituted by their estimators. Thus, the parametric bootstrap sample $\mathbf{x}_{i1}^{\mathcal{P}}, ..., \mathbf{x}_{in_i}^{\mathcal{P}} \sim GP_p(\mathbf{0}_p, \widehat{\boldsymbol{\Gamma}}_i), i \in \{1, ..., k\}$, conditionally on the functional data $(\mathbf{x}_{i1}, ..., \mathbf{x}_{in_i})_{i\in\{1,...,k\}}$ is obtained. In the following, we denote the parametric bootstrap counterparts of the statistics with a superscript $\mathcal{P}$. Then, we define *parametric bootstrap globalizing* *pointwise Hotelling's T^2-test statistic* for the multivariate functional data setup, that is,

$$T_n^{\mathcal{P}}(\mathbf{H}) := \int_{\mathcal{T}} \mathrm{PH}_{n,\mathbf{H}}^{\mathcal{P}}(t)\,\mathrm{d}t. \tag{S6}$$

The validity of the parametric bootstrap for the heteroscedastic linear hypothesis testing problem in the multivariate functional data setup is provided by the following theorem.

Theorem S2. *Let $\mathbf{y} \sim GP_r(\mathbf{0}_r, \mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top)$. Under (M1)–(M6), we have*

$$T_n^{\mathcal{P}}(\mathbf{H}) \xrightarrow{d^*} \int_{\mathcal{T}} \mathbf{y}^\top(t)(\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{y}(t)\,\mathrm{d}t$$

as $n \to \infty$.

Proposition S1. *Let $Q_{n,1-\alpha}^{\mathcal{P}}$ be as in Algorithm 1. Under null hypothesis in* (S2)*, (M1)–(M6), and $B = B(n) \to \infty$ as $n \to \infty$, we have* $\lim_{n\to\infty} \mathrm{P}_{\mathrm{out}}\left(T_n(\mathbf{H},\mathbf{c}) > Q_{n,1-\alpha}^{\mathcal{P}}\right) = \alpha$.

For the multiple testing problem, we interpret $\mathbf{H}$ as a block matrix, that is $\mathbf{H} = \left[\mathbf{H}_1^\top, \ldots, \mathbf{H}_R^\top\right]^\top$ for matrices $\mathbf{H}_\ell \in \mathbb{R}^{r_\ell \times k}$, and $\mathbf{c} = (\mathbf{c}_\ell)_{\ell\in\{1,\ldots,R\}} : \mathcal{T} \to \mathbb{R}^r$ as function with measurable component functions $\mathbf{c}_\ell : \mathcal{T} \to \mathbb{R}^{r_\ell}$ for all $\ell \in \{1,\ldots,R\}$, where $R, r_\ell \in \{1,\ldots,r\}$ such that $\sum_{l=1}^R r_\ell = r$. This leads to the multiple testing problem

$$\mathcal{H}_{0,\ell}: \ \mathbf{H}_\ell\boldsymbol{\eta}(t) = \mathbf{c}_\ell(t) \ \text{ for all } t \in \mathcal{T}, \qquad \text{for } \ell \in \{1,\ldots,R\}. \tag{S7}$$

To show the validity of the multiple testing procedure, we firstly need to investigate the joint distribution of different GPH statistics in more detail.

Theorem S3. *Let $\mathbf{y} = (\mathbf{y}_\ell)_{\ell\in\{1,\ldots,R\}} \sim GP_r(\mathbf{0}_r, \mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top)$. Under (M1)-(M4), (M6) and the null hypotheses in (S7), we have*

$$(T_n(\mathbf{H}_\ell, \mathbf{c}_\ell))_{\ell\in\{1,\ldots,R\}} \xrightarrow{d} \left(\int_{\mathcal{T}} \mathbf{y}_\ell^\top(t)\left(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top\right)^+\mathbf{y}_\ell(t)\,\mathrm{d}t\right)_{\ell\in\{1,\ldots,R\}}$$

as $n \to \infty$.

Furthermore, the following theorem ensures the parametric bootstrap consistency for the multiple testing problem.

Theorem S4. *Let $\mathbf{y} = (\mathbf{y}_\ell)_{\ell\in\{1,\ldots,R\}} \sim GP_r(\mathbf{0}_r, \mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top)$. Under (A1)-(A6), it holds*

$$\left(T_n^{\mathcal{P}}(\mathbf{H}_\ell)\right)_{\ell\in\{1,\ldots,R\}} \xrightarrow{d^*} \left(\int_{\mathcal{T}} \mathbf{y}_\ell^\top(t)\left(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top\right)^+\mathbf{y}_\ell(t)\,\mathrm{d}t\right)_{\ell\in\{1,\ldots,R\}}$$

as $n \to \infty$.

Based on these theoretical results, we can apply the same procedure as for the univariate functional data to approximate suitable critical values for the multiple testing problem. This yields an asymptotically valid multiple testing procedure, as shown by the following proposition.

Proposition S2. *Let $q_{\ell,\widetilde{\beta}}^{\mathcal{P}}$ be as in Section 5, $\mathcal{R} \subset \{1,\ldots,R\}$ denote the index set of true null hypotheses in* (S7) *and $B = B(n) \to \infty$ as $n \to \infty$. Under (A1)–(A6), we have*

$$\lim_{n\to\infty} \mathrm{P}_{\mathrm{out}}\left(\max_{\ell\in\mathcal{R}} T_n(\mathbf{H}_\ell, \mathbf{c}_\ell)/q_{\ell,\widetilde{\beta}}^{\mathcal{P}} > 1\right) \le \alpha.$$

S2. Auxiliary Theorems

In this section, some useful results are stated.
The first theorem provides a central limit theorem for triangular arrays of independent distributed random variables in a Hilbert space, which are not necessarily identical distributed. We will use this theorem in order to prove the consistency of the parametric bootstrap.

Theorem S5 (Kundu et al. [8])**.** *Let $\mathbb{H}$ be a Hilbert space with inner product $\langle .,. \rangle$, induced norm $||.||$ and countable complete orthonormal system $\{f_m \mid m \in M\}$ for a countable index set M. Furthermore, let $Z_{1,n}, \ldots, Z_{n,n}$ be independent random variables for all $n \in \mathbb{N}$ taking values in $\mathbb{H}$ and satisfying* $\mathrm{E}\left[||Z_{j,n}||^2\right] < \infty$ *for all $j \in \{1,\ldots,n\}, n \in \mathbb{N}$. Suppose*

(1) $\mathrm{E}\left[\langle Z_{j,n}, f_m\rangle\right] = 0$ *for all* $j \in \{1,...,n\}, n \in \mathbb{N}, m \in M$,

(2) $\sum\limits_{j=1}^{n} \mathrm{E}\left[\langle Z_{j,n}, f_m\rangle\langle Z_{j,n}, f_\ell\rangle\right] \to \sigma_{ml}^2$ *as* $n \to \infty$ *for some* $\sigma_{ml}^2 \geq 0$ *for all* $m, \ell \in M$,

(3) $\sum\limits_{m\in M}\sum\limits_{j=1}^{n} \mathrm{Var}\left(\langle Z_{j,n}, f_m\rangle\right) \to \sum\limits_{m\in M} \sigma_{mm}^2 < \infty$ *as* $n \to \infty$,

(4) $\sum_{j=1}^{n} \mathrm{E}\left[|\langle Z_{j,n}, f_m\rangle|^2 \mathbb{1}\{|\langle Z_{j,n}, f_m\rangle| > \varepsilon\}\right] \to 0$ *as* $n \to \infty$ *for all* $\varepsilon > 0, m \in M$.

Then, $\sum\limits_{j=1}^{n} Z_{j,n}$ *converges in distribution to a centered Gaussian variable* G *with covariance operator* C *as* $n \to \infty$, *where the covariance operator* C *is characterized by*

$$\langle Cf_\ell, f_m\rangle = \sigma_{ml}^2 \quad \textit{for all } m, l \in M. \tag{S8}$$

Proof of Theorem S5. This theorem follows immediately from Theorem 1.1 in [8, p. 266]. Therefore, note that the covariance operator C_n of $S_n := \sum\limits_{j=1}^{n} Z_{j,n}$ satisfies

$$\langle C_n f_m, f_\ell\rangle = \mathrm{E}\left[\langle S_n, f_m\rangle\langle S_n, f_\ell\rangle\right] = \sum_{j=1}^{n} \mathrm{E}\left[\langle Z_{j,n}, f_m\rangle\langle Z_{j,n}, f_\ell\rangle\right]$$

for all $n \in \mathbb{N}, m, \ell \in M$ by the independence and the centeredness of $Z_{1,n}, ..., Z_{n,n}$. □

Remark S2. *The Lindeberg's condition (4) in Theorem S5 can be replaced by the Lyapunov's condition: There exists a* $\delta > 0$ *such that*

$$\sum_{j=1}^{n} \mathrm{E}\left[|\langle Z_{j,n}, f_m\rangle|^{2+\delta}\right] \to 0 \quad \textit{as } n \to \infty \quad \textit{for all } m \in M.$$

In order to prove this, we firstly note that $|\langle Z_{j,n}, f_m\rangle| > \varepsilon$ *implies* $|\langle Z_{j,n}, f_m\rangle|^\delta/\varepsilon^\delta > 1$. *Hence, it follows*

$$\begin{aligned}\sum_{j=1}^{n} \mathrm{E}\left[|\langle Z_{j,n}, f_m\rangle|^2 \mathbb{1}\{|\langle Z_{j,n}, f_m\rangle| > \varepsilon\}\right] &\leq \frac{1}{\varepsilon^\delta}\sum_{j=1}^{n} \mathrm{E}\left[|\langle Z_{j,n}, f_m\rangle|^{2+\delta} \mathbb{1}\{|\langle Z_{j,n}, f_m\rangle| > \varepsilon\}\right] \\ &\leq \frac{1}{\varepsilon^\delta}\sum_{j=1}^{n} \mathrm{E}\left[|\langle Z_{j,n}, f_m\rangle|^{2+\delta}\right] \to 0\end{aligned}$$

as $n \to \infty$ *for all* $\varepsilon > 0, m \in M$.

In the following, useful results about Wishart matrices are stated.

Lemma S1. *Let* $p \in \mathbb{N}$ *with* $p \leq n$ *and* $\mathbf{W}_n \sim \mathcal{W}_p(\mathbf{I}_p, n)$. *Then, we have*

$$\lim_{n\to\infty} n^r \mathrm{E}\left[\mathrm{tr}\left(\mathbf{W}_n^{-1}\right)^r\right] = p^r$$

for all $r \in \mathbb{N}$. *Here and throughout,* $\mathcal{W}_p(\mathbf{I}_p, n)$ *denotes the Wishart distribution with scale matrix* $\mathbf{I}_p$ *and* n *degrees of freedom and* tr *denotes the trace of a matrix.*

Proof of Lemma S1. It holds

$$\mathrm{tr}\left(\mathbf{W}_n^{-1}\right)^r = \mathrm{tr}\left(\bigotimes^r \mathbf{W}_n^{-1}\right)$$

for all $n, r \in \mathbb{N}$, where $\bigotimes^r \mathbf{A} = \mathbf{A} \otimes ... \otimes \mathbf{A}$ denotes the r-fold Kronecker product. Hence, we aim to prove

$$n^r \mathrm{E}\left[\bigotimes^r \mathbf{W}_n^{-1}\right] \to \mathbf{I}_{p^r} \quad \text{as } n \to \infty \tag{S9}$$

for all $r \in \mathbb{N}$ by induction. From (S9), the statement of the lemma follows easily by

$$\operatorname{tr}\left(n^r \mathrm{E}\left[\bigotimes^r \mathbf{W}_n^{-1}\right]\right) \to p^r < \infty$$

as $n \to \infty$ for all $r \in \mathbb{N}$.
For the base case, let $r = 1$. By Theorem 3.1 in [16], it holds

$$n\mathrm{E}\left[\mathbf{W}_n^{-1}\right] = \frac{n}{n-p-1}\mathbf{I}_p$$

for $n > p + 1$. Hence, (S9) follows for $r = 1$.
For the induction step, assume that (S9) holds for some $r \in \mathbb{N}$. Let $\operatorname{vec} : \mathbb{R}^{p^r \times p^r} \to \mathbb{R}^{p^{2r}}$ denote the vectorization function. Then, (4.2) in [16] implies

$$\operatorname{vec}\left(n^{r+1}\mathrm{E}\left[\bigotimes^{r+1} \mathbf{W}_n^{-1}\right]\right) = \left[\frac{n-p-1}{n}\mathbf{I}_{p^{2(r+1)}} - \frac{1}{n}\sum_{i=1}^{r-1}\mathbf{P}_{p,r}(i)\right]^{-1} \operatorname{vec}\left(n^r \mathrm{E}\left[\bigotimes^r \mathbf{W}_n^{-1}\right] \otimes \mathbf{I}_p\right)$$

for $n > p + 2r + 1$. Here, $\mathbf{P}_{p,r}(i) := \mathbf{P}_1(i) + \mathbf{P}_2(i) \in \mathbb{R}^{p^{2(r+1)}}$ with $\mathbf{P}_1(i), \mathbf{P}_2(i)$ as in [16] for all $i \in \{1, ..., r\}$. Note that $\mathbf{P}_{p,r}(i)$ particularly do not depend on n for all $i \in \{1, ..., r\}$. Hence, it follows

$$\operatorname{vec}\left(n^{r+1}\mathrm{E}\left[\bigotimes^{r+1} \mathbf{W}_n^{-1}\right]\right) \to \operatorname{vec}\left(\mathbf{I}_{p^r} \otimes \mathbf{I}_p\right) = \operatorname{vec}\left(\mathbf{I}_{p^{r+1}}\right)$$

as $n \to \infty$ by the induction hypothesis. Thus, (S9) also holds for $r + 1$. □

Applying some basic inequalities for matrices yield an asymptotic result for the rth moment of the norm of an inverse Wishart matrix.

Corollary S1. *Let $p \in \mathbb{N}$ with $p \leq n$ and $\mathbf{W}_n \sim \mathcal{W}_p(\mathbf{I}_p, n)$. Then, we have*

$$\limsup_{n\to\infty} n^r \mathrm{E}\left[|||\mathbf{W}_n^{-1}|||^r\right] \leq p^r$$

for all $r \in \mathbb{N}$.

Proof of Corollary S1. First of all, we can bound the spectral norm by the frobenius norm, i.e.

$$|||\mathbf{W}_n^{-1}||| \leq \sqrt{\operatorname{tr}(\mathbf{W}_n^{-2})}$$

for all $n \in \mathbb{N}$. Let $\lambda_1(\mathbf{W}_n), ..., \lambda_p(\mathbf{W}_n)$ denote the eigenvalues of $\mathbf{W}_n$. Since W_n is almost surely positive definite, it holds

$$\operatorname{tr}(\mathbf{W}_n^{-2}) = \sum_{i=1}^p \lambda_i(\mathbf{W}_n)^{-2} \leq \left(\sum_{i=1}^p \lambda_i(\mathbf{W}_n)^{-1}\right)^2 = \operatorname{tr}\left(\mathbf{W}_n^{-1}\right)^2$$

almost surely for all $n \in \mathbb{N}$. Hence, it follows

$$|||\mathbf{W}_n^{-1}|||^r \leq \operatorname{tr}(\mathbf{W}_n^{-2})^{r/2} \leq \operatorname{tr}\left(\mathbf{W}_n^{-1}\right)^r$$

almost surely for all $n, r \in \mathbb{N}$. Thus, we have

$$\limsup_{n\to\infty} n^r \mathrm{E}\left[|||\mathbf{W}_n^{-1}|||^r\right] \leq \lim_{n\to\infty} n^r \mathrm{E}\left[\operatorname{tr}\left(\mathbf{W}_n^{-1}\right)^r\right] = p^r < \infty$$

for all $r \in \mathbb{N}$ by Lemma S1. □

Lemma S2. *Let $p \in \mathbb{N}$ and $\mathbf{W}_n \sim \mathcal{W}_p(\mathbf{I}_p, n)$, where here and throughout $\mathcal{W}_p(\mathbf{I}_p, n)$ denotes the Wishart distribution with scale matrix $\mathbf{I}_p$ and n degrees of freedom. Then, we have*

$$\lim_{n\to\infty} \mathrm{E}\left[|||\mathbf{W}_n/n - \mathbf{I}_p|||^4\right] = 0.$$

Proof of Lemma S2. For all symmetric $\mathbf{A} \in \mathbb{R}^{p\times p}$ with eigenvalues $\lambda_1(\mathbf{A}^2), ..., \lambda_p(\mathbf{A}^2)$ of $\mathbf{A}^2 = \mathbf{A}^\top\mathbf{A}$, we have

$$|||\mathbf{A}|||^4 = \max_{i\in\{1,\dots,p\}} \lambda_i(\mathbf{A}^2)^2 \leq \sum_{i=1}^{p} \lambda_i(\mathbf{A}^2)^2 = \sum_{i=1}^{p} \lambda_i(\mathbf{A}^4) = \operatorname{tr}(\mathbf{A}^4).$$

Hence, it follows

$$\mathrm{E}\left[|||\mathbf{W}_n/n - \mathbf{I}_p|||^4\right] \leq \operatorname{tr}\left(\mathrm{E}\left[(\mathbf{W}_n/n - \mathbf{I}_p)^4\right]\right)$$

for all $n \in \mathbb{N}$. By the binomial theorem, we get

$$\mathrm{E}\left[(\mathbf{W}_n/n - \mathbf{I}_p)^4\right] = \sum_{k=0}^{4}(-1)^{4-k}\binom{4}{k}n^{-k}\mathrm{E}\left[\mathbf{W}_n^k\right]$$

for all $n \in \mathbb{N}$. Section 2 of [7] provides $n^{-k}\mathrm{E}\left[\mathbf{W}_n^k\right] \to \mathbf{I}_p$ for all $k \in \{0, 1, 2, 3, 4\}$ and, therewith, it follows

$$\mathrm{E}\left[(\mathbf{W}_n/n - \mathbf{I}_p)^4\right] \to \sum_{k=0}^{4}(-1)^{4-k}\binom{4}{k}\mathbf{I}_p = \mathbf{0}_{p\times p}$$

as $n \to \infty$, where $\mathbf{0}_{p\times p}$ denotes the $(p \times p)$ zero matrix. Hence, the statement of the lemma follows. ☐

The following lemma is a rather technical result about the norm of a Moore-Penrose inverse.

Lemma S3. *Let $r, k \in \mathbb{N}$ and $\mathbf{H} \in \mathbb{R}^{r\times k}$. Furthermore, let $\mathbf{A} \in \mathbb{R}^{k\times k}$ be a symmetric positive definite matrix. Then, we have*

$$|||(\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+||| \leq |||\mathbf{H}^+|||^2|||\mathbf{A}^{-1}|||,$$

where here and throughout $|||.|||$ denotes the spectral norm.

Proof of Lemma S3. Obviously, $\mathbf{H}\mathbf{A}\mathbf{H}^\top$ and, therefore, its Moore-Penrose inverse is symmetric and positive semidefinite. Let $\lambda_1(\mathbf{B}) \geq \lambda_2(\mathbf{B}) \geq ...$ denote the ordered eigenvalues of a matrix $\mathbf{B}$. Then, it holds

$$|||(\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+||| = \lambda_1\left((\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+\right).$$

The nonzero eigenvalues of $(\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+$ are given as inverses of the nonzero eigenvalues of $(\mathbf{H}\mathbf{A}\mathbf{H}^\top)$. Hence, $\lambda_1\left((\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+\right)$ is the inverse of the smallest nonzero eigenvalue of $(\mathbf{H}\mathbf{A}\mathbf{H}^\top)$. Thus, for $h := \operatorname{rank}(\mathbf{H}) = \operatorname{rank}(\mathbf{H}\mathbf{A}\mathbf{H}^\top)$, it holds

$$|||(\mathbf{H}\mathbf{A}\mathbf{H}^\top)^+||| = 1/\lambda_h\left(\mathbf{H}\mathbf{A}\mathbf{H}^\top\right).$$

Furthermore, we have

$$\begin{aligned}
\lambda_h\left(\mathbf{H}\mathbf{A}\mathbf{H}^\top\right) &= \min_{\mathbf{x}\in\ker(\mathbf{H}^\top)^\perp\setminus\{\mathbf{0}_r\}} \frac{\mathbf{x}^\top\mathbf{H}\mathbf{A}\mathbf{H}^\top\mathbf{x}}{\mathbf{x}^\top\mathbf{x}} \\
&\geq \min_{\mathbf{y}\neq\mathbf{0}_k}\frac{\mathbf{y}^\top\mathbf{A}\mathbf{y}}{\mathbf{y}^\top\mathbf{y}} \min_{\mathbf{x}\in\ker(\mathbf{H}^\top)^\perp\setminus\{\mathbf{0}_r\}} \frac{\mathbf{x}^\top\mathbf{H}\mathbf{H}^\top\mathbf{x}}{\mathbf{x}^\top\mathbf{x}} \\
&= \lambda_k(\mathbf{A})\lambda_h(\mathbf{H}\mathbf{H}^\top).
\end{aligned}$$

Since $|||\mathbf{H}^+|||^2 = 1/\lambda_h(\mathbf{H}\mathbf{H}^\top)$ and $|||\mathbf{A}^{-1}||| = 1/\lambda_k(\mathbf{A})$, the lemma is proved. ☐

The next lemma provides that the distribution function of our limit distributions will be continuous and strictly monotone.

Lemma S4. *Let $\mathbf{x} \sim GP_r(\mathbf{0}_r, \mathbf{\Lambda})$ be a Gaussian process in $\mathcal{L}_2^r(\mathcal{T})$ with covariance function $\mathbf{\Lambda} : \mathcal{T}^2 \to \mathbb{R}^{r\times r}$ satisfying $\int_\mathcal{T} \operatorname{tr}(\mathbf{\Lambda}(t,t))\, \mathrm{d}t > 0$. Then, the distribution function of $||\mathbf{x}||_2^2 := \int_\mathcal{T} \mathbf{x}^\top(t)\mathbf{x}(t)\, \mathrm{d}t$ is continuous on $\mathbb{R}$ and strictly increasing on $[0, \infty)$.*

Proof of Lemma S4. We proceed similarly as in [3]. By Lemma 1.1 in [3], $\mathrm{P}^{\mathbf{x}}$ is log-concave. Let us define $A_u := \{\boldsymbol{x} \in \mathcal{L}_2^r(\mathcal{T}) \mid ||\boldsymbol{x}||_2^2 \leq u^2\}$ for $u \geq 0$. Then, we have $\lambda A_u + (1-\lambda)A_s \subset A_{\lambda u+(1-\lambda)s}$ for all $\lambda \in [0, 1], u, s \geq 0$ since

$$||\lambda\boldsymbol{y} + (1-\lambda)\boldsymbol{z}||_2^2 \leq (\lambda||\boldsymbol{y}||_2 + (1-\lambda)||\boldsymbol{z}||_2)^2 \leq (\lambda u + (1-\lambda)s)^2$$

for all $\boldsymbol{y} \in A_u, \boldsymbol{z} \in A_s, \lambda \in [0, 1], u, s \geq 0$. Moreover, we have $\mathrm{P}^{\mathbf{x}}(A_u) > 0$ for all $u > 0$ by [10, p. 61]. As in the proof of Lemma 1.2 in [3], we obtain that $(0, \infty) \ni u \mapsto F(u) := \mathrm{P}^{\mathbf{x}}(A_u)$ is log-concave. Hence, F is continuous on $(0,\infty)$. Since $F(u)=0$ for $u<0$, it remains to show $F(0)=0$ to obtain continuity on all of $\mathbb{R}$. Therefore, let $\{\mathbf{f}_m \mid m\in\mathbb{N}\}$ denote a countable and complete orthonormal system of $\mathcal{L}_2^r(\mathcal{T})$. Then, Parseval's identity [6, p. 245] yields

$$F(0)=\mathrm{P}(||\mathbf{x}||_2^2=0)=\mathrm{P}\left(\sum_{m=1}^{\infty}\langle\mathbf{x},\mathbf{f}_m\rangle^2=0\right)\leq\mathrm{P}\left(\langle\mathbf{x},\mathbf{f}_{\tilde{m}}\rangle=0\right)\quad\text{for all } \tilde{m}\in\mathbb{N}.$$

Due to the Gaussianity of $\mathbf{x}$, we have that $\langle\mathbf{x},\mathbf{f}_{\tilde{m}}\rangle$ is centered normal distributed with variance $\sigma_{\tilde{m}}^2:=\mathrm{E}[\langle\mathbf{x},\mathbf{f}_{\tilde{m}}\rangle^2]$. Parseval's identity [6, p. 245] and Fubini's theorem imply

$$\mathrm{E}\left[||\mathbf{x}||_2^2\right]=\mathrm{E}\left[\sum_{m=1}^{\infty}\langle\mathbf{x},\mathbf{f}_m\rangle^2\right]=\sum_{m=1}^{\infty}\sigma_m^2$$

Since Fubini's theorem also implies

$$\mathrm{E}\left[||\mathbf{x}||_2^2\right]=\mathrm{E}\left[\int_{\mathcal{T}}\mathbf{x}^\top(t)\mathbf{x}(t)\ \mathrm{d}t\right]=\int_{\mathcal{T}}\mathrm{E}\left[\mathbf{x}^\top(t)\mathbf{x}(t)\right]\ \mathrm{d}t=\int_{\mathcal{T}}\mathrm{tr}\left(\mathrm{E}\left[\mathbf{x}(t)\mathbf{x}^\top(t)\right]\right)\ \mathrm{d}t=\int_{\mathcal{T}}\mathrm{tr}\left(\mathbf{\Lambda}(t,t)\right)\ \mathrm{d}t>0,$$

there is at least one $\tilde{m}\in\mathbb{N}$ such that $\sigma_{\tilde{m}}^2>0$. By choosing this $\tilde{m}\in\mathbb{N}$, we obtain $F(0)\leq\mathrm{P}\left(\langle\mathbf{x},\mathbf{f}_{\tilde{m}}\rangle=0\right)=0$.

The strict monotony follows as in the proof of Corollary 1.3 in [3]. □

Finally, we show that FWER in Lemma S7 in the supplement of [14] is strictly increasing in the application in the proof of Proposition S2.

Lemma S5. *Let $T_\ell:=\int_{\mathcal{T}}(\mathbf{z}(t))^\top\boldsymbol{A}_\ell(t)\mathbf{z}(t)\ \mathrm{d}t, \ell\in\{1,...,R\}$, for a Gaussian process $\mathbf{z}\sim GP_r(\mathbf{0}_r,\mathbf{\Lambda})$ in $\mathcal{L}_2^r(\mathcal{T})$, where $\boldsymbol{A}_1,...,\boldsymbol{A}_R:\mathcal{T}\to\mathbb{R}^{r\times r}$ are essentially bounded measurable functions with $\boldsymbol{A}_\ell(t)$ being a symmetric positive semi-definite matrix satisfying $\int_{\mathcal{T}}\mathrm{tr}\left(\boldsymbol{A}_\ell(t)\mathbf{\Lambda}(t,t)\right)\ \mathrm{d}t>0$ for all $\ell\in\{1,...,R\}$. Moreover, let $F:\mathbb{R}^R\to[0,1]$ denote the cumulative distribution function of $(T_1,...,T_R)$ and $F_\ell:\mathbb{R}\to[0,1], \ell\in\{1,...,R\}$, denote the marginal distribution functions. Then,*

$$(0,1]\ni\zeta\mapsto\mathrm{FWER}(\zeta):=1-F\left(F_1^{-1}(1-\zeta),\ldots,F_L^{-1}(1-\zeta)\right)$$

is strictly increasing.

Proof of Lemma S5. First of all, note that $F_\ell:\mathbb{R}\to[0,1], \ell\in\{1,...,R\}$, are continuous and strictly increasing on $[0,\infty)$ by Lemma S4 with $\mathbf{x}(t)=\boldsymbol{A}_\ell^{1/2}(t)\mathbf{z}(t)$. Let $0<x<y\leq 1$. We aim to show $\mathrm{FWER}(y)-\mathrm{FWER}(x)>0$. We have

$$\begin{aligned}&\mathrm{FWER}(y)-\mathrm{FWER}(x)\\&=F\left(F_1^{-1}(1-x),...,F_R^{-1}(1-x)\right)-F\left(F_1^{-1}(1-y),...,F_R^{-1}(1-y)\right)\\&=\mathrm{P}\left(\forall\ell\in\{1,...,R\}:T_\ell\leq F_\ell^{-1}(1-x)\right)-\mathrm{P}\left(\forall\ell\in\{1,...,R\}:T_\ell\leq F_\ell^{-1}(1-y)\right).\end{aligned}$$

We proceed similarly to [3]. Lemma 1.1 in [3] implies that $\mathrm{P}^{\mathbf{z}}$ is log-concave. Let us define

$$A_u:=\left\{\boldsymbol{z}\in\mathcal{L}_2^r(\mathcal{T})\mid\forall\ell\in\{1,...,R\}:||\boldsymbol{A}_\ell^{1/2}\boldsymbol{z}||_2^2\ \mathrm{d}t\leq u^2F_\ell^{-1}(1-x)\right\}\quad\text{for all } u>0.$$

Now, we aim to show $\lambda A_u+(1-\lambda)A_s\subset A_{\lambda u+(1-\lambda)s}$ for all $u,s>0,\lambda\in[0,1]$. Therefore, let $\boldsymbol{x}\in A_u,\boldsymbol{y}\in A_s,\lambda\in[0,1]$. Then, we have

$$\begin{aligned}\left|\left|\boldsymbol{A}_\ell^{1/2}(\lambda\boldsymbol{x}+(1-\lambda)\boldsymbol{y})\right|\right|_2^2&\leq\left(\lambda\left|\left|\boldsymbol{A}_\ell^{1/2}\boldsymbol{x}\right|\right|_2+(1-\lambda)\left|\left|\boldsymbol{A}_\ell^{1/2}\boldsymbol{y}\right|\right|_2\right)^2\\&\leq(\lambda u+(1-\lambda)s)^2F_\ell^{-1}(1-x)\end{aligned}$$

for all $\ell\in\{1,...,R\}$. Hence, it follows $\lambda\boldsymbol{x}+(1-\lambda)\boldsymbol{y}\in A_{\lambda u+(1-\lambda)s}$. Moreover, we have $\mathrm{P}^{\mathbf{z}}\left(A_u\right)>0$ for all $u>0$ by [10, p. 61]. Hence, it follows that $(0,\infty)\ni u\mapsto\log(\mathrm{P}^{\mathbf{z}}\left(A_u\right))\in(-\infty,0]$ is concave as in Lemma 1.2 in [3]. Let $u:=\max_{\ell\in\{1,...,R\}}\sqrt{F_\ell^{-1}(1-y)/F_\ell^{-1}(1-x)}<1$. Assume that

$$\begin{aligned}0&=\mathrm{FWER}(y)-\mathrm{FWER}(x)\\&=\mathrm{P}\left(\forall\ell\in\{1,...,R\}:T_\ell\leq F_\ell^{-1}(1-x)\right)-\mathrm{P}\left(\forall\ell\in\{1,...,R\}:T_\ell\leq F_\ell^{-1}(1-y)\right).\end{aligned}$$

By the choice of u, it follows

$$0 = \mathrm{FWER}(y) - \mathrm{FWER}(x) \geq \mathrm{P}^{\mathbf{z}}(A_1) - \mathrm{P}^{\mathbf{z}}(A_u) \geq 0.$$

Hence, we have $\log(\mathrm{P}^{\mathbf{z}}(A_1)) = \log(\mathrm{P}^{\mathbf{z}}(A_u))$ and the concavity implies $\log(\mathrm{P}^{\mathbf{z}}(A_t)) \leq \log(\mathrm{P}^{\mathbf{z}}(A_1))$ for all $t > 1$. Thus, we obtain $\lim_{t\to\infty} \mathrm{P}^{\mathbf{z}}(A_t) \leq \mathrm{P}^{\mathbf{z}}(A_1)$. Note that

$$\mathrm{P}^{\mathbf{z}}(A_1) = \mathrm{P}\left(\forall \ell \in \{1, ..., R\} : T_\ell \leq F_\ell^{-1}(1-x)\right) \leq \mathrm{P}\left(T_1 \leq F_1^{-1}(1-x)\right) = F_1\left(F_1^{-1}(1-x)\right) = 1 - x < 1.$$

However, the essentially boundedness of $\boldsymbol{A}_1, ..., \boldsymbol{A}_R$ implies that $\lim_{t\to\infty} \mathrm{P}^{\mathbf{z}}(A_t) = 1$, which yields the contradiction. Hence, we obtain $\mathrm{FWER}(y) - \mathrm{FWER}(x) > 0$. □

S3. Proofs

In this section, the technical proofs of the stated theorems are presented. Note that the theorems and propositions of the main paper are special cases of the theorems and propositions of the multivariate case in Section S1 with $p = 1$. Thus, we only prove the statements of Section S1.
Throughout this section, we consider the Hilbert space $\mathcal{L}_2^p(\mathcal{T})$ with induced norm $||.||_2$ and countable complete orthonormal system $\{\mathbf{f}_m \mid m \in \mathbb{N}\}$.

First of all, we state an useful lemma, which gives us the convergence of the covariance function estimators uniformly over $\{(t,t) \in \mathcal{T}^2 \mid t \in \mathcal{T}\}$ in outer probability.

Lemma S6. *Under (M1), (M3) and (M6), we have*

$$\sup_{t\in\mathcal{T}} \left|\left|\left|\widehat{\mathbf{\Gamma}}_i(t,t) - \mathbf{\Gamma}_i(t,t)\right|\right|\right| \xrightarrow{P} 0 \quad \text{as } n_i \to \infty$$

for all $i \in \{1, ..., k\}$, where here and throughout $\xrightarrow{P}$ denotes convergence in outer probability.

Proof of Lemma S6. We aim to apply Theorem 2.1 in [15]. Let $i \in \{1, ..., k\}$ be fixed but arbitrary. The pointwise convergence

$$\widehat{\mathbf{\Gamma}}_i(t,t) \xrightarrow{P} \mathbf{\Gamma}_i(t,t) \quad \text{as } n \to \infty$$

is well known for all $t \in \mathcal{T}$ under (M1), which can be proven by applying the weak law of large numbers. Since $\mathcal{T}$ is a compact interval and $\mathcal{T} \ni t \mapsto \mathbf{\Gamma}_i(t,t)$ is continuous under (M3), it is also uniformly continuous and, thus, equicontinuous. Hence, the stochastic equicontinuity of $\mathcal{T} \ni t \mapsto \widehat{\mathbf{\Gamma}}_i(t,t)$ remains to show. For that purpose, let $\varepsilon, \eta > 0$ and $\delta > 0$ such that

$$\mathrm{E}_{\mathrm{out}}\left[\sup_{t,s\in\mathcal{T},\, |t-s|<\delta} |||\mathbf{v}_{i1}(t)\mathbf{v}_{ij}^\top(t) - \mathbf{v}_{i1}(s)\mathbf{v}_{ij}^\top(s)|||\right] \leq \frac{\eta\varepsilon}{6} \tag{S10}$$

holds for $j \in \{1, 2\}$. By (M6), such a $\delta > 0$ exists. In the following, we just write $|t - s| < \delta$ in the index of the supremum for the sake of clarity. Inserting the definition of the covariance function estimator (S3) and applying the triangular inequality yields

$$\begin{aligned}\sup_{|t-s|<\delta} \left|\left|\left|\widehat{\mathbf{\Gamma}}_i(t,t) - \widehat{\mathbf{\Gamma}}_i(s,s)\right|\right|\right| \leq{}& \frac{1}{n_i - 1}\sum_{j=1}^{n_i} \sup_{|t-s|<\delta} |||\mathbf{v}_{ij}(t)\mathbf{v}_{ij}^\top(t) - \mathbf{v}_{ij}(s)\mathbf{v}_{ij}^\top(s)||| \\ &+ \frac{1}{(n_i-1)n_i}\sum_{j_1,j_2=1}^{n_i} \sup_{|t-s|<\delta} |||\mathbf{v}_{ij_1}(t)\mathbf{v}_{ij_2}^\top(t) - \mathbf{v}_{ij_1}(s)\mathbf{v}_{ij_2}^\top(s)|||.\end{aligned}$$

By Markov's inequality, (M1) and (S10), it follows

$$\begin{aligned}\mathbb{P}_{\mathrm{out}}\left(\sup_{|t-s|<\delta} \left|\left|\left|\widehat{\mathbf{\Gamma}}_i(t,t) - \widehat{\mathbf{\Gamma}}_i(s,s)\right|\right|\right| > \varepsilon\right) &\leq \mathrm{E}_{\mathrm{out}}\left[\sup_{|t-s|<\delta} \left|\left|\left|\widehat{\mathbf{\Gamma}}_i(t,t) - \widehat{\mathbf{\Gamma}}_i(s,s)\right|\right|\right|\right] / \varepsilon \\ &\leq \frac{n_i}{n_i - 1}\frac{\eta}{6} + \frac{n_i^2}{(n_i-1)n_i}\frac{\eta}{6} \to \frac{\eta}{3} < \eta\end{aligned}$$

as $n \to \infty$. The lemma is proven by applying Theorem 2.1 in [15]. □

The following lemma ensures that $\mathbf{H}\widehat{\boldsymbol{\eta}}$ is asymptotically Gaussian.

Lemma S7. *Under (M1)-(M4), we have*

$$\sqrt{n}(\mathbf{H}\widehat{\boldsymbol{\eta}} - \mathbf{c}) \xrightarrow{d} \mathbf{y} \sim GP_r\left(\mathbf{0}_r, \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top\right) \quad \text{as } n \to \infty$$

in $\mathcal{L}_2^r(\mathcal{T})$.

Proof of Lemma S7. Let $i \in \{1, ..., k\}$ be arbitrary but fixed. Then, the subject-effect functions $\mathbf{v}_{i1}, ..., \mathbf{v}_{in_i}$ take values in $\mathcal{L}_2^p(\mathcal{T})$ under (M1) and (M2). Furthermore, it holds by Fubini‘s theorem

$$\mathrm{E}\left[||\mathbf{v}_{i1}||_2^2\right] = \int_\mathcal{T} \mathrm{E}\left[\mathrm{tr}\left(\mathbf{v}_{i1}^\top(t)\mathbf{v}_{i1}(t)\right)\right] \,\mathrm{d}t = \int_\mathcal{T} \mathrm{tr}\left(\mathrm{E}\left[\mathbf{v}_{i1}(t)\mathbf{v}_{i1}^\top(t)\right]\right) \,\mathrm{d}t = \int_\mathcal{T} \mathrm{tr}\left(\boldsymbol{\Gamma}_i(t,t)\right) \,\mathrm{d}t.$$

Since $\mathcal{T} \ni t \mapsto \mathrm{tr}(\boldsymbol{\Gamma}_i(t,t))$ is continuous on the compact set $\mathcal{T}$ under (M3), it attains its maximum. In particular, we get

$$\mathrm{E}\left[||\mathbf{v}_{i1}||_2^2\right] = \int_\mathcal{T} \mathrm{tr}\left(\boldsymbol{\Gamma}_i(t,t)\right) \,\mathrm{d}t < \infty. \tag{S11}$$

Hence, Theorem 7.5.1 in [9] provides

$$\sqrt{n_i}(\widehat{\boldsymbol{\eta}}_i - \boldsymbol{\eta}_i) = \frac{1}{\sqrt{n_i}} \sum_{j=1}^{n_i} \mathbf{v}_{ij} \xrightarrow{d} GP_p(\mathbf{0}_p, \boldsymbol{\Gamma}_i) \quad \text{as } n_i \to \infty$$

in $\mathcal{L}_2^p(\mathcal{T})$ under (M1)-(M3). By Slutsky‘s lemma as in [18, p. 20], it follows

$$\sqrt{n}(\widehat{\boldsymbol{\eta}}_i - \boldsymbol{\eta}_i) = \frac{\sqrt{n}}{\sqrt{n_i}} \sqrt{n_i}(\widehat{\boldsymbol{\eta}}_i - \boldsymbol{\eta}_i) \xrightarrow{d} GP_p(\mathbf{0}_p, \tau_i^{-1}\boldsymbol{\Gamma}_i) \quad \text{as } n \to \infty \tag{S12}$$

in $\mathcal{L}_2^p(\mathcal{T})$ under (A4). Due to the independence of the samples of the different groups and the continuous mapping theorem as in [18, p. 32], the statement of the lemma holds. □

With the two previous lemmas, we are now able to prove Theorem S1.

Proof of Theorem S1. Lemma S6 provides that $\widehat{\boldsymbol{\Lambda}}$ converges uniformly over $\{(t,t) \in \mathcal{T}^2 \mid t \in \mathcal{T}\}$ in probability to $\boldsymbol{\Lambda}$. In the following, we also write $\widehat{\boldsymbol{\Lambda}}(t)$ and $\boldsymbol{\Lambda}(t)$ instead of $\widehat{\boldsymbol{\Lambda}}(t,t)$ and $\boldsymbol{\Lambda}(t,t)$, respectively, for all $t \in \mathcal{T}$. Furthermore, Lemma S7 ensures the convergence in distribution of $\sqrt{n}(\mathbf{H}\widehat{\boldsymbol{\eta}} - \mathbf{c})$ to $\mathbf{y}$.
Let $\mathbb{D}$ denote the set of all continuous functions $\widetilde{\boldsymbol{\Lambda}} : \mathcal{T} \to \mathbb{R}^{pk \times pk}$ with $\widetilde{\boldsymbol{\Lambda}}(t)$ symmetric and positive semidefinite for all $t \in \mathcal{T}$ and be equipped with the $\mathcal{L}_\infty^{pk\times pk}(\mathcal{T})$-norm. Then, $\boldsymbol{\Lambda} \in \mathbb{D}$ holds due to (M3) and (M4). Moreover, similarly to Remark 1 (5), $\widehat{\boldsymbol{\Lambda}}$ takes values in $\mathbb{D}$ almost surely.
Furthermore, the map

$$h : \mathcal{L}_2^r(\mathcal{T}) \times \mathbb{D} \ni (\widetilde{\mathbf{x}}, \widetilde{\boldsymbol{\Lambda}}) \mapsto \int_\mathcal{T} \widetilde{\mathbf{x}}^\top(t) \left(\mathbf{H}\widetilde{\boldsymbol{\Lambda}}(t)\mathbf{H}^\top\right)^+ \widetilde{\mathbf{x}}(t) \,\mathrm{d}t \in \mathbb{R}_0^+ \cup \{\infty\}$$

is continuous on $\mathcal{L}_2^r(\mathcal{T}) \times \{\boldsymbol{\Lambda}\}$. By the continuous mapping theorem, it suffices to show that the map

$$h : \mathcal{L}_2^r(\mathcal{T}) \times \mathbb{D} \ni (\widetilde{\mathbf{x}}, \widetilde{\boldsymbol{\Lambda}}) \mapsto \int_\mathcal{T} \widetilde{\mathbf{x}}^\top(t) \left(\mathbf{H}\widetilde{\boldsymbol{\Lambda}}(t)\mathbf{H}^\top\right)^+ \widetilde{\mathbf{x}}(t) \,\mathrm{d}t \in \mathbb{R}_0^+ \cup \{\infty\}$$

is continuous on $\mathcal{L}_2^r(\mathcal{T}) \times \{\boldsymbol{\Lambda}\}$. Consequently, let $\widetilde{\mathbf{x}}_\infty, \widetilde{\mathbf{x}}_1, \widetilde{\mathbf{x}}_2, ... \in \mathcal{L}_2^r(\mathcal{T})$ and $\widetilde{\boldsymbol{\Lambda}}_1, \widetilde{\boldsymbol{\Lambda}}_2, ... \in \mathbb{D}$ be functions with $\widetilde{\mathbf{x}}_n \to \widetilde{\mathbf{x}}_\infty$ in $\mathcal{L}_2^r(\mathcal{T})$ and $\widetilde{\boldsymbol{\Lambda}}_n \to \boldsymbol{\Lambda}$ in $\mathcal{L}_\infty^{pk\times pk}(\mathcal{T})$ as $n \to \infty$. The triangular inequality implies

$$\begin{aligned}\left|h(\widetilde{\mathbf{x}}_n, \widetilde{\boldsymbol{\Lambda}}_n) - h(\widetilde{\mathbf{x}}_\infty, \boldsymbol{\Lambda})\right| \leq{}& \left|\left|\widetilde{\mathbf{x}}_n^\top \left(\left(\mathbf{H}\widetilde{\boldsymbol{\Lambda}}_n\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top\right)^+\right) \widetilde{\mathbf{x}}_n\right|\right|_1 \\ &+ \left|\left|\widetilde{\mathbf{x}}_n^\top \left(\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top\right)^+ (\widetilde{\mathbf{x}}_n - \widetilde{\mathbf{x}}_\infty)\right|\right|_1 + \left|\left|(\widetilde{\mathbf{x}}_n - \widetilde{\mathbf{x}}_\infty)^\top \left(\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top\right)^+ \widetilde{\mathbf{x}}_\infty\right|\right|_1\end{aligned}$$

for all $n \in \mathbb{N}$. We observe that

$$||\mathbf{a}^\top\mathbf{A}\mathbf{b}||_1 = \int_\mathcal{T} \left|\mathbf{a}^\top(t)\mathbf{A}(t)\mathbf{b}(t)\right| \,\mathrm{d}t \leq \int_\mathcal{T} ||\mathbf{a}(t)||\, ||\mathbf{b}(t)||\, |||\mathbf{A}(t)||| \,\mathrm{d}t \leq ||\mathbf{a}||_2||\mathbf{b}||_2|||\mathbf{A}|||_\infty$$

holds for all measurable functions $\mathbf{A} : \mathcal{T} \to \mathbb{R}^{r\times r}$ and $\mathbf{a}, \mathbf{b} \in \mathcal{L}_2^r(\mathcal{T})$ by the Cauchy-Schwarz inequality and the submultiplicativity of the spectral norm, where here and throughout $|||.|||_\infty$ denotes the $\mathcal{L}_\infty^{r\times r}(\mathcal{T})$-norm. The second triangular inequality yields $||\widetilde{\mathbf{x}}_n||_2 \to ||\widetilde{\mathbf{x}}_\infty||_2 < \infty$ as $n \to \infty$. Hence, it remains to show

$$\left|\left|\left|\left(\mathbf{H}\widetilde{\mathbf{\Lambda}}_n\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top\right)^+\right|\right|\right|_\infty \to 0 \quad \text{as } n \to \infty \tag{S13}$$

to conclude the continuity of h.
Since $\mathbf{\Lambda}(t)$ is invertible for all $t \in \mathcal{T}$ and $\mathbf{\Lambda}$ is continuous, we can find an $\varepsilon > 0$ such that every $\mathbf{K} \in \mathbb{R}^{r\times r}$ with $|||\mathbf{K} - \mathbf{\Lambda}(t)||| \leq \varepsilon$ for some $t \in \mathcal{T}$ is invertible. We consider the set

$$\mathcal{K} := \{\mathbf{H}\widetilde{\mathbf{\Lambda}}\mathbf{H}^\top \in \mathbb{R}^{r\times r} \mid \widetilde{\mathbf{\Lambda}} \in \mathbb{R}^{pk\times pk}, \inf_{t\in\mathcal{T}} |||\widetilde{\mathbf{\Lambda}} - \mathbf{\Lambda}(t)||| \leq \varepsilon\}.$$

Since the function $j : \mathbb{R}^{pk\times pk} \ni \widetilde{\mathbf{\Lambda}} \mapsto \inf_{t\in\mathcal{T}} |||\widetilde{\mathbf{\Lambda}} - \mathbf{\Lambda}(t)||| \in \mathbb{R}$ is continuous, it follows that the set $j^{-1}([0,\varepsilon])$ is a closed set. Moreover, it is bounded since $\sup_{t\in\mathcal{T}} |||\mathbf{\Lambda}(t)||| < \infty$. Thus, $\mathcal{K}$ is compact as continuous image of a compact set. Hence, the continuous map $\mathcal{K} \ni \mathbf{K} \mapsto \mathbf{K}^+ \in \mathbb{R}^{r\times r}$ is uniformly continuous on $\mathcal{K}$. Let $\eta > 0$ be arbitrary and $\delta > 0$ such that $|||\mathbf{K}_1^+ - \mathbf{K}_2^+||| < \eta$ holds for all $\mathbf{K}_1, \mathbf{K}_2 \in \mathcal{K}$ with $|||\mathbf{K}_1 - \mathbf{K}_2||| < \delta$. Furthermore, let $N \in \mathbb{N}$ such that $\left|\left|\left|\widetilde{\mathbf{\Lambda}}_n - \mathbf{\Lambda}\right|\right|\right|_\infty < \min\{\varepsilon, \delta/|||\mathbf{H}|||^2\}$ for all $n \geq N$. Hence, it holds

$$\left|\left|\left|\mathbf{H}\widetilde{\mathbf{\Lambda}}_n(t)\mathbf{H}^\top - \mathbf{H}\mathbf{\Lambda}(t)\mathbf{H}^\top\right|\right|\right| = \left|\left|\left|\mathbf{H}(\widetilde{\mathbf{\Lambda}}_n(t) - \mathbf{\Lambda}(t))\mathbf{H}^\top\right|\right|\right| \leq |||\mathbf{H}|||^2 \left|\left|\left|\widetilde{\mathbf{\Lambda}}_n(t) - \mathbf{\Lambda}(t)\right|\right|\right| < \delta$$

by the submultiplicativity of the spectral norm and $\mathbf{H}\widetilde{\mathbf{\Lambda}}_n(t)\mathbf{H}^\top \in \mathcal{K}$ for all $n \geq N$ and almost every $t \in \mathcal{T}$ and, therefore, $\left|\left|\left|(\mathbf{H}\widetilde{\mathbf{\Lambda}}_n\mathbf{H}^\top)^+ - (\mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top)^+\right|\right|\right|_\infty < \eta$ for all $n \geq N$. As a result, (S13) follows and the theorem is proven. □

In the following, we generally condition on the functional data

$$(\mathbf{x}_{i1}, \mathbf{x}_{i2}, ...)_{i\in\{1,...,k\}}, \tag{S14}$$

if not otherwise specified.

The following lemma gives an analogue of Lemma S7 for the parametric bootstrap counterpart of the sample mean function.

Lemma S8. *Under (M1)-(M5), we have*

$$\sqrt{n}\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}} \xrightarrow{d^*} \mathbf{y} \sim GP_r\left(\mathbf{0}_r, \mathbf{H}\mathbf{\Lambda}\mathbf{H}^\top\right) \quad \textit{as } n \to \infty$$

in $\mathcal{L}_2^r(\mathcal{T})$.

Proof of Lemma S8. Let $i \in \{1, ..., k\}$ be arbitrary but fixed. We aim to apply Theorem S5 conditionally on the functional data (S14) with $\mathbf{Z}_{j,n_i} := \frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}}$ for all $j \in \{1, ..., n_i\}$. Then, $\mathbf{Z}_{1,n_i}, ..., \mathbf{Z}_{n_i,n_i}$ are i.i.d. conditionally on (S14) by construction in Algorithm 1. Moreover, Fubini's theorem provides

$$\begin{aligned}
\mathrm{E}^*\left[||\mathbf{Z}_{1,n_i}||_2^2\right] &= \frac{1}{n_i}\mathrm{E}^*\left[\int_{\mathcal{T}} (\mathbf{x}_{i1}^{\mathcal{P}}(t))^\top \mathbf{x}_{i1}^{\mathcal{P}}(t)\,\mathrm{d}t\right] = \frac{1}{n_i}\int_{\mathcal{T}} \mathrm{E}^*\left[tr\left((\mathbf{x}_{i1}^{\mathcal{P}}(t))^\top \mathbf{x}_{i1}^{\mathcal{P}}(t)\right)\right]\,\mathrm{d}t \\
&= \frac{1}{n_i}\int_{\mathcal{T}} tr\left(\mathrm{E}^*\left[\mathbf{x}_{i1}^{\mathcal{P}}(t)(\mathbf{x}_{i1}^{\mathcal{P}}(t))^\top\right]\right)\,\mathrm{d}t = \frac{1}{n_i}\int_{\mathcal{T}} tr\left(\widehat{\mathbf{\Gamma}}_i(t,t)\right)\,\mathrm{d}t < \infty
\end{aligned}$$

almost surely since the functional data take values in $\mathcal{L}_2^p(\mathcal{T})$ under (M1), where here and throughout E^* denotes the conditional expectation given the functional data (S14). It should be noted that all following statements about conditional expectations hold just almost surely but we will not always add this throughout, for the sake of clarity. Hence, $\mathbf{Z}_{1,n_i}, ..., \mathbf{Z}_{n_i,n_i}$ take values in $\mathcal{L}_2^p(\mathcal{T})$ conditionally on the functional data (S14).
Assumption (1) of Theorem S5 is satisfied conditionally on (S14) since Fubini's theorem provides

$$\mathrm{E}^*\left[\left\langle \frac{1}{\sqrt{n_i}}\mathbf{x}_{i1}^{\mathcal{P}}, \mathbf{f}_m\right\rangle\right] = \int_{\mathcal{T}} \frac{1}{\sqrt{n_i}}\mathrm{E}^*\left[(\mathbf{x}_{i1}^{\mathcal{P}}(t))^\top\right]\mathbf{f}_m(t)\,\mathrm{d}t = 0$$

for all $m \in \mathbb{N}$.
Furthermore, due to the identical distribution of the parametric bootstrap observations and Fubini's theorem,

we have

$$
\begin{aligned}
\sum_{j=1}^{n_i} \mathrm{E}^* \left[\left\langle \frac{1}{\sqrt{n_i}} \mathbf{x}_{ij}^{\mathcal{P}}, \mathbf{f}_m \right\rangle \left\langle \frac{1}{\sqrt{n_i}} \mathbf{x}_{ij}^{\mathcal{P}}, \mathbf{f}_\ell \right\rangle \right] &= \mathrm{E}^* \left[\langle \mathbf{x}_{i1}^{\mathcal{P}}, \mathbf{f}_m \rangle \langle \mathbf{x}_{i1}^{\mathcal{P}}, \mathbf{f}_\ell \rangle \right] \\
&= \mathrm{E}^* \left[\int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \mathbf{x}_{i1}^{\mathcal{P}}(t) (\mathbf{x}_{i1}^{\mathcal{P}}(s))^\top \mathbf{f}_l(s) \,\mathrm{d}(t,s) \right] \\
&= \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \mathrm{E}^* \left[\mathbf{x}_{i1}^{\mathcal{P}}(t) (\mathbf{x}_{i1}^{\mathcal{P}}(s))^\top \right] \mathbf{f}_l(s) \,\mathrm{d}(t,s) \\
&= \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \widehat{\boldsymbol{\Gamma}}_i(t,s) \mathbf{f}_l(s) \,\mathrm{d}(t,s)
\end{aligned}
$$

for all $m, \ell \in \mathbb{N}$. It holds

$$
\begin{aligned}
&\int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \widehat{\boldsymbol{\Gamma}}_i(t,s) \mathbf{f}_l(s) \,\mathrm{d}(t,s) \\
&= \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) (\mathbf{x}_{ij} - \widehat{\boldsymbol{\eta}}_i)(t) (\mathbf{x}_{ij} - \widehat{\boldsymbol{\eta}}_i)^\top(s) \mathbf{f}_l(s) \,\mathrm{d}(t,s) \\
&= \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \langle \mathbf{f}_m, \mathbf{x}_{ij} - \widehat{\boldsymbol{\eta}}_i \rangle \langle \mathbf{f}_l, \mathbf{x}_{ij} - \widehat{\boldsymbol{\eta}}_i \rangle \\
&= \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \left(\langle \mathbf{f}_m, \mathbf{x}_{ij} \rangle - \frac{1}{n_i} \sum_{j_2=1}^{n_i} \langle \mathbf{f}_m, \mathbf{x}_{ij_2} \rangle \right) \left(\langle \mathbf{f}_l, \mathbf{x}_{ij} \rangle - \frac{1}{n_i} \sum_{j_2=1}^{n_i} \langle \mathbf{f}_l, \mathbf{x}_{ij_2} \rangle \right)
\end{aligned}
$$

for all $m, \ell \in \mathbb{N}$. Thus, the term $\int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \widehat{\boldsymbol{\Gamma}}_i(t,s) \mathbf{f}_l(s) \,\mathrm{d}(t,s)$ can be thought of as the sample covariance of $\langle \mathbf{f}_m, \mathbf{x}_{i1} \rangle$ and $\langle \mathbf{f}_l, \mathbf{x}_{i1} \rangle$. Hence, assumption (2) of Theorem S5 follows almost surely due to the strong consistency of the sample covariance, where σ_{ml}^2 denotes the covariance of $\langle \mathbf{f}_m, \mathbf{x}_{i1} \rangle$ and $\langle \mathbf{f}_l, \mathbf{x}_{i1} \rangle$ for all $m, \ell \in \mathbb{N}$. By Fubini‘s theorem, it holds

$$
\sigma_{ml}^2 = \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \boldsymbol{\Gamma}_i(t,s) \mathbf{f}_l(s) \,\mathrm{d}(t,s) \tag{S15}
$$

for all $m, \ell \in \mathbb{N}$ with similar calculations as above.
Analogously, we can calculate

$$
\begin{aligned}
\sum_{m \in \mathbb{N}} \sum_{j=1}^{n_i} \mathbb{V}ar^* \left(\left\langle \frac{1}{\sqrt{n_i}} \mathbf{x}_{ij}^{\mathcal{P}}, \mathbf{f}_m \right\rangle \right) &= \sum_{m \in \mathbb{N}} \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \widehat{\boldsymbol{\Gamma}}_i(t,s) \mathbf{f}_m(s) \,\mathrm{d}(t,s) \\
&= \sum_{m \in \mathbb{N}} \frac{1}{n_i - 1} \sum_{j=1}^{n_i} \langle \mathbf{f}_m, \mathbf{x}_{ij} - \widehat{\boldsymbol{\eta}}_i \rangle^2,
\end{aligned} \tag{S16}
$$

where here and throughout $\mathbb{V}ar^*$ denotes the conditional variance given the functional data (S14). Hence, it follows

$$
\begin{aligned}
\mathrm{E} \left[\sum_{m \in \mathbb{N}} \sum_{j=1}^{n_i} \mathbb{V}ar^* \left(\left\langle \frac{1}{\sqrt{n_i}} \mathbf{x}_{ij}^{\mathcal{P}}, \mathbf{f}_m \right\rangle \right) \right] &= \sum_{m \in \mathbb{N}} \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \boldsymbol{\Gamma}_i(t,s) \mathbf{f}_m(s) \,\mathrm{d}(t,s) \\
&= \sum_{m \in \mathbb{N}} \sigma_{mm}^2
\end{aligned} \tag{S17}
$$

by Fubini‘s theorem. The sum is finite since Fubini‘s theorem, the monotone convergence theorem and Parseval’s identity (6, p. 245) yield

$$
\begin{aligned}
\sum_{m \in \mathbb{N}} \sigma_{mm}^2 &= \sum_{m \in \mathbb{N}} \int_{\mathcal{T}^2} \mathbf{f}_m^\top(t) \mathrm{E} \left[\mathbf{v}_{i1}(t) \mathbf{v}_{i1}^\top(s) \right] \mathbf{f}_m(s) \,\mathrm{d}(t,s) \\
&= \mathrm{E} \left[\sum_{m \in \mathbb{N}} \langle \mathbf{f}_m, \mathbf{v}_{i1} \rangle^2 \right] \\
&= \mathrm{E} \left[||\mathbf{v}_{i1}||_2^2 \right] < \infty
\end{aligned} \tag{S18}
$$

due to (S11). In order to prove assumption (3) of Theorem S5 in probability conditionally on (S14), it remains to show that the variance of (S16) vanishes asymptotically. For the second moment of (S16), we have

$$\mathrm{E}\left[\left(\sum_{m\in\mathbb{N}}\sum_{j=1}^{n_i}\mathbb{V}ar^*\left(\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle\right)\right)^2\right]=\mathrm{E}\left[\left(\sum_{m\in\mathbb{N}}\frac{1}{n_i-1}\sum_{j=1}^{n_i}\langle\mathbf{f}_m,\mathbf{x}_{ij}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right)^2\right].$$

Rearranging the summands yields

$$\mathrm{E}\left[\left(\sum_{m\in\mathbb{N}}\sum_{j=1}^{n_i}\mathrm{Var}^*\left(\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle\right)\right)^2\right]=\frac{1}{(n_i-1)^2}\sum_{j_1,j_2=1}^{n_i}\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{ij_2}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right].$$

Here, the rearrangement is valid since all summands are non-negative. By using the Parseval's identity [6, p. 245] and (A1), the expectations on the right side can be expressed for $j_1=j_2\in\{1,...,n_i\}$ by

$$\mathrm{E}\left[\left(\sum_{m\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right)^2\right]=\mathrm{E}\left[||\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i||_2^4\right]=\mathrm{E}\left[||\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i||_2^4\right].\tag{S19}$$

By applying the triangular inequality, we get

$$||\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i||_2=\left|\left|\frac{n_i-1}{n_i}(\mathbf{x}_{i1}-\boldsymbol{\eta}_i)-\frac{1}{n_i}\sum_{j=2}^{n_i}(\mathbf{x}_{ij}-\boldsymbol{\eta}_i)\right|\right|_2\le\sum_{j=1}^{n_i}a_j||\mathbf{x}_{ij}-\boldsymbol{\eta}_i||_2,$$

where $a_1:=(n_i-1)/n_i$ and $a_j:=1/n_i$ for all $j\in\{2,...,n_i\}$. Under (A1) and (A5), Hölder‘s inequality implies

$$\mathrm{E}\left[||\mathbf{x}_{ij_1}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_2}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_3}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_4}-\boldsymbol{\eta}_i||_2\right]\le\mathrm{E}\left[||\mathbf{x}_{i1}-\boldsymbol{\eta}_i||_2^4\right]<\infty$$

for all $j_1,j_2,j_3,j_4\in\{1,...,n_i\}$. Hence, the expectation in (S19) can be bounded uniformly over n_i since

$$\begin{aligned}\mathrm{E}\left[||\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i||_2^4\right]&\le\sum_{j_1,j_2,j_3,j_4=1}^{n_i}a_{j_1}a_{j_2}a_{j_3}a_{j_4}\mathrm{E}\left[||\mathbf{x}_{ij_1}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_2}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_3}-\boldsymbol{\eta}_i||_2\,||\mathbf{x}_{ij_4}-\boldsymbol{\eta}_i||_2\right]\\&\le\left(\sum_{j=1}^{n_i}a_j\right)^4\mathrm{E}\left[||\mathbf{x}_{i1}-\boldsymbol{\eta}_i||_2^4\right]\le 2^4\mathrm{E}\left[||\mathbf{x}_{i1}-\boldsymbol{\eta}_i||_2^4\right]=16\mathrm{E}\left[||\mathbf{x}_{i1}-\boldsymbol{\eta}_i||_2^4\right]\end{aligned}$$

by definition of $a_1,...,a_{n_i}$. Thus, it follows that the sum of the expectations with $j_1=j_2\in\{1,...,n_i\}$, that is

$$\frac{1}{(n_i-1)^2}\sum_{j_1=1}^{n_i}\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right]=\frac{n_i}{(n_i-1)^2}\mathrm{E}\left[||\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i||_2^4\right],\tag{S20}$$

vanishes asymptotically as $n\to\infty$.
Furthermore, we have for the expectations with $j_1\neq j_2, j_1,j_2\in\{1,...,n_i\}$,

$$\begin{aligned}\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{ij_1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{ij_2}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right]&=\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{i2}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right]\\&=\sum_{m,\ell\in\mathbb{N}}\mathrm{E}\left[\langle\mathbf{f}_m,\mathbf{v}_{i1}-(\widehat{\boldsymbol{\eta}}_i-\boldsymbol{\eta}_i)\rangle^2\langle\mathbf{f}_l,\mathbf{v}_{i2}-(\widehat{\boldsymbol{\eta}}_i-\boldsymbol{\eta}_i)\rangle^2\right]\end{aligned}$$

due to (M1) and the monotone convergence theorem. By simplifying and applying the Parseval‘s identity [6,

p. 245], one can show

$$\begin{aligned}
\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{i2}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right] &\to \sum_{m,\ell\in\mathbb{N}}\mathrm{E}\left[\langle\mathbf{f}_m,\mathbf{v}_{i1}\rangle^2\langle\mathbf{f}_l,\mathbf{v}_{i2}\rangle^2\right]\\
&=\left(\mathrm{E}\left[\sum_{m\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{v}_{i1}\rangle^2\right]\right)^2 \qquad \text{(S21)}\\
&=\left(\sum_{m\in\mathbb{N}}\sigma_{mm}^2\right)^2
\end{aligned}$$

as $n\to\infty$ under (M5), where the last equality follows as in (S18). For the sake of clarity, we will not describe the calculation further.

Summing up the calculations above, we get

$$\begin{aligned}
\mathrm{E}\left[\left(\sum_{m\in\mathbb{N}}\sum_{j=1}^{n_i}\mathrm{Var}^*\left(\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle\right)\right)^2\right] &= o(1)+\frac{n_i}{n_i-1}\mathrm{E}\left[\sum_{m,\ell\in\mathbb{N}}\langle\mathbf{f}_m,\mathbf{x}_{i1}-\widehat{\boldsymbol{\eta}}_i\rangle^2\langle\mathbf{f}_l,\mathbf{x}_{i2}-\widehat{\boldsymbol{\eta}}_i\rangle^2\right]\\
&\to\left(\sum_{m\in\mathbb{N}}\sigma_{mm}^2\right)^2
\end{aligned}$$

as $n\to\infty$ by (S20) and (S21), where and throughout $o(1)$ denotes convergence to 0. Since

$$\left(\sum_{m\in\mathbb{N}}\sigma_{mm}^2\right)^2=\mathrm{E}\left[\sum_{m\in\mathbb{N}}\sum_{j=1}^{n_i}\mathrm{Var}^*\left(\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle\right)\right]^2$$

holds by (S17), we can conclude that the variance of (S16) vanishes asymptotically. Hence, assumption (3) of Theorem S5 holds in probability conditionally on (S14).

Finally, we aim to show assumption (4) of Theorem S5 by using Remark S2 with $\delta=2$. By applying the Cauchy-Schwarz inequality several times and Fubini‘s theorem, one can show

$$\sum_{j=1}^{n_i}\mathrm{E}^*\left[\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle^4\right]\le\frac{1}{n_i}\left(\sum_{v=1}^{p}\int_{\mathcal{T}}\sqrt{\mathrm{E}^*\left[(\mathbf{x}_{i1,v}^{\mathcal{P}}(t))^4\right]}\,\mathrm{d}t\right)^2$$

with $\mathbf{x}_{ij}^{\mathcal{P}}=\left(\mathbf{x}_{ij,v}^{\mathcal{P}}\right)_{v\in\{1,\dots,p\}}$ for all $m\in\mathbb{N}$. Since $\mathbf{x}_{i1,v}^{\mathcal{P}}(t)$ is normally distributed with mean 0 and variance $\widehat{\boldsymbol{\Gamma}}_{i,vv}(t,t)$ conditionally on (S14) for all $v\in\{1,\dots,p\}$, it follows

$$\sum_{j=1}^{n_i}\mathrm{E}^*\left[\left\langle\frac{1}{\sqrt{n_i}}\mathbf{x}_{ij}^{\mathcal{P}},\mathbf{f}_m\right\rangle^4\right]\le\frac{1}{n_i}\left(\sqrt{3}\sum_{v=1}^{p}\int_{\mathcal{T}}\widehat{\boldsymbol{\Gamma}}_{i,vv}(t,t)\,\mathrm{d}t\right)^2$$

for all $m\in\mathbb{N}$. Hence, Lemma S6 and (M3) yield Lyapunov‘s condition in Remark S2 in probability conditionally on (S14) with $\delta=2$.

Assumptions (3) and (4) of Theorem S5 hold in probability conditionally on (S14). Thus, we can find for every subsequence of increasing sample sizes n_i a further subsequence such that (3) and (4) hold almost surely along the latter subsequence by applying the subsequence criterion. Consequently, Theorem S5 provides that $\sqrt{n_i}\widehat{\boldsymbol{\eta}}_i^{\mathcal{P}}=\sum_{j=1}^{n_i}\mathbf{Z}_{j,n_i}$ converges in distribution to a centered Gaussian process with covariance function $\boldsymbol{\Gamma}_i$ conditionally on (S14) almost surely along the latter subsequence as $n\to\infty$. Here, the covariance structure of the limiting process follows from the characterization (S8) and equation (S15).

Since $\sqrt{n_i}\widehat{\boldsymbol{\eta}}_i^{\mathcal{P}}, i\in\{1,\dots,k\}$, are independent conditionally on (S14), it follows with Slutsky's lemma that

$$\sqrt{n}\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}=\mathbf{H}\left(\frac{\sqrt{n}}{\sqrt{n_i}}\sqrt{n_i}\widehat{\boldsymbol{\eta}}_i^{\mathcal{P}}\right)_{i\in\{1,\dots,k\}}$$

converges in distribution to $\mathbf{y}\sim GP_r(\mathbf{0}_r,\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^\top)$ conditionally on (S14) almost surely along the specified subsequence as $n \to \infty$. Hence, by applying the subsequence criterion again, it holds

$$\sqrt{n}\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}} \xrightarrow{d^*} \mathbf{y}$$

as $n \to \infty$. $\square$

Theorem S2 can now be proved by using Lemma S8.

Proof of Theorem S2. By Lemma S8 and the subsequence criterion, every subsequence has a further subsequence such that $\sqrt{n}\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}$ converges in distribution almost surely along the subsequence to $\mathbf{y}$ in $\mathcal{L}_2^r(\mathcal{T})$. Applying the continuous mapping theorem yields that

$$\int_{\mathcal{T}} n \left(\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+ \mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t$$

converges in distribution almost surely along the latter subsequence to $\int_{\mathcal{T}} \mathbf{y}^\top(t)(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{y}(t)\,\mathrm{d}t$. Hence, by applying the subsequence criterion again, it follows

$$\int_{\mathcal{T}} n \left(\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+ \mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t \xrightarrow{d^*} \int_{\mathcal{T}} \mathbf{y}^\top(t)(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{y}(t)\,\mathrm{d}t$$

as $n \to \infty$.
Consequently, it remains to show

$$D_n^{\mathcal{P}}(\mathbf{H}) := \left| T_n^{\mathcal{P}}(\mathbf{H}) - \int_{\mathcal{T}} n \left(\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+ \mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t \right| \xrightarrow{P^*} 0 \tag{S22}$$

as $n \to \infty$, where $\xrightarrow{P^*}$ denotes convergence in outer probability conditionally on (S14). By the triangular inequality, Hölder's inequality and the submultiplicativity, one can show

$$\begin{aligned} D_n^{\mathcal{P}}(\mathbf{H}) &\leq \int_{\mathcal{T}} n \left| \left(\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top \left(\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+ \right) \mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t) \right| \,\mathrm{d}t \\ &\leq |||\mathbf{H}|||^2 \,||\sqrt{n}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}||_4^2 \left(\int_{\mathcal{T}} \left|\left|\left| \left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+ \right|\right|\right|^2 \,\mathrm{d}t \right)^{1/2}. \end{aligned}$$

Hence, by Markov's inequality and the Cauchy-Schwarz inequality, we have

$$\mathrm{P}^*_{\mathrm{out}}\left(D_n^{\mathcal{P}}(\mathbf{H}) > \varepsilon\right) \leq \varepsilon^{-1}\,|||\mathbf{H}|||^2 \sqrt{\mathrm{E}^*_{\mathrm{out}}\left[||\sqrt{n}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}||_4^4\right]}\cdot \tag{S23}$$

$$\sqrt{\mathrm{E}^*_{\mathrm{out}}\left[\int_{\mathcal{T}} \left|\left|\left| \left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - (\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top)^+ \right|\right|\right|^2 \,\mathrm{d}t\right]} \tag{S24}$$

for all $\varepsilon > 0$, where here and throughout $\mathrm{P}^*_{\mathrm{out}}$ denotes the outer probability and $\mathrm{E}^*_{\mathrm{out}}$ denotes the outer expectation given the functional data (S14), respectively. For all $t \in \mathcal{T}$, $i \in \{1,...,k\}$, it holds $\widehat{\boldsymbol{\eta}}_i^{\mathcal{P}}(t) \sim \mathcal{N}_p\left(\mathbf{0}_p, \widehat{\boldsymbol{\Gamma}}_i(t,t)/n_i\right)$ conditional on the data (S14). In the following, let $\widehat{w}_{i,1}(t), ..., \widehat{w}_{i,p}(t)$ denote the eigenvalues of $\widehat{\boldsymbol{\Gamma}}_i(t,t)/n_i$ for all $i \in \{1,...,k\}$. Then,

$$||\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)||^4 = \left(\sum_{i=1}^k ||\widehat{\boldsymbol{\eta}}_i^{\mathcal{P}}(t)||^2\right)^2 \stackrel{d}{=} \left(\sum_{i=1}^k \sum_{v=1}^p \widehat{w}_{i,v}(t) Z_{i,v}^2\right)^2$$

conditional on the data (S14), where $Z_{1,1}, ..., Z_{k,p}$ are independent standard normal distributed and independent of the data. Thus, $||\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)||^2$ follows a generalized chi-square distribution. Hence, the expectation in (S23) can be calculated as

$$
\begin{aligned}
\mathrm{E}^*_{\text{out}}\left[\left|\left|\sqrt{n}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}\right|\right|_4^4\right] &= n^2 \int_{\mathcal{T}} \mathrm{E}^*\left[\left|\left|\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right|\right|^4\right] \,\mathrm{d}t \\
&= n^2 \int_{\mathcal{T}} \mathrm{E}^*\left[\left(\sum_{i=1}^{k}\sum_{v=1}^{p}\widehat{w}_{i,v}(t)Z_{i,v}^2\right)^2\right] \,\mathrm{d}t \\
&= n^2 \int_{\mathcal{T}} 2\sum_{i=1}^{k}\sum_{v=1}^{p}\widehat{w}_{i,v}(t)^2 + \left(\sum_{i=1}^{k}\sum_{v=1}^{p}\widehat{w}_{i,v}(t)\right)^2 \,\mathrm{d}t \qquad \text{(S25)}\\
&= n^2 \int_{\mathcal{T}} 2\sum_{i=1}^{k}\mathrm{tr}\left(\widehat{\boldsymbol{\Gamma}}_i(t,t)^2\right)/n_i^2 + \left(\sum_{i=1}^{k}\mathrm{tr}\left(\widehat{\boldsymbol{\Gamma}}_i(t,t)\right)/n_i\right)^2 \,\mathrm{d}t \\
&\xrightarrow{P} \int_{\mathcal{T}} 2\sum_{i=1}^{k}\mathrm{tr}\left(\widehat{\boldsymbol{\Gamma}}_i(t,t)^2\right)/\tau_i^2 + \left(\sum_{i=1}^{k}\mathrm{tr}\left(\widehat{\boldsymbol{\Gamma}}_i(t,t)\right)/\tau_i\right)^2 \,\mathrm{d}t < \infty
\end{aligned}
$$

as $n \to \infty$ by Fubini's theorem and Lemma S6. Thus, it remains to show that the expectation in (S24) vanishes asymptotically in probability. By Theorem 4.1 of [19, p. 221], it holds

$$
\begin{aligned}
&\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right| \\
&\leq \frac{1+\sqrt{5}}{2}\max\left\{\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^2, \left|\left|\left|\left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^2\right\}\left|\left|\left|\mathbf{H}\left(\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t) - \boldsymbol{\Lambda}(t,t)\right)\mathbf{H}^\top\right|\right|\right|
\end{aligned}
$$

for all $t \in \mathcal{T}$.
Since $\mathcal{T} \ni t \mapsto |||(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top)^+|||^2$ is a continuous function, it attains its maximum over the compact set $\mathcal{T}$. Hence, there exists a $C > 0$ such that $\left|\left|\left|\left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^2 \leq C$ for all $t \in \mathcal{T}$.
Fubini's theorem, the submultiplicativity and the Cauchy-Schwarz inequality imply

$$
\begin{aligned}
&\mathrm{E}^*_{\text{out}}\left[\int_{\mathcal{T}}\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\boldsymbol{\Lambda}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^2 \,\mathrm{d}t\right] \\
&\leq \left(\frac{1+\sqrt{5}}{2}\right)^2 |||\mathbf{H}|||^4 \sqrt{\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^8\right] + C^4 \,\mathrm{d}t}\cdot \qquad \text{(S26)}\\
&\sqrt{\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t) - \boldsymbol{\Lambda}(t,t)\right|\right|\right|^4\right] \,\mathrm{d}t}.
\end{aligned}
$$

Let

$$
B_n := \left\{\inf_{t\in\mathcal{T}}\min_{v\in\{1,\dots,p\}}\widehat{w}_{i,v}(t) > 0 \quad \forall i \in \{1,\dots,k\}\right\}.
$$

By Lemma S3, it holds

$$
\begin{aligned}
\mathrm{E}^*\left[\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^8\right] &\leq |||\mathbf{H}^+|||^{16}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)^{-1}\right|\right|\right|^8\right] \\
&\leq |||\mathbf{H}^+|||^{16}\sum_{i=1}^{k}\left(\frac{n_i}{n}\right)^8\mathrm{E}^*\left[|||\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)^{-1}|||^8\right]
\end{aligned}
$$

on B_n for all $t \in \mathcal{T}$ since $\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)$ is almost surely positive definite on B_n. It is well known that

$$
(n_i - 1)\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t) \sim \mathcal{W}_p\left(\mathbf{I}_p, n_i - 1\right) \qquad \text{(S27)}
$$

conditional on the data (S14) for all $t \in \mathcal{T}, i \in \{1,\dots,k\}$ with $\widehat{\boldsymbol{\Gamma}}_i(t,t)$ positive definite. Let $\mathbf{W}_{n_i-1} \sim$

$\mathcal{W}_p\left(\mathbf{I}_p, n_i-1\right)$. Then, we have

$$\begin{aligned}\mathrm{E}^*\left[|||\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)^{-1}|||^8\right] &\leq (n_i-1)^8|||\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)|||^2\mathrm{E}^*\left[\left|\left|\left|\left((n_i-1)\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)\right)^{-1}\right|\right|\right|^8\right]\\ &= (n_i-1)^8|||\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)|||^2\mathrm{E}^*\left[\left|\left|\left|\mathbf{W}_{n_i-1}^{-1}\right|\right|\right|^8\right]\end{aligned}$$

for all $t\in\mathcal{T}, i\in\{1,...,k\}$ on B_n. Therefore, we have

$$\begin{aligned}&\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^8\right]\,\mathrm{d}t\\ &\leq |||\mathbf{H}^+|||^{16}\sum_{i=1}^k\left(\frac{n_i}{n}\right)^8(n_i-1)^8\mathrm{E}^*\left[\left|\left|\left|\mathbf{W}_{n_i-1}^{-1}\right|\right|\right|^8\right]\int_{\mathcal{T}}|||\widehat{\boldsymbol{\Gamma}}_i^{-1/2}(t,t)|||^2\,\mathrm{d}t\end{aligned}$$

on B_n. Set $G:=|||\mathbf{H}^+|||^{16}\sum\limits_{i=1}^k\tau_i^8p^8\int_{\mathcal{T}}|||\boldsymbol{\Gamma}_i^{-1/2}(t,t)|||^2\,\mathrm{d}t$. Due to (M3), the smallest eigenvalue $\lambda_{\min}(t)$ of $\boldsymbol{\Gamma}_i(t,t)$ attains its minimum $\lambda_{\min}(t_0)>0$ for some $t_0\in\mathcal{T}$ and, thus, $|||\boldsymbol{\Gamma}_i^{-1/2}(t,t)|||^2=1/\lambda_{\min}(t)\leq 1/\lambda_{\min}(t_0)$. Hence, we have $G<\infty$. By Lemma S6, it holds $\mathrm{P}_{\mathrm{out}}\left(B_n\right)\to 1$. Hence, it follows

$$\mathrm{P}_{\mathrm{out}}\left(\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\left(\mathbf{H}\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^8\right]\,\mathrm{d}t>G+\delta\right)\to 0 \tag{S28}$$

as $n\to\infty$ for all $\delta>0$ by Corollary S1.
Furthermore, it can be shown

$$\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\boldsymbol{\Lambda}(t,t)\right|\right|\right|^4\right]\leq 16\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Lambda}}(t,t)\right|\right|\right|^4\right]+16\left|\left|\left|\widehat{\boldsymbol{\Lambda}}(t,t)-\boldsymbol{\Lambda}(t,t)\right|\right|\right|^4 \tag{S29}$$

for all $t\in\mathcal{T}$, where the second summand vanishes uniformly over $\mathcal{T}$ in probability as $n\to\infty$ due to Lemma S6. Moreover, we have

$$\begin{aligned}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Lambda}}(t,t)\right|\right|\right|^4\right]&\leq\sum_{i=1}^k\left(\frac{n}{n_i}\right)^4\mathrm{E}^*\left[|||\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Gamma}}_i(t,t)|||^4\right]\\ &\leq\sum_{i=1}^k\left(\frac{n}{n_i}\right)^4\mathrm{E}^*\left[|||\widehat{\boldsymbol{\Gamma}}_i(t,t)^{-1/2}\widehat{\boldsymbol{\Gamma}}_i^{\mathcal{P}}(t,t)\widehat{\boldsymbol{\Gamma}}_i(t,t)^{-1/2}-\mathbf{I}_p|||^4\right]|||\widehat{\boldsymbol{\Gamma}}_i(t,t)^{1/2}|||^8\end{aligned}$$

for all $t\in\mathcal{T}$ on B_n. Recalling (S27) yields

$$\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Lambda}}(t,t)\right|\right|\right|^4\right]\,\mathrm{d}t\leq\sum_{i=1}^k\left(\frac{n}{n_i}\right)^4\mathrm{E}^*\left[|||\mathbf{W}_{n_i-1}/(n_i-1)-\mathbf{I}_p|||^4\right]\int_{\mathcal{T}}|||\widehat{\boldsymbol{\Gamma}}_i(t,t)^{1/2}|||^8\,\mathrm{d}t$$

for all $t\in\mathcal{T}$ on B_n. Lemma S2, Lemma S6 and $\mathrm{P}(B_n)\to 1$ as $n\to\infty$ imply

$$\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Lambda}}(t,t)\right|\right|\right|^4\right]\,\mathrm{d}t\xrightarrow{P}0$$

as $n\to\infty$ and, thus,

$$\begin{aligned}&\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\boldsymbol{\Lambda}(t,t)\right|\right|\right|^4\right]\\ &\leq 16\int_{\mathcal{T}}\mathrm{E}^*\left[\left|\left|\left|\widehat{\boldsymbol{\Lambda}}^{\mathcal{P}}(t,t)-\widehat{\boldsymbol{\Lambda}}(t,t)\right|\right|\right|^4\right]\,\mathrm{d}t+16\int_{\mathcal{T}}\left|\left|\left|\widehat{\boldsymbol{\Lambda}}(t,t)-\boldsymbol{\Lambda}(t,t)\right|\right|\right|^4\,\mathrm{d}t\\ &\xrightarrow{P}0\end{aligned} \tag{S30}$$

as $n\to\infty$ by (S29).

By combining (S28) and (S30) in (S26), we get

$$\mathrm{E}^*_{\mathrm{out}}\left[\int_{\mathcal{T}} \left|\left|\left|\left(\mathbf{H}\widehat{\mathbf{\Lambda}}^{\mathcal{P}}(t,t)\mathbf{H}^\top\right)^+ - \left(\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top\right)^+\right|\right|\right|^2 \,\mathrm{d}t\right] \xrightarrow{P} 0$$

as $n \to \infty$. Together with (S23)–(S25), it follows

$$\mathrm{P}^*_{\mathrm{out}}\left(D_n^{\mathcal{P}}(\mathbf{H}) > \varepsilon\right) \xrightarrow{P} 0$$

as $n \to \infty$ for all $\varepsilon > 0$ and, thus, (S22). □

The proof of Proposition S1 uses the same arguments as the proofs in the supplements of [11, 14].

Proof of Proposition S1. We aim to apply Lemma S3 in the supplement of [11], which provides convergence of quantile functions, with $\mathcal{F}_n$ being the empirical distribution function of $T^{\mathcal{P}}_{n;1}, ..., T^{\mathcal{P}}_{n;B(n)}$. With $\mathbf{x}(t) := \left((\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\right)^{1/2}\mathbf{y}(t)$, Lemma S4 implies that $\mathcal{F}_0$ is continuous on $\mathbb{R}$ and strictly increasing on $[0,\infty)$, where $\mathcal{F}_0$ denotes the distribution function of $\int_{\mathcal{T}} \mathbf{y}^\top(t)(\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{y}(t)\,\mathrm{d}t$. Therefore, note that the assumption on the covariance function is satisfied due to

$$\begin{aligned}\mathrm{tr}\left(\left((\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\right)^{1/2}\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top\left((\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\right)^{1/2}\right) &= \mathrm{tr}\left((\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top\right)\\ &= \mathrm{rank}\left((\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top)^+\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top\right)\\ &= \mathrm{rank}\left(\mathbf{H}\mathbf{\Lambda}(t,t)\mathbf{H}^\top\right) = \mathrm{rank}\left(\mathbf{H}\right) > 0\end{aligned}$$

for all $t \in \mathcal{T}$. Lemma S7 in the supplement of [14] and Theorem S2 imply that $\mathcal{F}_n(w) \xrightarrow{\mathrm{P}} \mathcal{F}_0(w)$ for all $w \in \mathbb{R}$. By applying the subsequence criterion (Lemma 1.9.2 (ii) in [18]), Lemma S3 in the supplement of [11] provides $Q^{\mathcal{P}}_{n,1-\alpha} \xrightarrow{\mathrm{P}} \mathcal{F}_0^{-1}(1-\alpha)$. As in the proof of Proposition 1 in [14], this completes the proof by Theorem 1. □

With minor adjustments, the proofs of Theorem S3 and S4 can be obtained analogously as the proofs of Theorem S1 and S2, respectively.

Proof of Theorem 3. Lemma S6 provides that $\widehat{\mathbf{\Lambda}}$ converges uniformly over $\{(t,t) \in \mathcal{T}^2 \mid t \in \mathcal{T}\}$ in probability to $\mathbf{\Lambda}$. Furthermore, Lemma S7 ensures the convergence in distribution of $\sqrt{n}(\mathbf{H}\widehat{\boldsymbol{\eta}} - \mathbf{c})$ to $\mathbf{z}$. By the continuous mapping theorem, it suffices to show that the map

$$\mathbf{g} : \mathcal{L}_2^r(\mathcal{T}) \times \mathbb{D} \to (\mathbb{R}_0^+ \cup \{\infty\})^R, \quad \mathbf{g}(\widetilde{\mathbf{x}}, \widetilde{\mathbf{\Lambda}}) := \left(\int_{\mathcal{T}} \widetilde{\mathbf{x}}_\ell^\top(t)\left(\mathbf{H}_\ell\widetilde{\mathbf{\Lambda}}(t)\mathbf{H}_\ell^\top\right)^+ \widetilde{\mathbf{x}}_\ell(t)\,\mathrm{d}t\right)_{\ell \in \{1,...,R\}}$$

is continuous on $\mathcal{L}_2^r(\mathcal{T}) \times \{\mathbf{\Lambda}\}$ with $\mathbb{D}$ as in the proof of Theorem S1. This holds whenever its component functions are continuous. The continuity of the component functions follows analogously as in the proof of Theorem S1. □

Proof of Theorem 4. By Lemma S8 and the subsequence criterion, every subsequence has a further subsequence such that $\sqrt{n}\mathbf{H}\widehat{\boldsymbol{\eta}}^{\mathcal{P}}$ converges in distribution almost surely along the subsequence to $\mathbf{z}$ in $\mathcal{L}_2^r(\mathcal{T})$. Applying the continuous mapping theorem yields that

$$\left(\int_{\mathcal{T}} n\left(\mathbf{H}_\ell\boldsymbol{\eta}^{\mathcal{P}}(t)\right)^\top\left(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top\right)^+\mathbf{H}_\ell\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t\right)_{\ell \in \{1,...,R\}}$$

converges in distribution almost surely along the latter subsequence to $\left(\int_{\mathcal{T}} \mathbf{z}_\ell^\top(t)(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top)^+\mathbf{z}_\ell(t)\,\mathrm{d}t\right)_{\ell \in \{1,...,R\}}$. Hence, by applying the subsequence criterion again, it follows

$$\begin{aligned}&\left(\int_{\mathcal{T}} n\left(\mathbf{H}_\ell\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top\left(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top\right)^+\mathbf{H}_\ell\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t\right)_{\ell \in \{1,...,R\}}\\ &\xrightarrow{d^*} \left(\int_{\mathcal{T}} \mathbf{z}_\ell^\top(t)(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top)^+\mathbf{z}_\ell(t)\,\mathrm{d}t\right)_{\ell \in \{1,...,R\}}\end{aligned}$$

as $n \to \infty$.
Consequently, it remains to show

$$D_n^{\mathcal{P}}(\mathbf{H}_\ell) = \left|T_n^{\mathcal{P}}(\mathbf{H}_\ell) - \int_{\mathcal{T}} n\left(\mathbf{H}_\ell\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\right)^\top\left(\mathbf{H}_\ell\mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top\right)^+\mathbf{H}_\ell\widehat{\boldsymbol{\eta}}^{\mathcal{P}}(t)\,\mathrm{d}t\right| \xrightarrow{P^*} 0$$

as $n \to \infty$ for all $\ell \in \{1, ..., R\}$. This can be shown analogously as in the proof of Theorem S2. $\square$

The proof of Proposition S2 is an application of Lemma S8 in the supplement of [14], which yields the convergence of the critical values.

Proof of Proposition S2. We aim to apply Lemma S8 in the supplement of [14] with $L = R$, F_n being the empirical distribution function of $\left(T_n^{b,\mathcal{P}}(\mathbf{H}_1), ..., T_n^{b,\mathcal{P}}(\mathbf{H}_R)\right), b \in \{1, ..., B(n)\}$, and $F_{n,1}, ..., F_{n,R}$ being its marginal distribution functions. Furthermore, let F denote the distribution function of

$$\left(\int_{\mathcal{T}} \mathbf{y}_\ell^\top(t) \left(\mathbf{H}_\ell \mathbf{\Lambda}(t,t) \mathbf{H}_\ell^\top\right)^+ \mathbf{y}_\ell(t) \, \mathrm{d}t\right)_{\ell \in \{1,...,R\}}$$

and $F_1, ..., F_R$ denote its marginal distribution functions. With $\mathbf{x}(t) := \left((\mathbf{H}_\ell \mathbf{\Lambda}(t,t)\mathbf{H}_\ell^\top)^+\right)^{1/2} \mathbf{H}_\ell \mathbf{z}(t)$ for $\mathbf{z} \sim GP_k(\mathbf{0}_k, \mathbf{\Lambda})$, Lemma S4 implies that $F_1, ..., F_R$ are continuous on $\mathbb{R}$ and strictly increasing on $[0, \infty)$, where the assumption on the covariance function is satisfied as in the proof of Proposition S1. Lemma S7 and Remark 1 in the supplement of [14] and Theorem S4 guarantee that the assumptions (S10) and (S11) are satisfied. Moreover, Lemma S5 implies that FWER in Lemma S7 in the supplement of [14] is strictly increasing on $(0, 1]$ with $t \mapsto \boldsymbol{A}_\ell(t) := \boldsymbol{H}_\ell^\top(\boldsymbol{H}_\ell \mathbf{\Lambda}(t,t)\boldsymbol{H}_\ell^\top)^+ \boldsymbol{H}_\ell$ and $\int_{\mathcal{T}} \mathrm{tr}\left(\boldsymbol{A}_\ell(t)\mathbf{\Lambda}(t,t)\right) \, \mathrm{d}t = \int_{\mathcal{T}} \mathrm{rank}\left(\mathbf{H}_\ell\right) \, \mathrm{d}t > 0$. Thus, Lemma S8 in the supplement of [14] provides $q_{l,\widetilde{\beta}}^{\mathcal{P}} \xrightarrow{\mathrm{P}} F_\ell^{-1}(1 - \mathrm{FWER}^{-1}(\alpha))$ for all $\ell \in \{1, ..., R\}$. As in the proof of Proposition S1 in [14], this completes the proof by Theorem S3. $\square$

S4. More Details on the Data Examples

In this section, we first present the plots of the empirical covariance functions for Example 1 based on the chlorine concentration data (Section S4.1) and for Example 2 based on the electrocardiogram data (Section S4.2). For the chlorine concentration data, the sample covariance functions seem to be very similar, which can not be said for the electrocardiogram data. To verify these issues, we used the tests for equality of covariance functions by [4, 5]. The results are presented in Table S1 and confirm our observations.

Table S1: P-values of tests for equality of covariance functions by Guo et al. (2018, $T_{\max,rp}$) and by Guo et al. (2019, GPF_{nv}, GPF_{rp}, $F_{\max,rp}$) for chlorine concentration and electrocardiogram data

Data set	$T_{\max,rp}$	GPF_{nv}	GPF_{rp}	$F_{\max,rp}$
Chlorine concentration	0.515	0.310	0.401	0.427
Electrocardiogram	0.000	0.000	0.000	0.000

Now we describe and present the results of the simulation studies based on both real data examples. To mimic the data given in the sets, we generated the simulation data from the multivariate normal distribution with the following specifications:

- the samples sizes from the data example, i.e., $n_i = 25$, $i = 1, 2, 3$ for the chlorine concentration data, and $n_1 = 292$, $n_2 = 177$, $n_3 = 10$, and $n_4 = 19$ for the electrocardiogram data,
- the covariance matrix in the i-th group was equal to the sample covariance function for the i-th sample from the data set.
- for checking the type-I error control, in each group, the mean function was set to the sample mean function of the pooled data,
- for power investigation, the mean function in the i-th group was equal to the sample mean function for the i-th sample from the data set.

We have calculated the empirical FWER, sizes and powers for multiple contrast tests. We considered the Tukey-type contrast matrix. The results of these simulation studies are presented in Sections S4.3-S4.4.

S4.1. Plots of the Empirical Covariances – Example 1

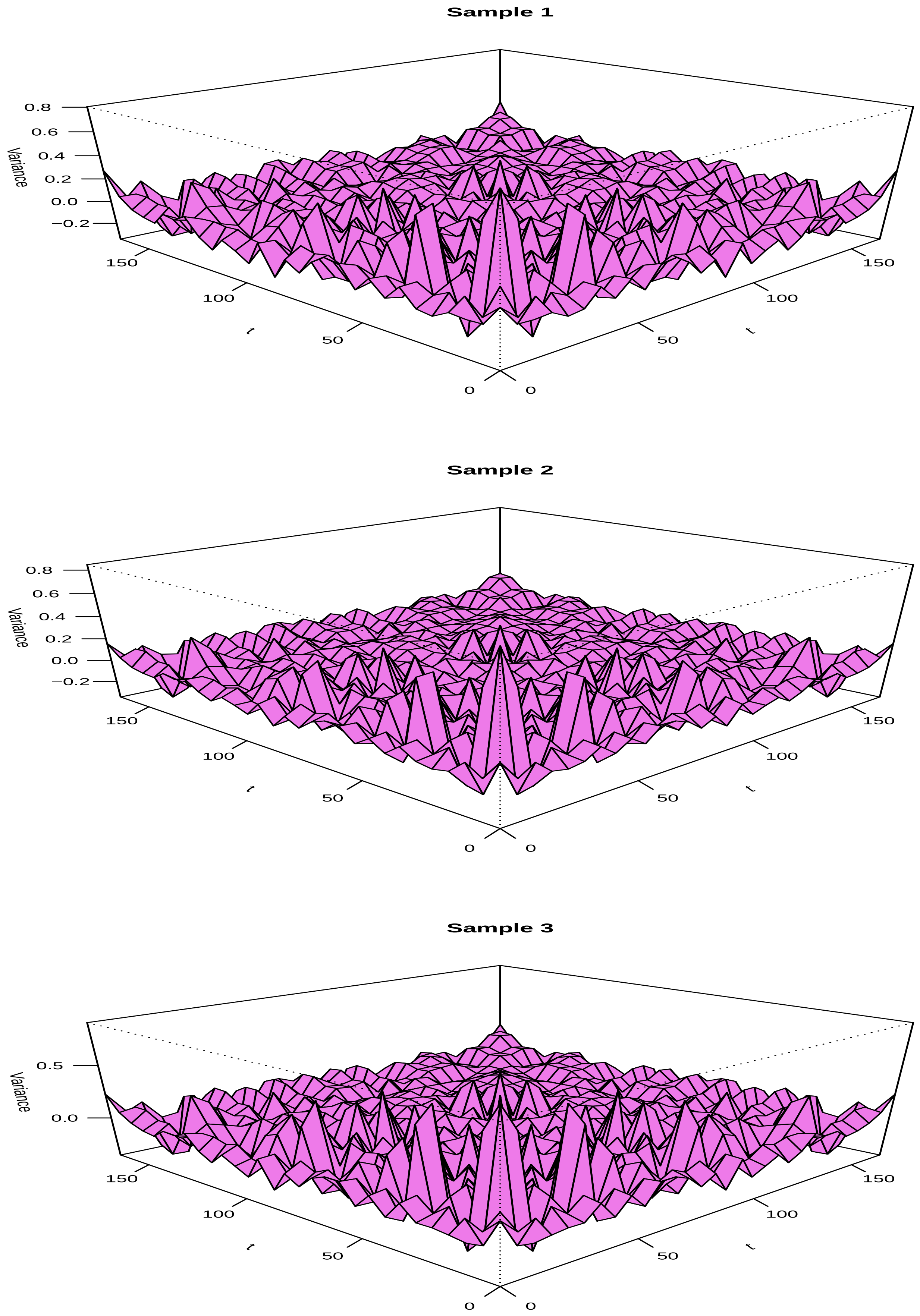

Fig. S1: Empirical covariance functions for the chlorine concentration data

S4.2. Plots of the Empirical Covariances – Example 2

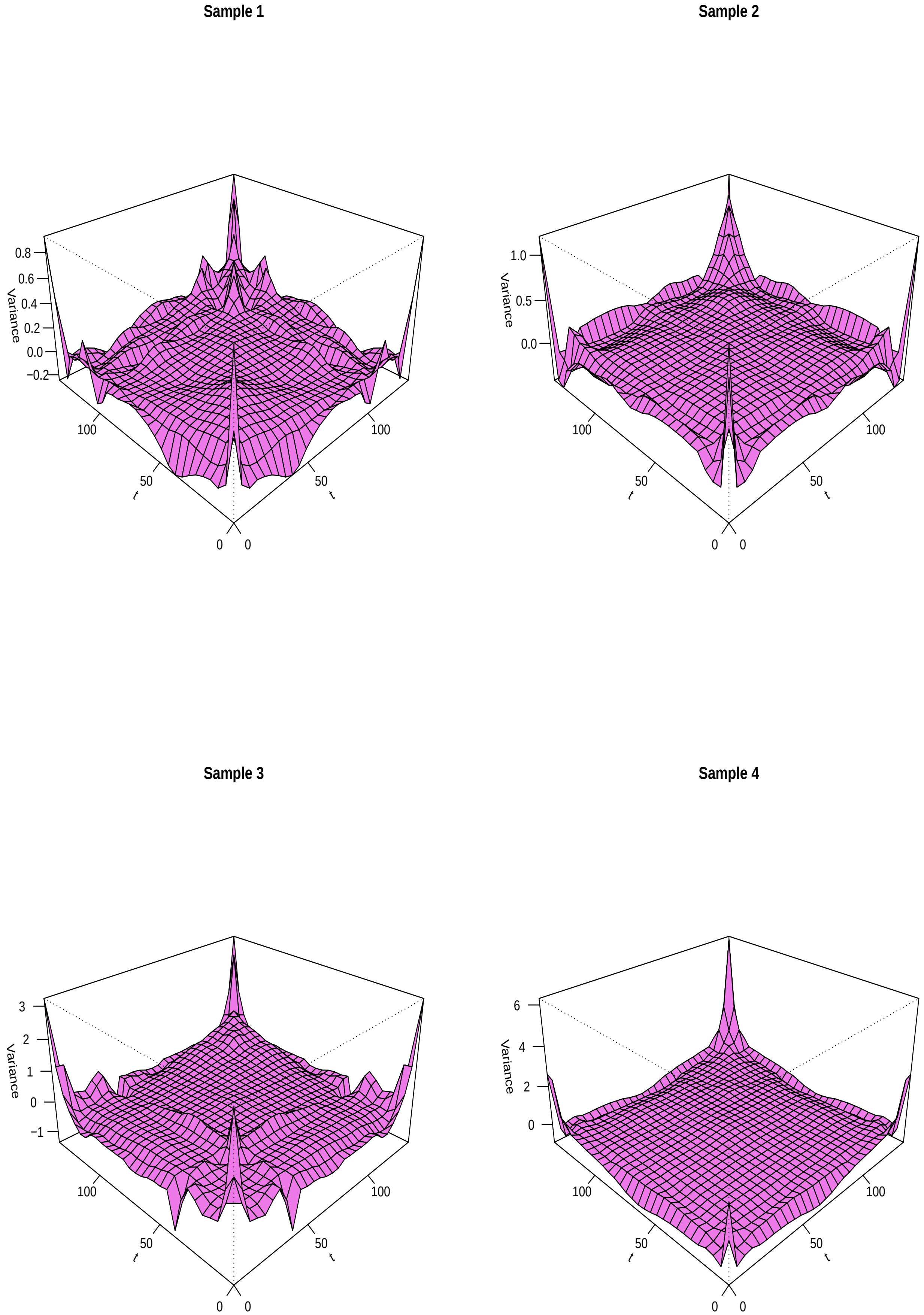

Fig. S2: Empirical covariance functions for the electrocardiogram data

S4.3. Simulation Study inspired by Example 1

Table S2: Empirical FWER, sizes and powers (as percentages) of all tests obtained for the Tukey contrasts in the simulation based on the chlorine concentration data ($\mathcal{H}_{0,1} : \eta_1 \equiv \eta_2$, $\mathcal{H}_{0,2} : \eta_1 \equiv \eta_3$, $\mathcal{H}_{0,3} : \eta_2 \equiv \eta_3$)

		Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
FWER		4.60	4.30	5.60	3.95	4.15	4.25	4.45	4.70	5.05
Empirical sizes	$\mathcal{H}_{0,1}$	2.15	2.3	2.45	1.80	1.75	1.82	1.78	1.90	1.85
	$\mathcal{H}_{0,2}$	1.65	1.2	2.05	1.35	1.50	1.25	1.50	1.50	1.95
	$\mathcal{H}_{0,3}$	1.15	1.2	1.80	1.35	1.05	1.20	1.20	1.30	1.35
Empirical powers	$\mathcal{H}_{0,1}$	70.65	66.55	42.20	36.3	54.30	66.80	69.35	67.90	70.65
	$\mathcal{H}_{0,2}$	25.70	15.15	9.15	7.1	16.20	15.05	24.45	16.00	25.85
	$\mathcal{H}_{0,3}$	29.35	27.30	27.40	23.7	19.85	27.30	28.45	28.45	29.70

S4.4. Simulation Study inspired by Example 2

Table S3: Empirical FWER, sizes and powers (as percentages) of all tests obtained for the Tukey contrasts in the simulation based on the electrocardiogram data ($\mathcal{H}_{0,1} : \eta_1 \equiv \eta_2$, $\mathcal{H}_{0,2} : \eta_1 \equiv \eta_3$, $\mathcal{H}_{0,3} : \eta_1 \equiv \eta_4$, $\mathcal{H}_{0,4} : \eta_2 \equiv \eta_3$, $\mathcal{H}_{0,5} : \eta_2 \equiv \eta_4$, $\mathcal{H}_{0,6} : \eta_3 \equiv \eta_4$)

		Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
FWER		55.35	33.25	6.90	2.50	4.70	3.75	3.70	5.00	4.75
Empirical sizes	$\mathcal{H}_{0,1}$	0.80	0.85	0.75	0.55	0.55	1.15	0.75	1.15	1.00
	$\mathcal{H}_{0,2}$	26.05	17.15	2.35	0.45	1.35	0.85	1.00	1.20	1.20
	$\mathcal{H}_{0,3}$	23.65	12.85	1.70	0.90	0.90	1.20	0.55	1.20	1.00
	$\mathcal{H}_{0,4}$	37.30	20.50	2.30	0.60	1.25	1.00	1.05	1.30	1.10
	$\mathcal{H}_{0,5}$	13.00	9.95	1.65	0.85	0.90	1.10	0.75	1.25	1.00
	$\mathcal{H}_{0,6}$	2.75	1.45	1.65	0.75	1.20	0.80	1.15	1.15	1.15
Empirical powers	$\mathcal{H}_{0,1}$	100.00	100.00	100.0	100.00	100.00	100.00	100.0	100.00	100.00
	$\mathcal{H}_{0,2}$	100.00	100.00	100.0	99.75	100.00	98.85	100.0	99.70	100.00
	$\mathcal{H}_{0,3}$	100.00	100.00	100.0	99.55	100.00	100.00	100.0	100.00	100.00
	$\mathcal{H}_{0,4}$	100.00	100.00	85.8	44.95	98.00	57.30	99.5	67.50	99.70
	$\mathcal{H}_{0,5}$	93.85	89.25	70.4	58.80	68.35	55.60	66.8	61.80	71.65
	$\mathcal{H}_{0,6}$	99.90	60.10	74.4	51.55	96.10	43.20	97.7	51.15	98.55

S5. Detailed Results from the Simulation Study

In this section, we present all alternative hypotheses considered and the results of the simulation studies. The scenarios in the six alternative hypotheses are as follows:

A1. $\mu_1(t) = \mu_2(t) = \mu_3(t) = 0$ and $\mu_4(t) = 2t$,

A2. $\mu_1(t) = \mu_2(t) = \mu_3(t) = 0$ and $\mu_4(t) = 1.5$,

A3. $\mu_1(t) = 0, \mu_2(t) = 0.75, \mu_3(t) = 1.5$ and $\mu_4(t) = 2.25$,

A4. $\mu_1(t) = 0, \mu_2(t) = t, \mu_3(t) = 2t$ and $\mu_4(t) = 3t$,

A5. $\mu_1(t) = \mu_2(t) = \mu_3(t) = 0$ and $\mu_4(t) = 2(1 - \frac{1}{J})(1 - t)$,

A6. $\mu_1(t) = 0, \mu_2(t) = (1 - \frac{1}{J})(1 - t), \mu_3(t) = 2(1 - \frac{1}{J})(1 - t)$ and $\mu_4(t) = 3(1 - \frac{1}{J})(1 - t)$

for all $t \in [0, 1]$.

The results of the simulation studies are presented in tables and figures. The lists of them are given on the next page (Sections S5.1-S5.2). The results for the usual simulation (i.e., without scaling function) are depicted in Section S5.3, while those with scaling function are given in Section S5.4. In column $\mathcal{H}$ in the tables, $\mathcal{H}_i$ represents the i-th hypothesis for given kind of contrasts, i.e.,

- for Dunnett contrasts: $\mathcal{H}_{0,1} : \eta_1 \equiv \eta_2$, $\mathcal{H}_{0,2} : \eta_1 \equiv \eta_3$, $\mathcal{H}_{0,3} : \eta_2 \equiv \eta_3$,
- for Tukey contrasts: $\mathcal{H}_{0,1} : \eta_1 \equiv \eta_2$, $\mathcal{H}_{0,2} : \eta_1 \equiv \eta_3$, $\mathcal{H}_{0,3} : \eta_1 \equiv \eta_4$, $\mathcal{H}_{0,4} : \eta_2 \equiv \eta_3$, $\mathcal{H}_{0,5} : \eta_2 \equiv \eta_4$, $\mathcal{H}_{0,6} : \eta_3 \equiv \eta_4$.

Moreover, the column D denotes the distribution of the generated data, i.e., N - the normal distribution, t - the t_5-distribution and χ^2 - the χ^2_5-distribution.

S5.1. List of Tables

List of Tables

S5.2. List of Figures

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S5.3. Results without Scaling Function

Table S4: Empirical FWER and sizes (as percentages) of all tests obtained for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.35	3.75	5.55	3.30	6.10	3.05	4.05	4.30	5.20
			$\mathcal{H}_{0,1}$	1.45	1.20	1.90	1.00	1.90	0.85	1.40	1.30	1.85
			$\mathcal{H}_{0,2}$	2.05	1.45	2.30	1.35	2.30	1.20	2.05	1.75	2.20
			$\mathcal{H}_{0,3}$	1.35	1.75	2.30	1.30	2.55	1.55	1.35	1.95	1.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.65	1.60	6.15	4.10	6.65	4.55	5.75	5.10	5.90
			$\mathcal{H}_{0,1}$	1.60	0.95	1.95	1.20	2.35	1.40	1.90	1.60	1.90
			$\mathcal{H}_{0,2}$	0.70	0.40	2.45	1.40	2.40	1.75	1.90	1.85	2.10
			$\mathcal{H}_{0,3}$	0.40	0.25	2.25	1.55	2.25	1.70	2.30	1.95	2.50
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	11.75	10.00	4.15	2.30	6.10	2.35	3.65	3.50	5.00
			$\mathcal{H}_{0,1}$	2.25	1.85	2.05	1.35	1.85	1.15	1.55	1.85	2.30
			$\mathcal{H}_{0,2}$	4.35	3.15	1.85	0.85	2.50	1.20	1.65	1.70	2.35
			$\mathcal{H}_{0,3}$	8.45	8.20	1.65	0.75	2.25	0.70	1.65	1.25	1.85
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	3.80	4.00	4.15	3.45	4.60	3.95	3.95	4.50	4.80
			$\mathcal{H}_{0,1}$	1.65	1.30	1.50	1.10	2.00	1.15	1.50	1.65	1.65
			$\mathcal{H}_{0,2}$	1.35	1.35	1.65	1.25	1.65	1.45	1.35	1.60	1.70
			$\mathcal{H}_{0,3}$	1.25	1.85	1.90	1.75	1.45	1.90	1.80	2.00	2.15
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	1.80	1.55	5.15	4.00	4.35	3.55	4.55	4.15	5.10
			$\mathcal{H}_{0,1}$	1.15	1.05	2.15	1.55	1.60	1.50	1.65	1.80	1.90
			$\mathcal{H}_{0,2}$	0.45	0.50	1.70	1.45	1.40	1.10	1.60	1.35	1.65
			$\mathcal{H}_{0,3}$	0.25	0.05	1.65	1.30	1.45	1.15	1.45	1.45	1.70
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.35	10.15	3.55	3.00	4.60	3.15	3.50	4.25	4.45
			$\mathcal{H}_{0,1}$	1.70	1.55	1.35	1.25	1.45	1.20	1.15	1.60	1.65
			$\mathcal{H}_{0,2}$	3.15	3.30	1.50	1.30	1.65	1.25	1.25	1.65	1.60
			$\mathcal{H}_{0,3}$	9.15	8.05	1.85	1.35	2.05	1.55	1.75	2.25	2.15
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.10	4.05	4.45	4.15	4.40	3.75	3.90	4.65	4.40
			$\mathcal{H}_{0,1}$	1.70	1.55	1.85	1.55	1.50	1.65	1.45	1.90	1.75
			$\mathcal{H}_{0,2}$	1.35	1.45	1.75	1.60	1.50	1.25	1.30	1.70	1.45
			$\mathcal{H}_{0,3}$	1.65	1.90	1.90	1.90	1.65	1.75	1.65	2.25	1.85
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	1.80	1.75	4.30	4.05	4.50	3.90	5.10	4.10	5.30
			$\mathcal{H}_{0,1}$	1.25	1.20	1.75	1.65	2.00	1.45	1.95	1.55	2.00
			$\mathcal{H}_{0,2}$	0.40	0.45	1.40	1.30	1.35	1.35	1.90	1.40	1.95
			$\mathcal{H}_{0,3}$	0.25	0.10	1.45	1.40	1.30	1.30	1.45	1.35	1.55
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.20	11.55	4.25	3.90	4.90	3.95	4.25	5.35	5.15
			$\mathcal{H}_{0,1}$	2.40	2.60	2.00	1.80	1.75	1.80	1.95	2.55	2.25
			$\mathcal{H}_{0,2}$	3.80	4.10	1.75	1.60	2.00	1.95	1.60	2.20	2.00
			$\mathcal{H}_{0,3}$	9.25	8.75	1.80	1.60	1.75	1.35	1.80	2.15	2.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.10	3.05	3.95	2.50	5.20	2.55	4.70	3.50	5.65
			$\mathcal{H}_{0,1}$	1.70	1.20	1.60	0.95	1.90	1.15	1.45	1.60	1.80
			$\mathcal{H}_{0,2}$	1.95	1.40	1.90	1.20	1.70	1.10	2.10	1.50	2.50
			$\mathcal{H}_{0,3}$	1.80	1.10	1.25	0.70	2.10	0.90	1.60	1.10	1.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.55	1.35	4.20	2.85	5.20	2.55	4.95	3.30	5.40
			$\mathcal{H}_{0,1}$	1.40	0.95	1.75	1.05	1.55	1.05	1.80	1.30	1.95
			$\mathcal{H}_{0,2}$	1.05	0.25	1.50	1.15	1.80	0.70	1.85	0.90	2.10
			$\mathcal{H}_{0,3}$	0.30	0.25	1.30	0.90	1.90	1.00	1.65	1.30	1.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	11.15	8.50	3.15	1.60	5.20	1.70	3.40	2.75	4.75
			$\mathcal{H}_{0,1}$	1.90	1.60	1.90	0.95	1.50	0.90	1.45	1.60	2.00
			$\mathcal{H}_{0,2}$	3.50	2.35	1.30	0.60	2.15	0.80	1.10	1.00	1.65
			$\mathcal{H}_{0,3}$	8.00	6.75	0.80	0.40	2.05	0.45	1.35	0.85	1.95
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.25	3.60	4.25	3.40	4.50	3.85	3.30	4.50	4.30
			$\mathcal{H}_{0,1}$	1.80	1.10	1.50	1.10	1.75	1.30	1.65	1.55	2.00
			$\mathcal{H}_{0,2}$	1.10	1.25	1.60	1.25	1.70	1.30	0.85	1.55	1.10

Table S4: Empirical FWER and sizes (as percentages) of all tests obtained for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
			$\mathcal{H}_{0,3}$	1.65	1.85	2.10	1.70	1.40	1.60	1.15	2.15	1.65
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	2.20	1.45	4.30	3.35	4.55	3.75	4.95	4.40	5.35
			$\mathcal{H}_{0,1}$	1.25	0.95	1.80	1.40	1.60	1.60	1.50	1.75	1.60
			$\mathcal{H}_{0,2}$	0.65	0.45	1.35	1.05	1.80	1.30	1.70	1.40	1.85
			$\mathcal{H}_{0,3}$	0.30	0.10	1.50	1.25	1.25	1.15	1.90	1.55	2.05
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.85	9.90	3.80	2.90	4.55	3.25	3.80	4.15	5.05
			$\mathcal{H}_{0,1}$	2.15	1.75	1.60	1.35	1.40	1.35	1.55	1.70	2.10
			$\mathcal{H}_{0,2}$	3.75	3.60	1.95	1.30	1.65	1.50	1.70	1.90	2.10
			$\mathcal{H}_{0,3}$	9.05	8.05	1.55	1.30	1.90	1.25	1.35	1.75	1.95
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.05	3.75	4.20	3.90	3.60	3.60	3.85	4.45	4.40
			$\mathcal{H}_{0,1}$	1.30	1.40	1.50	1.45	0.95	1.20	1.20	1.60	1.40
			$\mathcal{H}_{0,2}$	1.30	1.40	1.70	1.50	1.25	1.45	1.15	1.75	1.45
			$\mathcal{H}_{0,3}$	1.95	1.75	1.85	1.65	1.65	1.60	2.00	1.95	2.25
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	2.15	1.00	4.30	3.90	4.20	4.35	4.65	4.75	4.95
			$\mathcal{H}_{0,1}$	1.25	0.70	1.55	1.20	1.40	1.45	1.85	1.60	2.05
			$\mathcal{H}_{0,2}$	0.60	0.30	1.50	1.35	1.10	1.45	1.75	1.65	1.80
			$\mathcal{H}_{0,3}$	0.35	0.15	1.50	1.60	1.80	1.70	1.25	1.80	1.30
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.85	10.90	3.95	3.45	3.95	3.60	4.10	5.20	5.70
			$\mathcal{H}_{0,1}$	2.15	2.50	2.00	1.75	1.65	1.65	1.80	2.60	2.20
			$\mathcal{H}_{0,2}$	4.75	3.60	1.75	1.60	1.10	1.65	1.95	2.30	2.95
			$\mathcal{H}_{0,3}$	9.30	8.45	1.65	1.45	1.65	1.55	1.10	2.10	1.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.70	3.00	4.50	2.80	5.50	2.95	4.55	3.65	5.35
			$\mathcal{H}_{0,1}$	1.65	0.95	1.30	0.90	1.80	0.90	1.65	1.15	1.80
			$\mathcal{H}_{0,2}$	1.95	1.40	2.40	1.45	2.25	1.30	1.75	1.60	2.30
			$\mathcal{H}_{0,3}$	1.55	1.05	1.40	0.80	1.75	1.10	1.70	1.40	1.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.15	1.30	5.00	2.85	5.75	3.15	5.15	3.90	5.35
			$\mathcal{H}_{0,1}$	0.90	1.00	2.05	1.35	2.05	1.30	1.85	1.65	1.85
			$\mathcal{H}_{0,2}$	1.05	0.35	1.65	0.90	2.15	1.15	2.25	1.40	2.35
			$\mathcal{H}_{0,3}$	0.20	0.05	1.50	0.80	1.80	0.95	1.40	1.15	1.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	12.05	9.40	3.90	1.95	5.35	2.25	3.95	3.85	5.40
			$\mathcal{H}_{0,1}$	2.10	1.70	2.05	0.90	1.60	0.90	1.60	1.70	2.20
			$\mathcal{H}_{0,2}$	4.10	2.85	1.55	0.70	1.90	0.80	1.50	1.20	2.15
			$\mathcal{H}_{0,3}$	8.80	7.15	1.50	0.70	2.30	1.05	1.90	1.80	2.65
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.60	4.25	4.50	3.50	5.20	3.85	4.80	4.50	5.70
			$\mathcal{H}_{0,1}$	2.00	1.95	2.35	1.70	2.10	1.90	1.95	2.10	2.20
			$\mathcal{H}_{0,2}$	1.60	1.30	1.35	1.10	1.10	1.05	1.65	1.40	2.20
			$\mathcal{H}_{0,3}$	2.00	1.80	2.10	1.60	2.35	1.70	2.05	2.05	2.35
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	2.75	1.70	4.30	3.65	4.80	3.90	5.40	4.15	5.75
			$\mathcal{H}_{0,1}$	1.90	1.35	1.75	1.55	2.45	1.65	2.30	1.80	2.50
			$\mathcal{H}_{0,2}$	0.70	0.30	1.40	1.10	0.95	1.10	1.60	1.15	1.70
			$\mathcal{H}_{0,3}$	0.25	0.15	1.45	1.30	1.50	1.60	1.75	1.80	1.80
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	10.70	9.65	4.00	3.00	4.55	2.70	3.80	4.00	4.70
			$\mathcal{H}_{0,1}$	1.75	1.55	1.65	1.30	1.85	1.10	1.55	2.05	1.90
			$\mathcal{H}_{0,2}$	3.30	3.45	1.75	1.40	1.20	1.35	1.50	1.80	1.90
			$\mathcal{H}_{0,3}$	8.65	7.80	1.70	1.25	1.95	1.20	1.65	1.70	2.20
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	5.05	4.90	4.80	4.15	4.70	4.75	4.90	5.30	5.80
			$\mathcal{H}_{0,1}$	2.20	2.10	2.15	1.95	1.75	2.05	1.85	2.35	2.25
			$\mathcal{H}_{0,2}$	1.55	1.80	1.80	1.55	1.70	1.50	1.65	1.90	1.75
			$\mathcal{H}_{0,3}$	2.05	2.10	1.95	1.60	1.75	1.95	2.20	2.30	2.75
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	1.90	2.15	5.85	5.10	4.60	4.65	4.70	5.25	5.05
			$\mathcal{H}_{0,1}$	1.25	1.65	2.85	2.45	1.90	2.50	1.95	2.80	2.20
			$\mathcal{H}_{0,2}$	0.60	0.40	1.75	1.70	1.40	1.20	1.50	1.35	1.60
			$\mathcal{H}_{0,3}$	0.05	0.10	1.60	1.30	1.40	1.35	1.45	1.50	1.45
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	11.55	11.00	3.85	3.70	4.80	3.55	3.80	5.25	4.80
			$\mathcal{H}_{0,1}$	2.35	2.40	1.75	1.65	2.15	1.65	1.80	2.35	2.35
			$\mathcal{H}_{0,2}$	3.40	3.30	1.35	1.25	1.70	1.45	1.40	2.00	1.80

Table S4: Empirical FWER and sizes (as percentages) of all tests obtained for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
			$\mathcal{H}_{0,3}$	8.85	8.90	1.60	1.50	1.60	1.25	1.75	2.10	2.10

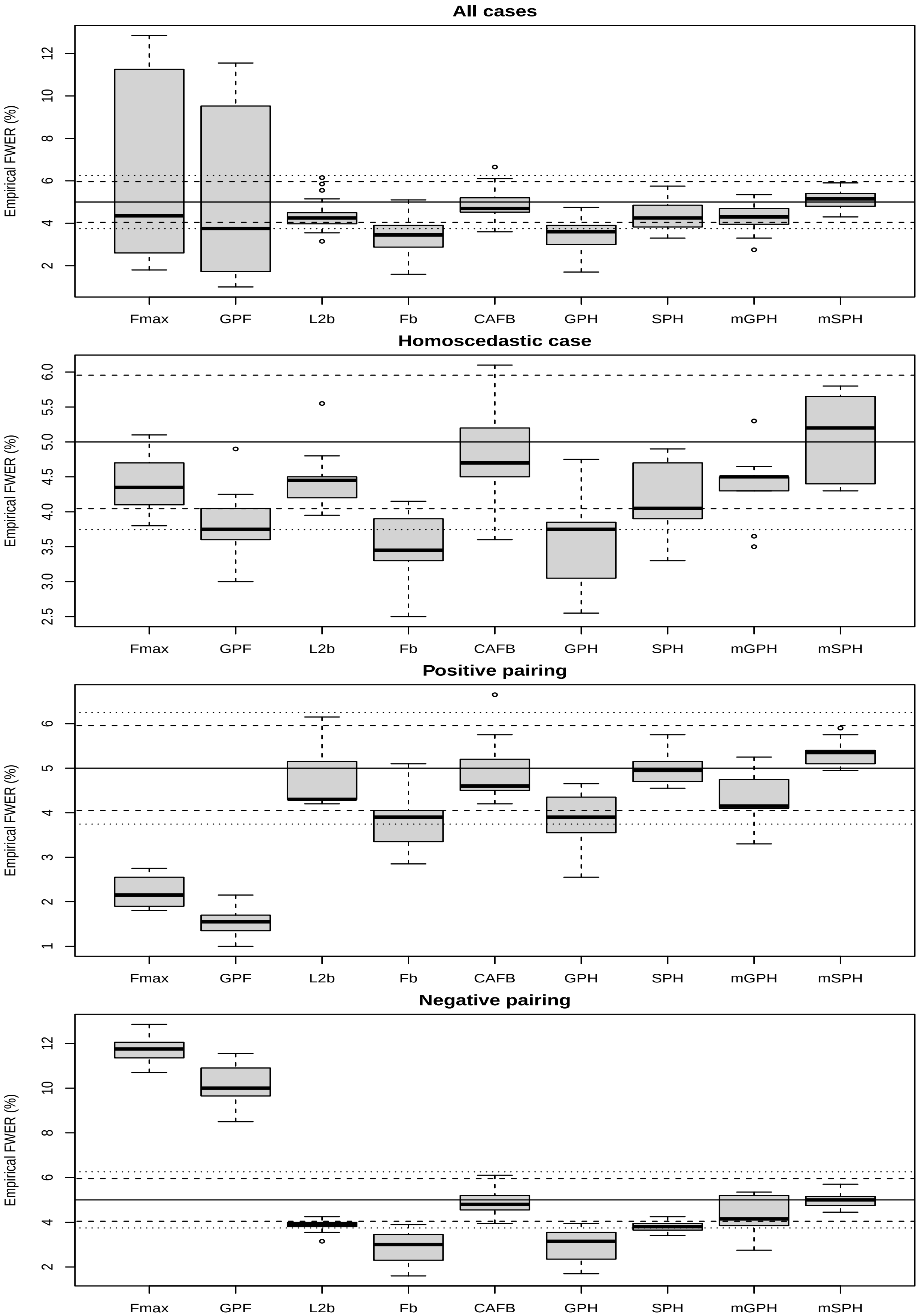


Fig. S3: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts for homoscedastic and heteroscedastic cases

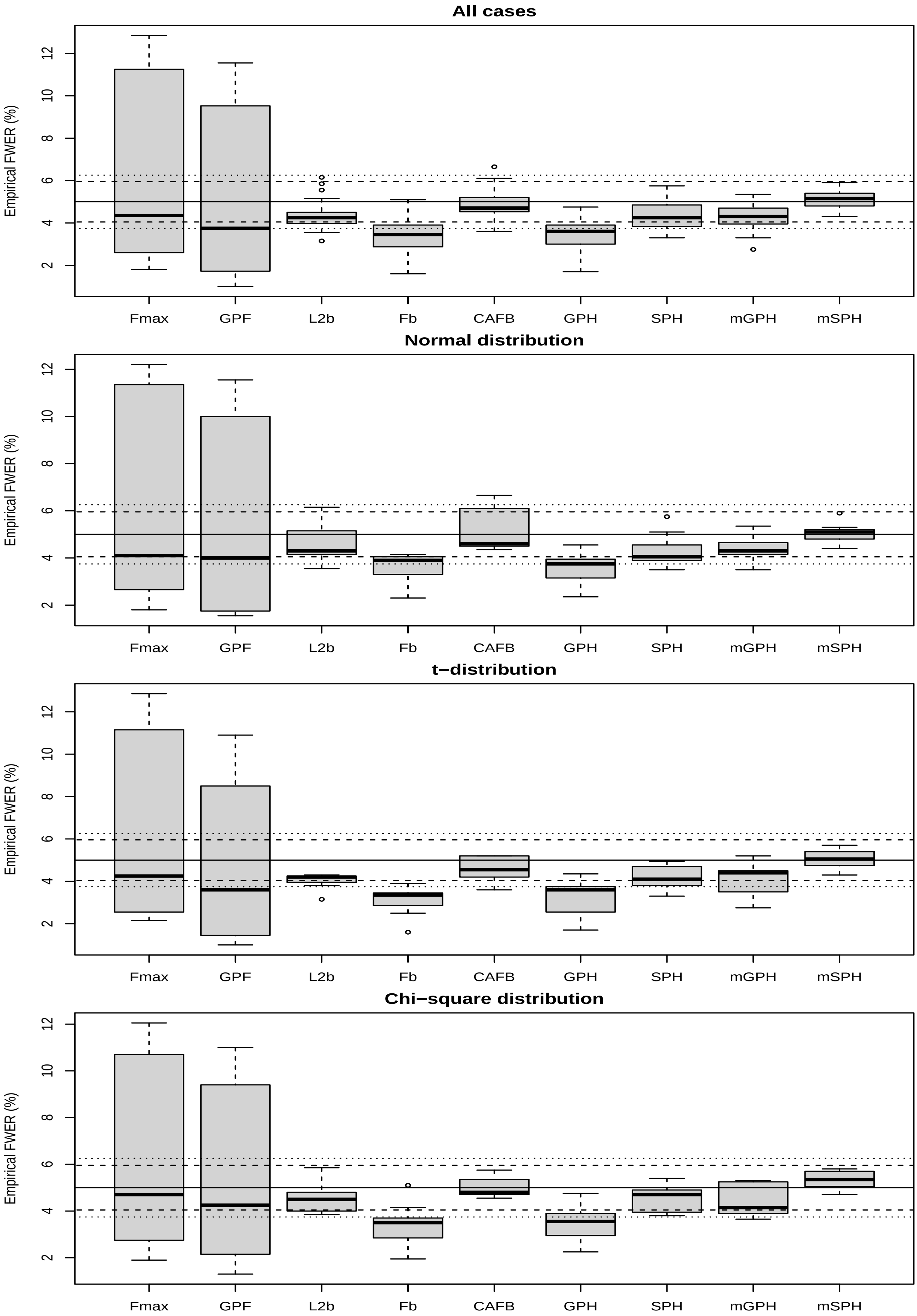


Fig. S4: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts for different distributions

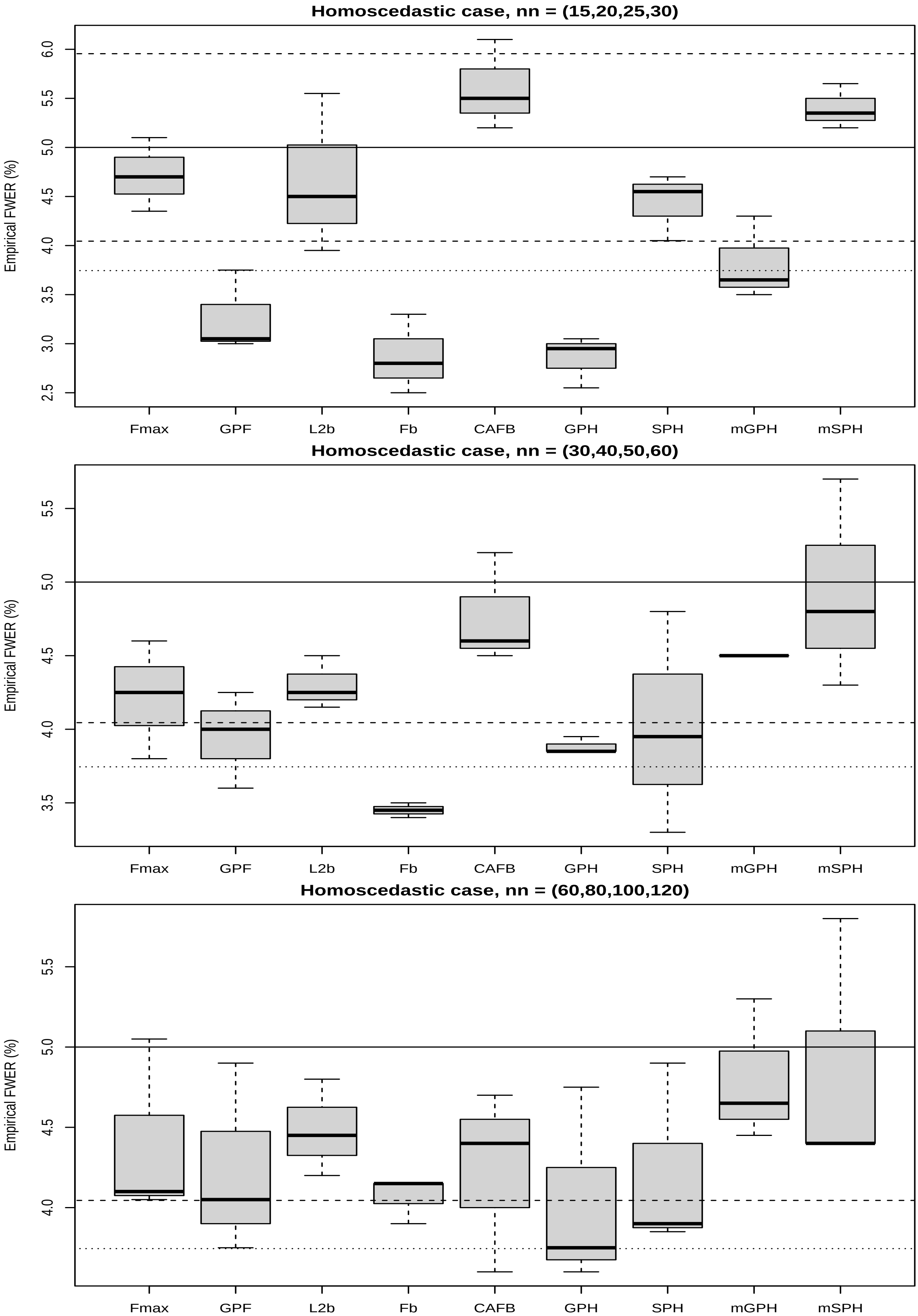


Fig. S5: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes and homoscedastic case

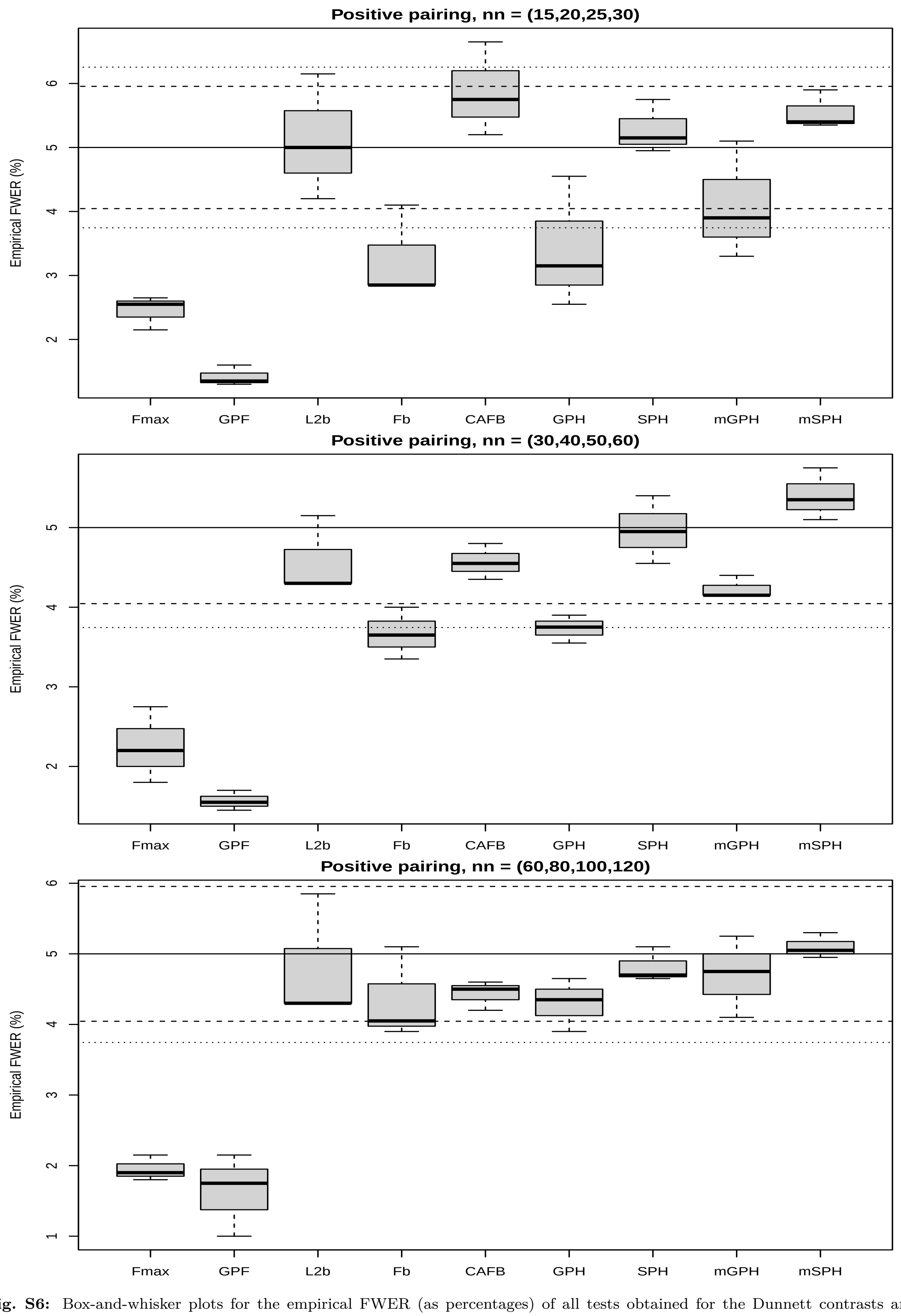


Fig. S6: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes and positive pairing

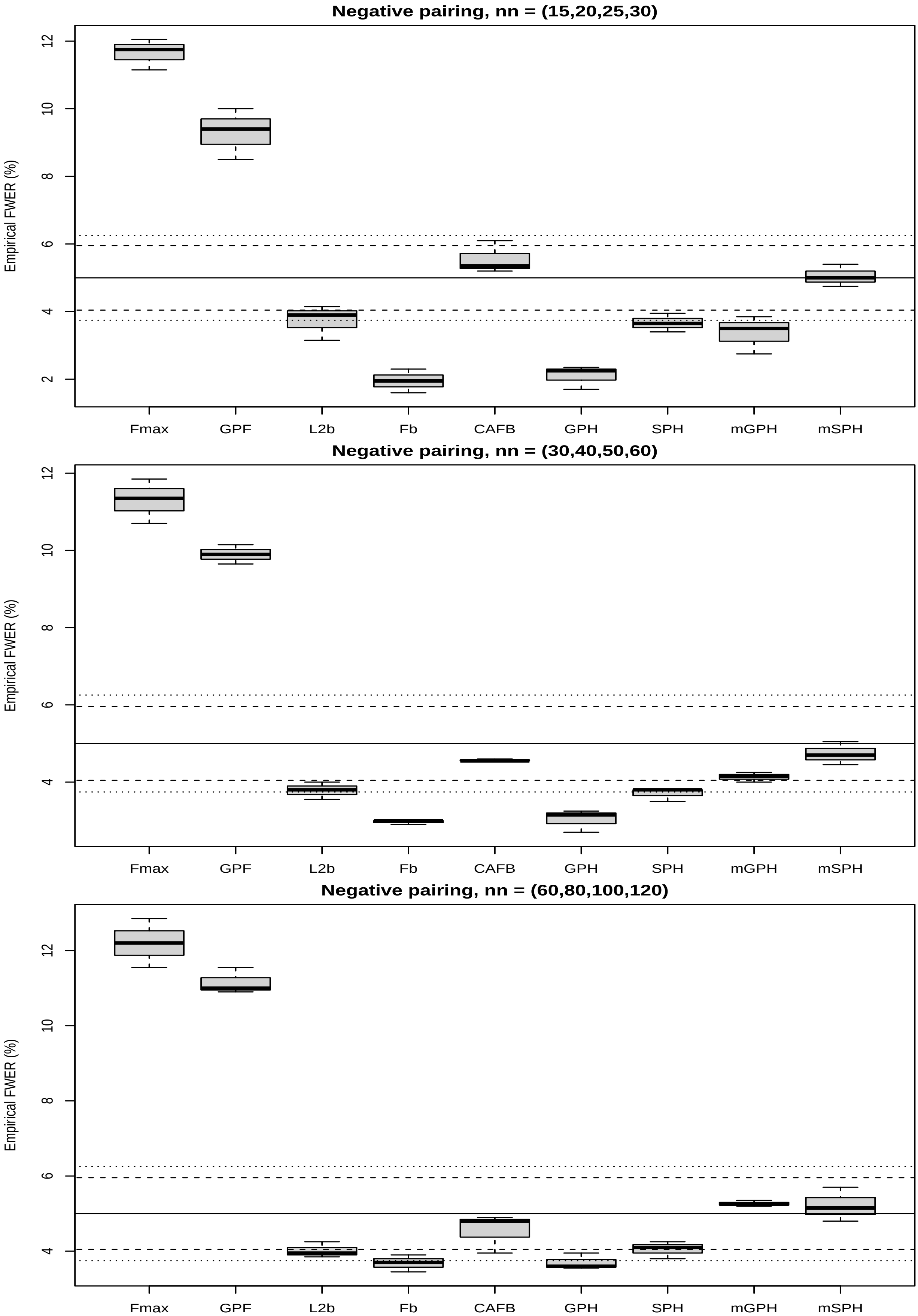


Fig. S7: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes and negative pairing

Table S5: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.90	1.45	1.80	1.15	1.75	1.40	2.15	1.95	2.30
			$\mathcal{H}_{0,2}$	1.60	0.70	1.65	0.90	2.10	1.00	1.90	1.30	2.25
			$\mathcal{H}_{0,3}$	81.30	87.35	85.35	79.05	73.85	85.50	79.45	87.70	82.00
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.40	1.00	1.90	1.15	2.15	1.30	1.85	1.40	1.95
			$\mathcal{H}_{0,2}$	0.70	0.30	1.90	1.05	2.15	1.55	1.65	1.75	1.85
			$\mathcal{H}_{0,3}$	23.05	25.75	57.20	50.00	44.25	58.90	49.30	60.90	50.85
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.75	1.80	2.25	1.20	1.70	1.35	1.45	2.10	2.00
			$\mathcal{H}_{0,2}$	3.95	3.35	1.35	0.80	2.05	1.15	2.15	1.80	2.40
			$\mathcal{H}_{0,3}$	66.10	69.15	34.50	24.65	27.75	31.90	27.85	39.30	32.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.90	1.25	1.80	1.05	1.80	0.90	1.60	1.30	2.20
			$\mathcal{H}_{0,2}$	1.60	1.20	1.45	0.90	2.20	1.10	1.35	1.20	1.60
			$\mathcal{H}_{0,3}$	53.35	60.20	55.05	47.15	46.05	55.65	50.65	60.20	54.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.25	1.05	1.90	1.05	1.45	1.30	1.50	1.40	1.60
			$\mathcal{H}_{0,2}$	0.55	0.35	1.65	1.05	1.50	1.10	1.65	1.15	1.75
			$\mathcal{H}_{0,3}$	10.00	8.70	29.60	24.05	23.90	28.65	26.55	30.55	27.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.25	1.50	1.55	0.90	1.55	0.95	1.50	1.55	2.10
			$\mathcal{H}_{0,2}$	4.20	2.55	1.20	0.70	2.20	0.75	1.75	1.00	2.60
			$\mathcal{H}_{0,3}$	45.40	45.50	17.20	11.00	15.70	15.15	16.35	19.90	18.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	2.10	1.10	1.50	0.65	1.80	1.00	2.10	1.30	2.40
			$\mathcal{H}_{0,2}$	1.95	1.55	2.40	1.45	1.95	1.70	2.00	2.10	2.45
			$\mathcal{H}_{0,3}$	82.00	87.90	83.15	76.80	72.90	83.50	79.10	86.95	81.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.35	1.15	2.25	1.20	2.50	1.50	1.65	1.60	1.80
			$\mathcal{H}_{0,2}$	0.70	0.35	1.65	1.20	1.50	1.25	1.65	1.60	1.95
			$\mathcal{H}_{0,3}$	22.65	25.40	54.70	48.30	45.45	56.50	49.50	58.20	51.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.85	1.20	1.45	0.85	2.20	1.05	1.20	1.30	2.05
			$\mathcal{H}_{0,2}$	3.35	2.35	1.75	0.75	1.85	1.05	1.30	1.60	1.70
			$\mathcal{H}_{0,3}$	67.00	69.00	36.50	28.20	31.15	35.70	33.35	41.70	37.65

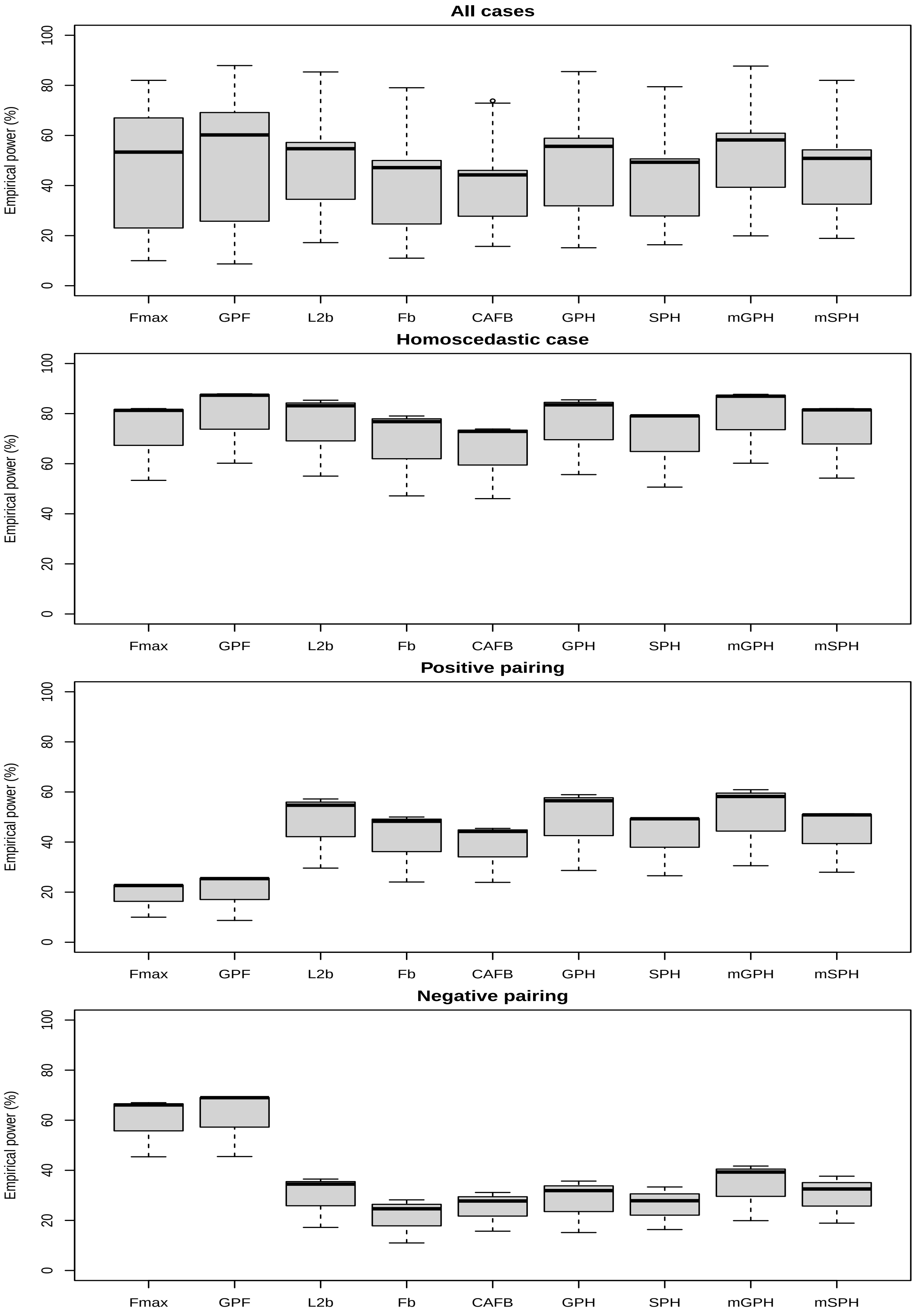


Fig. S8: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

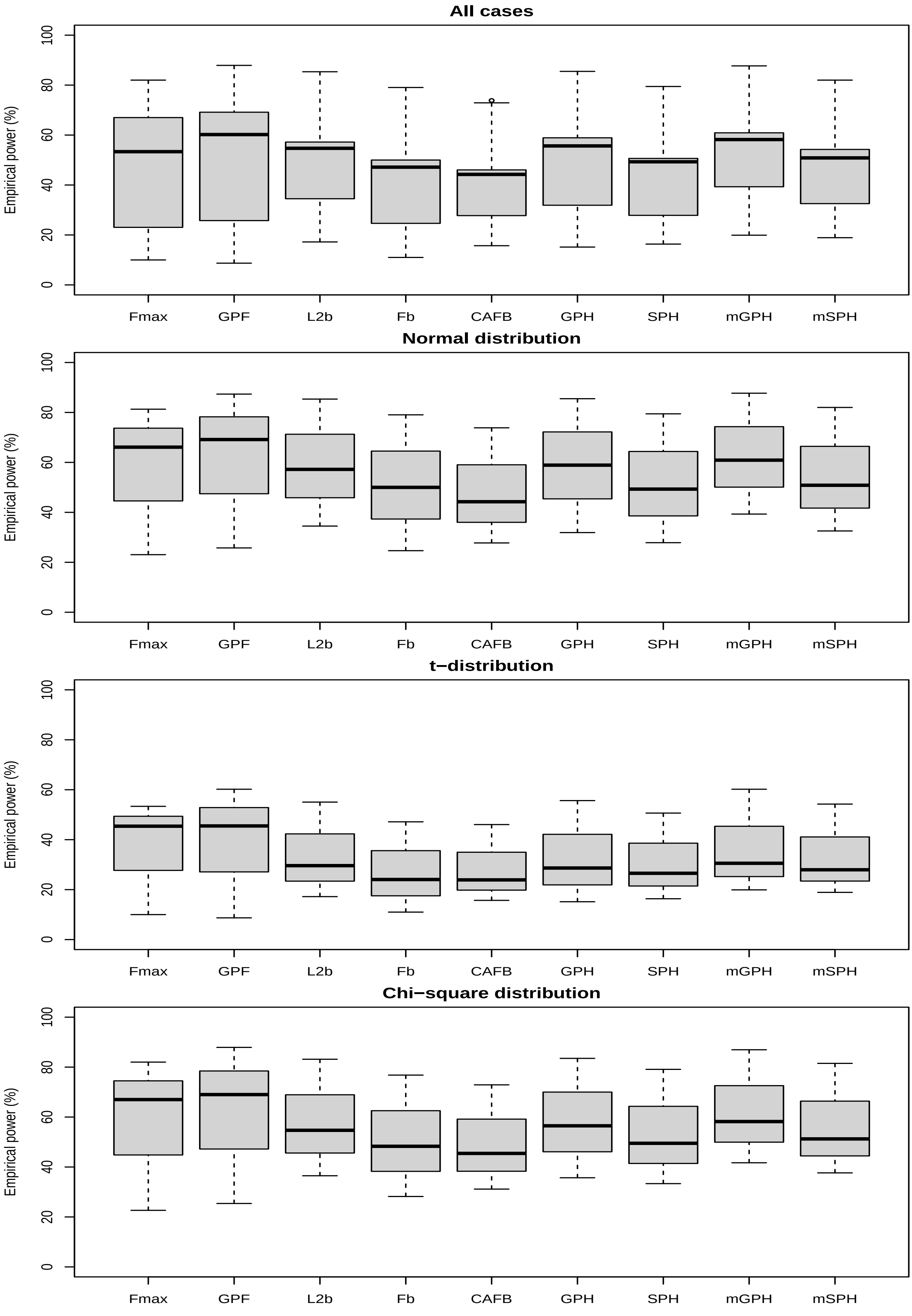


Fig. S9: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different distributions

Table S6: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	2.10	1.30	2.00	1.05	2.10	1.50	2.30	1.70	2.70
			$\mathcal{H}_{0,2}$	1.65	1.10	1.85	1.10	1.75	1.25	1.90	1.65	2.15
			$\mathcal{H}_{0,3}$	80.95	98.50	97.95	96.50	90.70	98.25	77.75	98.65	80.60
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.05	0.80	1.90	1.05	2.05	1.15	1.45	1.35	1.75
			$\mathcal{H}_{0,2}$	0.60	0.30	1.40	0.95	1.70	1.15	1.50	1.40	1.65
			$\mathcal{H}_{0,3}$	21.40	57.60	84.25	79.95	68.30	84.15	49.70	85.35	51.45
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.40	1.45	1.65	1.00	2.10	0.85	1.70	1.75	2.30
			$\mathcal{H}_{0,2}$	4.70	3.80	2.20	0.90	2.60	1.40	1.65	2.00	2.25
			$\mathcal{H}_{0,3}$	70.90	85.60	61.75	50.90	45.25	58.95	30.55	65.65	35.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.45	0.70	1.80	0.75	1.75	0.95	1.50	1.30	2.15
			$\mathcal{H}_{0,2}$	1.35	0.75	1.65	0.70	1.80	0.75	1.15	1.10	1.20
			$\mathcal{H}_{0,3}$	47.15	82.15	79.20	73.15	65.25	78.80	46.25	82.05	49.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.15	0.85	1.75	1.00	1.60	1.00	1.35	1.20	1.45
			$\mathcal{H}_{0,2}$	0.90	0.25	1.30	0.85	1.50	0.80	1.80	0.90	1.90
			$\mathcal{H}_{0,3}$	8.90	25.45	54.90	48.55	38.70	53.50	26.70	55.45	28.35
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.65	1.80	1.75	0.95	1.60	1.15	1.85	2.00	2.50
			$\mathcal{H}_{0,2}$	4.15	2.85	1.20	0.60	2.80	0.95	2.15	1.40	2.80
			$\mathcal{H}_{0,3}$	48.75	64.10	33.85	24.75	26.15	30.85	17.90	36.95	22.35
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.85	1.10	1.60	0.80	1.85	1.00	1.85	1.20	2.05
			$\mathcal{H}_{0,2}$	1.80	1.05	2.00	1.00	2.05	1.25	1.45	1.50	1.95
			$\mathcal{H}_{0,3}$	81.40	98.25	95.85	94.10	89.60	96.70	78.60	97.45	81.40
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.15	0.60	1.35	0.80	1.65	0.85	1.25	1.00	1.40
			$\mathcal{H}_{0,2}$	0.60	0.55	2.00	1.50	2.50	1.70	1.60	2.00	1.85
			$\mathcal{H}_{0,3}$	18.55	59.65	83.70	80.25	68.05	84.35	49.80	85.80	51.55
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.90	1.70	1.65	0.70	1.70	0.85	1.15	1.70	1.60
			$\mathcal{H}_{0,2}$	4.35	3.30	2.05	1.30	2.65	1.35	1.85	2.10	2.80
			$\mathcal{H}_{0,3}$	71.05	85.15	60.80	52.35	47.20	59.10	34.85	64.30	39.60

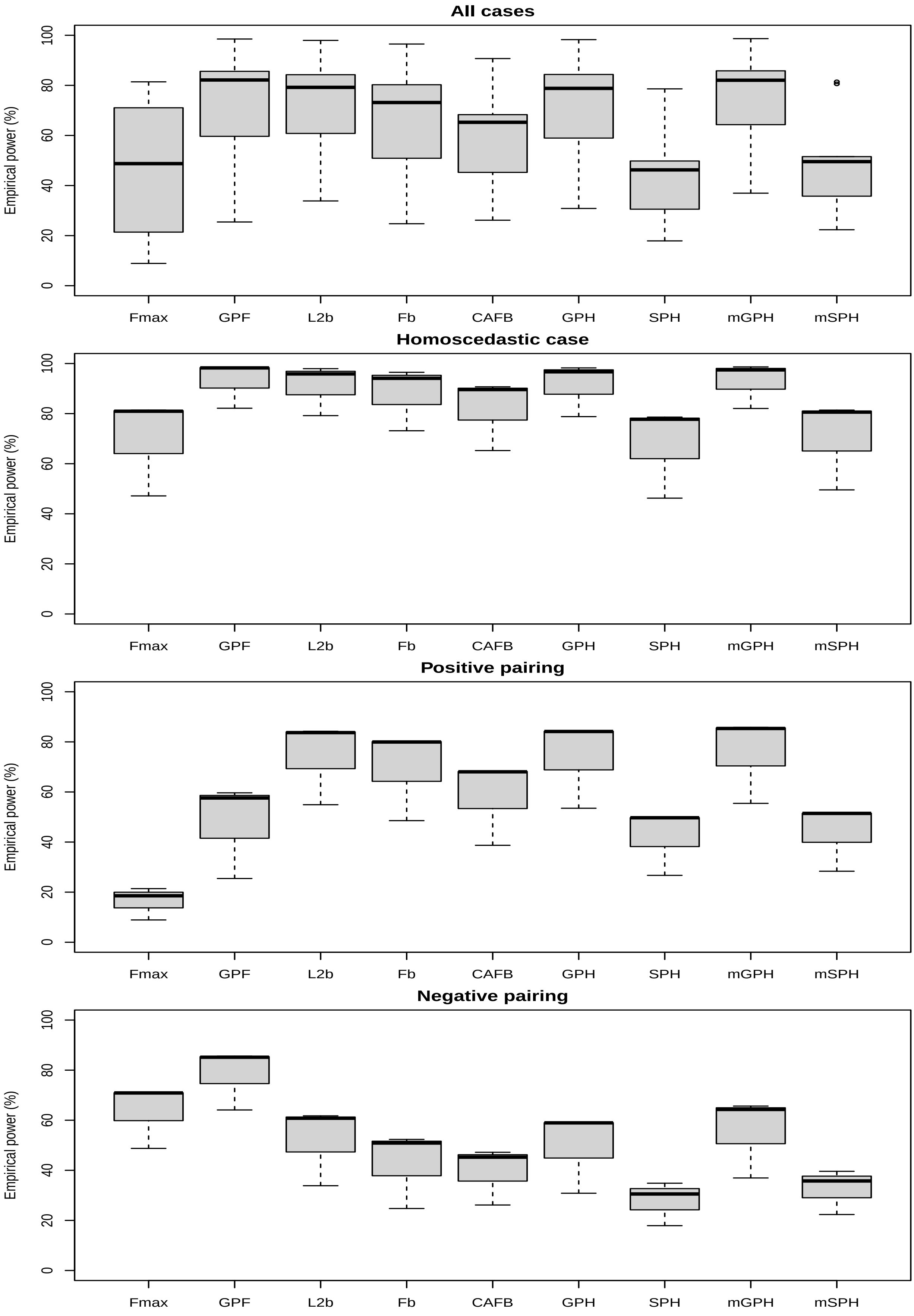


Fig. S10: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

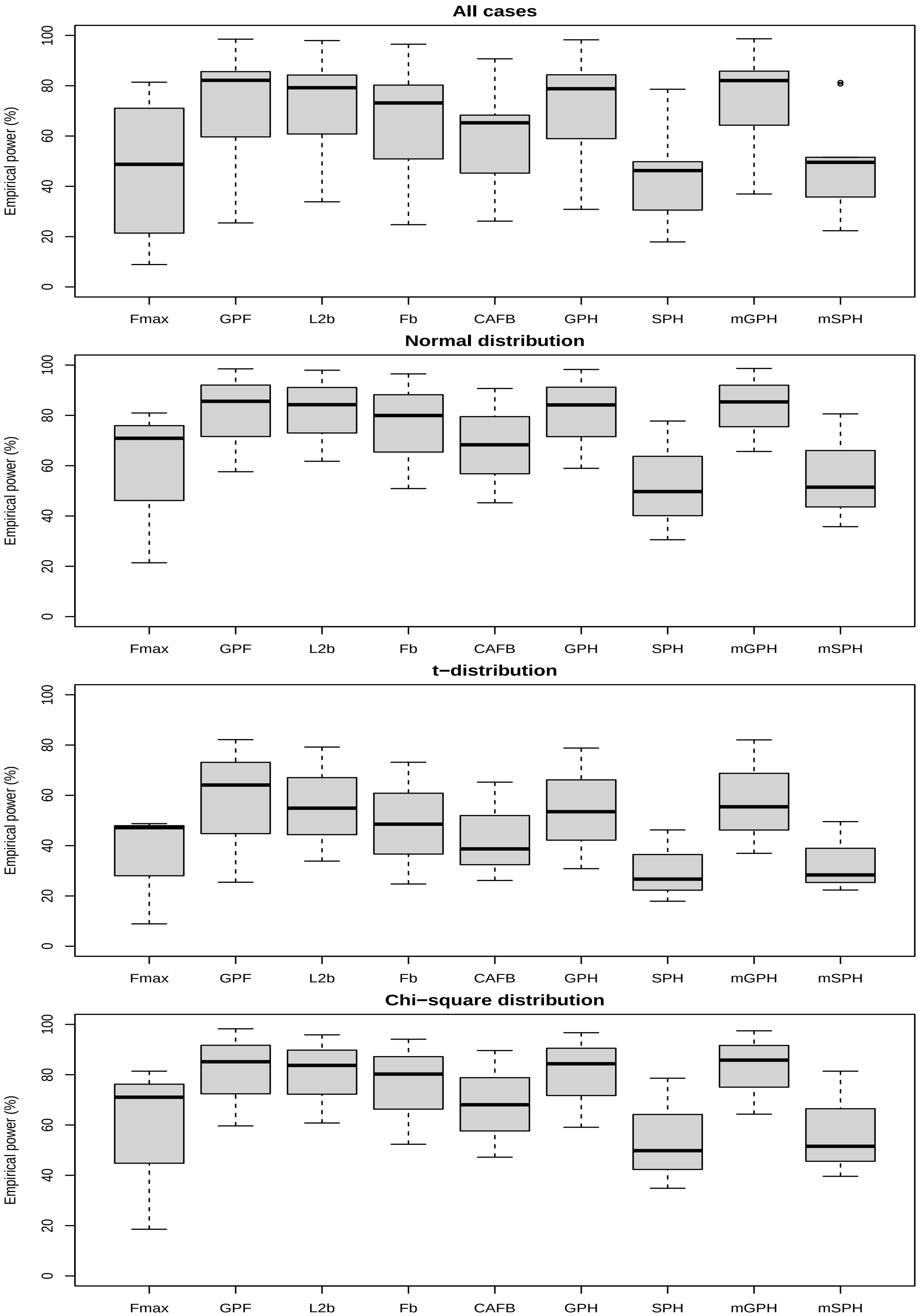


Fig. S11: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts and different distributions

Table S7: Empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	13.80	26.55	29.30	22.90	19.55	26.10	13.65	30.10	15.45
			$\mathcal{H}_{0,2}$	75.15	97.75	97.20	95.20	88.25	97.25	73.65	97.75	76.75
			$\mathcal{H}_{0,3}$	99.85	100.00	100.00	100.00	99.95	100.00	99.70	100.00	99.90
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	9.05	16.50	23.05	17.35	16.30	19.80	10.50	20.95	11.40
			$\mathcal{H}_{0,2}$	35.85	75.60	86.00	82.05	69.95	86.80	52.25	87.60	53.75
			$\mathcal{H}_{0,3}$	68.00	98.45	99.75	99.70	98.30	99.85	92.15	99.85	93.10
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	7.50	8.80	8.75	6.00	6.90	7.00	6.30	8.95	8.05
			$\mathcal{H}_{0,2}$	45.05	69.25	56.20	46.20	40.30	53.80	28.35	59.35	33.60
			$\mathcal{H}_{0,3}$	95.70	99.65	96.20	93.70	85.85	96.20	72.65	97.25	77.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	9.40	13.55	15.05	10.65	11.55	12.85	8.60	14.60	9.75
			$\mathcal{H}_{0,2}$	48.25	80.65	78.40	72.85	65.25	78.10	46.35	81.30	49.50
			$\mathcal{H}_{0,3}$	93.25	99.65	99.15	98.90	97.10	99.35	91.45	99.65	92.90
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	6.45	9.15	14.35	10.05	10.30	11.10	7.30	12.05	7.70
			$\mathcal{H}_{0,2}$	19.85	43.15	60.15	52.75	44.15	59.05	30.45	61.30	32.25
			$\mathcal{H}_{0,3}$	37.35	81.35	94.20	92.55	85.35	94.60	69.35	95.35	71.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	5.65	5.15	5.40	3.20	4.40	3.95	4.25	5.55	5.25
			$\mathcal{H}_{0,2}$	28.85	44.90	31.30	23.75	23.85	27.55	15.40	33.95	19.70
			$\mathcal{H}_{0,3}$	81.50	93.85	78.10	69.55	63.65	76.50	45.70	81.10	51.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	15.45	28.40	30.80	24.85	20.75	27.90	16.20	30.75	17.70
			$\mathcal{H}_{0,2}$	78.10	97.75	95.90	94.05	88.25	96.05	76.90	97.30	79.45
			$\mathcal{H}_{0,3}$	99.85	100.00	99.90	99.90	99.80	100.00	99.40	100.00	99.60
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.85	16.35	22.65	18.10	15.70	20.20	11.10	21.30	11.80
			$\mathcal{H}_{0,2}$	35.75	77.20	86.15	82.20	71.60	87.10	53.90	88.30	55.70
			$\mathcal{H}_{0,3}$	69.95	99.15	99.75	99.60	98.25	99.95	92.95	99.95	94.00
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.80	9.70	10.50	7.20	9.35	8.55	5.70	11.25	7.00
			$\mathcal{H}_{0,2}$	48.50	72.35	56.60	48.30	45.55	55.00	32.05	61.90	36.20
			$\mathcal{H}_{0,3}$	96.50	99.35	93.70	89.50	85.60	94.15	74.35	95.90	79.25

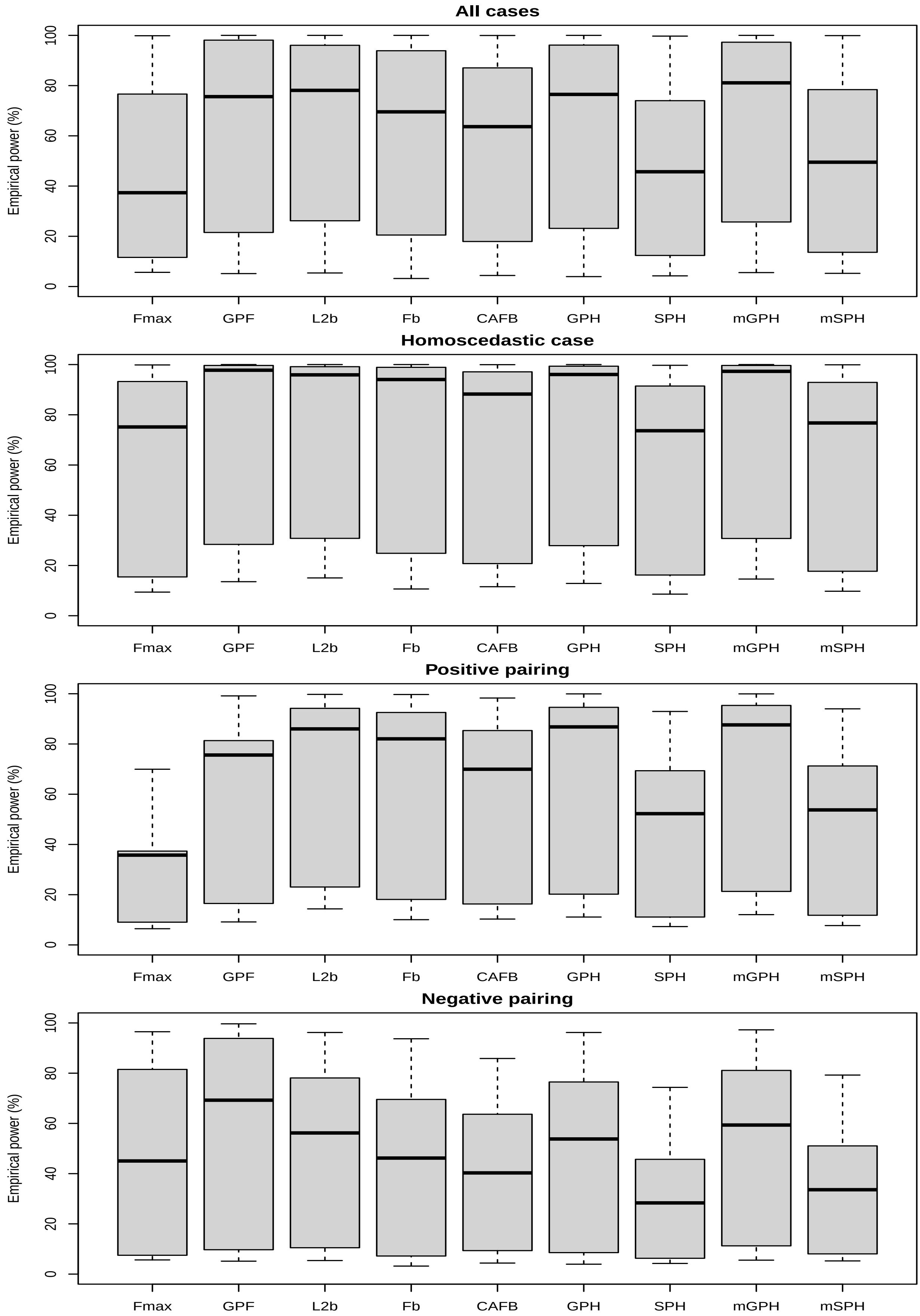


Fig. S12: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

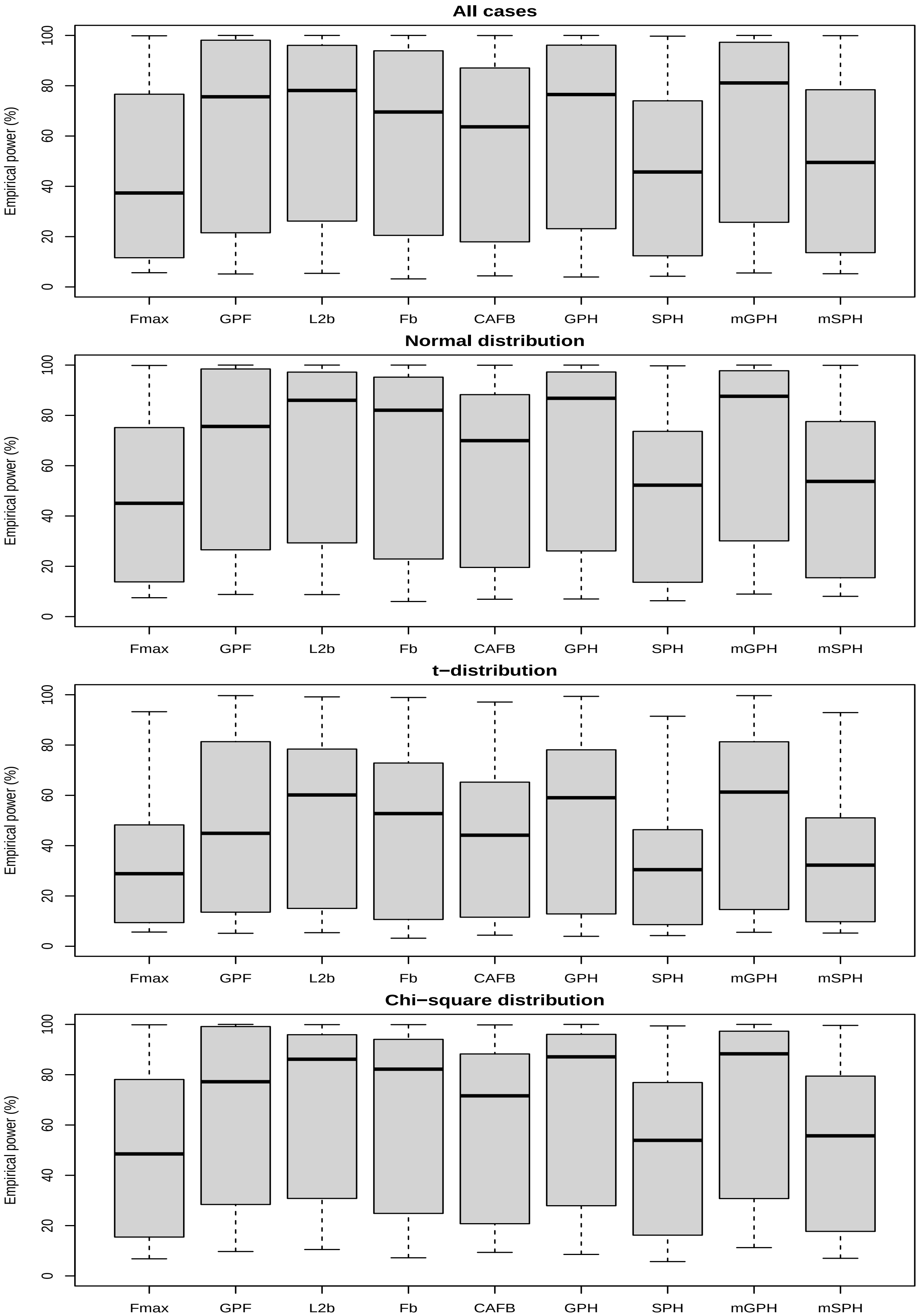


Fig. S13: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts and different distributions

Table S8: Empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.85	13.75	14.70	10.95	11.50	13.30	12.65	15.60	14.45
			$\mathcal{H}_{0,2}$	77.70	84.75	80.85	73.75	70.00	82.30	75.60	85.35	77.75
			$\mathcal{H}_{0,3}$	100.00	99.95	99.95	99.80	98.80	99.90	99.90	99.95	100.00
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	9.05	8.65	12.40	8.80	9.65	10.95	11.30	11.95	11.75
			$\mathcal{H}_{0,2}$	37.50	44.30	60.30	52.00	48.10	60.65	52.80	61.90	53.95
			$\mathcal{H}_{0,3}$	77.00	85.50	95.60	93.90	88.60	96.30	94.30	96.80	94.75
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.10	5.90	6.35	3.90	4.95	4.50	4.55	6.10	6.05
			$\mathcal{H}_{0,2}$	42.90	47.25	32.25	22.60	24.90	28.70	25.80	35.25	30.35
			$\mathcal{H}_{0,3}$	95.65	97.15	82.20	71.75	66.35	81.05	72.20	86.05	77.35
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.15	6.50	7.00	4.55	6.50	5.85	7.05	7.25	8.30
			$\mathcal{H}_{0,2}$	48.35	55.40	51.30	43.50	43.45	52.35	47.45	56.35	50.00
			$\mathcal{H}_{0,3}$	93.90	97.10	93.80	91.70	89.05	96.15	92.70	96.85	94.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.25	4.80	6.50	4.55	5.55	5.40	5.90	6.10	6.15
			$\mathcal{H}_{0,2}$	20.40	19.85	32.60	26.95	27.65	33.35	30.30	34.80	31.90
			$\mathcal{H}_{0,3}$	40.50	47.05	72.55	67.20	62.85	76.40	69.40	77.80	70.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	5.20	3.90	3.35	2.05	3.45	2.55	4.00	3.85	4.80
			$\mathcal{H}_{0,2}$	23.30	25.10	15.00	10.45	13.85	13.75	13.50	16.90	15.95
			$\mathcal{H}_{0,3}$	80.05	82.65	48.95	37.40	41.25	47.15	44.55	55.05	49.15
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	13.85	14.35	16.55	11.45	12.85	14.80	13.00	17.15	14.50
			$\mathcal{H}_{0,2}$	78.75	85.45	80.05	73.75	71.45	82.40	76.05	86.00	78.25
			$\mathcal{H}_{0,3}$	99.80	100.00	99.70	99.55	98.35	99.90	99.55	99.95	99.65
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	7.55	6.55	10.30	6.75	9.60	8.70	9.05	9.60	9.95
			$\mathcal{H}_{0,2}$	36.70	43.80	59.15	51.65	49.70	62.65	54.40	64.45	56.00
			$\mathcal{H}_{0,3}$	78.35	86.30	95.20	93.35	87.55	96.40	94.10	96.95	94.45
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.00	4.85	5.30	3.50	5.55	3.95	4.65	5.10	6.20
			$\mathcal{H}_{0,2}$	42.90	46.60	32.05	25.00	27.45	30.95	28.00	35.20	31.70
			$\mathcal{H}_{0,3}$	94.95	95.50	76.60	67.70	66.65	77.15	71.15	82.45	75.60

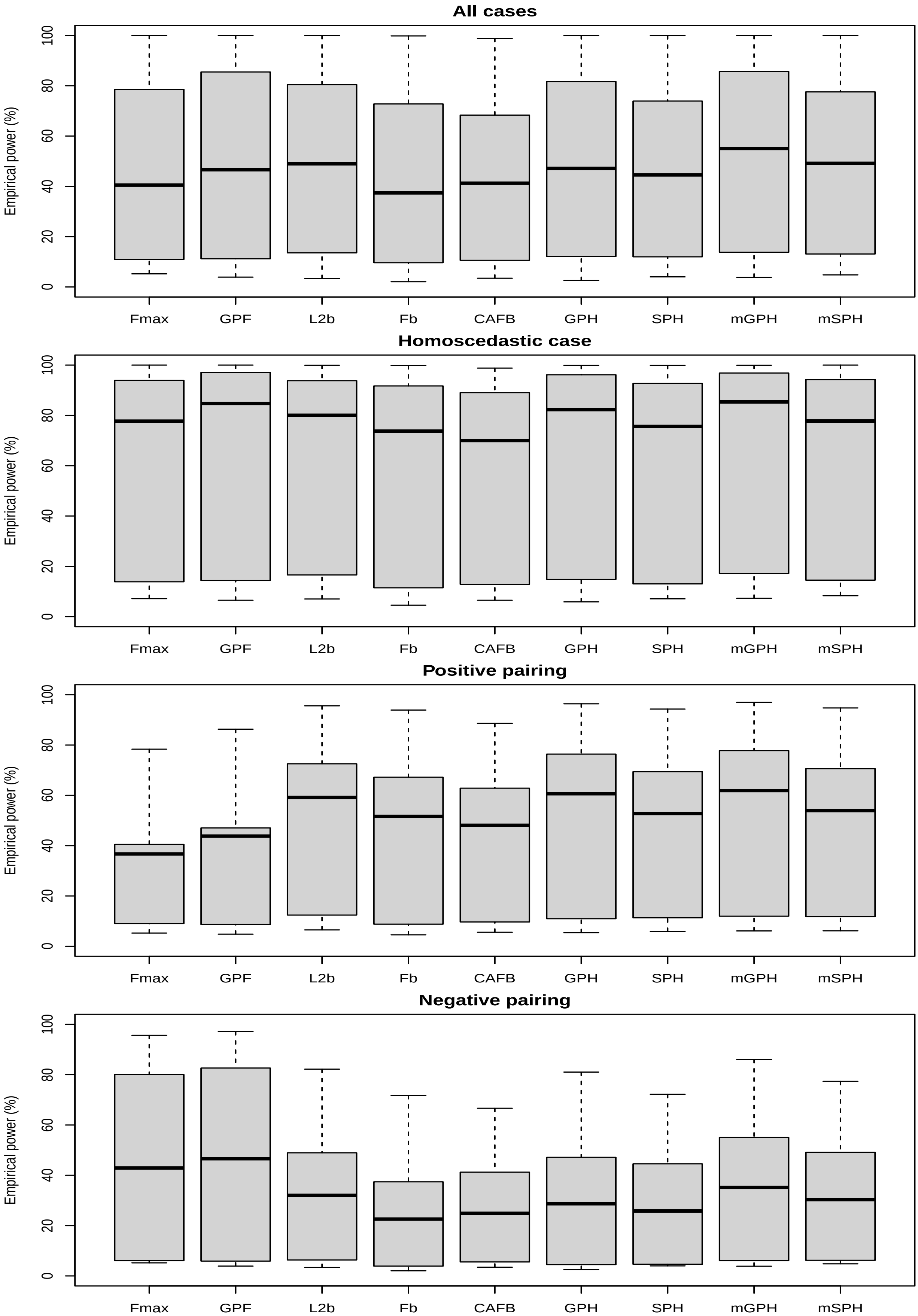


Fig. S14: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

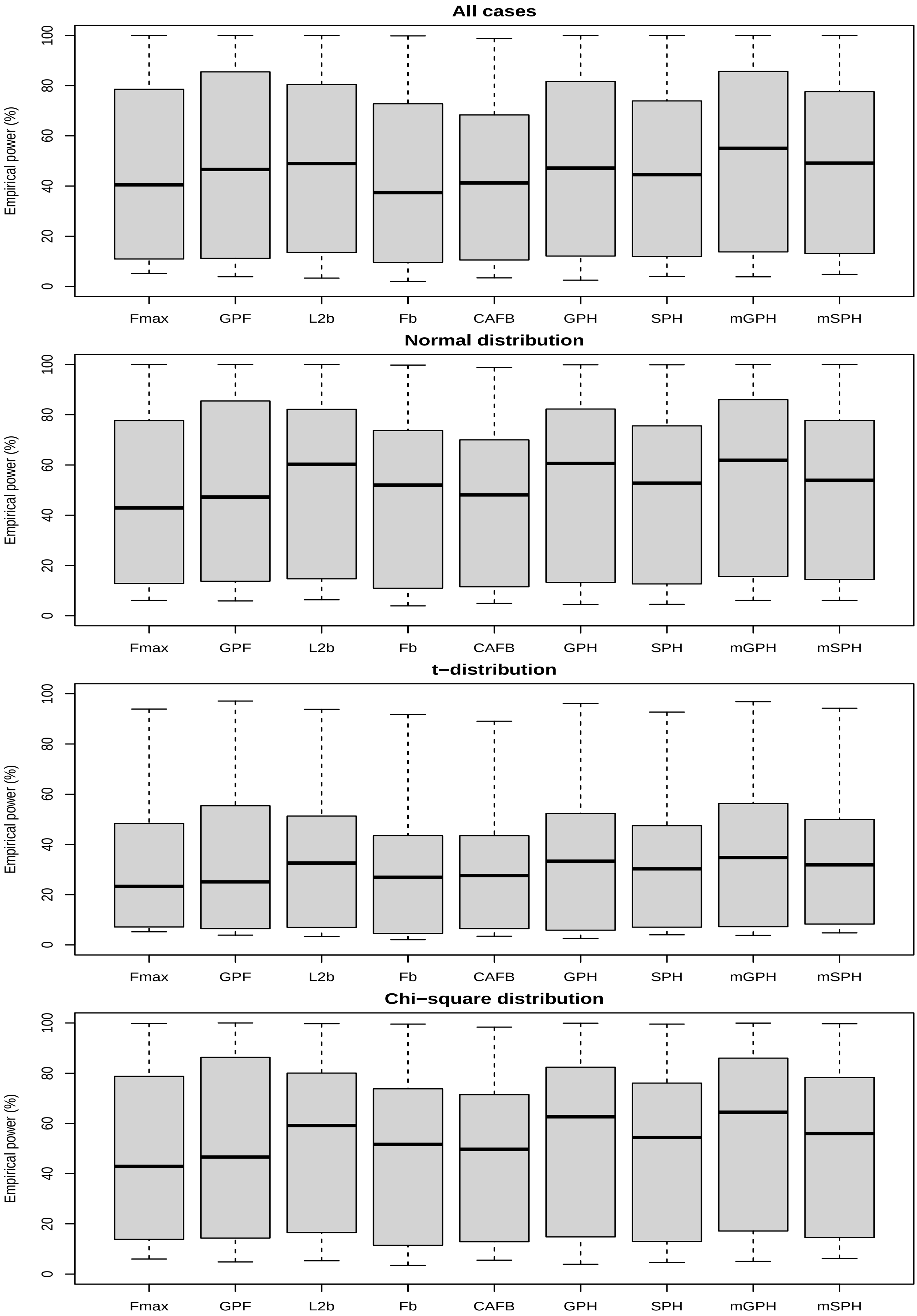


Fig. S15: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts and different distributions

Table S9: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.85	1.65	2.20	1.45	2.25	1.65	1.95	2.10	2.20
			$\mathcal{H}_{0,2}$	1.85	1.15	1.60	0.85	2.25	1.05	1.85	1.40	2.15
			$\mathcal{H}_{0,3}$	79.15	86.40	82.40	75.65	70.55	83.65	76.30	86.55	78.65
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.10	0.70	1.75	0.85	1.75	1.10	1.50	1.25	1.75
			$\mathcal{H}_{0,2}$	1.05	0.65	2.45	1.60	2.45	1.75	1.90	1.85	2.05
			$\mathcal{H}_{0,3}$	24.25	25.25	53.15	46.15	42.20	54.40	48.10	56.60	49.45
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.75	1.65	2.15	0.95	1.70	1.30	2.45	2.00	2.65
			$\mathcal{H}_{0,2}$	4.05	3.50	2.05	1.20	2.90	1.40	2.15	2.00	2.60
			$\mathcal{H}_{0,3}$	62.60	66.00	31.85	22.10	25.65	29.20	29.50	35.30	33.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.50	1.05	1.70	0.95	1.65	1.00	1.05	1.15	1.50
			$\mathcal{H}_{0,2}$	1.60	1.10	1.40	0.85	1.55	0.80	1.40	1.25	1.65
			$\mathcal{H}_{0,3}$	49.90	57.75	55.15	45.40	44.65	53.90	46.05	59.35	48.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.00	0.65	1.40	0.70	1.75	0.75	1.25	1.00	1.35
			$\mathcal{H}_{0,2}$	0.55	0.20	1.65	1.05	2.05	1.10	1.25	1.35	1.40
			$\mathcal{H}_{0,3}$	9.10	7.90	27.35	22.40	22.80	27.30	24.60	28.70	25.85
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.25	1.70	1.90	1.05	2.15	1.20	1.50	1.85	2.00
			$\mathcal{H}_{0,2}$	3.65	2.70	1.55	0.85	1.80	0.75	1.60	1.10	1.95
			$\mathcal{H}_{0,3}$	42.95	43.65	15.85	10.05	15.55	13.85	14.35	18.35	17.55
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.50	1.20	1.80	0.80	2.00	0.85	1.45	1.35	1.95
			$\mathcal{H}_{0,2}$	1.70	1.05	1.45	0.90	2.15	1.00	1.75	1.45	2.00
			$\mathcal{H}_{0,3}$	81.60	87.55	79.65	72.90	74.05	82.20	77.10	85.95	79.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.80	0.60	1.70	0.85	1.55	0.95	2.20	1.00	2.30
			$\mathcal{H}_{0,2}$	0.80	0.10	1.45	0.80	1.50	0.70	1.20	1.00	1.50
			$\mathcal{H}_{0,3}$	20.20	22.90	52.90	47.30	43.65	53.90	47.35	56.60	48.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.45	1.25	1.10	0.65	1.90	0.95	0.95	1.35	1.50
			$\mathcal{H}_{0,2}$	3.50	2.30	1.30	0.65	2.20	0.75	1.60	1.05	2.45
			$\mathcal{H}_{0,3}$	65.60	67.30	35.25	25.95	32.00	34.50	31.90	40.10	36.70

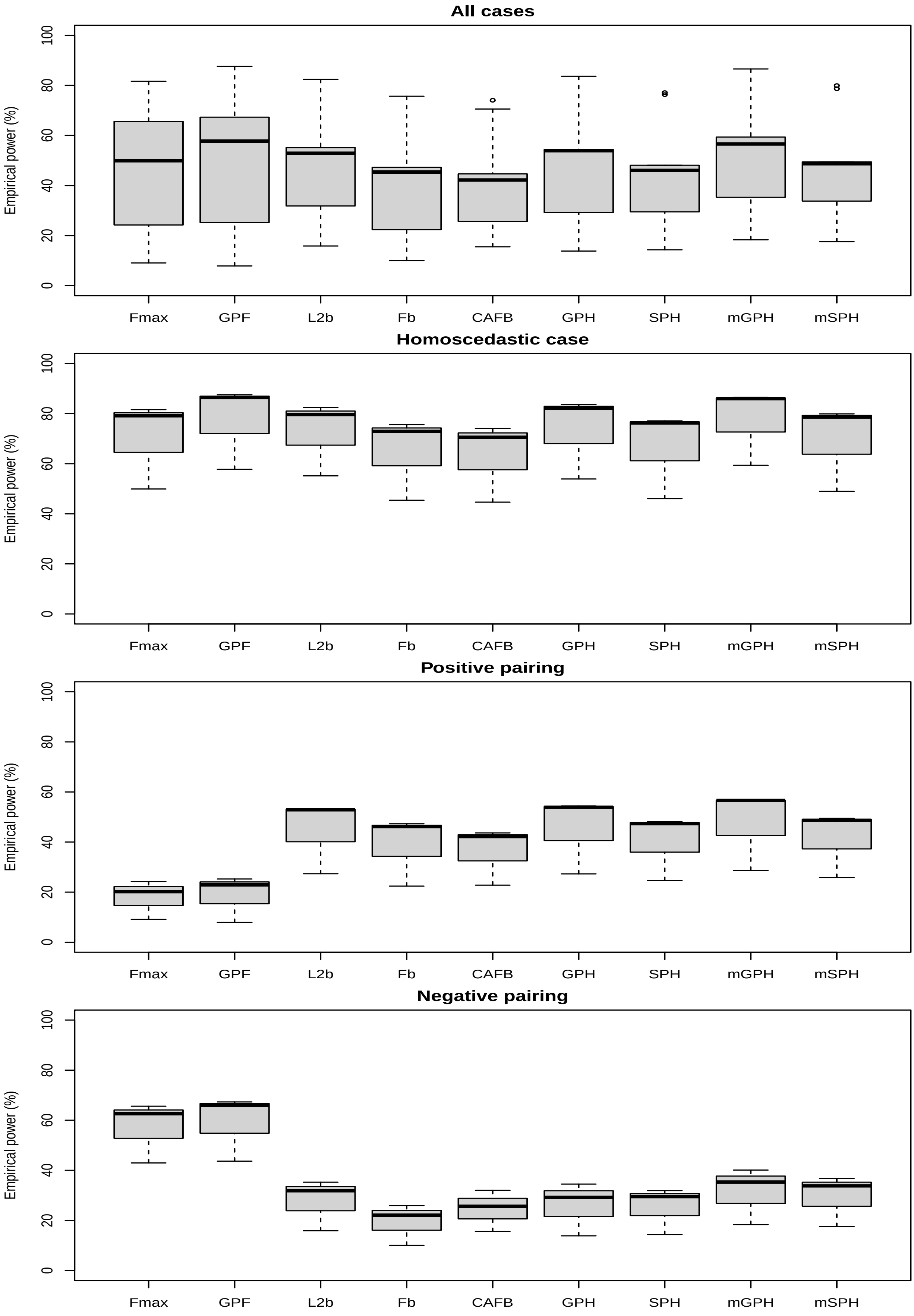


Fig. S16: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

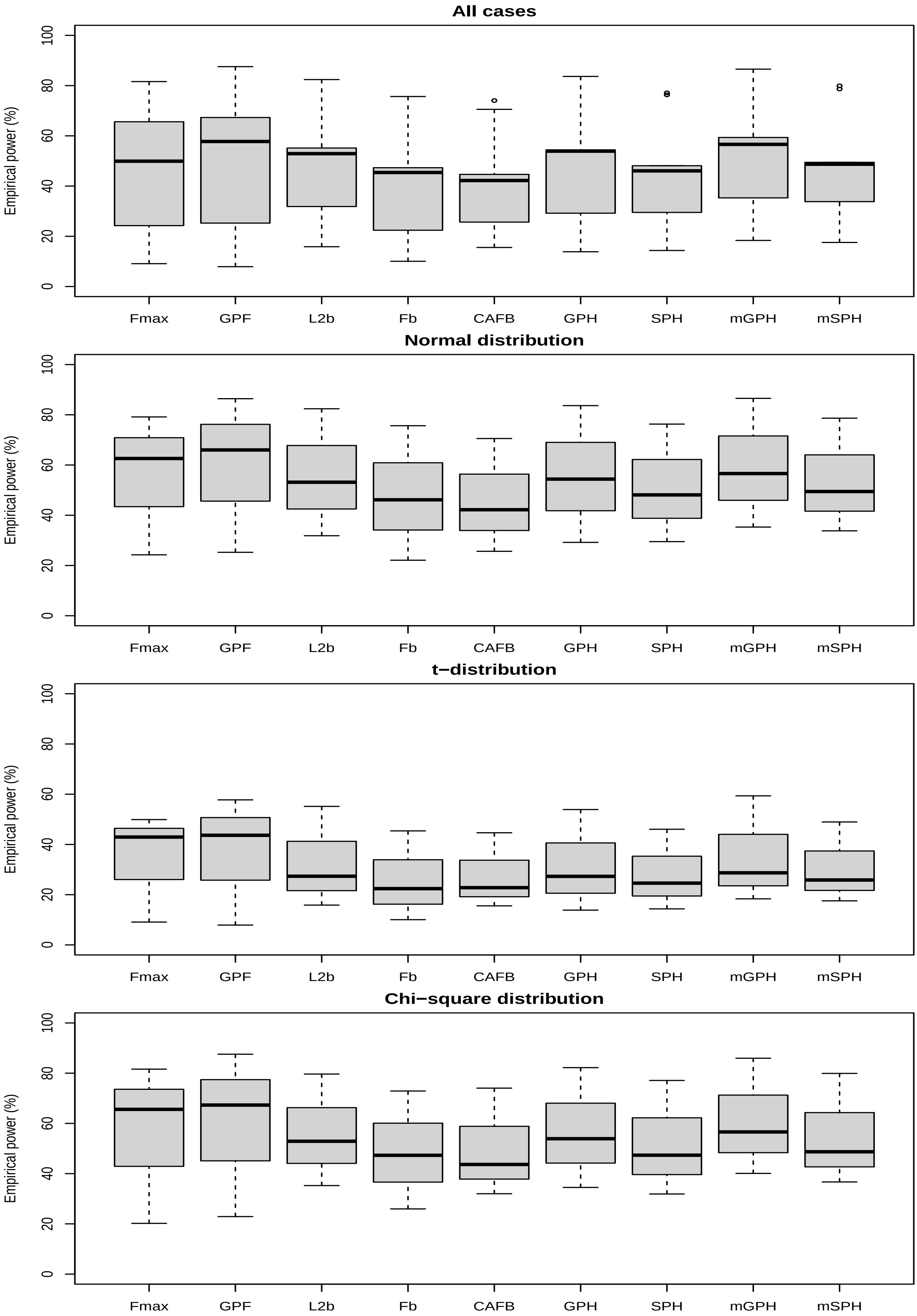


Fig. S17: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts and different distributions

Table S10: Empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.80	12.95	14.65	10.85	12.05	12.45	12.15	14.85	13.65
			$\mathcal{H}_{0,2}$	73.75	82.00	80.15	72.05	65.75	80.75	71.45	83.50	74.60
			$\mathcal{H}_{0,3}$	99.75	99.95	99.85	99.70	98.25	100.00	99.60	100.00	99.75
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	6.90	6.80	9.75	6.45	8.50	9.05	8.55	9.40	9.10
			$\mathcal{H}_{0,2}$	34.40	41.80	57.85	49.85	45.00	58.70	49.50	60.60	50.95
			$\mathcal{H}_{0,3}$	76.10	82.95	96.10	93.60	87.25	97.40	93.25	97.60	93.75
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	5.55	5.05	5.50	3.50	5.00	3.90	4.35	5.35	5.70
			$\mathcal{H}_{0,2}$	41.15	43.70	28.35	20.15	23.00	26.30	23.85	31.80	28.70
			$\mathcal{H}_{0,3}$	94.35	96.95	78.85	67.85	62.35	78.10	70.05	83.85	74.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.35	7.15	8.50	6.10	7.90	6.95	7.35	8.60	8.40
			$\mathcal{H}_{0,2}$	46.00	52.75	50.90	41.80	41.35	49.60	44.00	54.35	46.95
			$\mathcal{H}_{0,3}$	93.25	96.55	93.70	90.95	87.60	94.75	91.35	96.20	93.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.00	3.65	5.30	3.70	4.85	4.55	6.05	5.25	6.50
			$\mathcal{H}_{0,2}$	18.15	17.75	31.15	24.85	25.80	31.00	28.20	32.90	29.15
			$\mathcal{H}_{0,3}$	39.85	44.95	71.10	66.75	61.25	74.50	67.60	75.90	68.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	4.40	3.70	3.75	2.50	3.20	2.90	3.70	3.95	4.45
			$\mathcal{H}_{0,2}$	24.90	24.90	15.45	10.25	14.45	13.30	13.45	16.80	15.90
			$\mathcal{H}_{0,3}$	78.75	81.65	47.40	36.65	37.55	46.30	41.20	53.40	46.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.15	13.10	15.20	11.00	13.15	13.70	12.30	15.80	14.10
			$\mathcal{H}_{0,2}$	75.05	83.85	78.75	71.90	70.85	80.70	73.55	84.20	76.10
			$\mathcal{H}_{0,3}$	99.65	99.90	99.35	98.80	98.30	99.70	99.35	99.80	99.45
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.25	6.60	10.20	7.30	8.55	9.15	10.10	9.90	10.65
			$\mathcal{H}_{0,2}$	34.95	40.65	56.55	49.25	45.45	58.15	51.20	61.00	52.80
			$\mathcal{H}_{0,3}$	74.30	84.70	94.50	92.40	87.55	95.90	92.20	96.20	92.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	5.25	5.00	5.75	3.70	6.25	3.75	4.85	5.50	6.15
			$\mathcal{H}_{0,2}$	42.35	45.85	32.60	25.05	28.65	29.90	27.50	35.20	31.25
			$\mathcal{H}_{0,3}$	95.15	96.00	76.80	67.95	67.35	77.80	72.15	82.65	77.25

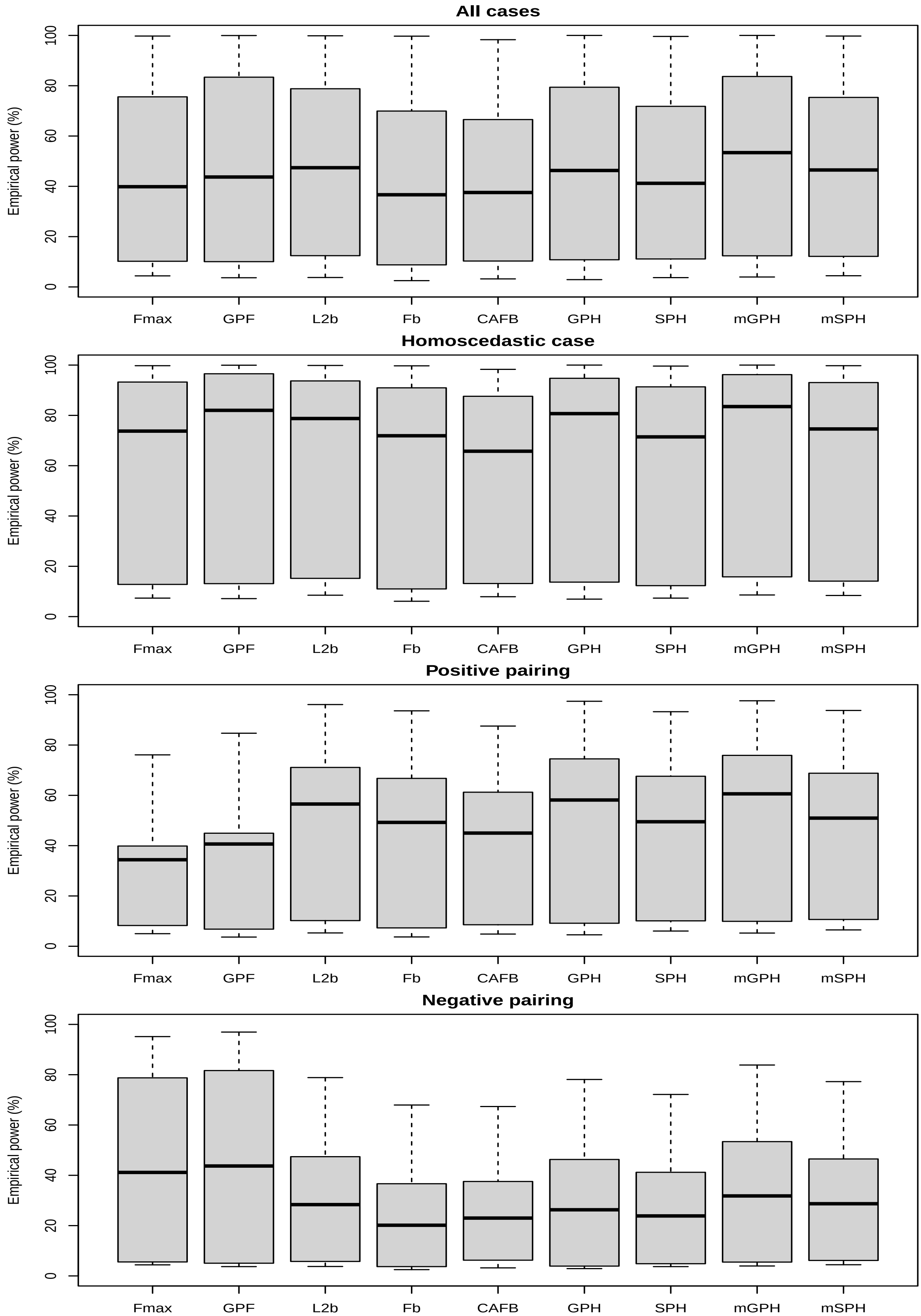


Fig. S18: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts and homoscedastic and heteroscedastic cases

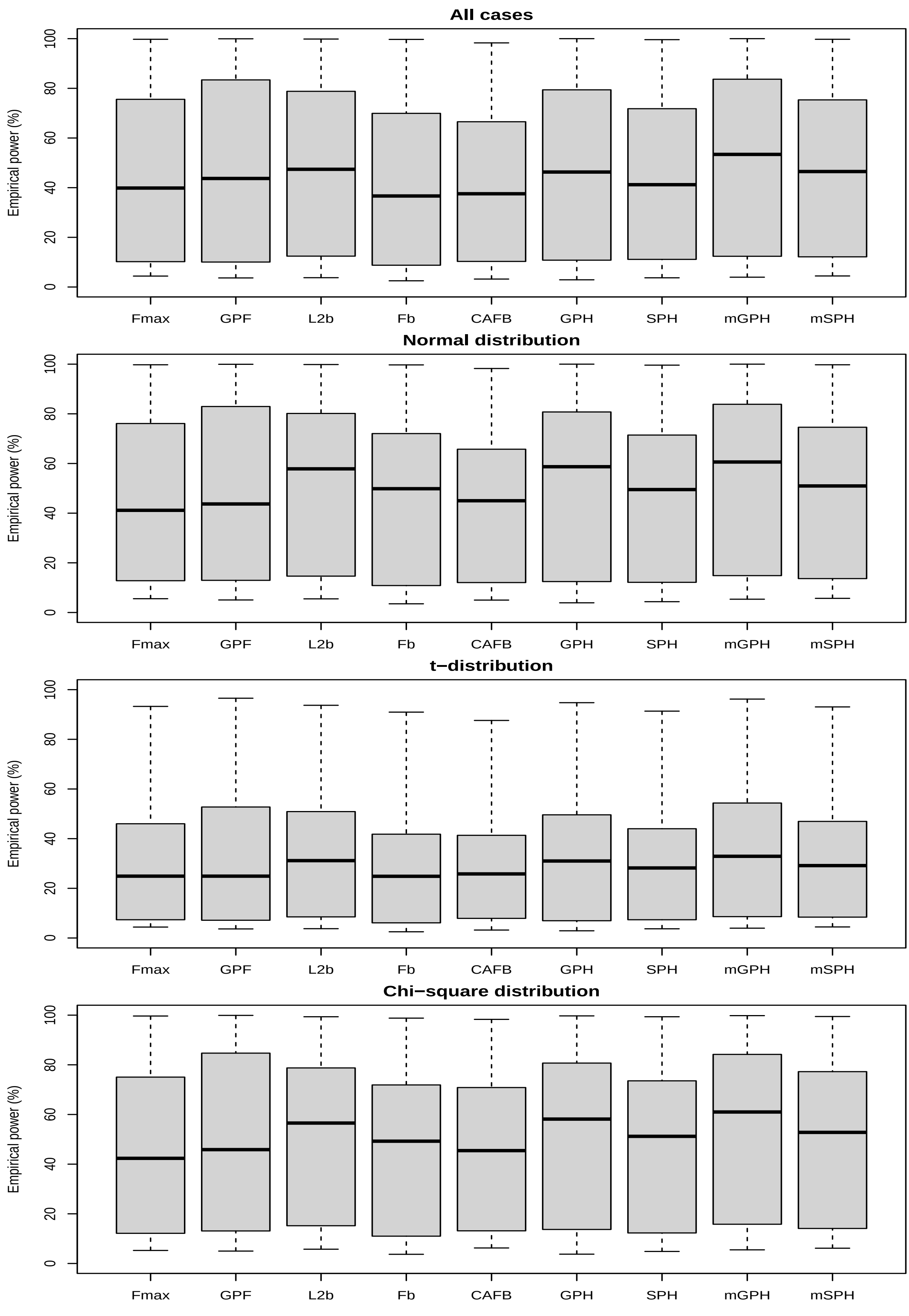


Fig. S19: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts and different distributions

Table S11: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.05	3.75	5.10	3.30	5.40	3.80	4.65	4.40	5.45
			$\mathcal{H}_{0,1}$	0.85	0.50	0.85	0.25	0.90	0.35	0.85	0.40	1.10
			$\mathcal{H}_{0,2}$	1.20	0.60	1.35	0.50	0.90	0.50	1.10	0.80	1.30
			$\mathcal{H}_{0,3}$	0.65	1.00	1.20	0.70	1.20	0.85	0.70	1.05	0.75
			$\mathcal{H}_{0,4}$	0.95	1.00	0.95	0.65	0.80	1.05	0.85	1.25	1.10
			$\mathcal{H}_{0,5}$	0.55	0.35	0.70	0.35	1.00	0.60	0.80	0.80	0.90
			$\mathcal{H}_{0,6}$	0.90	0.95	1.40	1.00	0.95	1.05	0.85	1.20	0.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	5.40	2.65	5.85	4.40	6.60	4.40	5.05	5.10	5.75
			$\mathcal{H}_{0,1}$	0.75	0.45	1.25	0.65	1.20	0.80	1.15	1.05	1.25
			$\mathcal{H}_{0,2}$	0.50	0.15	1.20	0.55	1.20	0.70	0.90	0.90	1.00
			$\mathcal{H}_{0,3}$	0.15	0.15	1.25	0.65	1.60	0.90	0.95	0.95	1.05
			$\mathcal{H}_{0,4}$	0.90	0.85	1.40	0.80	1.25	0.80	1.00	0.95	1.10
			$\mathcal{H}_{0,5}$	0.70	0.70	1.05	0.90	1.25	1.00	0.90	1.15	0.95
			$\mathcal{H}_{0,6}$	0.80	0.85	1.60	1.00	1.10	1.20	1.00	1.45	1.00
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	10.70	8.45	4.70	3.50	5.55	3.40	4.45	3.70	5.30
			$\mathcal{H}_{0,1}$	0.85	1.05	1.35	0.75	1.10	0.75	1.00	0.85	1.10
			$\mathcal{H}_{0,2}$	2.35	1.60	0.85	0.55	0.95	0.45	0.80	0.60	0.90
			$\mathcal{H}_{0,3}$	5.20	4.80	0.90	0.40	1.25	0.40	0.80	0.45	1.05
			$\mathcal{H}_{0,4}$	1.05	0.95	0.95	0.65	0.95	0.70	0.80	0.80	0.90
			$\mathcal{H}_{0,5}$	2.75	1.75	0.80	0.55	1.10	0.50	1.15	0.90	1.30
			$\mathcal{H}_{0,6}$	1.10	1.10	1.25	0.80	1.10	0.75	0.90	1.00	1.05
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.20	4.25	4.90	4.05	4.75	4.35	3.85	4.85	4.50
			$\mathcal{H}_{0,1}$	0.85	0.65	0.85	0.55	0.95	0.70	0.90	0.80	0.95
			$\mathcal{H}_{0,2}$	0.65	0.60	0.95	0.65	0.85	0.75	0.55	0.85	0.75
			$\mathcal{H}_{0,3}$	0.85	1.20	1.35	1.15	0.60	1.20	0.80	1.30	0.85
			$\mathcal{H}_{0,4}$	0.60	0.60	0.90	0.75	0.85	0.65	0.45	0.75	0.60
			$\mathcal{H}_{0,5}$	0.60	0.75	0.95	0.75	0.95	0.80	0.75	0.90	0.85
			$\mathcal{H}_{0,6}$	1.05	0.65	1.00	0.90	0.95	0.80	1.00	0.95	1.00
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.05	1.70	4.10	3.40	4.25	3.70	4.50	4.45	5.10
			$\mathcal{H}_{0,1}$	0.45	0.50	1.15	0.75	0.85	0.75	0.95	0.95	1.05
			$\mathcal{H}_{0,2}$	0.25	0.30	0.90	0.65	0.85	0.75	0.55	0.85	0.85
			$\mathcal{H}_{0,3}$	0.00	0.05	0.85	0.70	0.60	0.70	0.90	0.80	0.90
			$\mathcal{H}_{0,4}$	0.60	0.60	0.80	0.75	0.85	0.75	0.75	0.80	0.90
			$\mathcal{H}_{0,5}$	0.60	0.15	0.65	0.50	0.70	0.60	0.80	0.90	0.85
			$\mathcal{H}_{0,6}$	0.40	0.40	0.65	0.60	0.60	0.70	0.65	0.70	0.70
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	10.75	9.30	4.95	3.65	4.65	3.95	4.75	4.60	5.65
			$\mathcal{H}_{0,1}$	0.80	0.60	0.75	0.65	0.65	0.50	0.65	0.75	0.75
			$\mathcal{H}_{0,2}$	1.60	1.90	0.80	0.55	0.85	0.45	0.55	0.75	0.55
			$\mathcal{H}_{0,3}$	5.50	4.90	0.80	0.50	1.00	0.75	0.95	0.85	1.15
			$\mathcal{H}_{0,4}$	1.05	1.10	1.05	0.80	0.65	0.90	0.90	1.15	1.05
			$\mathcal{H}_{0,5}$	2.50	1.90	1.20	0.85	0.90	0.90	1.20	1.05	1.55
			$\mathcal{H}_{0,6}$	1.15	1.60	1.05	1.05	1.15	1.10	1.05	1.30	1.15
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.85	4.25	4.45	4.25	3.95	4.00	3.80	4.65	4.25
			$\mathcal{H}_{0,1}$	0.75	0.65	0.95	0.95	0.75	0.85	0.65	1.10	0.80
			$\mathcal{H}_{0,2}$	0.65	0.60	0.75	0.70	0.60	0.65	0.65	0.75	0.70
			$\mathcal{H}_{0,3}$	0.75	1.10	1.10	1.05	1.00	0.90	0.80	1.05	0.95
			$\mathcal{H}_{0,4}$	0.40	0.85	0.75	0.85	0.45	0.75	0.35	0.80	0.45
			$\mathcal{H}_{0,5}$	0.65	0.90	0.90	0.80	1.00	0.85	0.95	1.00	0.95
			$\mathcal{H}_{0,6}$	1.00	0.90	0.65	0.50	0.75	0.95	1.00	0.95	1.10
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	4.25	2.35	4.35	3.75	4.05	4.20	4.80	4.95	5.25
			$\mathcal{H}_{0,1}$	0.75	0.55	1.10	0.80	1.00	0.90	0.95	1.00	1.05
			$\mathcal{H}_{0,2}$	0.15	0.25	0.70	0.60	0.55	0.80	0.75	1.00	0.95
			$\mathcal{H}_{0,3}$	0.05	0.00	0.70	0.70	0.60	0.70	0.55	0.90	0.60
			$\mathcal{H}_{0,4}$	0.80	0.60	0.90	0.75	0.85	0.95	1.00	1.25	1.25
			$\mathcal{H}_{0,5}$	0.65	0.50	1.15	0.90	0.80	0.90	1.20	1.00	1.30
			$\mathcal{H}_{0,6}$	0.60	0.80	0.70	0.55	0.75	0.95	0.60	0.95	0.80

Table S11: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	11.10	10.25	4.20	3.75	5.05	3.90	4.60	4.70	5.25
			$\mathcal{H}_{0,1}$	1.15	1.35	1.05	0.95	0.95	1.05	1.10	1.35	1.20
			$\mathcal{H}_{0,2}$	2.15	2.30	1.20	1.00	1.10	0.90	1.00	1.20	1.10
			$\mathcal{H}_{0,3}$	5.45	5.15	0.85	0.65	1.00	0.60	0.90	0.90	1.00
			$\mathcal{H}_{0,4}$	1.40	1.15	0.95	0.85	0.75	0.90	1.05	1.05	1.15
			$\mathcal{H}_{0,5}$	2.05	1.95	0.80	0.75	0.80	0.70	0.90	0.75	0.95
			$\mathcal{H}_{0,6}$	1.10	0.90	0.85	0.85	0.80	0.60	0.80	0.85	0.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.05	3.30	4.20	2.70	4.70	2.35	3.85	3.40	4.65
			$\mathcal{H}_{0,1}$	0.80	0.40	0.70	0.35	0.70	0.30	0.70	0.55	0.95
			$\mathcal{H}_{0,2}$	0.85	0.65	0.95	0.45	0.60	0.55	0.95	0.55	0.95
			$\mathcal{H}_{0,3}$	0.55	0.50	0.60	0.35	1.25	0.30	0.80	0.35	0.95
			$\mathcal{H}_{0,4}$	0.65	0.70	0.85	0.65	1.05	0.65	0.50	0.80	0.65
			$\mathcal{H}_{0,5}$	0.70	0.85	1.15	0.50	0.90	0.55	0.85	0.85	1.00
			$\mathcal{H}_{0,6}$	0.75	0.60	0.65	0.55	0.45	0.35	0.80	0.55	0.85
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	4.65	2.35	4.15	3.45	4.65	3.40	4.10	3.70	4.80
			$\mathcal{H}_{0,1}$	0.60	0.30	0.90	0.60	0.95	0.40	0.75	0.55	1.05
			$\mathcal{H}_{0,2}$	0.60	0.10	0.85	0.50	0.90	0.40	1.00	0.45	1.25
			$\mathcal{H}_{0,3}$	0.15	0.15	0.70	0.50	0.90	0.50	0.65	0.60	0.65
			$\mathcal{H}_{0,4}$	0.55	0.40	1.05	0.65	0.95	0.45	0.70	0.55	0.80
			$\mathcal{H}_{0,5}$	0.50	0.40	0.85	0.80	0.80	0.60	0.80	0.75	0.85
			$\mathcal{H}_{0,6}$	1.20	0.55	0.95	0.65	1.40	0.70	1.15	0.85	1.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	10.65	7.55	3.10	1.80	4.30	2.60	3.50	2.80	4.25
			$\mathcal{H}_{0,1}$	1.10	0.70	0.90	0.50	0.80	0.40	0.70	0.45	0.85
			$\mathcal{H}_{0,2}$	1.70	1.10	0.50	0.10	1.10	0.35	0.65	0.40	0.75
			$\mathcal{H}_{0,3}$	5.00	3.30	0.40	0.15	0.85	0.20	0.60	0.25	0.75
			$\mathcal{H}_{0,4}$	1.40	0.90	0.65	0.45	0.90	0.50	0.65	0.65	0.90
			$\mathcal{H}_{0,5}$	1.85	1.50	0.55	0.25	0.75	0.15	0.75	0.55	0.90
			$\mathcal{H}_{0,6}$	1.35	0.65	0.60	0.35	1.20	0.60	0.70	0.65	0.95
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	5.25	4.40	4.25	3.90	4.40	4.00	4.35	4.40	4.85
			$\mathcal{H}_{0,1}$	1.15	0.60	1.00	0.80	0.85	0.60	1.10	0.65	1.15
			$\mathcal{H}_{0,2}$	0.55	0.75	0.80	0.60	0.95	0.65	0.50	0.75	0.50
			$\mathcal{H}_{0,3}$	0.95	1.05	0.80	0.75	0.65	0.65	0.65	0.90	0.75
			$\mathcal{H}_{0,4}$	1.15	0.85	0.80	0.65	0.70	0.60	1.00	0.75	1.10
			$\mathcal{H}_{0,5}$	0.70	0.80	0.70	0.65	0.60	0.60	0.80	0.85	0.90
			$\mathcal{H}_{0,6}$	0.80	0.90	1.10	0.95	1.25	0.95	0.90	1.20	1.05
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.90	2.35	4.35	4.35	5.15	3.90	5.00	4.85	5.45
			$\mathcal{H}_{0,1}$	0.65	0.55	0.85	0.80	0.80	0.80	0.90	1.05	1.00
			$\mathcal{H}_{0,2}$	0.40	0.10	0.60	0.70	0.90	0.55	0.70	0.65	0.75
			$\mathcal{H}_{0,3}$	0.15	0.00	0.70	0.60	0.85	0.60	0.95	0.80	1.05
			$\mathcal{H}_{0,4}$	0.60	0.40	0.75	0.70	0.70	0.60	0.80	0.85	0.95
			$\mathcal{H}_{0,5}$	0.35	0.30	1.05	0.90	0.80	0.65	0.65	1.00	0.75
			$\mathcal{H}_{0,6}$	1.10	1.10	1.25	1.30	1.00	1.25	1.15	1.40	1.25
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.20	9.20	3.85	3.20	4.25	2.80	3.95	3.85	4.75
			$\mathcal{H}_{0,1}$	0.90	0.85	0.75	0.50	0.60	0.30	0.90	0.55	1.10
			$\mathcal{H}_{0,2}$	2.45	2.30	0.95	0.75	0.75	0.65	0.75	1.10	0.90
			$\mathcal{H}_{0,3}$	5.90	4.80	0.75	0.60	0.95	0.50	0.65	0.80	0.90
			$\mathcal{H}_{0,4}$	1.10	0.75	0.65	0.45	0.95	0.55	0.95	0.70	0.95
			$\mathcal{H}_{0,5}$	2.30	2.15	1.10	0.95	0.90	0.80	0.85	0.90	0.95
			$\mathcal{H}_{0,6}$	0.90	0.60	0.55	0.40	0.40	0.40	0.55	0.55	0.85
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.20	4.30	4.95	4.85	4.20	3.95	4.40	5.35	4.80
			$\mathcal{H}_{0,1}$	0.75	0.55	0.80	0.70	0.50	0.60	0.65	0.70	0.70
			$\mathcal{H}_{0,2}$	0.50	0.60	0.90	0.95	0.45	0.60	0.50	0.75	0.60
			$\mathcal{H}_{0,3}$	1.15	0.95	1.05	1.05	1.10	0.95	1.00	1.15	1.15
			$\mathcal{H}_{0,4}$	1.10	1.15	1.05	1.00	0.90	0.85	0.95	1.15	1.05
			$\mathcal{H}_{0,5}$	0.95	0.90	1.15	1.05	0.95	1.05	0.95	1.30	1.05
			$\mathcal{H}_{0,6}$	0.95	0.65	0.70	0.65	0.45	0.80	1.10	0.80	1.15

Table S11: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	3.55	2.35	4.35	3.70	4.15	4.05	4.25	4.45	4.70
			$\mathcal{H}_{0,1}$	0.75	0.40	0.75	0.65	0.65	0.65	1.00	0.75	1.00
			$\mathcal{H}_{0,2}$	0.25	0.15	0.85	0.70	0.65	0.55	0.80	0.80	0.95
			$\mathcal{H}_{0,3}$	0.05	0.00	1.00	0.90	0.70	1.15	0.80	1.25	0.80
			$\mathcal{H}_{0,4}$	0.55	0.65	0.95	0.90	0.85	0.65	0.80	0.70	0.90
			$\mathcal{H}_{0,5}$	0.25	0.50	0.80	0.65	0.70	0.80	0.50	0.90	0.60
			$\mathcal{H}_{0,6}$	0.80	0.75	0.80	0.80	0.90	0.80	0.75	1.00	0.75
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.30	10.00	4.15	3.90	4.30	3.40	4.60	4.90	5.55
			$\mathcal{H}_{0,1}$	1.15	1.20	0.90	0.90	0.60	0.70	0.85	1.00	1.10
			$\mathcal{H}_{0,2}$	2.80	1.95	1.10	1.10	0.50	1.05	1.05	1.05	1.35
			$\mathcal{H}_{0,3}$	6.15	5.65	0.75	0.80	1.10	0.50	0.35	1.10	0.55
			$\mathcal{H}_{0,4}$	1.70	1.05	0.95	0.80	0.80	0.80	1.20	1.15	1.35
			$\mathcal{H}_{0,5}$	2.30	1.95	0.70	0.65	0.90	0.70	0.85	0.85	1.00
			$\mathcal{H}_{0,6}$	1.15	1.10	0.85	0.70	0.50	1.00	0.95	1.15	1.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.30	3.20	4.20	3.20	5.25	3.70	4.75	3.65	5.30
			$\mathcal{H}_{0,1}$	0.90	0.50	0.65	0.45	1.05	0.60	0.80	0.65	0.90
			$\mathcal{H}_{0,2}$	1.05	0.80	1.30	0.75	1.20	0.70	1.00	0.70	1.10
			$\mathcal{H}_{0,3}$	0.75	0.55	0.75	0.50	1.05	0.60	1.10	0.80	1.10
			$\mathcal{H}_{0,4}$	0.95	0.65	0.95	0.80	0.95	0.80	0.80	0.95	0.90
			$\mathcal{H}_{0,5}$	0.85	0.55	0.75	0.50	0.75	0.45	0.85	0.60	0.95
			$\mathcal{H}_{0,6}$	0.80	0.80	1.05	0.60	1.20	0.70	0.75	0.85	1.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	4.35	2.10	4.40	2.90	4.80	3.05	4.55	4.05	5.20
			$\mathcal{H}_{0,1}$	0.50	0.50	1.15	0.55	0.95	0.70	0.65	0.85	0.80
			$\mathcal{H}_{0,2}$	0.60	0.10	0.60	0.35	1.15	0.35	1.30	0.60	1.35
			$\mathcal{H}_{0,3}$	0.15	0.00	0.80	0.35	0.75	0.40	0.70	0.60	0.80
			$\mathcal{H}_{0,4}$	0.80	0.60	0.95	0.45	0.90	0.65	0.95	0.90	1.05
			$\mathcal{H}_{0,5}$	0.30	0.20	0.95	0.75	0.60	0.35	0.70	0.55	0.95
			$\mathcal{H}_{0,6}$	0.90	0.70	0.95	0.60	0.85	0.70	0.95	0.75	1.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	11.00	8.65	3.50	2.25	5.55	3.00	4.45	3.30	5.25
			$\mathcal{H}_{0,1}$	1.25	0.55	0.70	0.20	0.90	0.45	0.95	0.65	1.05
			$\mathcal{H}_{0,2}$	2.20	1.35	0.60	0.25	1.25	0.35	1.00	0.35	1.00
			$\mathcal{H}_{0,3}$	5.55	4.45	0.70	0.35	1.45	0.55	0.95	0.75	1.20
			$\mathcal{H}_{0,4}$	1.05	0.85	0.85	0.50	0.60	0.50	0.65	0.85	0.95
			$\mathcal{H}_{0,5}$	2.70	1.70	0.60	0.50	1.40	0.40	1.10	0.55	1.25
			$\mathcal{H}_{0,6}$	1.25	0.50	0.65	0.40	1.05	0.35	0.60	0.50	0.75
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	5.20	3.85	4.25	3.50	4.90	4.05	4.25	4.75	4.85
			$\mathcal{H}_{0,1}$	1.25	1.15	1.25	1.00	1.25	1.15	1.20	1.30	1.35
			$\mathcal{H}_{0,2}$	0.90	0.45	0.70	0.65	0.55	0.55	0.80	0.55	0.90
			$\mathcal{H}_{0,3}$	1.25	0.85	1.25	0.95	1.30	0.95	1.10	1.20	1.25
			$\mathcal{H}_{0,4}$	0.70	0.75	0.75	0.60	0.60	0.70	0.55	0.80	0.65
			$\mathcal{H}_{0,5}$	0.75	0.70	0.65	0.50	1.10	0.75	0.80	0.95	0.95
			$\mathcal{H}_{0,6}$	0.75	0.70	0.55	0.55	0.60	0.50	0.90	0.50	1.10
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.85	2.55	4.15	3.75	5.25	4.00	5.30	4.90	5.80
			$\mathcal{H}_{0,1}$	0.95	0.70	1.25	1.00	1.25	1.05	1.65	1.20	1.75
			$\mathcal{H}_{0,2}$	0.20	0.20	0.55	0.45	0.60	0.65	0.90	0.80	0.95
			$\mathcal{H}_{0,3}$	0.10	0.15	0.70	0.75	0.65	0.90	1.00	1.05	1.05
			$\mathcal{H}_{0,4}$	0.70	0.50	0.65	0.55	1.20	0.50	0.70	0.75	0.75
			$\mathcal{H}_{0,5}$	0.70	0.35	0.80	0.55	1.10	0.80	0.80	0.90	0.95
			$\mathcal{H}_{0,6}$	0.90	0.90	1.20	0.90	0.70	0.85	1.00	0.95	1.10
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	9.95	8.60	4.45	3.45	4.50	3.75	3.95	4.50	4.65
			$\mathcal{H}_{0,1}$	0.95	0.75	0.65	0.70	1.15	0.65	0.75	0.75	0.80
			$\mathcal{H}_{0,2}$	1.75	1.95	0.85	0.65	0.50	0.85	0.75	1.00	0.85
			$\mathcal{H}_{0,3}$	5.05	4.55	0.95	0.55	1.00	0.70	1.00	0.75	1.00
			$\mathcal{H}_{0,4}$	1.25	0.95	1.15	0.80	0.90	0.65	0.90	0.95	1.20
			$\mathcal{H}_{0,5}$	1.80	1.85	0.85	0.65	0.85	0.60	0.60	0.90	0.80
			$\mathcal{H}_{0,6}$	1.10	0.90	0.75	0.50	0.90	0.65	1.00	0.90	1.10

Table S11: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.10	4.60	4.90	4.60	4.65	4.80	5.55	5.30	5.65
			$\mathcal{H}_{0,1}$	1.05	1.30	1.30	1.10	0.85	1.30	1.10	1.45	1.45
			$\mathcal{H}_{0,2}$	0.90	0.90	0.95	0.90	0.95	1.05	1.15	1.25	1.15
			$\mathcal{H}_{0,3}$	1.35	1.05	1.10	0.90	1.05	0.85	1.25	1.00	1.45
			$\mathcal{H}_{0,4}$	0.75	0.60	0.65	0.60	0.75	0.60	0.90	0.70	1.00
			$\mathcal{H}_{0,5}$	1.00	0.95	0.90	0.95	0.75	1.00	0.95	1.10	1.00
			$\mathcal{H}_{0,6}$	0.85	0.95	0.85	0.80	0.70	0.85	0.90	1.05	1.10
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	3.55	2.30	4.25	3.80	4.70	4.40	4.40	4.70	4.95
			$\mathcal{H}_{0,1}$	0.60	1.15	1.40	1.20	1.20	1.35	0.85	1.55	1.05
			$\mathcal{H}_{0,2}$	0.10	0.10	0.65	0.65	0.65	0.70	0.90	0.75	1.00
			$\mathcal{H}_{0,3}$	0.05	0.05	0.75	0.70	0.95	0.70	0.60	0.80	0.60
			$\mathcal{H}_{0,4}$	0.60	0.45	0.45	0.45	0.65	0.60	0.80	0.65	0.90
			$\mathcal{H}_{0,5}$	0.30	0.30	0.70	0.50	0.75	0.75	0.75	0.95	0.80
			$\mathcal{H}_{0,6}$	0.40	0.80	0.75	0.70	0.85	0.90	0.45	1.10	0.65
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	10.70	10.45	4.55	4.00	4.30	4.05	3.75	4.70	4.35
			$\mathcal{H}_{0,1}$	1.35	1.20	0.80	0.75	0.85	0.95	0.75	1.20	0.80
			$\mathcal{H}_{0,2}$	2.25	2.00	0.85	0.70	0.80	0.85	0.95	0.95	1.05
			$\mathcal{H}_{0,3}$	5.70	5.75	0.75	0.60	0.85	0.50	0.65	0.75	0.90
			$\mathcal{H}_{0,4}$	1.25	1.10	0.95	0.90	0.80	0.90	0.85	1.15	0.90
			$\mathcal{H}_{0,5}$	2.10	2.70	0.95	1.00	0.95	1.15	0.65	1.25	0.80
			$\mathcal{H}_{0,6}$	0.70	1.10	0.85	0.85	0.50	0.90	0.50	1.00	0.70

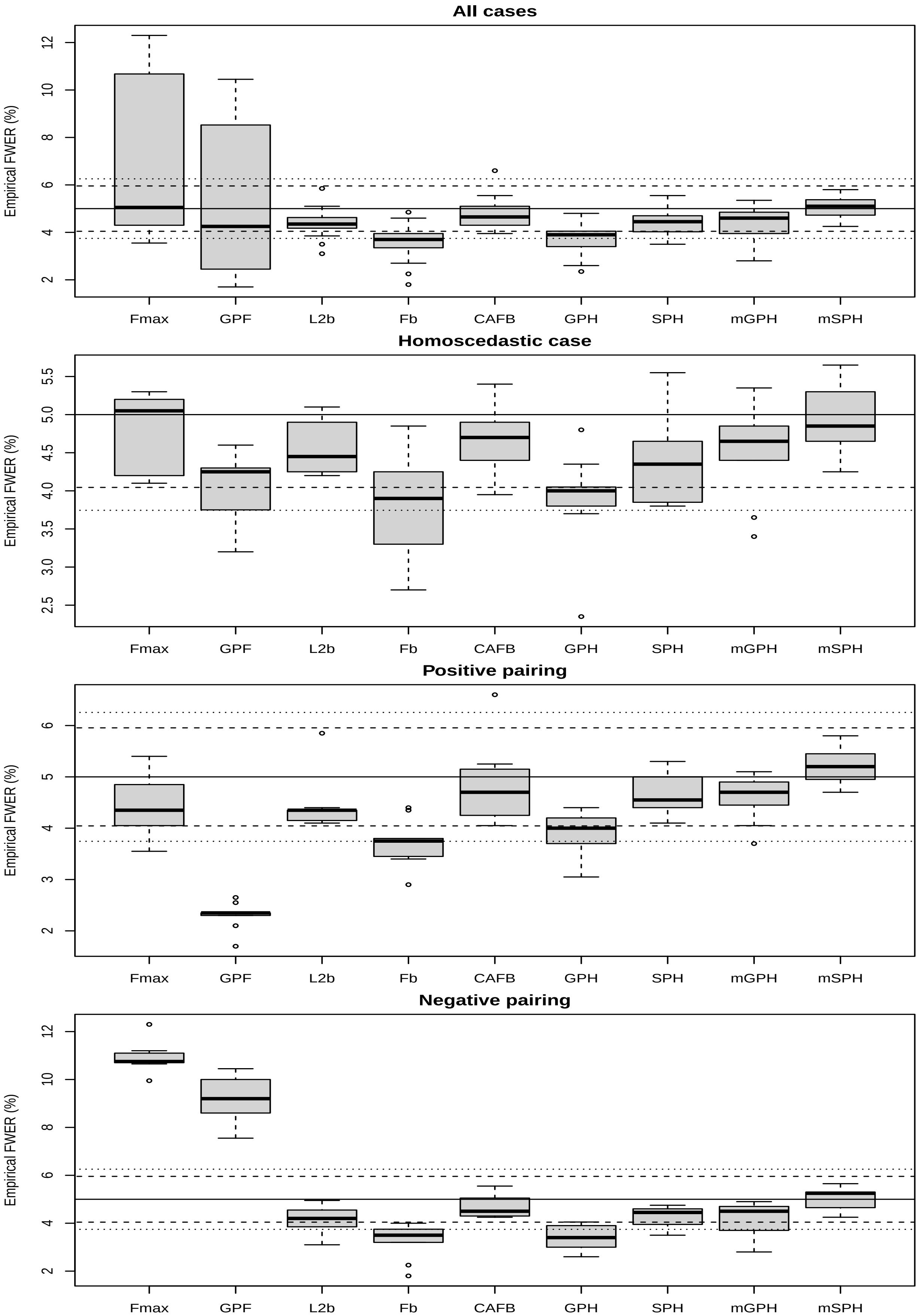


Fig. S20: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and homoscedastic and heteroscedastic cases

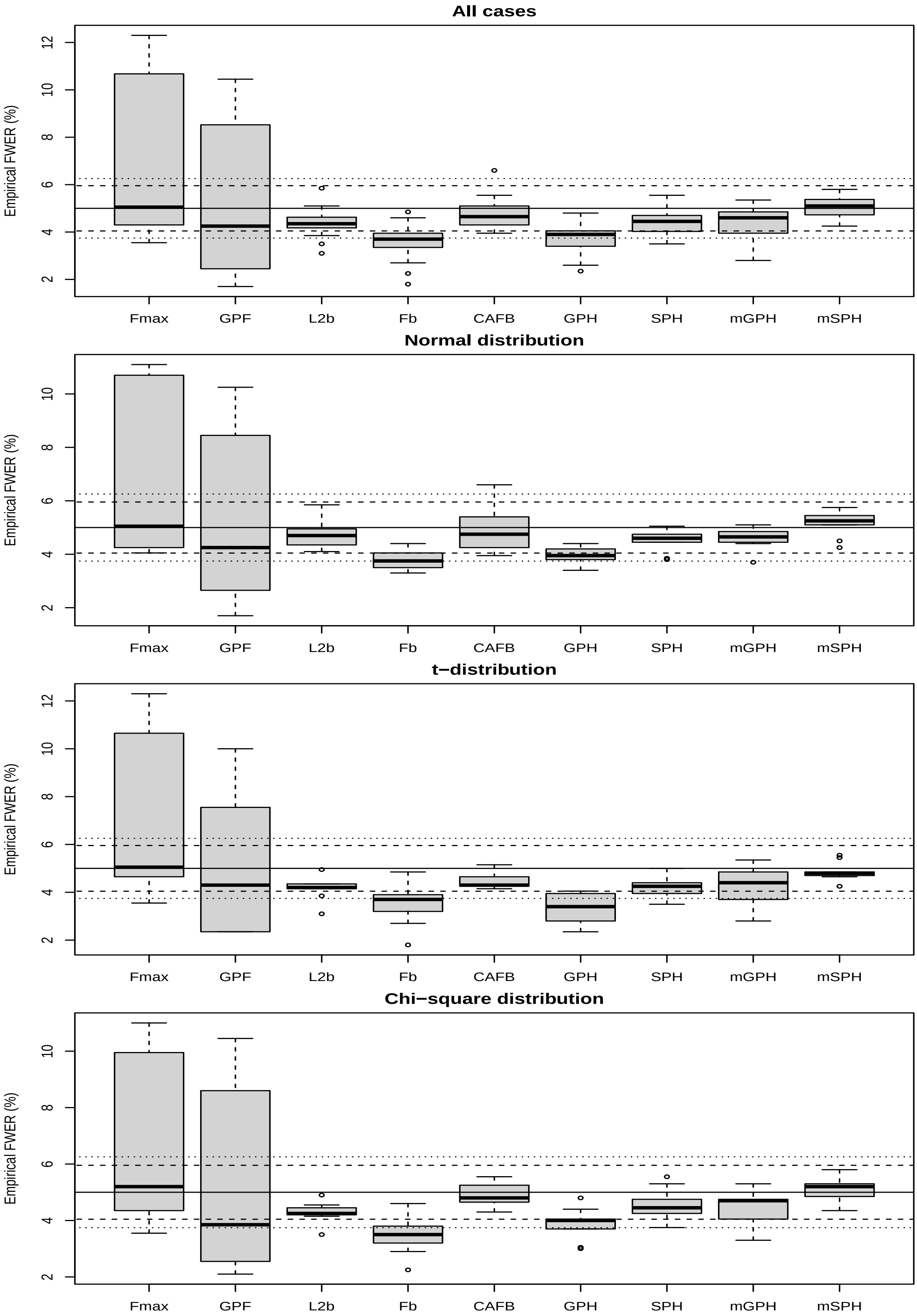


Fig. S21: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different distributions

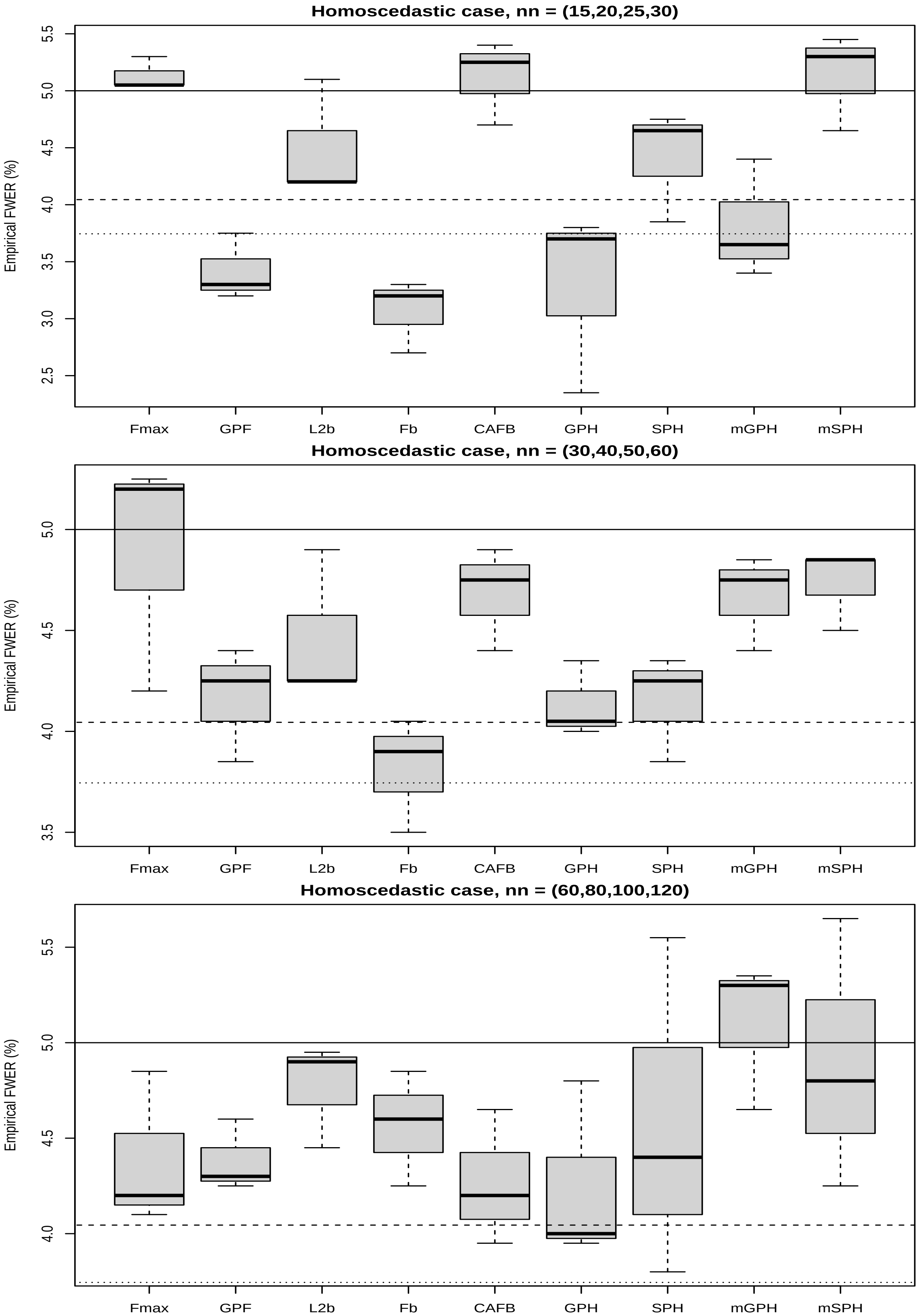


Fig. S22: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes and homoscedastic case

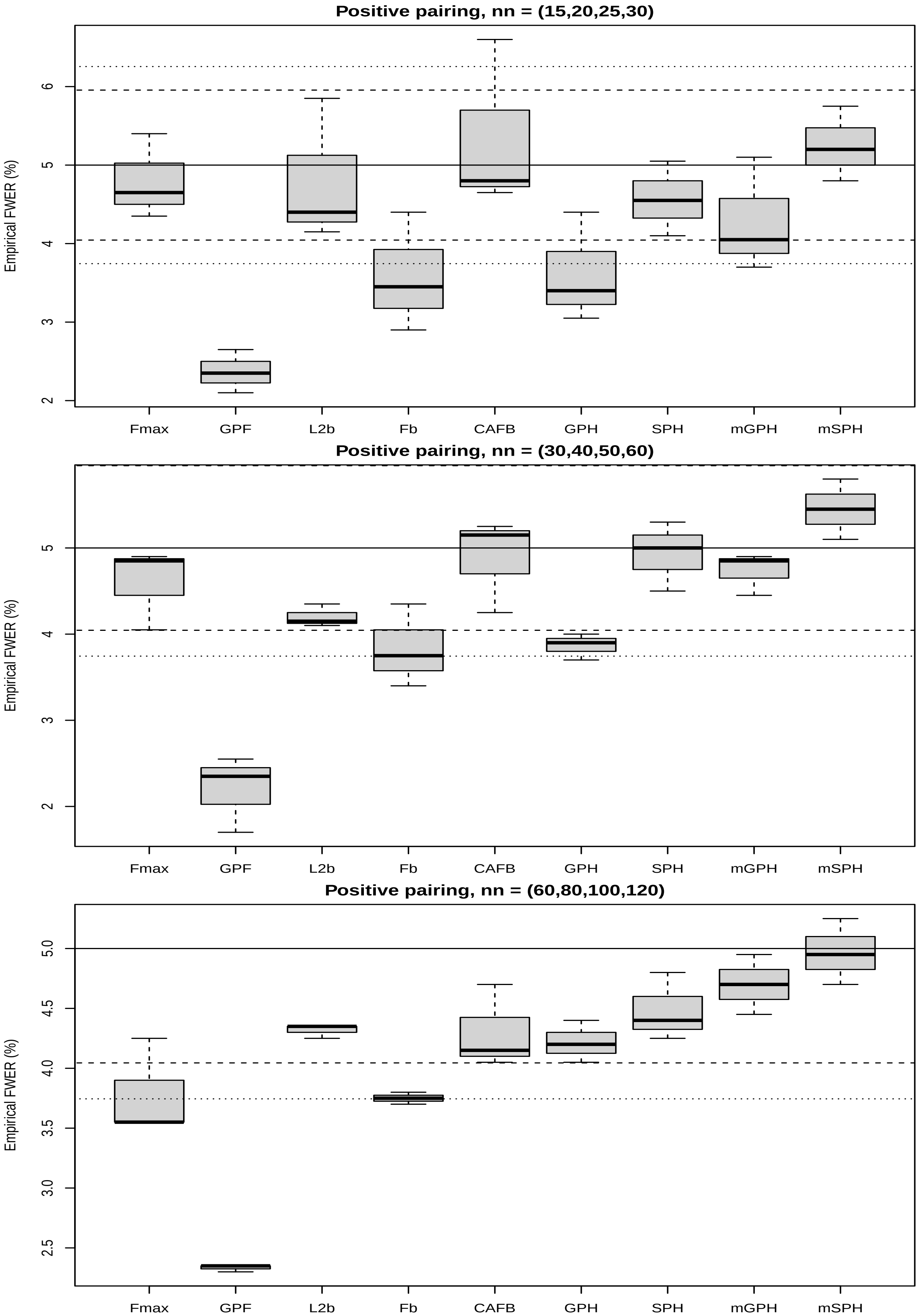


Fig. S23: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes and positive pairing

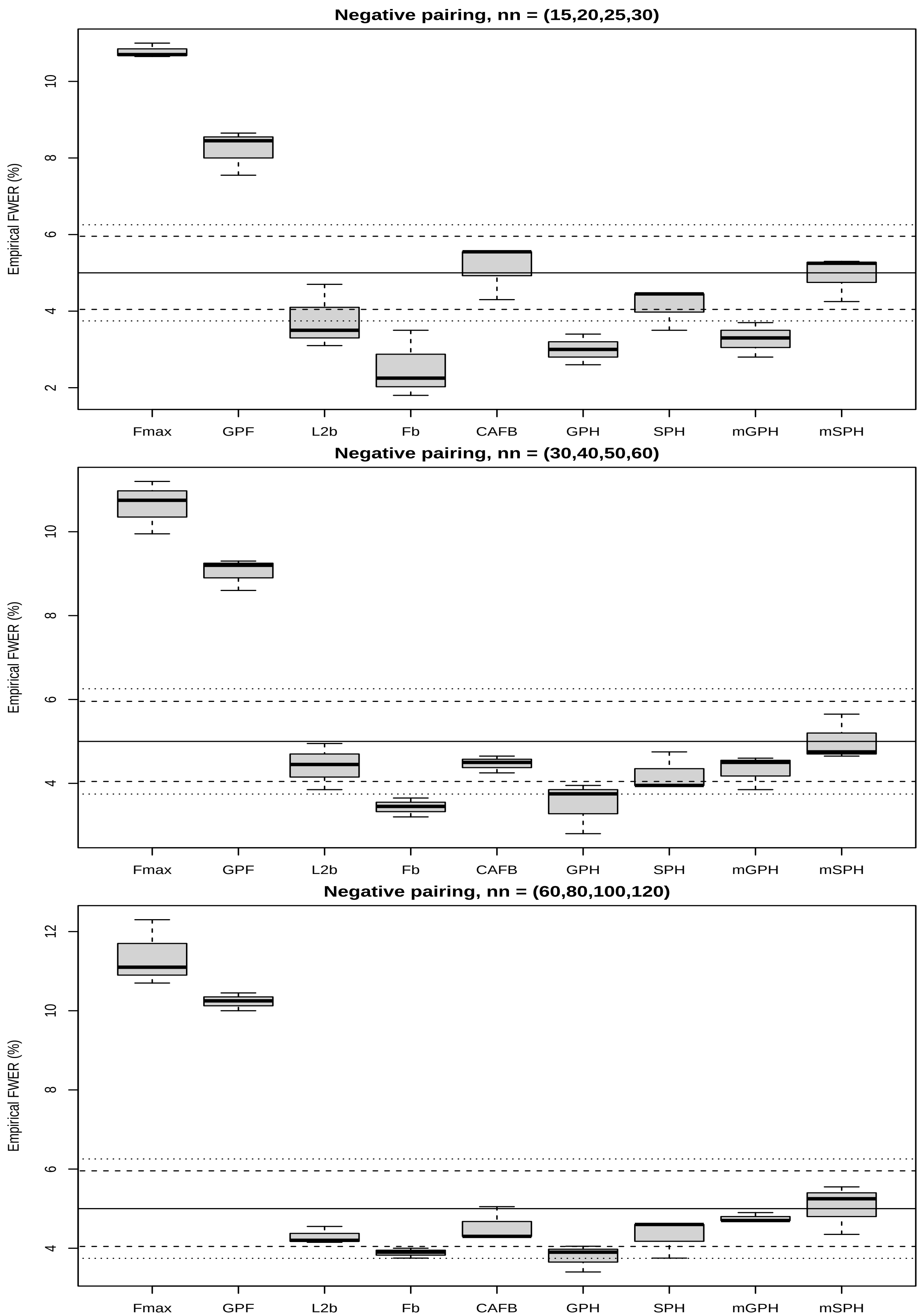


Fig. S24: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes and negative pairing

Table S12: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$, $\mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}$, $\mathcal{H}_{0,5}$, $\mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.05	0.55	1.15	0.55	1.10	0.60	1.00	0.80	1.15
			$\mathcal{H}_{0,2}$	0.70	0.30	0.75	0.20	1.15	0.35	0.55	0.40	0.80
			$\mathcal{H}_{0,3}$	72.70	80.90	78.00	68.25	63.00	78.80	69.35	81.55	71.05
			$\mathcal{H}_{0,4}$	0.80	0.65	1.10	0.60	1.15	0.85	0.90	1.00	0.95
			$\mathcal{H}_{0,5}$	85.00	91.85	88.10	83.95	77.20	90.85	84.00	92.35	85.20
			$\mathcal{H}_{0,6}$	90.10	95.15	92.25	90.15	83.05	94.90	90.80	95.50	91.40
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.65	0.60	0.95	0.60	1.00	0.80	0.85	0.95	1.00
			$\mathcal{H}_{0,2}$	0.30	0.25	0.95	0.55	0.95	0.65	0.85	0.80	0.90
			$\mathcal{H}_{0,3}$	16.00	17.70	45.25	37.35	34.15	46.80	38.25	50.85	40.05
			$\mathcal{H}_{0,4}$	0.65	0.60	1.15	0.75	1.00	0.70	0.70	0.85	0.70
			$\mathcal{H}_{0,5}$	25.40	30.50	40.60	34.55	30.55	43.45	34.45	45.95	36.10
			$\mathcal{H}_{0,6}$	32.10	39.60	38.70	32.80	29.55	41.80	33.85	44.00	35.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.80	0.85	1.30	0.60	0.85	0.55	0.65	0.80	0.65
			$\mathcal{H}_{0,2}$	2.50	2.10	0.90	0.30	0.90	0.40	1.35	0.60	1.55
			$\mathcal{H}_{0,3}$	56.40	58.95	25.65	15.70	20.20	22.25	18.95	25.90	21.25
			$\mathcal{H}_{0,4}$	0.60	0.95	1.10	0.55	0.70	0.50	0.40	0.80	0.55
			$\mathcal{H}_{0,5}$	66.00	73.45	52.50	44.40	42.45	54.45	47.45	58.95	50.50
			$\mathcal{H}_{0,6}$	82.75	88.65	82.00	77.50	69.70	86.20	78.30	88.45	81.15
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.90	0.55	0.90	0.50	0.85	0.45	0.90	0.55	1.00
			$\mathcal{H}_{0,2}$	0.70	0.65	0.65	0.45	1.20	0.55	0.80	0.65	0.85
			$\mathcal{H}_{0,3}$	41.85	48.35	44.55	36.45	35.65	44.45	38.70	47.75	40.55
			$\mathcal{H}_{0,4}$	0.65	0.45	0.80	0.40	0.75	0.50	0.50	0.60	0.55
			$\mathcal{H}_{0,5}$	54.40	62.10	56.45	51.55	47.25	60.15	52.40	63.50	54.75
			$\mathcal{H}_{0,6}$	62.25	71.45	65.10	61.45	55.95	70.05	60.00	72.80	62.70
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.60	0.45	0.85	0.55	0.75	0.60	0.80	0.80	0.90
			$\mathcal{H}_{0,2}$	0.25	0.20	0.90	0.55	0.60	0.60	0.95	0.80	1.00
			$\mathcal{H}_{0,3}$	6.15	5.40	20.40	16.55	17.10	20.55	18.25	22.40	19.50
			$\mathcal{H}_{0,4}$	0.95	0.55	0.65	0.60	1.00	0.55	0.80	0.60	0.95
			$\mathcal{H}_{0,5}$	10.65	13.00	19.55	16.20	15.20	19.50	16.55	21.60	18.15
			$\mathcal{H}_{0,6}$	13.20	15.65	15.75	13.50	12.90	17.50	13.75	19.20	14.70
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.30	0.80	0.65	0.40	0.80	0.45	0.85	0.60	0.95
			$\mathcal{H}_{0,2}$	2.45	1.35	0.60	0.20	0.95	0.35	0.75	0.50	0.85
			$\mathcal{H}_{0,3}$	36.00	35.85	11.30	6.90	11.00	10.05	10.15	11.95	11.45
			$\mathcal{H}_{0,4}$	1.10	0.90	0.80	0.55	1.20	0.70	0.80	0.90	1.05
			$\mathcal{H}_{0,5}$	38.05	41.50	24.55	19.30	21.40	24.65	21.85	28.40	23.70
			$\mathcal{H}_{0,6}$	49.75	57.00	47.65	43.75	39.30	51.80	44.40	54.65	47.20
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.05	0.45	0.55	0.30	0.65	0.50	1.20	0.55	1.40
			$\mathcal{H}_{0,2}$	1.20	0.70	1.15	0.75	1.05	0.80	1.00	1.00	1.10
			$\mathcal{H}_{0,3}$	72.25	80.75	74.80	66.70	62.85	76.15	69.25	78.85	71.35
			$\mathcal{H}_{0,4}$	0.55	0.50	0.60	0.50	1.10	0.45	0.50	0.55	0.55
			$\mathcal{H}_{0,5}$	85.80	92.05	86.45	82.30	76.60	90.15	84.20	91.30	85.35
			$\mathcal{H}_{0,6}$	91.75	95.85	92.05	90.40	85.95	95.40	90.20	96.15	91.55
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.80	0.75	1.25	0.75	1.20	0.75	1.10	0.90	1.15
			$\mathcal{H}_{0,2}$	0.35	0.10	0.80	0.35	0.85	0.45	0.55	0.55	0.75
			$\mathcal{H}_{0,3}$	14.30	15.00	43.65	38.05	35.15	46.25	38.05	48.80	39.75
			$\mathcal{H}_{0,4}$	0.75	0.65	1.15	0.90	1.25	0.65	0.95	0.80	1.00
			$\mathcal{H}_{0,5}$	25.95	29.65	38.80	34.45	32.30	40.60	36.75	44.05	38.05
			$\mathcal{H}_{0,6}$	31.65	38.65	37.45	33.65	30.00	41.00	32.95	44.05	34.75
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.95	0.75	0.85	0.20	1.15	0.40	0.70	0.60	0.70
			$\mathcal{H}_{0,2}$	1.65	1.35	0.90	0.30	1.10	0.55	0.70	0.70	0.90
			$\mathcal{H}_{0,3}$	57.15	59.55	27.65	18.50	23.35	25.30	23.45	29.15	25.10
			$\mathcal{H}_{0,4}$	1.50	0.75	0.50	0.35	1.25	0.65	1.20	0.65	1.30
			$\mathcal{H}_{0,5}$	65.15	71.45	52.50	45.40	42.30	54.60	47.10	58.65	50.05
			$\mathcal{H}_{0,6}$	81.70	87.10	79.40	75.50	69.20	83.10	77.30	85.90	79.35

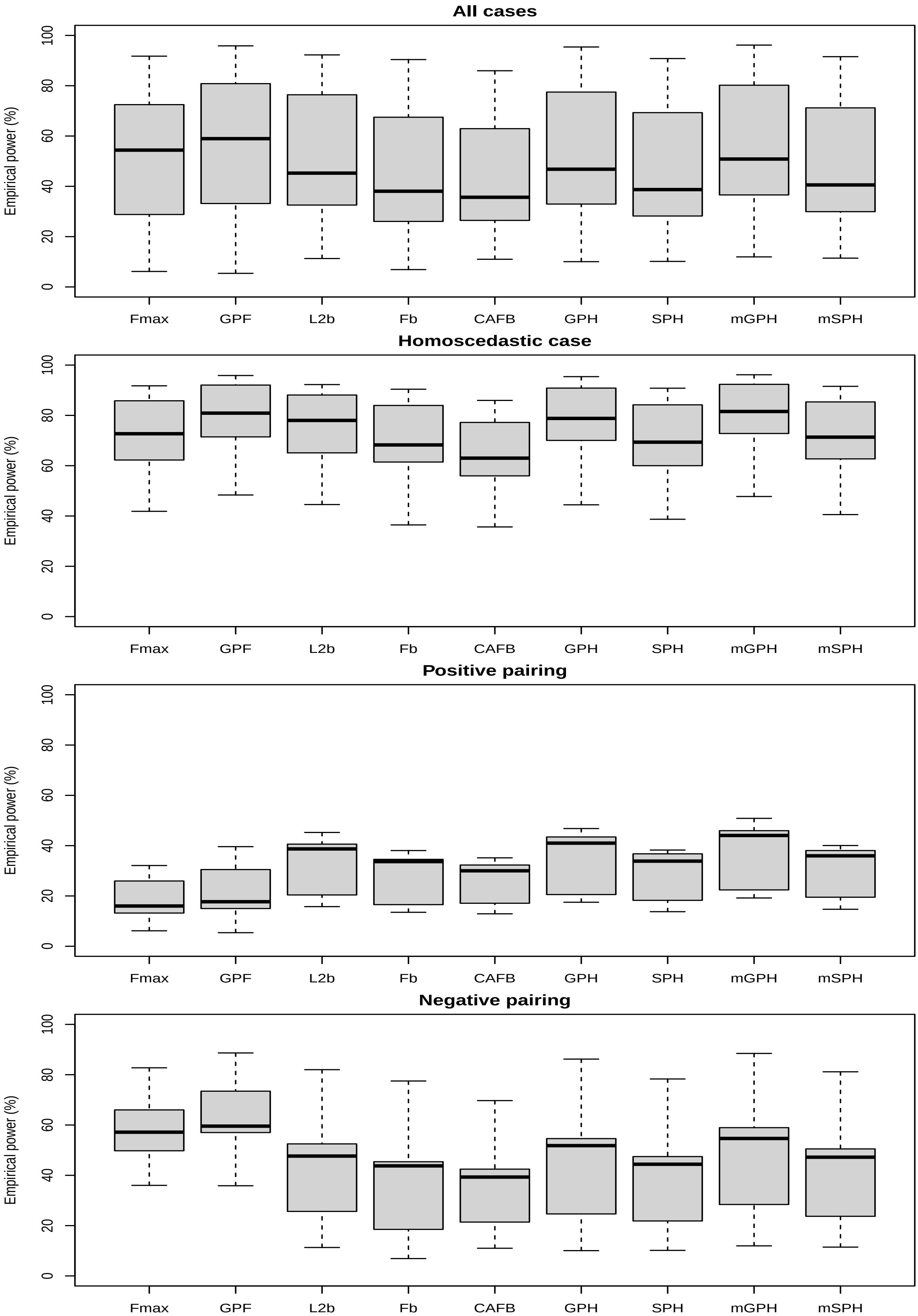


Fig. S25: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and homoscedastic and heteroscedastic cases

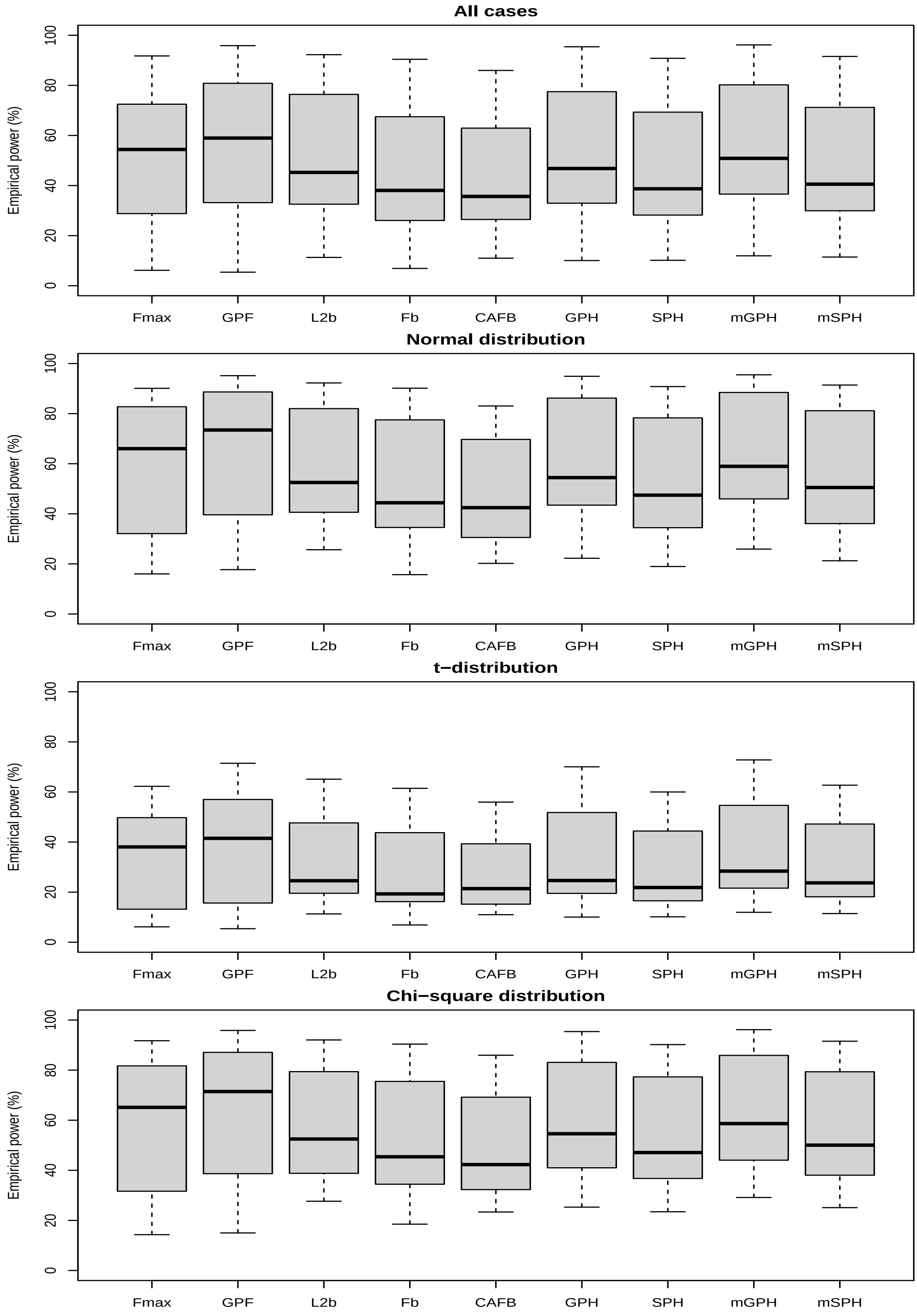


Fig. S26: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different distributions

Table S13: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$, $\mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}$, $\mathcal{H}_{0,5}$, $\mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.20	0.90	1.25	0.65	0.90	0.60	1.10	0.80	1.20
			$\mathcal{H}_{0,2}$	0.80	0.55	1.05	0.65	0.85	0.50	1.00	0.85	1.25
			$\mathcal{H}_{0,3}$	69.15	97.45	96.00	93.45	85.25	96.95	66.00	97.50	68.05
			$\mathcal{H}_{0,4}$	0.85	0.80	1.15	0.65	0.70	0.85	0.75	0.95	0.80
			$\mathcal{H}_{0,5}$	82.40	99.20	98.95	98.25	94.10	99.10	81.90	99.25	83.90
			$\mathcal{H}_{0,6}$	89.90	99.85	99.75	99.55	97.75	99.80	89.80	99.80	90.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.50	0.35	0.85	0.50	0.90	0.55	0.40	0.70	0.65
			$\mathcal{H}_{0,2}$	0.30	0.20	0.70	0.50	1.10	0.55	0.75	0.60	0.75
			$\mathcal{H}_{0,3}$	14.10	47.05	77.20	71.50	57.55	76.95	36.50	79.25	39.60
			$\mathcal{H}_{0,4}$	1.20	0.50	0.85	0.50	1.20	0.60	1.15	0.75	1.20
			$\mathcal{H}_{0,5}$	25.45	64.25	72.40	66.95	52.45	73.70	35.40	76.00	37.35
			$\mathcal{H}_{0,6}$	30.80	68.00	68.90	63.95	50.30	71.10	33.00	73.35	34.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.30	0.80	0.95	0.55	1.50	0.45	0.65	0.55	0.85
			$\mathcal{H}_{0,2}$	2.65	2.55	1.10	0.45	1.45	0.45	1.05	0.60	1.30
			$\mathcal{H}_{0,3}$	59.65	79.85	51.85	38.80	34.90	47.75	21.15	51.80	23.85
			$\mathcal{H}_{0,4}$	0.85	0.55	0.60	0.45	0.70	0.50	0.70	0.65	0.90
			$\mathcal{H}_{0,5}$	65.35	92.40	82.05	76.30	65.05	82.75	45.20	84.95	49.10
			$\mathcal{H}_{0,6}$	78.75	98.55	97.65	96.40	89.55	98.30	73.30	98.45	76.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.55	0.30	0.55	0.35	0.90	0.40	0.80	0.45	0.85
			$\mathcal{H}_{0,2}$	0.60	0.25	0.45	0.35	1.05	0.30	0.70	0.40	0.75
			$\mathcal{H}_{0,3}$	36.15	75.05	71.10	64.60	55.65	71.85	35.50	74.50	37.85
			$\mathcal{H}_{0,4}$	0.70	0.55	1.00	0.65	1.00	0.60	0.70	0.85	0.80
			$\mathcal{H}_{0,5}$	49.25	88.30	84.90	82.25	70.20	86.90	47.45	88.45	50.40
			$\mathcal{H}_{0,6}$	59.65	93.10	90.35	87.85	77.60	92.15	58.20	93.40	60.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.55	0.35	0.85	0.45	0.80	0.45	0.80	0.55	0.85
			$\mathcal{H}_{0,2}$	0.55	0.10	0.55	0.40	0.65	0.30	0.90	0.45	1.20
			$\mathcal{H}_{0,3}$	5.20	16.25	43.25	37.45	29.80	42.70	18.15	44.70	19.65
			$\mathcal{H}_{0,4}$	0.75	0.35	0.60	0.30	0.60	0.45	0.80	0.50	0.85
			$\mathcal{H}_{0,5}$	11.45	30.15	38.95	33.85	27.20	39.90	17.55	42.40	18.70
			$\mathcal{H}_{0,6}$	16.35	35.80	35.30	31.60	24.05	37.25	16.55	40.35	17.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.80	0.90	0.75	0.40	0.90	0.45	1.10	0.65	1.25
			$\mathcal{H}_{0,2}$	2.45	1.45	0.50	0.30	1.35	0.35	1.00	0.45	1.25
			$\mathcal{H}_{0,3}$	38.90	56.75	24.60	16.45	18.10	21.40	11.55	24.75	13.00
			$\mathcal{H}_{0,4}$	1.35	1.05	1.00	0.60	1.55	0.75	1.00	0.90	1.20
			$\mathcal{H}_{0,5}$	41.90	70.30	51.50	45.75	38.10	52.45	24.90	56.80	27.65
			$\mathcal{H}_{0,6}$	48.00	85.30	78.55	75.00	62.00	81.75	41.90	84.20	45.15
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.10	0.60	0.70	0.40	1.10	0.60	1.05	0.75	1.10
			$\mathcal{H}_{0,2}$	0.95	0.50	1.00	0.60	0.85	0.50	0.90	0.60	0.90
			$\mathcal{H}_{0,3}$	70.70	96.65	93.55	90.60	84.20	94.20	68.50	95.00	70.25
			$\mathcal{H}_{0,4}$	0.95	0.65	0.70	0.55	0.65	0.60	0.80	0.75	0.95
			$\mathcal{H}_{0,5}$	82.25	99.35	97.90	96.95	92.40	98.60	80.50	98.90	81.95
			$\mathcal{H}_{0,6}$	89.35	99.75	99.20	98.95	97.25	99.55	88.95	99.65	90.15
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.40	0.45	0.60	0.35	0.85	0.40	0.65	0.40	0.75
			$\mathcal{H}_{0,2}$	0.30	0.25	1.30	0.65	1.30	0.80	0.70	0.85	0.70
			$\mathcal{H}_{0,3}$	11.00	47.15	76.45	71.85	57.35	77.20	36.15	79.15	38.50
			$\mathcal{H}_{0,4}$	0.90	0.45	1.25	0.85	1.10	0.65	0.90	0.90	0.95
			$\mathcal{H}_{0,5}$	24.80	64.25	72.65	67.85	53.85	73.95	35.70	76.05	37.75
			$\mathcal{H}_{0,6}$	31.75	69.35	68.45	63.85	50.95	70.10	34.10	72.45	35.80
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.05	0.55	0.60	0.30	0.70	0.50	0.85	0.60	0.90
			$\mathcal{H}_{0,2}$	2.55	2.10	1.40	0.50	1.40	0.70	1.00	0.85	1.40
			$\mathcal{H}_{0,3}$	61.00	79.45	52.25	42.65	38.60	49.80	25.45	53.35	27.20
			$\mathcal{H}_{0,4}$	1.25	0.85	1.10	0.75	1.00	0.70	0.60	0.80	0.80
			$\mathcal{H}_{0,5}$	66.35	90.65	78.50	73.60	63.60	80.05	46.05	82.75	49.05
			$\mathcal{H}_{0,6}$	79.30	98.10	96.00	95.10	89.15	97.00	74.30	97.60	77.25

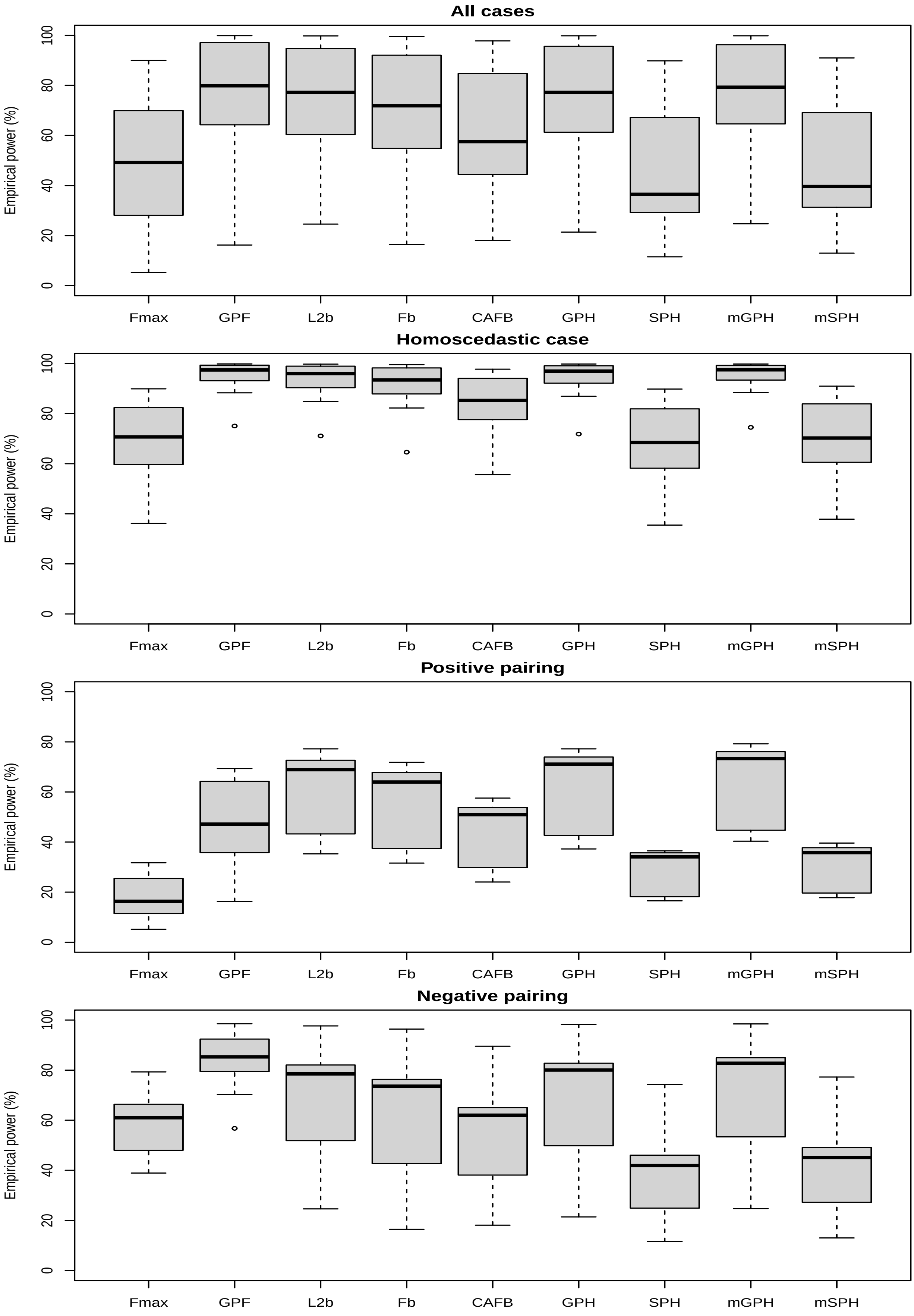


Fig. S27: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts and homoscedastic and heteroscedastic cases

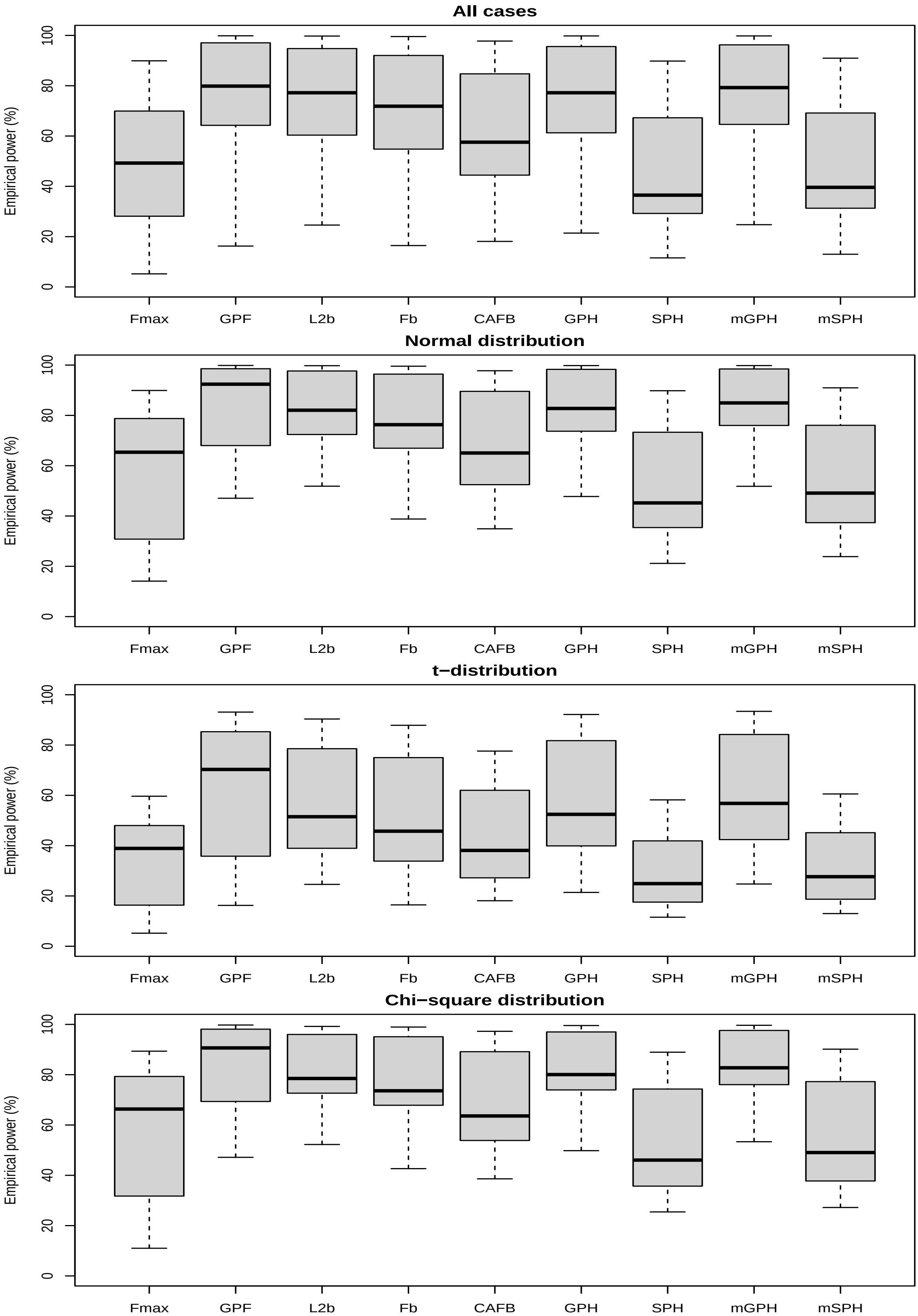


Fig. S28: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts and different distributions

Table S14: Empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.60	18.90	21.60	15.40	13.00	18.15	8.15	20.20	9.05
			$\mathcal{H}_{0,2}$	63.30	95.85	94.60	91.05	81.25	94.65	60.45	95.45	63.70
			$\mathcal{H}_{0,3}$	99.45	100.00	100.00	100.00	99.90	100.00	98.70	100.00	98.95
			$\mathcal{H}_{0,4}$	12.90	29.90	29.85	24.15	18.20	29.80	12.80	32.55	14.25
			$\mathcal{H}_{0,5}$	81.90	99.20	98.95	98.25	94.15	99.40	80.75	99.50	82.55
			$\mathcal{H}_{0,6}$	19.85	45.40	44.35	38.35	27.95	45.30	18.75	48.20	20.45
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.50	10.75	16.45	11.60	11.05	12.85	6.40	14.45	6.95
			$\mathcal{H}_{0,2}$	26.05	65.85	79.65	74.00	59.90	80.40	40.25	82.15	42.15
			$\mathcal{H}_{0,3}$	55.05	97.15	99.60	99.20	96.10	99.80	84.85	99.85	86.60
			$\mathcal{H}_{0,4}$	4.95	10.30	13.75	10.40	8.70	12.30	6.05	13.80	6.65
			$\mathcal{H}_{0,5}$	25.35	63.00	72.45	67.20	53.35	74.10	34.50	76.15	37.00
			$\mathcal{H}_{0,6}$	4.70	8.60	11.00	8.85	6.75	9.65	4.95	10.45	5.80
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.90	5.35	5.70	3.50	3.90	4.15	3.55	4.75	3.90
			$\mathcal{H}_{0,2}$	34.25	60.90	45.85	36.05	30.95	42.75	19.55	47.45	21.50
			$\mathcal{H}_{0,3}$	92.40	99.05	93.70	88.70	79.25	92.40	60.90	94.35	64.70
			$\mathcal{H}_{0,4}$	7.70	13.10	12.00	8.80	8.25	10.50	6.00	12.85	6.95
			$\mathcal{H}_{0,5}$	64.60	91.70	82.45	76.70	62.95	82.90	43.85	85.15	47.00
			$\mathcal{H}_{0,6}$	14.50	31.60	27.80	24.10	18.30	28.30	11.55	30.65	12.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	5.70	8.60	9.30	6.45	7.70	7.90	5.10	9.45	5.55
			$\mathcal{H}_{0,2}$	37.15	72.40	70.70	63.80	54.60	70.00	35.10	72.60	37.00
			$\mathcal{H}_{0,3}$	87.95	99.25	98.95	98.10	94.60	98.85	84.65	99.20	86.25
			$\mathcal{H}_{0,4}$	7.30	14.75	14.70	12.20	10.45	12.95	7.05	14.50	7.70
			$\mathcal{H}_{0,5}$	52.00	88.40	85.15	82.45	70.90	86.95	49.85	89.25	51.95
			$\mathcal{H}_{0,6}$	10.30	23.65	21.40	19.25	13.65	23.30	10.55	25.40	11.45
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	3.50	5.75	8.95	5.90	6.15	6.70	3.75	7.75	4.25
			$\mathcal{H}_{0,2}$	12.45	33.65	48.65	42.40	32.70	48.00	22.40	51.35	23.90
			$\mathcal{H}_{0,3}$	25.40	73.45	90.00	87.10	77.35	91.25	55.00	92.45	57.85
			$\mathcal{H}_{0,4}$	3.00	5.35	6.50	5.45	4.30	5.85	3.50	6.95	4.05
			$\mathcal{H}_{0,5}$	12.80	32.25	41.60	36.95	28.75	41.95	18.15	44.80	19.65
			$\mathcal{H}_{0,6}$	3.50	4.90	5.60	4.55	3.85	5.15	3.55	5.85	3.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.40	3.20	2.75	1.65	2.65	1.90	2.25	2.35	2.70
			$\mathcal{H}_{0,2}$	19.60	35.50	22.80	15.65	16.50	20.05	9.70	22.70	11.25
			$\mathcal{H}_{0,3}$	73.70	90.65	68.90	58.05	53.05	67.20	32.95	71.45	36.00
			$\mathcal{H}_{0,4}$	3.70	6.40	5.80	4.45	3.60	4.75	2.80	5.75	3.45
			$\mathcal{H}_{0,5}$	40.55	68.40	51.95	45.05	37.95	51.20	22.75	55.25	25.50
			$\mathcal{H}_{0,6}$	9.05	16.55	14.00	11.70	9.20	13.95	7.15	15.80	8.40
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	10.30	19.05	23.20	17.40	14.45	19.65	9.70	21.40	10.50
			$\mathcal{H}_{0,2}$	66.50	95.80	92.70	89.90	81.60	93.75	63.75	94.60	65.95
			$\mathcal{H}_{0,3}$	99.25	100.00	99.90	99.90	99.50	100.00	98.55	100.00	98.70
			$\mathcal{H}_{0,4}$	14.25	33.65	33.15	27.65	22.60	33.35	14.05	35.95	15.70
			$\mathcal{H}_{0,5}$	83.35	99.20	98.40	97.75	92.95	98.70	80.85	98.90	82.80
			$\mathcal{H}_{0,6}$	19.00	42.30	42.15	37.45	28.55	42.45	19.45	45.30	21.10
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.40	10.40	16.50	11.90	10.00	13.90	6.60	15.30	7.10
			$\mathcal{H}_{0,2}$	23.40	66.90	79.90	74.10	61.05	80.00	41.05	81.95	43.05
			$\mathcal{H}_{0,3}$	54.85	98.30	99.50	99.25	96.70	99.80	86.10	99.85	87.75
			$\mathcal{H}_{0,4}$	5.10	8.75	12.70	9.60	8.85	10.95	6.30	12.50	6.90
			$\mathcal{H}_{0,5}$	25.00	65.00	72.45	68.65	54.40	74.75	36.50	77.00	39.05
			$\mathcal{H}_{0,6}$	4.40	8.10	10.15	7.85	5.85	9.55	4.70	10.70	5.00
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	4.05	6.10	6.85	4.40	5.50	5.05	3.50	6.00	4.00
			$\mathcal{H}_{0,2}$	37.40	63.90	48.15	39.35	35.60	45.80	23.30	50.35	25.70
			$\mathcal{H}_{0,3}$	93.35	98.70	88.95	84.20	79.10	89.90	62.65	92.35	66.00
			$\mathcal{H}_{0,4}$	8.50	14.70	13.15	10.90	8.95	13.10	7.65	14.50	8.20
			$\mathcal{H}_{0,5}$	66.00	90.90	78.40	73.45	65.70	80.65	48.35	83.00	51.00
			$\mathcal{H}_{0,6}$	17.60	35.95	30.45	26.70	22.20	31.20	14.65	35.45	16.60

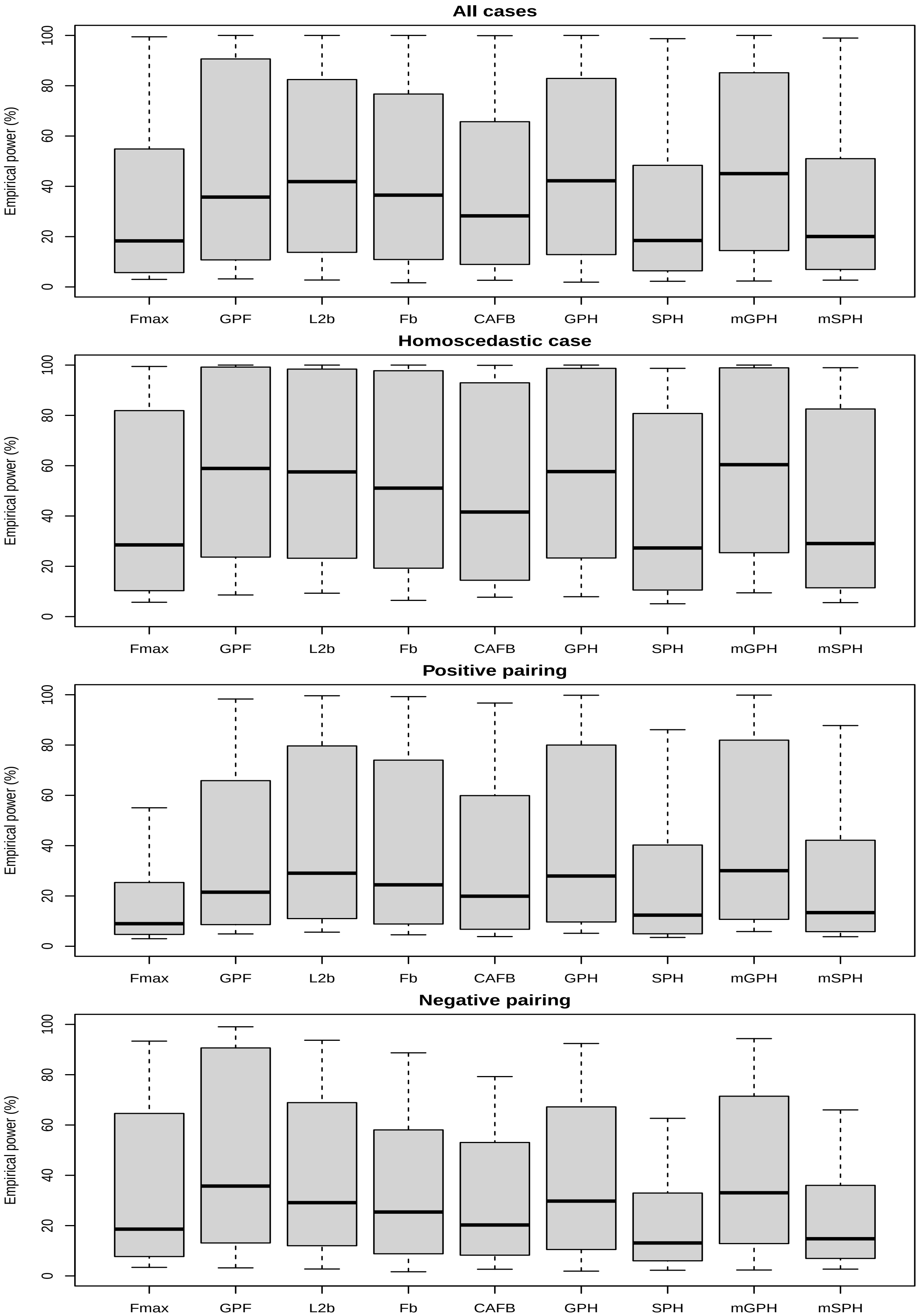


Fig. S29: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts and homoscedastic and heteroscedastic cases

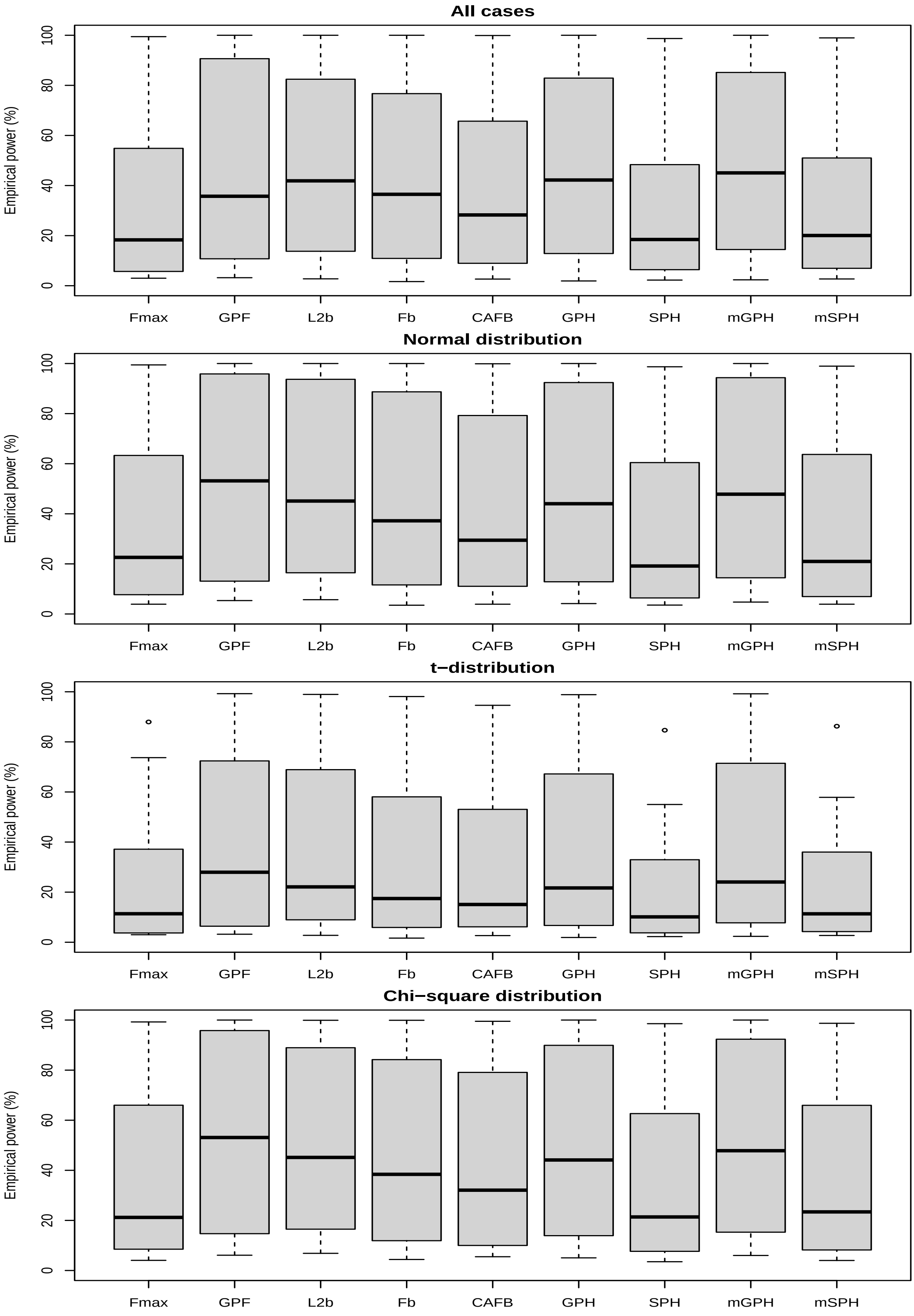


Fig. S30: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts and different distributions

Table S15: Empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.10	8.85	10.25	6.40	7.35	8.70	7.80	9.85	8.65
			$\mathcal{H}_{0,2}$	68.30	76.50	72.55	62.85	60.30	73.80	65.95	76.55	67.85
			$\mathcal{H}_{0,3}$	99.65	99.85	99.60	99.40	97.80	99.85	99.35	99.90	99.50
			$\mathcal{H}_{0,4}$	11.90	12.85	13.90	10.05	11.55	13.50	11.85	14.75	12.85
			$\mathcal{H}_{0,5}$	83.20	91.10	88.10	83.25	75.30	89.90	82.40	90.90	84.00
			$\mathcal{H}_{0,6}$	17.20	20.25	20.50	16.80	15.45	19.80	17.00	21.80	18.70
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.60	5.10	8.25	5.35	6.25	6.75	7.45	7.75	7.90
			$\mathcal{H}_{0,2}$	27.80	33.30	48.80	41.35	37.95	50.70	42.10	53.25	44.05
			$\mathcal{H}_{0,3}$	65.45	76.40	92.90	90.05	82.20	93.70	89.45	94.35	90.15
			$\mathcal{H}_{0,4}$	3.95	4.40	6.15	4.85	4.80	5.05	4.95	5.80	5.35
			$\mathcal{H}_{0,5}$	27.05	31.10	39.60	33.40	31.50	42.90	36.65	45.05	38.45
			$\mathcal{H}_{0,6}$	5.25	4.50	5.25	3.90	4.00	5.25	5.45	5.90	5.85
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.35	3.45	4.00	2.20	3.30	2.35	2.30	2.95	2.85
			$\mathcal{H}_{0,2}$	32.30	36.60	22.35	13.85	18.00	19.95	17.45	23.40	19.10
			$\mathcal{H}_{0,3}$	92.25	94.80	73.45	59.70	56.35	71.90	60.75	75.25	63.30
			$\mathcal{H}_{0,4}$	6.50	7.00	6.30	4.30	4.40	4.70	4.95	5.75	5.70
			$\mathcal{H}_{0,5}$	65.25	70.40	51.25	43.35	37.85	53.15	44.60	57.85	48.85
			$\mathcal{H}_{0,6}$	14.05	16.35	13.50	10.20	10.75	13.40	11.35	15.95	12.70
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	4.15	3.45	4.00	2.60	4.05	3.30	3.70	4.15	4.00
			$\mathcal{H}_{0,2}$	38.20	45.05	40.65	33.35	33.95	41.10	36.70	44.65	38.80
			$\mathcal{H}_{0,3}$	89.25	94.55	90.10	85.80	82.70	92.55	86.35	94.20	87.85
			$\mathcal{H}_{0,4}$	6.05	6.90	7.10	5.20	6.55	6.45	5.85	7.55	6.40
			$\mathcal{H}_{0,5}$	53.00	62.45	55.45	49.50	46.50	60.00	51.55	62.80	53.40
			$\mathcal{H}_{0,6}$	9.30	9.15	8.25	7.10	6.80	8.65	8.90	9.80	9.90
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	3.05	2.60	4.05	2.65	3.00	3.00	3.50	3.80	4.00
			$\mathcal{H}_{0,2}$	14.40	13.25	23.85	18.95	19.25	23.75	21.30	26.10	23.10
			$\mathcal{H}_{0,3}$	30.85	34.15	62.25	56.90	53.55	66.90	57.30	69.30	60.15
			$\mathcal{H}_{0,4}$	2.75	3.05	3.70	2.55	3.20	3.10	3.20	3.60	3.45
			$\mathcal{H}_{0,5}$	10.70	12.00	17.95	15.55	14.60	18.65	15.55	20.95	16.80
			$\mathcal{H}_{0,6}$	3.05	2.55	2.75	2.05	2.40	2.40	2.90	2.80	3.35
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.20	1.95	1.75	1.10	1.95	1.35	2.20	1.90	2.80
			$\mathcal{H}_{0,2}$	16.80	17.65	9.90	6.60	9.30	9.05	8.90	10.40	9.60
			$\mathcal{H}_{0,3}$	71.05	75.30	37.05	28.15	31.75	35.80	32.70	40.20	34.90
			$\mathcal{H}_{0,4}$	4.25	3.95	3.20	2.25	2.25	2.75	3.15	3.60	3.65
			$\mathcal{H}_{0,5}$	38.85	41.55	25.10	19.40	20.95	25.10	22.20	28.75	25.00
			$\mathcal{H}_{0,6}$	6.65	6.15	5.00	3.85	5.35	4.75	4.95	5.60	5.85
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.40	8.90	10.40	7.20	7.90	8.80	8.10	9.95	9.20
			$\mathcal{H}_{0,2}$	68.00	77.35	71.80	63.60	60.80	74.45	66.50	77.20	68.15
			$\mathcal{H}_{0,3}$	99.45	99.95	99.30	98.95	96.95	99.70	98.90	99.80	99.00
			$\mathcal{H}_{0,4}$	13.75	15.05	15.20	12.90	12.20	14.95	13.95	16.85	15.15
			$\mathcal{H}_{0,5}$	84.35	89.75	84.70	81.85	76.05	88.00	81.70	89.80	83.55
			$\mathcal{H}_{0,6}$	17.35	20.85	19.35	15.55	15.90	20.20	17.25	22.65	18.65
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.85	3.55	6.05	4.20	5.45	5.55	5.80	6.35	6.15
			$\mathcal{H}_{0,2}$	26.35	31.30	47.95	40.25	38.85	50.90	42.15	54.25	44.15
			$\mathcal{H}_{0,3}$	66.50	76.90	91.85	89.30	82.15	93.60	90.25	94.40	90.95
			$\mathcal{H}_{0,4}$	4.10	4.65	6.10	4.70	5.50	5.70	5.15	6.75	5.70
			$\mathcal{H}_{0,5}$	26.30	31.65	40.30	35.60	32.50	43.30	36.80	46.15	39.05
			$\mathcal{H}_{0,6}$	4.15	3.45	4.40	3.20	4.35	4.00	4.95	4.90	5.40
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.05	2.70	3.35	2.00	2.60	2.35	2.30	3.00	2.75
			$\mathcal{H}_{0,2}$	33.40	36.65	23.85	17.05	19.70	22.30	20.40	25.45	22.55
			$\mathcal{H}_{0,3}$	92.10	92.60	68.00	57.75	57.05	68.75	61.50	72.30	64.40
			$\mathcal{H}_{0,4}$	7.15	6.90	6.70	4.60	6.65	5.65	5.75	7.05	6.65
			$\mathcal{H}_{0,5}$	66.10	72.60	53.20	46.00	45.75	55.70	47.90	60.10	51.05
			$\mathcal{H}_{0,6}$	14.40	15.50	13.25	10.80	11.10	13.80	11.85	16.10	13.15

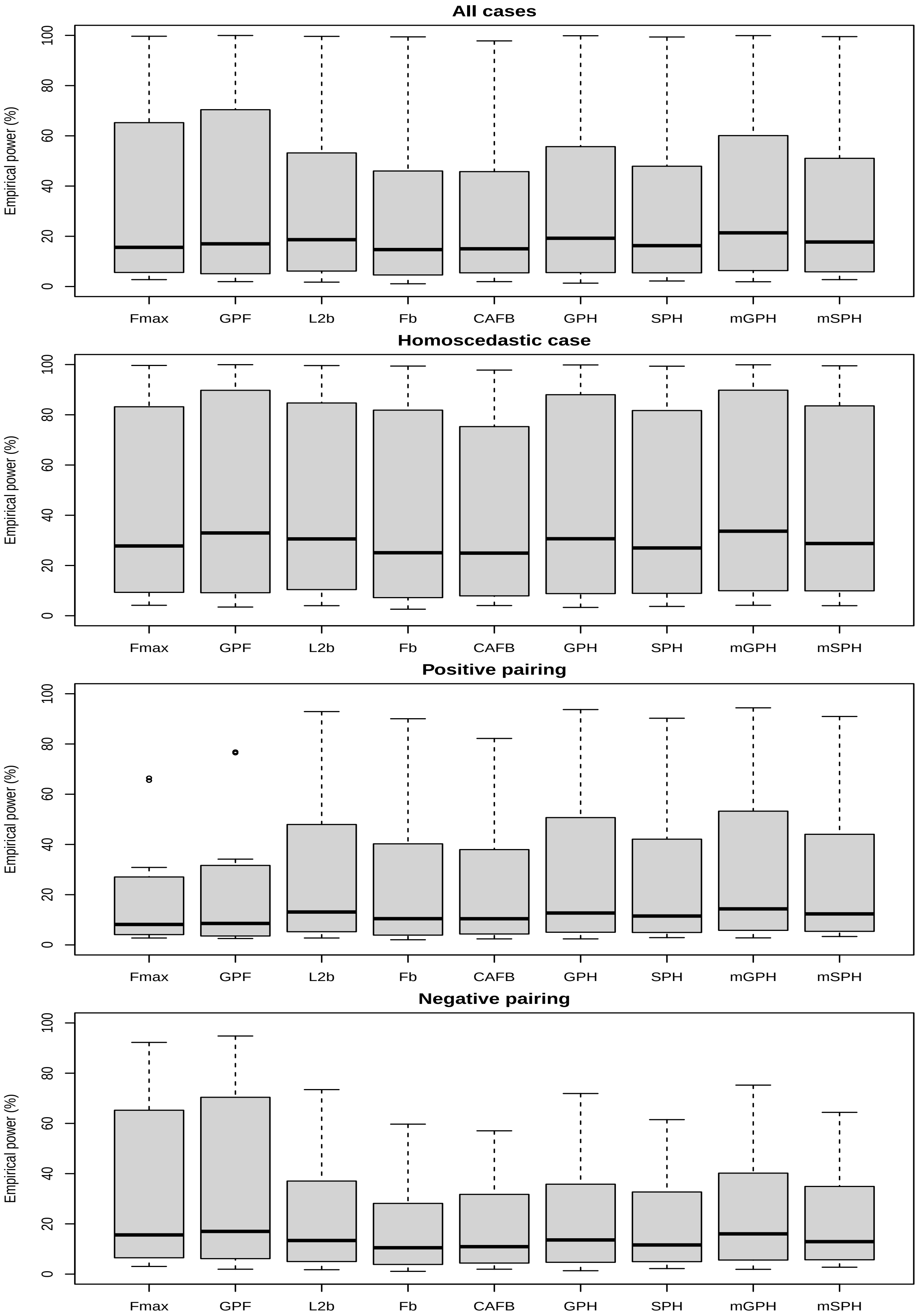


Fig. S31: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts and homoscedastic and heteroscedastic cases

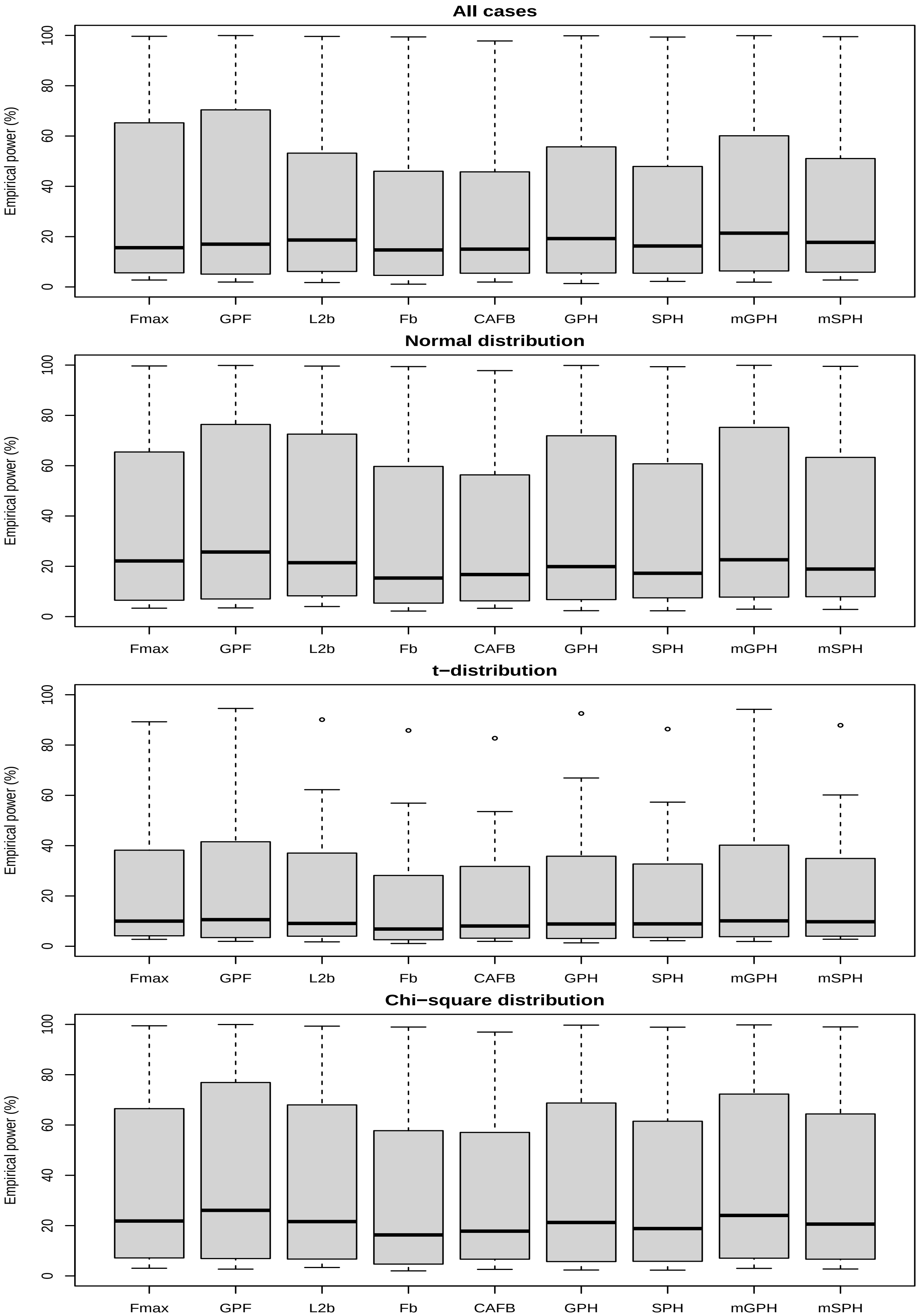


Fig. S32: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts and different distributions

Table S16: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$, $\mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}$, $\mathcal{H}_{0,5}$, $\mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.25	1.05	1.40	0.80	1.25	0.90	1.30	1.00	1.45
			$\mathcal{H}_{0,2}$	0.95	0.55	0.80	0.40	1.30	0.55	1.00	0.70	1.20
			$\mathcal{H}_{0,3}$	69.35	79.20	74.60	64.85	59.00	74.85	65.25	77.60	67.95
			$\mathcal{H}_{0,4}$	1.10	0.65	0.85	0.50	0.80	0.85	0.85	0.95	0.85
			$\mathcal{H}_{0,5}$	82.35	90.60	87.65	82.30	75.85	90.00	81.45	91.75	82.80
			$\mathcal{H}_{0,6}$	88.30	94.80	91.90	89.80	81.10	94.75	87.60	95.90	88.75
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.80	0.35	0.80	0.35	0.70	0.50	0.75	0.60	0.75
			$\mathcal{H}_{0,2}$	0.40	0.25	1.50	0.85	1.35	0.90	1.15	1.00	1.20
			$\mathcal{H}_{0,3}$	16.45	17.10	42.20	35.45	32.60	44.15	37.60	47.15	39.15
			$\mathcal{H}_{0,4}$	0.80	0.50	1.10	0.70	1.10	0.80	1.15	1.00	1.40
			$\mathcal{H}_{0,5}$	26.80	30.35	38.65	31.85	29.90	41.40	35.10	45.15	37.15
			$\mathcal{H}_{0,6}$	30.20	35.15	35.55	31.05	27.95	37.75	32.35	40.90	34.15
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.45	0.75	1.00	0.55	0.80	0.50	1.00	0.65	1.35
			$\mathcal{H}_{0,2}$	2.65	2.00	1.15	0.45	2.05	0.80	1.05	0.90	1.30
			$\mathcal{H}_{0,3}$	53.85	55.75	23.40	14.35	17.75	20.45	20.00	23.75	22.55
			$\mathcal{H}_{0,4}$	1.10	0.90	1.25	0.80	1.00	0.75	0.90	0.95	1.05
			$\mathcal{H}_{0,5}$	60.45	68.25	47.55	39.70	38.15	48.95	41.55	53.30	44.55
			$\mathcal{H}_{0,6}$	78.60	85.20	78.05	73.70	65.75	81.60	73.60	84.45	76.00
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.70	0.40	0.85	0.35	0.80	0.35	0.75	0.35	0.75
			$\mathcal{H}_{0,2}$	0.65	0.55	0.70	0.50	0.75	0.45	0.75	0.50	0.75
			$\mathcal{H}_{0,3}$	38.25	46.50	43.00	33.70	35.35	42.00	35.20	45.90	37.60
			$\mathcal{H}_{0,4}$	0.85	0.50	0.70	0.40	0.80	0.40	0.80	0.50	0.95
			$\mathcal{H}_{0,5}$	49.10	57.40	52.70	47.40	44.40	55.45	46.75	57.85	49.00
			$\mathcal{H}_{0,6}$	59.50	68.25	62.00	56.95	51.80	67.25	57.85	70.70	59.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.60	0.40	0.55	0.45	0.70	0.40	0.75	0.55	0.95
			$\mathcal{H}_{0,2}$	0.40	0.10	0.75	0.40	1.15	0.55	0.80	0.70	0.90
			$\mathcal{H}_{0,3}$	5.65	4.55	19.35	15.00	15.85	18.85	17.00	21.10	18.25
			$\mathcal{H}_{0,4}$	0.45	0.25	0.70	0.45	0.85	0.30	0.65	0.45	0.75
			$\mathcal{H}_{0,5}$	10.05	11.65	17.90	14.80	14.40	18.00	15.30	20.65	16.25
			$\mathcal{H}_{0,6}$	14.10	16.00	16.45	13.60	13.45	17.60	15.60	19.80	16.90
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.30	0.75	0.95	0.30	1.15	0.15	1.00	0.35	1.05
			$\mathcal{H}_{0,2}$	2.00	1.30	0.85	0.50	0.95	0.50	0.70	0.55	0.95
			$\mathcal{H}_{0,3}$	32.95	33.70	10.00	5.70	10.40	8.30	9.30	10.00	10.55
			$\mathcal{H}_{0,4}$	1.30	0.70	0.50	0.30	1.00	0.30	0.90	0.35	1.10
			$\mathcal{H}_{0,5}$	37.20	41.15	23.80	19.80	20.50	24.70	21.60	28.10	23.70
			$\mathcal{H}_{0,6}$	46.75	52.40	43.75	38.25	35.55	46.95	40.85	50.50	43.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.00	0.65	0.65	0.40	1.10	0.50	0.90	0.55	1.25
			$\mathcal{H}_{0,2}$	0.85	0.50	0.85	0.50	1.40	0.40	0.90	0.50	0.95
			$\mathcal{H}_{0,3}$	71.70	80.75	71.45	65.15	65.00	74.65	67.60	77.10	69.75
			$\mathcal{H}_{0,4}$	1.20	1.15	1.25	0.95	1.50	0.90	0.85	1.15	1.00
			$\mathcal{H}_{0,5}$	81.15	89.80	81.75	78.10	75.15	86.65	80.50	88.35	81.35
			$\mathcal{H}_{0,6}$	89.45	93.90	89.50	87.80	82.20	93.55	88.70	94.50	90.30
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.05	0.35	0.60	0.25	0.80	0.40	1.05	0.45	1.20
			$\mathcal{H}_{0,2}$	0.25	0.10	0.75	0.40	0.75	0.35	0.95	0.45	0.95
			$\mathcal{H}_{0,3}$	13.60	13.95	42.95	36.30	33.15	43.80	36.90	46.85	38.95
			$\mathcal{H}_{0,4}$	0.85	0.60	1.15	0.70	1.05	0.60	1.05	0.75	1.20
			$\mathcal{H}_{0,5}$	23.70	29.35	38.65	33.95	32.00	42.25	34.70	44.90	36.00
			$\mathcal{H}_{0,6}$	28.80	35.10	35.90	31.70	28.45	37.15	30.15	40.25	32.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.60	0.55	0.60	0.25	0.85	0.35	0.45	0.45	0.60
			$\mathcal{H}_{0,2}$	2.25	1.30	0.60	0.25	1.55	0.20	0.90	0.55	1.05
			$\mathcal{H}_{0,3}$	56.90	57.85	26.15	17.55	24.15	24.15	23.65	27.95	25.60
			$\mathcal{H}_{0,4}$	1.15	0.85	0.80	0.50	0.95	0.60	0.85	0.95	0.95
			$\mathcal{H}_{0,5}$	63.30	70.70	50.75	43.95	44.85	53.05	48.00	57.25	50.95
			$\mathcal{H}_{0,6}$	79.90	85.85	77.25	73.25	69.80	82.05	75.50	84.45	77.70

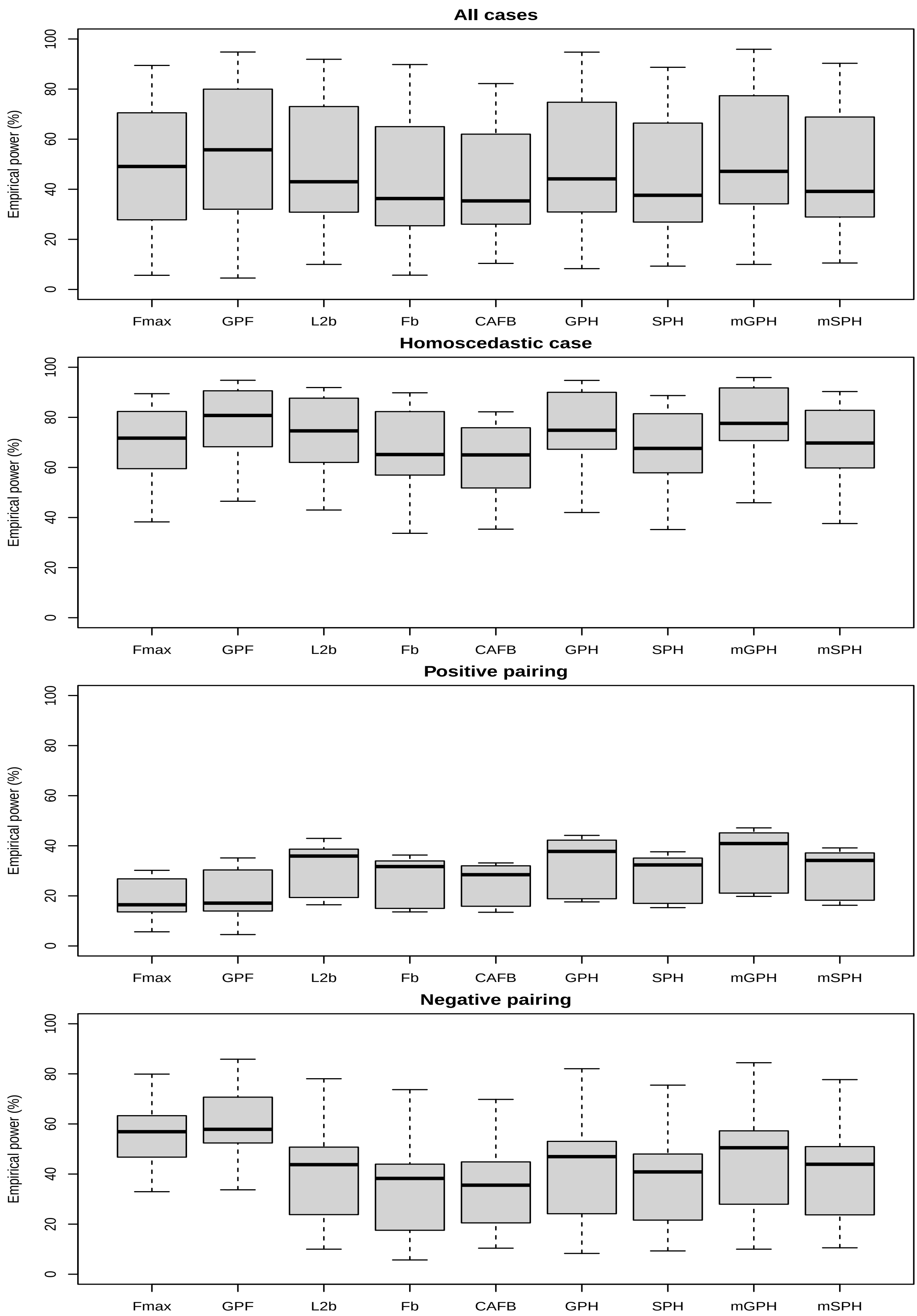


Fig. S33: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts and homoscedastic and heteroscedastic cases

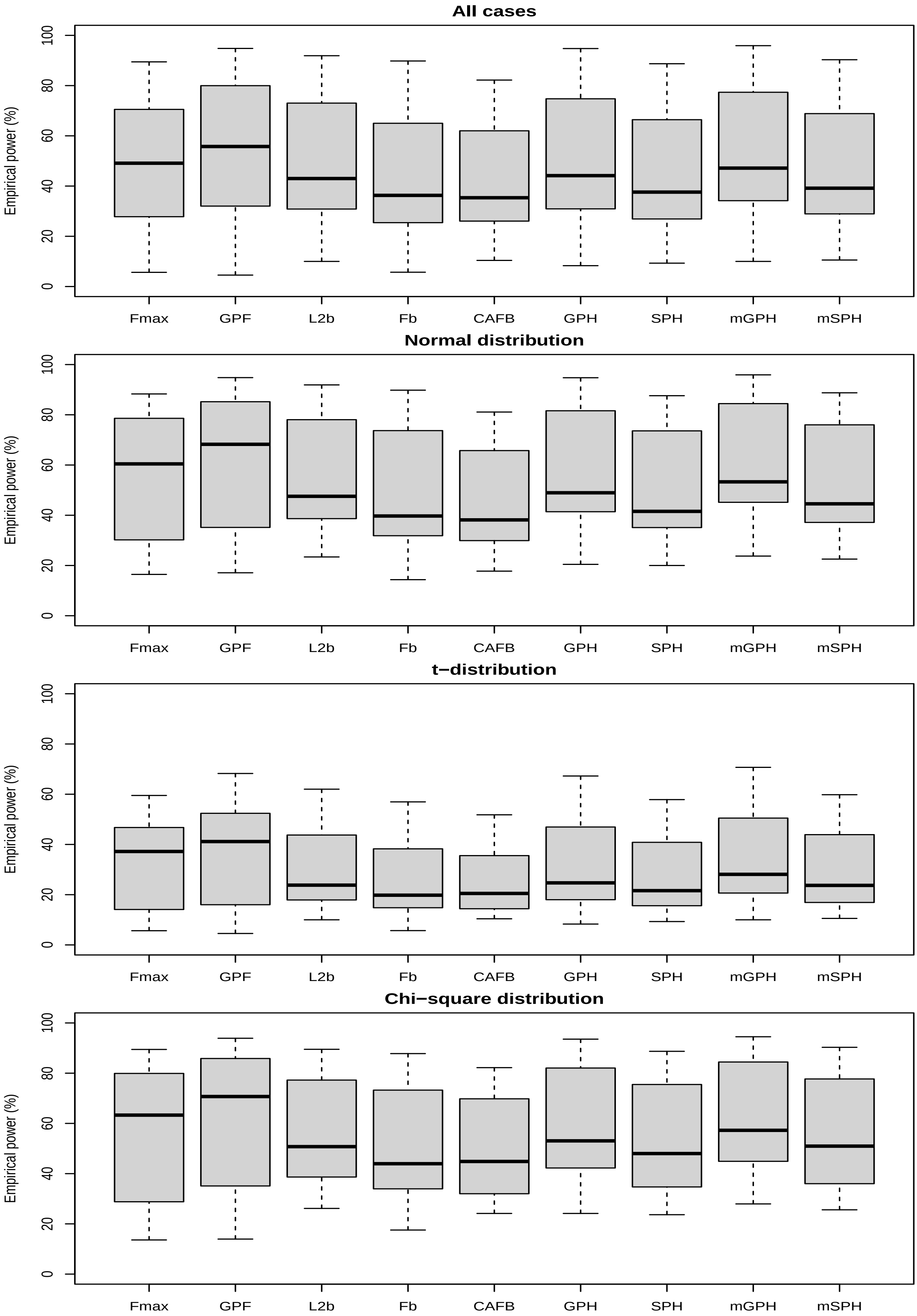


Fig. S34: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts and different distributions

Table S17: Empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.70	8.30	9.90	6.00	8.00	7.50	8.30	8.70	9.15
			$\mathcal{H}_{0,2}$	61.55	73.35	70.15	60.45	56.00	70.30	59.15	73.20	61.80
			$\mathcal{H}_{0,3}$	99.35	99.85	99.65	99.15	96.60	99.80	98.95	99.85	99.20
			$\mathcal{H}_{0,4}$	11.70	12.55	13.50	10.30	9.65	12.15	11.00	13.30	11.70
			$\mathcal{H}_{0,5}$	83.10	89.10	85.15	80.60	74.30	88.50	81.45	90.20	82.85
			$\mathcal{H}_{0,6}$	17.05	19.40	18.30	15.15	14.90	19.40	16.70	21.90	18.10
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.05	4.50	5.95	3.95	4.65	5.50	5.05	6.45	5.60
			$\mathcal{H}_{0,2}$	24.00	31.00	46.70	39.55	34.40	48.00	38.75	50.55	40.55
			$\mathcal{H}_{0,3}$	64.30	74.05	91.95	87.90	79.90	93.95	87.45	95.45	88.60
			$\mathcal{H}_{0,4}$	3.75	3.45	5.40	4.05	4.65	4.05	4.15	4.95	4.55
			$\mathcal{H}_{0,5}$	25.70	30.75	38.85	34.10	29.15	41.25	35.85	44.35	37.60
			$\mathcal{H}_{0,6}$	4.10	3.95	5.20	4.05	4.00	4.65	4.05	5.15	4.80
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.20	3.25	3.25	1.60	2.95	2.30	2.45	2.80	2.85
			$\mathcal{H}_{0,2}$	31.00	34.15	20.30	13.20	15.05	17.50	16.80	20.95	18.35
			$\mathcal{H}_{0,3}$	90.35	94.40	68.70	55.25	51.50	67.15	58.55	71.70	61.85
			$\mathcal{H}_{0,4}$	6.75	6.65	6.40	4.30	5.25	5.35	5.45	6.60	5.95
			$\mathcal{H}_{0,5}$	62.50	69.20	48.50	40.85	39.30	50.75	41.90	55.50	44.95
			$\mathcal{H}_{0,6}$	12.55	15.40	13.20	10.55	11.90	12.45	10.15	15.30	11.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	4.35	4.00	5.50	3.55	4.85	4.15	4.20	4.90	4.65
			$\mathcal{H}_{0,2}$	34.05	41.45	39.50	31.00	31.60	38.45	32.50	42.10	35.35
			$\mathcal{H}_{0,3}$	88.20	93.40	89.70	85.50	81.15	90.90	84.90	92.75	86.40
			$\mathcal{H}_{0,4}$	6.90	6.20	7.65	5.20	5.40	6.30	6.35	7.20	7.15
			$\mathcal{H}_{0,5}$	49.70	59.00	51.95	46.55	43.05	56.70	48.25	60.15	50.80
			$\mathcal{H}_{0,6}$	8.10	8.60	8.40	6.90	7.90	8.25	8.00	9.10	8.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	2.70	2.10	3.15	1.80	2.65	2.20	3.15	2.70	3.60
			$\mathcal{H}_{0,2}$	12.30	11.90	21.55	17.00	17.15	22.20	20.60	24.65	22.00
			$\mathcal{H}_{0,3}$	29.20	32.10	61.40	55.75	50.55	65.30	56.55	68.10	58.85
			$\mathcal{H}_{0,4}$	2.65	1.60	2.40	1.55	1.75	1.95	2.60	2.55	3.00
			$\mathcal{H}_{0,5}$	11.40	11.30	17.30	14.70	13.85	18.50	16.05	19.80	17.65
			$\mathcal{H}_{0,6}$	2.35	2.35	2.60	1.95	2.60	2.55	2.65	3.05	2.85
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.65	2.10	2.20	1.15	1.70	1.35	1.85	1.90	2.10
			$\mathcal{H}_{0,2}$	17.00	16.90	9.70	6.00	9.20	8.20	8.35	10.40	9.80
			$\mathcal{H}_{0,3}$	70.80	73.70	36.45	26.50	29.05	34.15	30.75	38.70	32.95
			$\mathcal{H}_{0,4}$	4.00	3.50	3.15	2.10	2.95	2.35	2.70	2.80	3.25
			$\mathcal{H}_{0,5}$	35.75	40.20	23.40	17.70	20.05	22.90	21.00	26.20	22.85
			$\mathcal{H}_{0,6}$	7.55	6.25	5.30	3.85	4.85	4.75	5.85	5.95	6.75
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.60	8.20	10.10	6.50	8.75	7.55	7.60	8.95	8.25
			$\mathcal{H}_{0,2}$	65.20	75.10	70.00	61.30	60.75	71.70	63.15	74.85	65.75
			$\mathcal{H}_{0,3}$	99.10	99.80	98.45	97.80	96.70	99.25	98.40	99.45	98.80
			$\mathcal{H}_{0,4}$	13.10	15.20	15.25	12.05	11.60	15.85	13.25	17.80	14.25
			$\mathcal{H}_{0,5}$	83.35	90.50	84.95	81.80	77.00	88.45	82.55	89.80	84.05
			$\mathcal{H}_{0,6}$	17.95	20.20	20.00	17.05	14.30	20.90	18.55	23.65	19.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.65	3.40	6.80	4.30	5.25	5.50	6.50	6.55	6.85
			$\mathcal{H}_{0,2}$	24.30	28.35	45.40	38.50	34.75	46.85	39.40	49.45	41.80
			$\mathcal{H}_{0,3}$	61.50	74.40	90.35	87.75	80.90	93.10	86.80	94.15	88.25
			$\mathcal{H}_{0,4}$	4.90	3.85	6.10	4.50	4.80	5.50	5.70	6.05	6.30
			$\mathcal{H}_{0,5}$	24.95	29.40	38.55	33.35	30.00	40.65	34.55	43.30	36.45
			$\mathcal{H}_{0,6}$	3.40	3.65	4.85	3.95	3.90	4.80	4.00	5.50	4.55
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.05	2.70	3.35	2.35	3.30	2.50	2.60	2.85	3.10
			$\mathcal{H}_{0,2}$	32.95	36.15	23.75	17.10	21.10	22.70	19.40	25.35	21.30
			$\mathcal{H}_{0,3}$	91.60	93.40	68.20	58.40	58.30	68.20	61.65	73.15	64.60
			$\mathcal{H}_{0,4}$	5.85	6.45	5.60	4.20	5.90	5.75	5.60	6.55	6.05
			$\mathcal{H}_{0,5}$	64.25	69.15	49.40	42.25	42.50	51.50	45.50	55.40	48.25
			$\mathcal{H}_{0,6}$	14.55	16.00	14.10	11.65	11.45	13.85	12.00	16.00	13.70

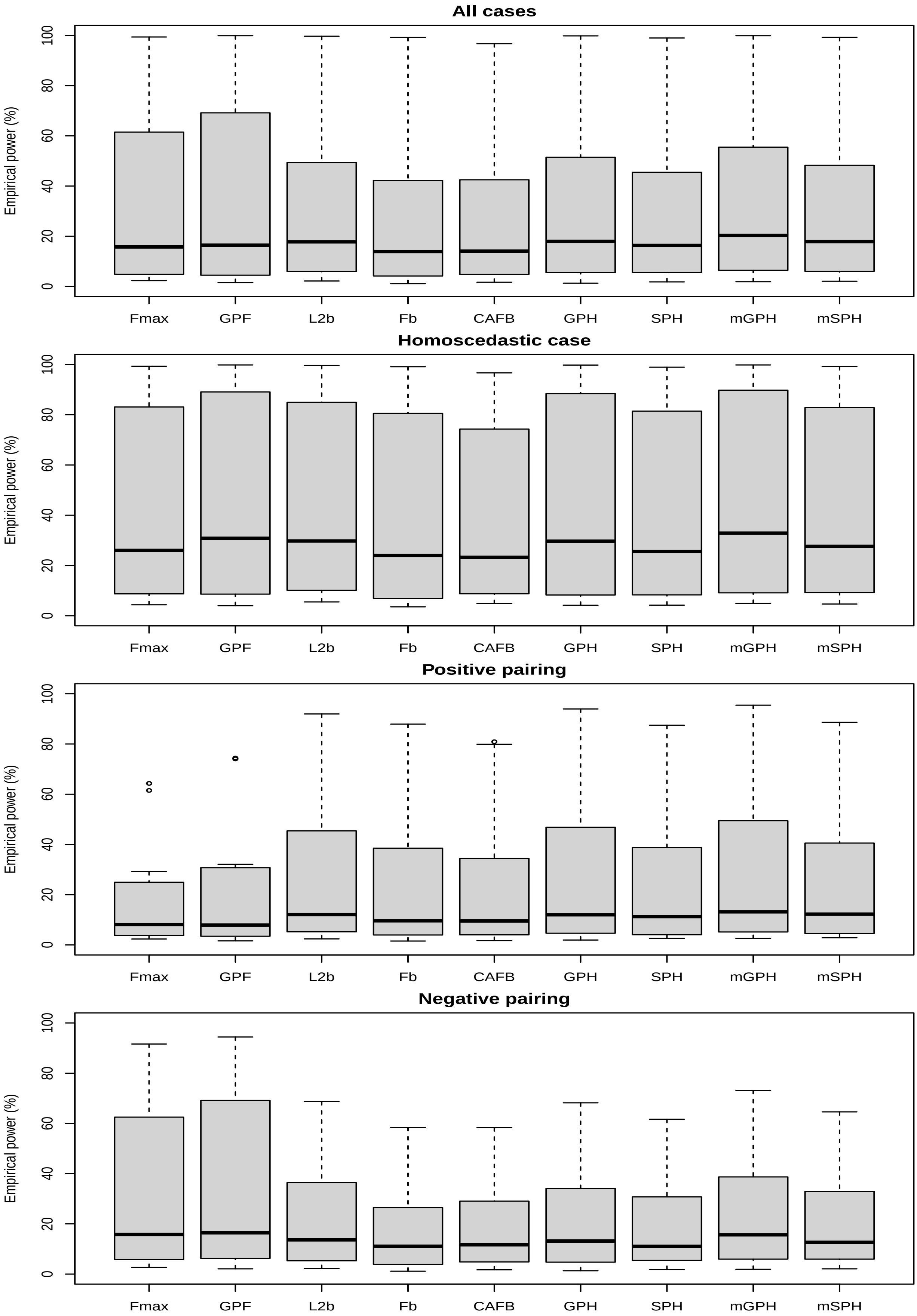


Fig. S35: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts and homoscedastic and heteroscedastic cases

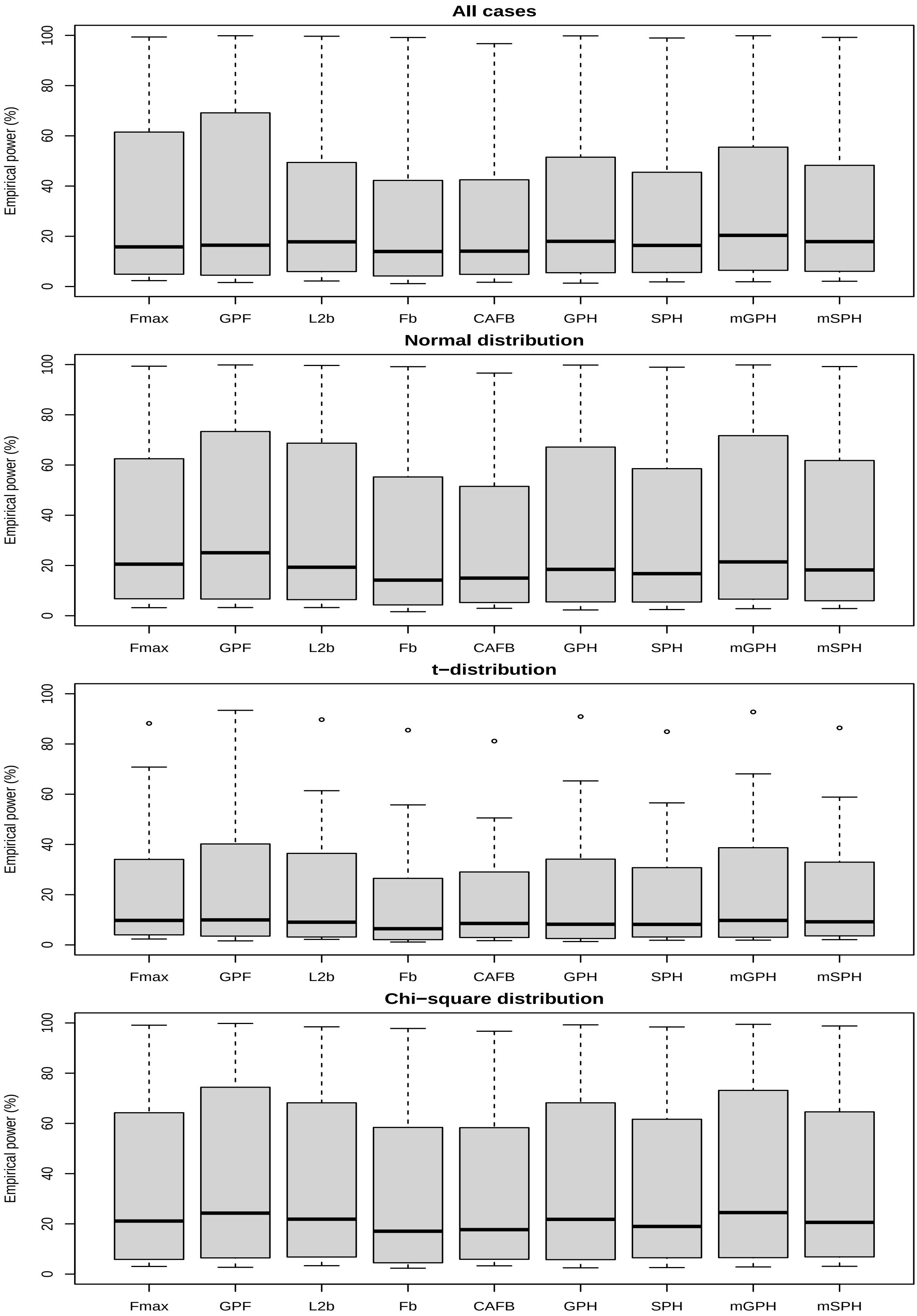


Fig. S36: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts and different distributions

S5.4. Results with Scaling Function

Table S18: Empirical FWER and sizes (as percentages) of all tests obtained for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.15	4.25	5.25	3.75	3.55	3.80	3.80	4.50	4.80
			$\mathcal{H}_{0,1}$	1.90	1.85	2.15	1.50	1.15	1.75	1.55	2.05	2.05
			$\mathcal{H}_{0,2}$	1.20	1.25	1.80	1.20	1.10	1.20	1.05	1.40	1.30
			$\mathcal{H}_{0,3}$	1.60	1.45	2.75	2.15	1.80	1.35	1.60	1.60	2.05
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.00	1.25	6.20	4.40	4.05	3.60	4.30	4.15	4.75
			$\mathcal{H}_{0,1}$	1.40	1.05	2.35	1.55	1.15	1.60	1.65	1.70	1.70
			$\mathcal{H}_{0,2}$	0.30	0.20	2.10	1.45	1.35	0.75	1.30	0.85	1.65
			$\mathcal{H}_{0,3}$	0.30	0.00	2.40	1.65	1.55	1.40	1.40	1.75	1.50
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	12.55	9.65	6.60	4.00	3.90	2.70	3.95	3.75	5.45
			$\mathcal{H}_{0,1}$	2.00	1.70	2.95	1.85	2.00	1.35	1.55	2.05	2.20
			$\mathcal{H}_{0,2}$	4.00	2.85	3.10	1.80	1.25	1.05	1.85	1.20	2.55
			$\mathcal{H}_{0,3}$	9.75	7.55	3.10	1.60	1.55	0.85	1.75	1.25	2.15
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.20	3.80	4.65	4.05	3.45	3.35	4.15	4.25	5.35
			$\mathcal{H}_{0,1}$	1.70	1.50	1.80	1.65	1.85	1.45	2.00	1.95	2.35
			$\mathcal{H}_{0,2}$	1.30	1.05	2.55	2.05	1.05	0.90	1.00	1.05	1.55
			$\mathcal{H}_{0,3}$	1.50	1.55	1.50	1.45	0.85	1.35	1.50	1.65	2.00
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	1.80	1.00	5.30	4.45	3.70	3.70	5.00	4.05	5.40
			$\mathcal{H}_{0,1}$	1.00	0.80	2.15	1.85	1.40	1.10	1.60	1.35	1.85
			$\mathcal{H}_{0,2}$	0.55	0.20	2.10	1.60	1.25	1.00	1.20	1.20	1.40
			$\mathcal{H}_{0,3}$	0.30	0.05	2.00	1.90	1.40	1.75	2.45	1.90	2.55
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	13.80	11.00	5.15	4.25	4.00	3.15	4.00	4.75	5.60
			$\mathcal{H}_{0,1}$	2.35	1.90	2.15	1.95	1.35	1.20	1.85	2.00	2.60
			$\mathcal{H}_{0,2}$	4.70	3.65	2.55	1.90	1.65	1.55	1.50	2.00	2.55
			$\mathcal{H}_{0,3}$	10.10	8.55	2.50	2.00	1.50	1.30	1.65	2.00	2.05
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.70	3.55	3.75	3.50	2.70	3.30	4.50	3.95	5.30
			$\mathcal{H}_{0,1}$	1.85	1.45	1.65	1.65	1.40	1.20	1.75	1.45	2.05
			$\mathcal{H}_{0,2}$	2.05	1.50	1.20	1.05	0.80	1.60	1.95	2.05	2.25
			$\mathcal{H}_{0,3}$	1.30	1.15	1.75	1.60	0.80	1.05	1.30	1.20	1.55
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	2.00	2.15	4.90	4.45	3.45	4.60	5.00	4.95	5.20
			$\mathcal{H}_{0,1}$	1.45	1.55	1.70	1.50	1.10	2.00	2.05	2.00	2.15
			$\mathcal{H}_{0,2}$	0.40	0.55	2.00	1.60	1.40	1.55	1.70	1.85	1.70
			$\mathcal{H}_{0,3}$	0.15	0.15	1.70	1.85	1.25	1.60	1.50	1.70	1.60
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.95	12.05	4.70	4.35	3.25	4.50	4.60	5.90	5.55
			$\mathcal{H}_{0,1}$	2.50	2.60	2.20	1.85	1.35	1.80	1.65	2.65	2.20
			$\mathcal{H}_{0,2}$	4.95	5.00	1.80	1.65	1.25	2.25	2.50	3.15	2.85
			$\mathcal{H}_{0,3}$	9.10	8.95	2.35	2.25	1.20	1.65	1.55	2.20	2.10
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.40	3.60	6.85	5.10	3.85	2.80	4.15	3.80	4.80
			$\mathcal{H}_{0,1}$	1.60	1.25	2.30	1.80	1.45	1.00	1.50	1.35	1.80
			$\mathcal{H}_{0,2}$	1.60	1.45	3.45	2.40	1.50	1.05	1.55	1.60	1.90
			$\mathcal{H}_{0,3}$	1.80	1.45	2.75	1.85	1.30	1.15	1.80	1.50	2.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.90	1.50	8.35	5.50	3.60	2.90	3.85	3.60	4.20
			$\mathcal{H}_{0,1}$	1.70	1.05	3.35	2.05	1.30	1.15	1.50	1.25	1.60
			$\mathcal{H}_{0,2}$	1.00	0.40	3.10	1.95	1.50	1.10	1.45	1.55	1.70
			$\mathcal{H}_{0,3}$	0.30	0.15	2.70	1.90	1.05	0.75	1.10	1.00	1.20
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	12.85	9.70	6.05	4.05	4.65	2.40	4.60	3.55	5.80
			$\mathcal{H}_{0,1}$	2.35	1.75	2.80	1.95	1.55	1.10	2.15	1.75	2.60
			$\mathcal{H}_{0,2}$	4.80	3.15	2.80	1.80	1.80	0.95	1.90	1.50	2.65
			$\mathcal{H}_{0,3}$	9.50	7.55	3.05	1.50	2.00	0.80	2.15	1.20	2.65
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	5.55	4.45	5.70	4.60	3.90	4.05	5.05	5.10	5.85
			$\mathcal{H}_{0,1}$	2.40	1.95	2.65	2.10	1.75	1.60	2.35	2.15	2.75
			$\mathcal{H}_{0,2}$	1.80	1.25	2.45	1.80	1.40	1.40	1.60	1.65	1.80
			$\mathcal{H}_{0,3}$	1.90	2.10	1.85	1.40	1.15	1.65	1.60	2.15	1.85
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	1.70	1.30	5.60	4.75	2.90	3.45	4.65	3.90	4.85

Table S18: Empirical FWER and sizes (as percentages) of all tests obtained for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
			$\mathcal{H}_{0,1}$	1.00	1.05	2.35	1.90	0.80	1.25	1.60	1.30	1.65
			$\mathcal{H}_{0,2}$	0.55	0.25	1.75	1.65	1.20	1.25	1.45	1.55	1.50
			$\mathcal{H}_{0,3}$	0.15	0.10	1.85	1.55	1.05	1.25	1.75	1.40	1.85
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.35	9.10	4.75	3.90	3.05	2.95	3.90	3.90	4.75
			$\mathcal{H}_{0,1}$	1.90	1.60	2.15	1.80	1.50	1.30	1.35	1.65	1.85
			$\mathcal{H}_{0,2}$	3.85	3.10	2.20	1.85	1.25	1.35	1.80	1.80	1.95
			$\mathcal{H}_{0,3}$	8.30	7.30	2.15	1.65	0.85	1.40	1.30	1.80	1.65
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.45	3.95	4.75	4.55	3.45	3.55	4.55	4.20	5.35
			$\mathcal{H}_{0,1}$	1.50	1.85	1.80	1.70	1.25	1.55	1.50	1.85	1.70
			$\mathcal{H}_{0,2}$	1.75	1.25	1.80	1.70	1.30	1.35	1.75	1.50	2.05
			$\mathcal{H}_{0,3}$	1.65	1.05	2.05	1.90	1.20	1.00	1.45	1.20	1.95
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	1.80	1.40	5.10	4.65	2.95	3.30	4.45	3.85	5.00
			$\mathcal{H}_{0,1}$	1.25	0.85	1.85	1.80	1.50	1.30	1.50	1.55	1.85
			$\mathcal{H}_{0,2}$	0.50	0.50	2.20	2.00	0.95	1.35	1.80	1.55	1.95
			$\mathcal{H}_{0,3}$	0.05	0.10	1.45	1.25	0.65	1.00	1.40	1.10	1.45
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	13.35	11.10	3.80	3.05	2.45	2.80	4.55	4.40	5.95
			$\mathcal{H}_{0,1}$	2.00	1.65	1.30	1.15	0.95	1.20	1.50	1.80	2.00
			$\mathcal{H}_{0,2}$	5.15	3.75	2.00	1.45	0.80	1.30	2.15	2.25	3.00
			$\mathcal{H}_{0,3}$	10.00	8.85	2.05	1.75	1.15	1.25	1.75	1.55	2.30
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.90	3.65	6.20	3.90	3.55	3.35	5.00	4.05	5.90
			$\mathcal{H}_{0,1}$	2.00	1.50	2.40	1.40	1.20	1.15	2.20	1.45	2.50
			$\mathcal{H}_{0,2}$	1.80	1.35	2.60	1.55	1.05	1.40	1.60	1.85	2.00
			$\mathcal{H}_{0,3}$	1.50	1.40	2.60	1.55	1.50	1.35	1.60	1.50	2.10
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	2.75	1.40	6.80	4.85	3.60	3.25	4.75	4.05	5.05
			$\mathcal{H}_{0,1}$	1.70	1.05	2.60	1.55	1.40	1.15	1.70	1.40	1.85
			$\mathcal{H}_{0,2}$	0.60	0.20	2.55	1.40	0.70	1.10	1.35	1.35	1.40
			$\mathcal{H}_{0,3}$	0.50	0.25	2.80	2.35	1.70	1.30	1.80	1.60	1.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	11.30	8.80	6.75	3.95	4.90	2.30	4.30	3.20	5.45
			$\mathcal{H}_{0,1}$	2.35	1.30	2.90	1.50	1.80	0.95	1.65	1.50	2.20
			$\mathcal{H}_{0,2}$	3.85	2.90	3.50	2.05	1.95	0.90	1.95	1.25	2.40
			$\mathcal{H}_{0,3}$	8.50	7.15	2.85	1.60	1.65	0.90	1.60	1.20	2.10
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	5.55	4.55	4.95	3.90	3.30	4.45	5.20	5.25	5.70
			$\mathcal{H}_{0,1}$	2.15	1.55	1.85	1.35	0.90	1.35	2.10	1.80	2.30
			$\mathcal{H}_{0,2}$	2.05	1.90	2.15	1.70	1.50	2.05	2.20	2.40	2.50
			$\mathcal{H}_{0,3}$	2.00	1.65	1.70	1.40	1.15	1.60	1.95	1.95	2.40
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	2.05	1.95	4.95	4.00	3.55	5.15	4.80	5.45	5.45
			$\mathcal{H}_{0,1}$	1.05	1.15	1.80	1.35	1.35	1.75	1.65	1.85	1.90
			$\mathcal{H}_{0,2}$	0.50	0.65	1.65	1.40	1.20	1.85	1.50	2.15	1.65
			$\mathcal{H}_{0,3}$	0.65	0.20	1.85	1.55	1.05	1.70	1.95	1.75	2.20
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	12.45	10.60	4.75	3.65	2.75	3.10	4.75	4.30	5.90
			$\mathcal{H}_{0,1}$	2.25	1.55	2.15	1.55	1.00	1.20	1.60	1.70	1.95
			$\mathcal{H}_{0,2}$	4.35	3.70	2.00	1.60	1.20	1.50	2.25	2.10	2.70
			$\mathcal{H}_{0,3}$	9.75	8.65	2.30	1.75	1.05	1.35	2.30	1.85	2.90
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.70	4.35	4.90	4.50	3.30	4.55	4.75	5.10	5.65
			$\mathcal{H}_{0,1}$	1.95	1.70	1.70	1.50	1.15	1.85	1.85	2.05	2.30
			$\mathcal{H}_{0,2}$	1.85	1.85	2.15	2.00	1.00	1.75	1.95	2.00	2.35
			$\mathcal{H}_{0,3}$	1.50	1.65	1.90	1.75	1.55	1.75	1.70	1.95	1.95
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	1.45	1.30	5.75	5.40	2.95	3.85	4.10	4.20	4.45
			$\mathcal{H}_{0,1}$	0.90	0.90	2.05	1.95	1.25	1.30	1.25	1.40	1.35
			$\mathcal{H}_{0,2}$	0.50	0.40	2.25	2.05	0.80	1.45	1.50	1.60	1.75
			$\mathcal{H}_{0,3}$	0.10	0.00	1.75	1.70	1.00	1.35	1.50	1.45	1.55
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.05	9.30	3.80	3.20	2.70	3.00	4.20	4.30	5.35
			$\mathcal{H}_{0,1}$	2.20	2.15	1.55	1.40	1.25	1.40	1.70	2.15	2.35
			$\mathcal{H}_{0,2}$	3.15	3.05	1.80	1.50	1.00	1.05	1.55	1.60	2.00
			$\mathcal{H}_{0,3}$	9.10	6.95	1.90	1.55	0.90	1.20	1.65	1.70	2.25

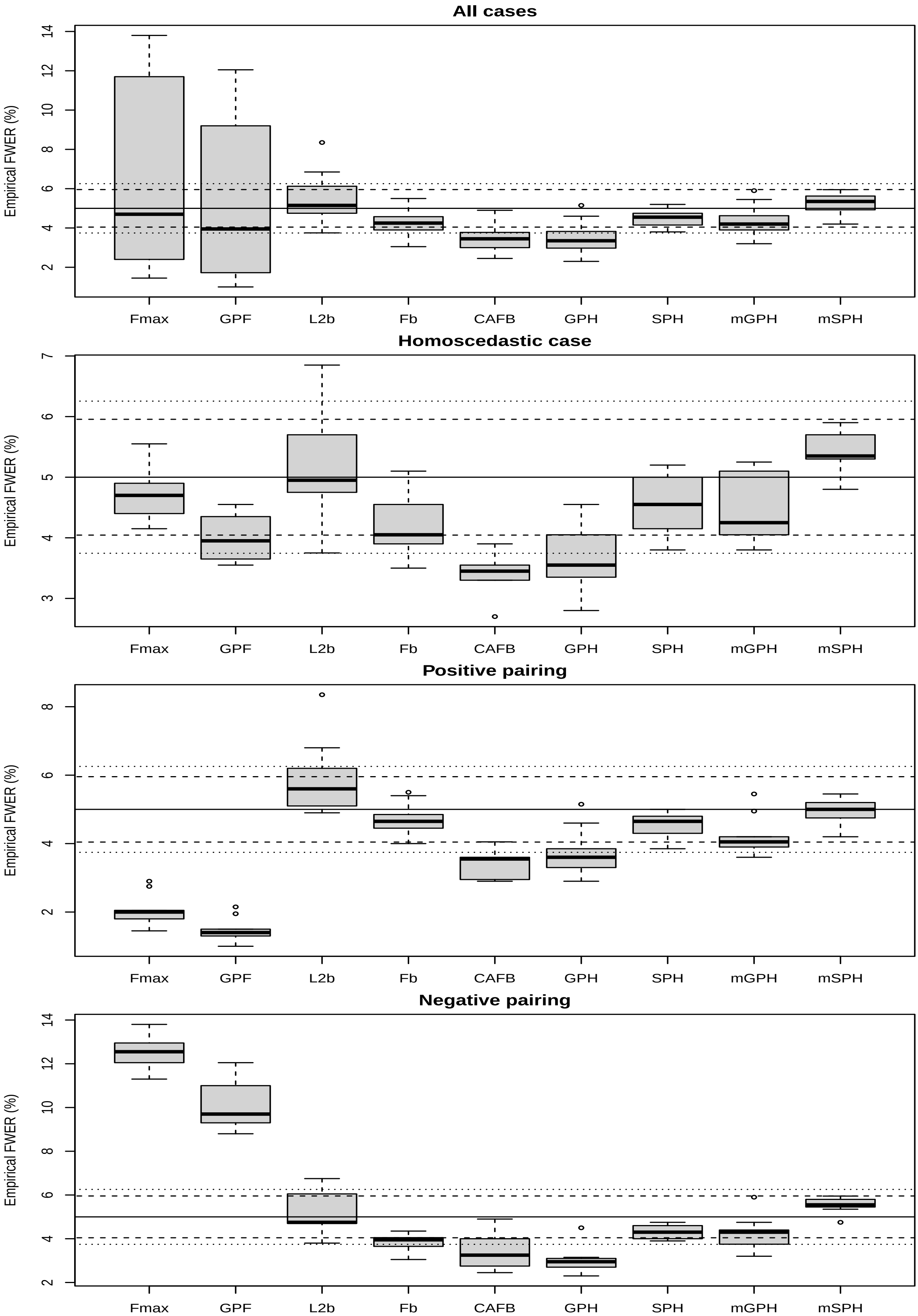


Fig. S37: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts with scaling function for homoscedastic and heteroscedastic cases

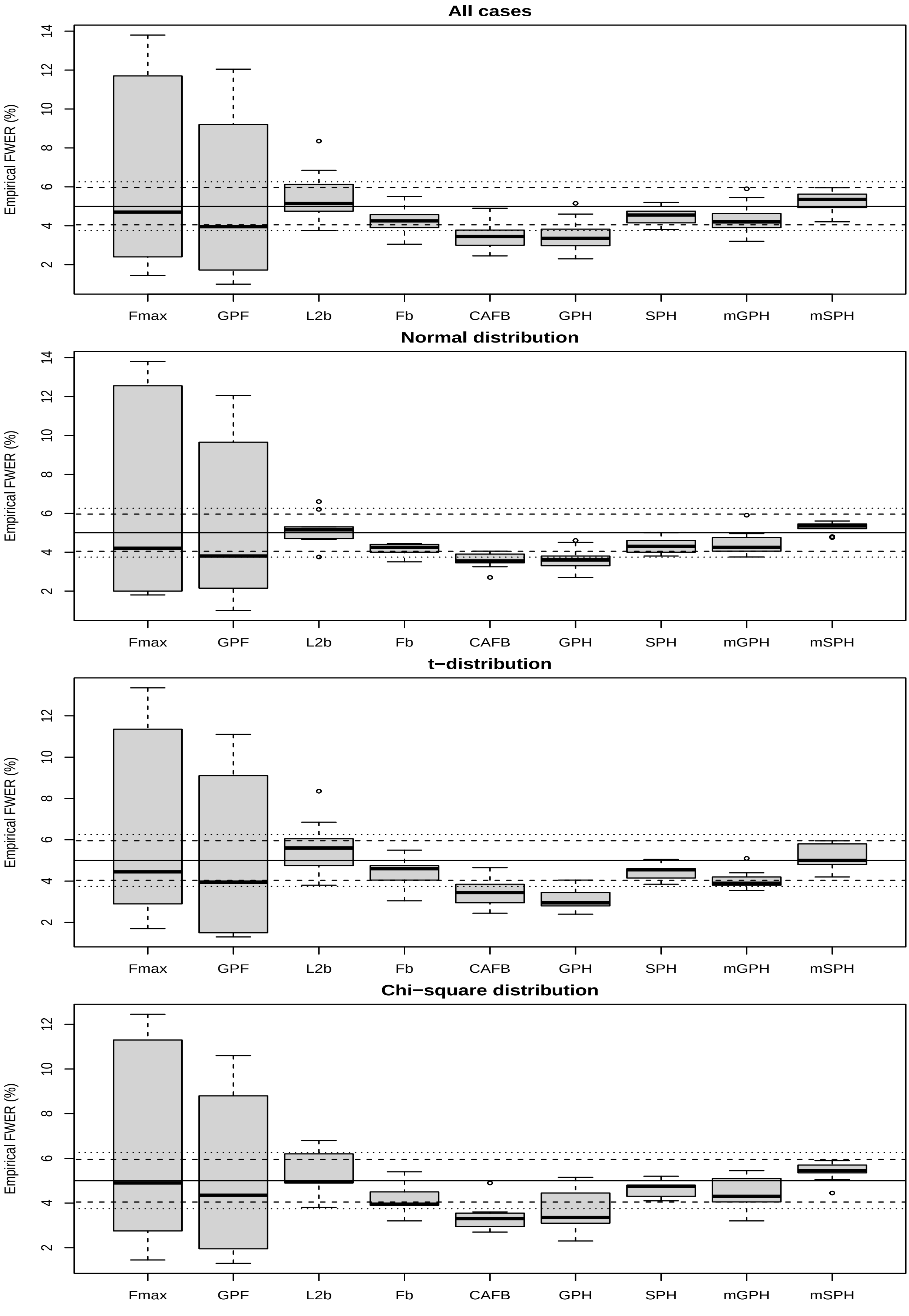


Fig. S38: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts with scaling function for different distributions

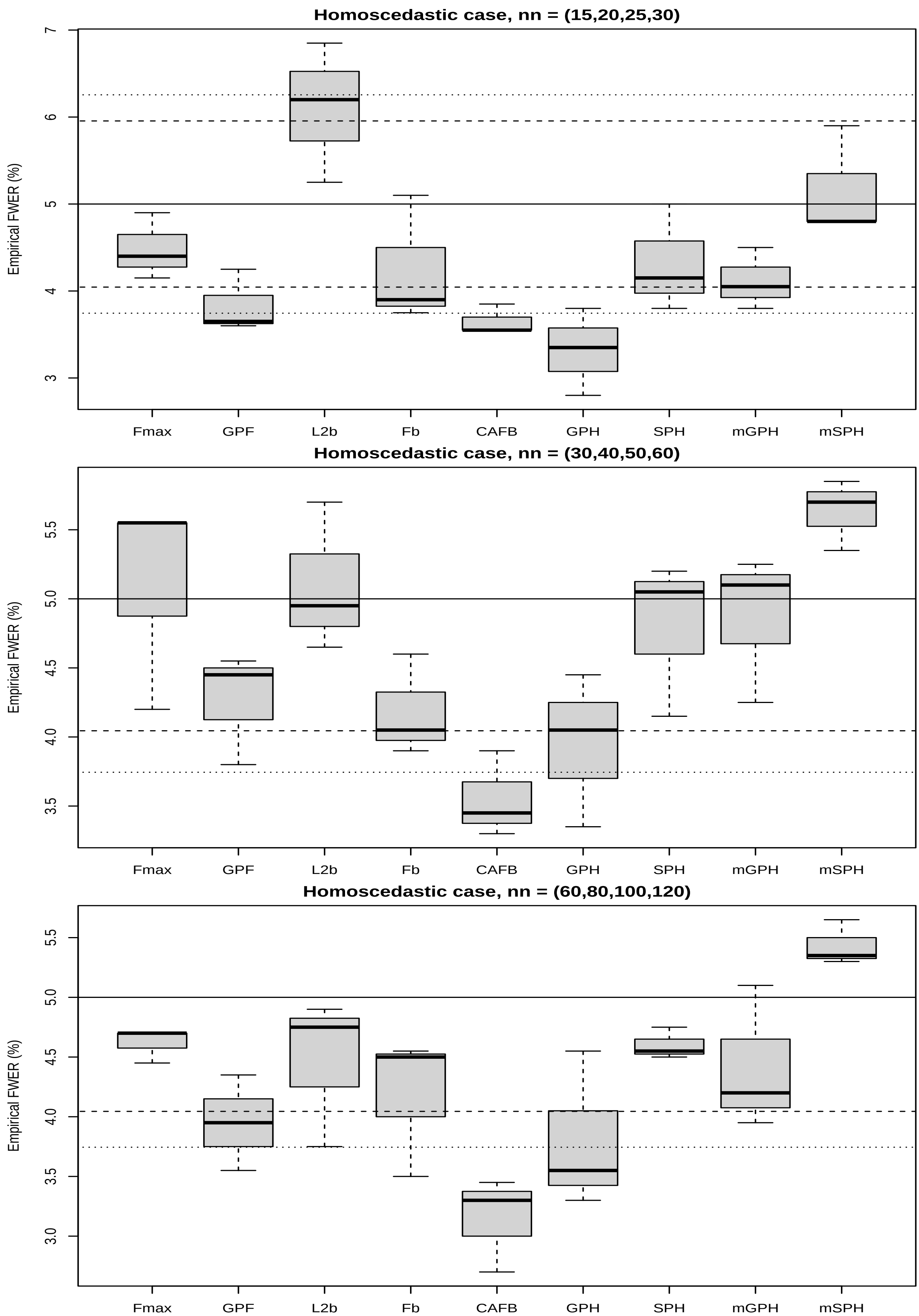


Fig. S39: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes with scaling function and homoscedastic case

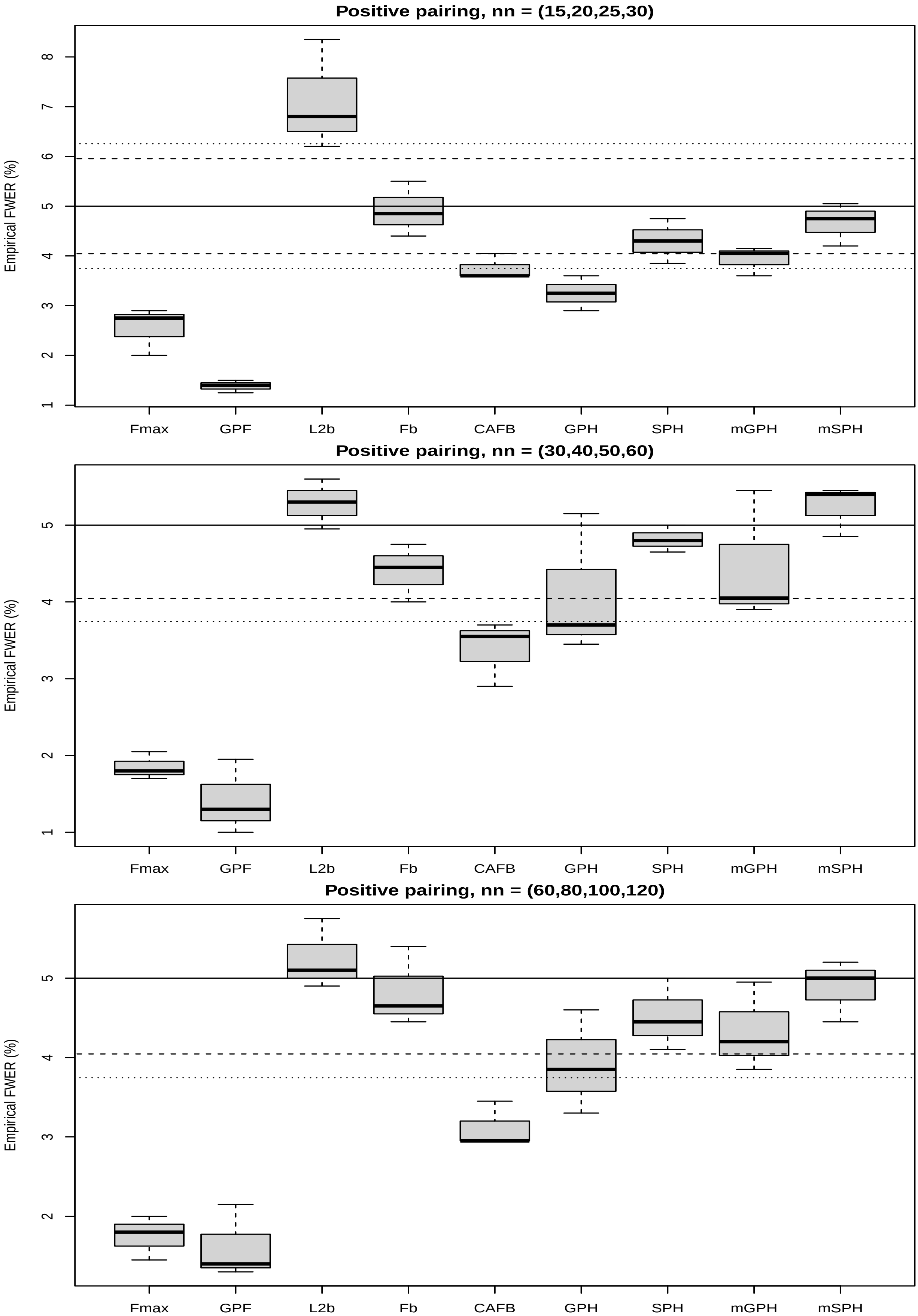


Fig. S40: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes with scaling function and positive pairing

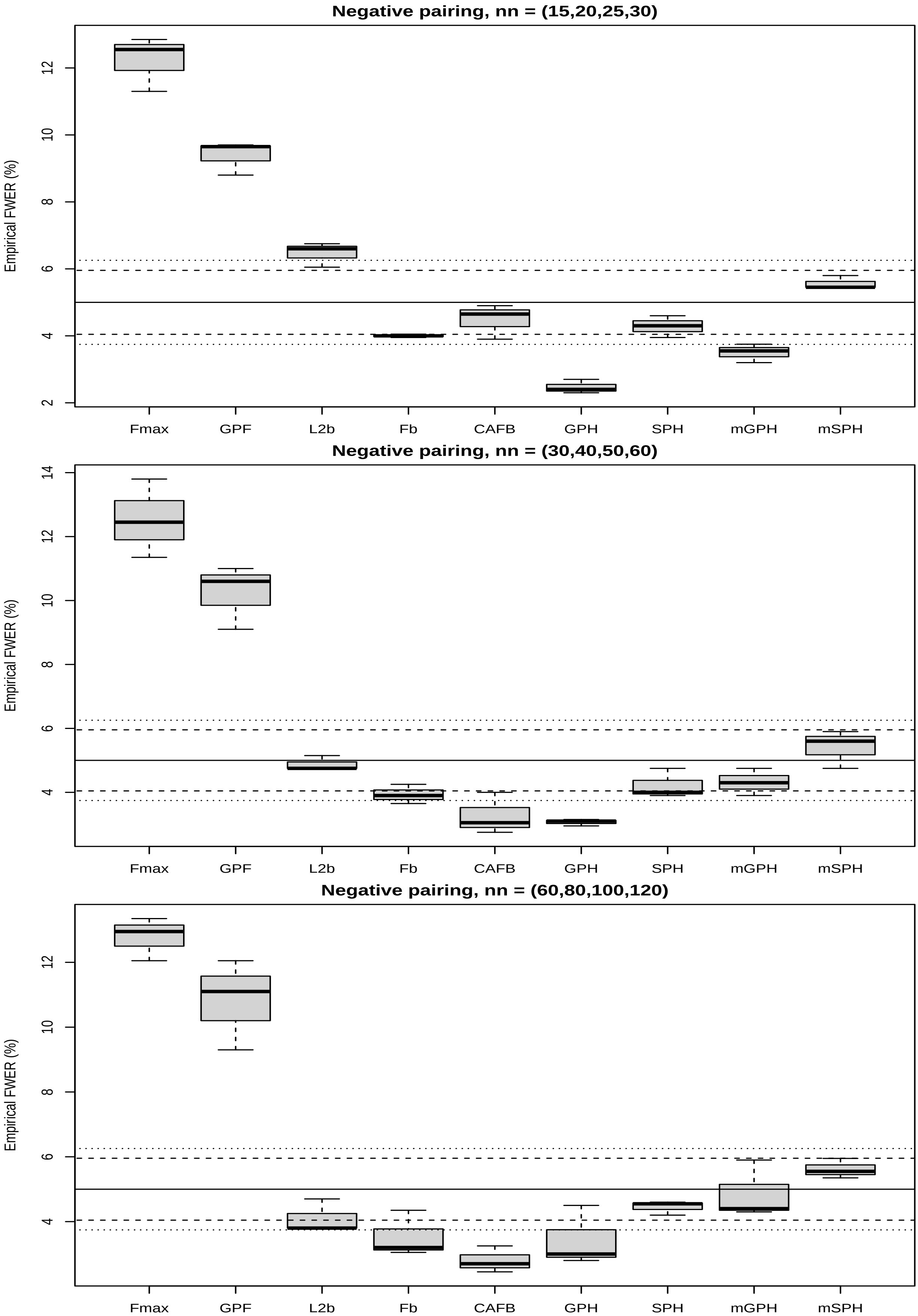


Fig. S41: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts and different sample sizes with scaling function and negative pairing

Table S19: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.85	1.35	2.65	1.80	1.20	1.15	1.85	1.60	2.25
			$\mathcal{H}_{0,2}$	1.65	0.90	2.70	1.90	1.60	0.70	1.40	1.00	1.80
			$\mathcal{H}_{0,3}$	83.65	88.30	2.60	1.75	6.20	85.90	79.50	88.20	82.10
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.10	0.50	2.60	1.90	1.85	0.75	1.20	0.95	1.35
			$\mathcal{H}_{0,2}$	1.10	0.95	2.20	1.55	1.65	1.70	2.35	1.85	2.50
			$\mathcal{H}_{0,3}$	22.65	25.20	2.55	1.70	3.30	56.45	49.80	58.40	51.20
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.85	2.00	2.80	1.90	1.35	1.35	2.20	1.85	2.85
			$\mathcal{H}_{0,2}$	4.15	2.90	1.75	1.10	1.15	1.25	2.00	2.00	2.50
			$\mathcal{H}_{0,3}$	64.80	66.75	3.20	1.90	3.35	30.25	28.60	37.50	32.90
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	2.15	1.20	2.45	1.35	1.20	1.20	2.15	1.45	2.40
			$\mathcal{H}_{0,2}$	1.95	1.45	2.95	1.90	1.60	1.20	1.65	1.80	2.05
			$\mathcal{H}_{0,3}$	51.90	58.75	2.65	1.55	3.00	54.55	48.75	59.30	52.50
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.25	0.65	2.30	1.45	1.25	1.20	1.50	1.35	1.75
			$\mathcal{H}_{0,2}$	0.95	0.25	1.80	1.20	1.00	1.25	1.70	1.45	1.80
			$\mathcal{H}_{0,3}$	8.20	8.95	2.10	1.30	1.85	28.85	25.60	30.85	26.90
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.85	1.45	3.10	2.10	1.70	0.90	1.60	1.45	2.15
			$\mathcal{H}_{0,2}$	3.15	2.60	3.20	1.95	1.85	0.70	1.40	1.25	1.80
			$\mathcal{H}_{0,3}$	45.15	46.05	3.80	2.20	3.65	15.85	15.00	20.75	18.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.70	1.10	2.75	1.70	1.45	1.15	1.85	1.40	2.00
			$\mathcal{H}_{0,2}$	1.70	1.45	2.40	1.45	1.50	1.30	1.45	1.55	1.70
			$\mathcal{H}_{0,3}$	82.75	88.60	3.30	2.00	8.00	85.05	79.35	87.45	81.65
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.05	0.90	2.85	1.80	1.20	1.25	1.50	1.30	1.55
			$\mathcal{H}_{0,2}$	0.95	0.40	3.00	2.05	1.75	1.30	1.80	1.45	2.10
			$\mathcal{H}_{0,3}$	21.90	25.05	2.70	2.10	3.55	55.70	50.30	57.30	51.75
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.55	1.30	2.40	1.55	1.40	0.95	1.10	1.80	1.65
			$\mathcal{H}_{0,2}$	4.50	3.60	2.80	2.05	2.50	1.20	2.15	1.95	2.75
			$\mathcal{H}_{0,3}$	67.70	68.30	3.60	2.05	4.65	35.20	32.75	41.40	37.25

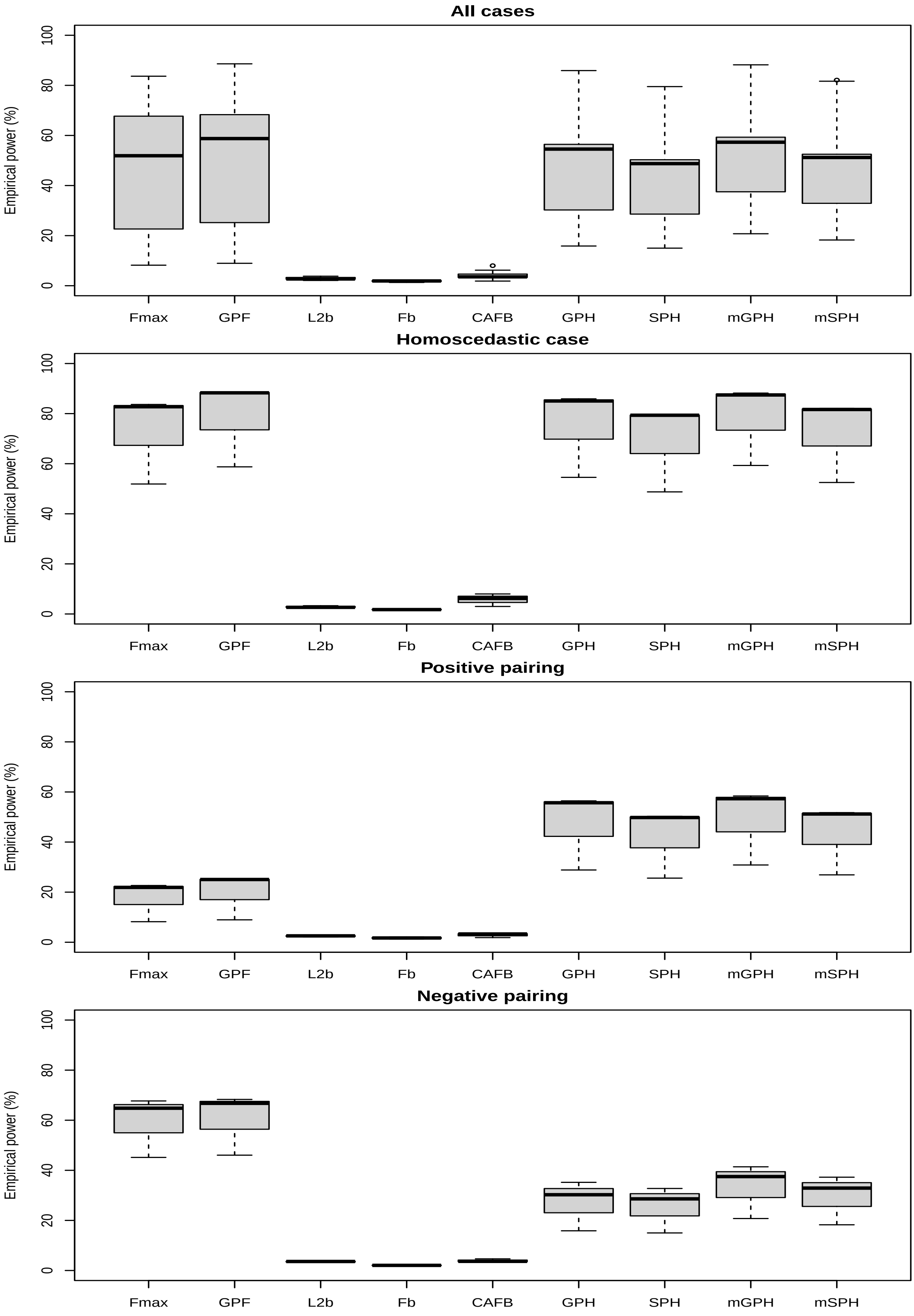


Fig. S42: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

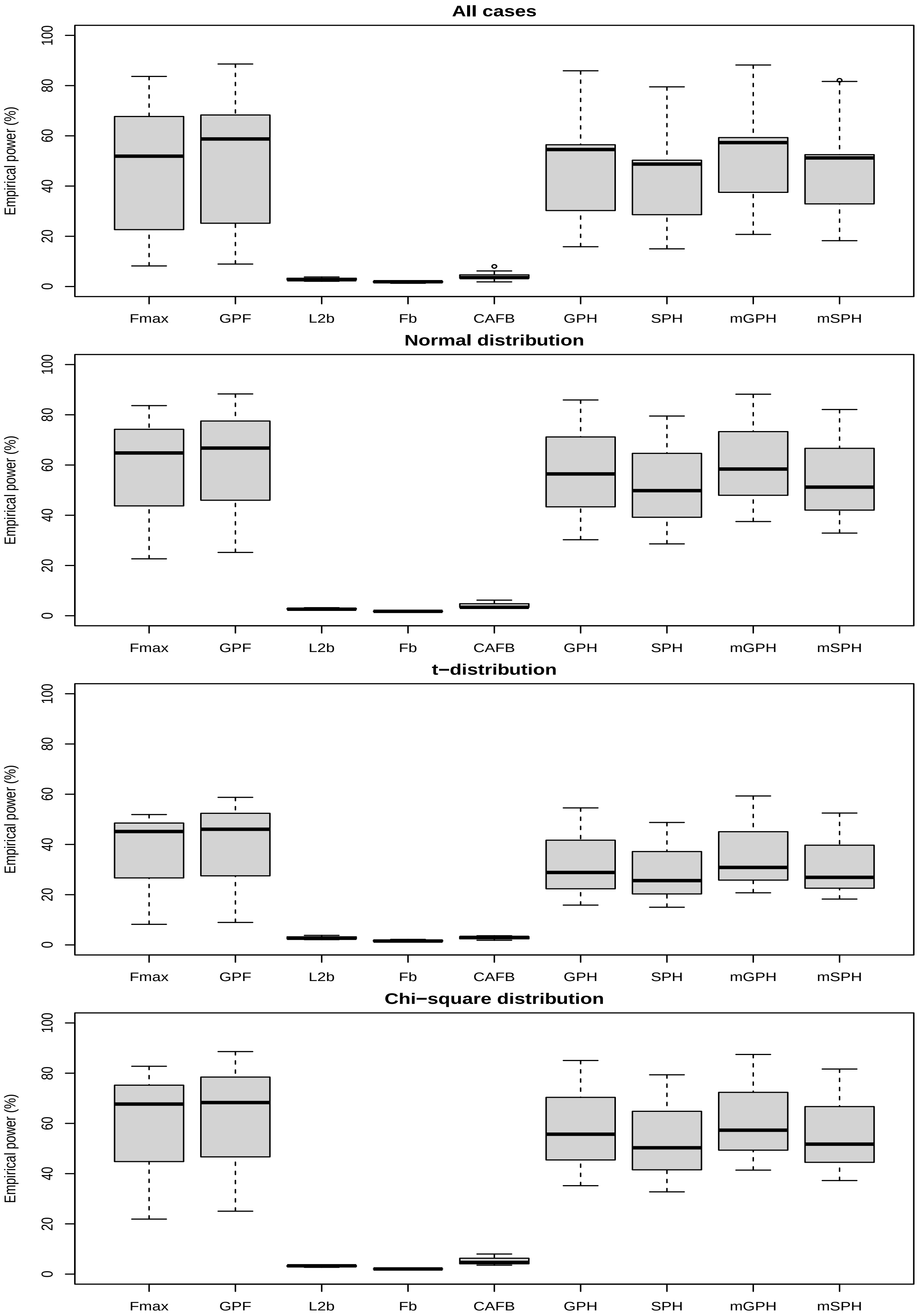


Fig. S43: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts with scaling function and different distributions

Table S20: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.50	1.20	2.60	1.45	1.15	1.05	1.90	1.50	2.20
			$\mathcal{H}_{0,2}$	1.55	1.15	2.60	1.60	1.60	1.05	1.40	1.45	1.75
			$\mathcal{H}_{0,3}$	81.10	98.30	81.35	73.65	85.75	98.00	79.35	98.55	82.40
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.80	1.00	2.85	1.90	1.90	1.40	2.00	1.50	2.10
			$\mathcal{H}_{0,2}$	0.85	0.20	2.55	1.85	1.60	1.15	1.75	1.30	1.90
			$\mathcal{H}_{0,3}$	23.10	57.35	53.00	47.60	57.95	83.25	50.05	84.70	51.50
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.90	1.55	3.10	1.85	1.45	1.15	1.45	1.75	2.00
			$\mathcal{H}_{0,2}$	4.20	3.40	3.20	2.25	2.20	1.05	1.70	1.75	2.55
			$\mathcal{H}_{0,3}$	69.95	86.15	39.65	31.15	36.90	58.10	30.35	64.60	35.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.45	1.15	2.65	1.70	1.05	1.20	1.25	1.45	1.55
			$\mathcal{H}_{0,2}$	1.35	1.05	2.45	1.75	1.55	0.90	1.50	1.20	1.60
			$\mathcal{H}_{0,3}$	52.30	85.05	54.50	46.85	58.25	82.10	49.45	84.45	53.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.00	1.05	2.55	1.80	1.35	1.55	1.00	1.55	1.05
			$\mathcal{H}_{0,2}$	0.55	0.25	3.05	2.30	1.35	1.15	1.50	1.25	1.55
			$\mathcal{H}_{0,3}$	9.75	26.75	33.70	30.00	34.30	54.65	27.25	56.75	28.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.15	1.15	2.90	2.00	1.40	0.80	1.60	1.20	2.20
			$\mathcal{H}_{0,2}$	3.50	2.75	2.45	1.60	1.30	0.75	1.50	1.35	2.00
			$\mathcal{H}_{0,3}$	50.40	63.30	25.35	17.75	20.55	30.50	17.05	36.90	20.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.60	0.90	2.25	1.25	1.05	0.85	1.65	1.15	1.95
			$\mathcal{H}_{0,2}$	1.55	1.15	2.45	1.40	1.30	1.10	1.60	1.50	2.10
			$\mathcal{H}_{0,3}$	81.35	98.70	79.60	73.05	84.45	96.80	78.95	97.75	81.60
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.55	0.90	2.25	1.40	1.45	1.05	1.85	1.20	1.95
			$\mathcal{H}_{0,2}$	0.70	0.45	2.90	1.95	1.30	1.15	1.85	1.20	1.95
			$\mathcal{H}_{0,3}$	20.30	59.30	53.20	46.30	56.55	84.25	49.35	84.90	51.10
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.55	1.65	2.20	1.35	0.85	1.20	1.55	1.65	1.80
			$\mathcal{H}_{0,2}$	3.80	3.10	2.85	1.85	2.25	0.85	1.90	1.50	2.15
			$\mathcal{H}_{0,3}$	71.75	85.15	44.10	34.65	44.50	59.35	36.00	65.85	42.00

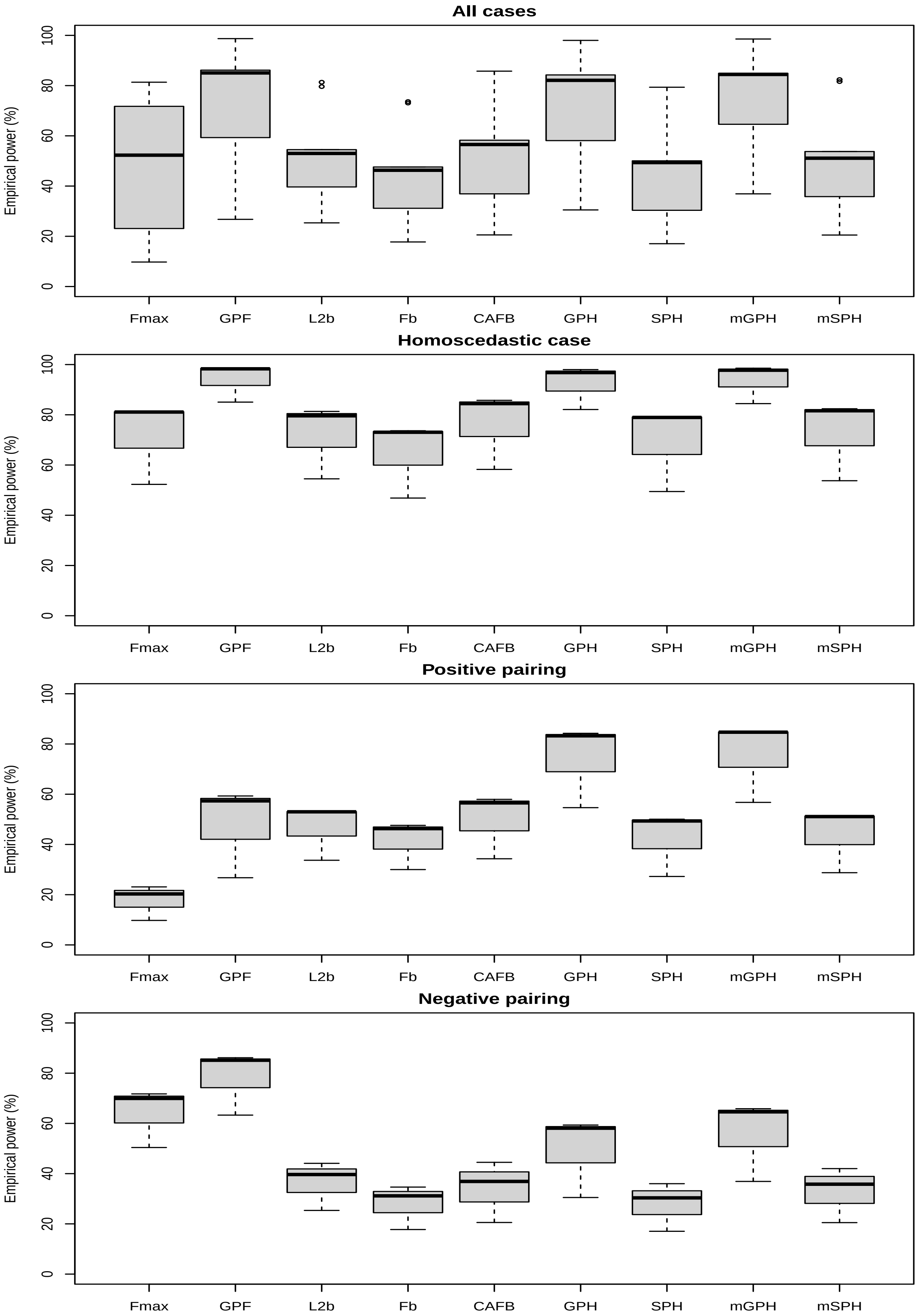


Fig. S44: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

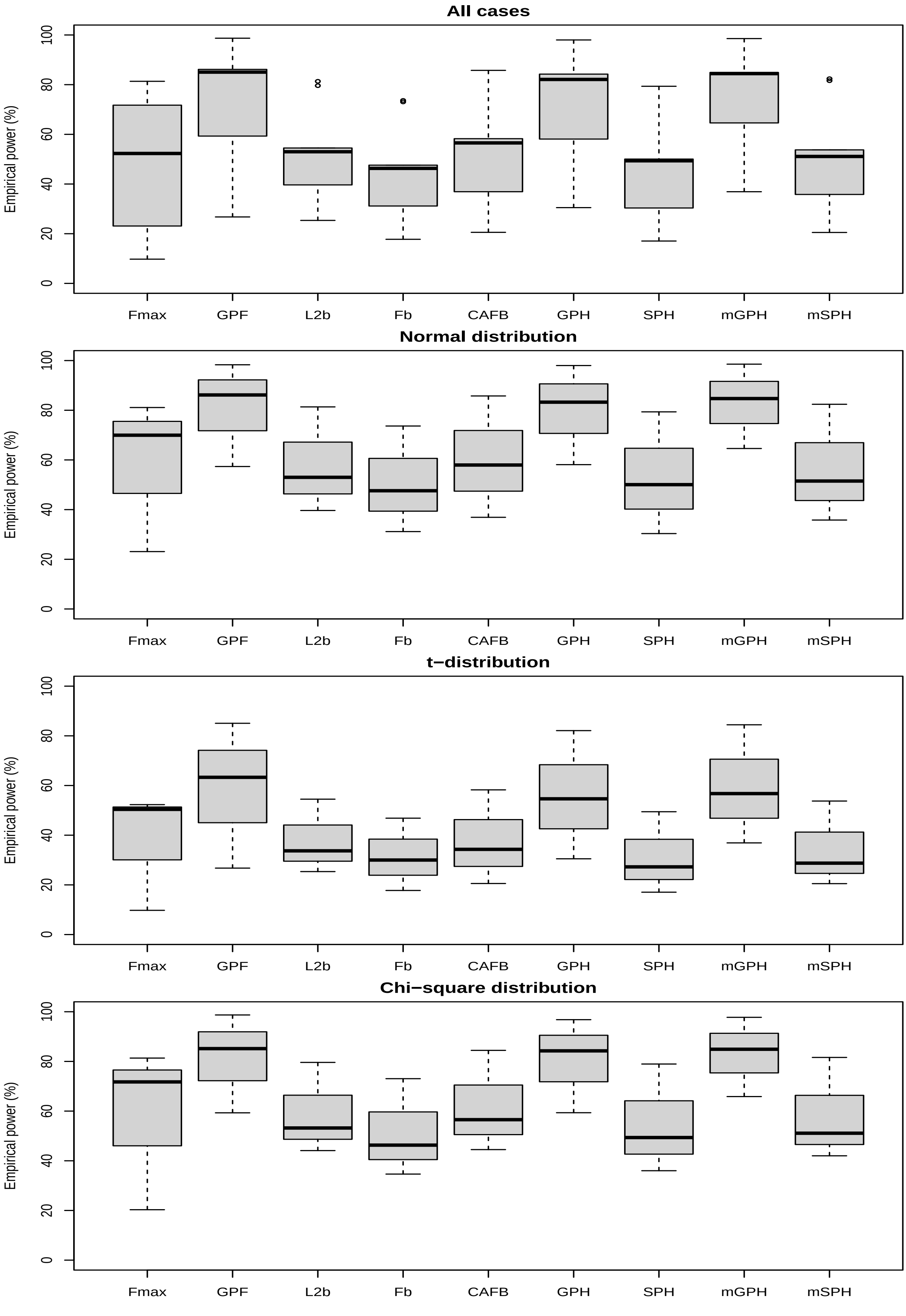


Fig. S45: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Dunnett contrasts with scaling function and different distributions

Table S21: Empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	16.45	29.00	20.45	15.25	18.45	27.75	15.50	30.80	17.70
			$\mathcal{H}_{0,2}$	75.35	98.30	76.65	69.20	82.15	98.15	74.00	98.50	77.60
			$\mathcal{H}_{0,3}$	99.75	100.00	99.40	98.75	99.85	100.00	99.65	100.00	99.80
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	9.10	17.00	16.00	12.25	11.80	20.50	11.15	21.95	12.05
			$\mathcal{H}_{0,2}$	35.10	75.00	58.30	50.90	62.30	86.80	52.65	88.25	54.45
			$\mathcal{H}_{0,3}$	71.90	98.95	91.90	89.00	96.15	99.95	93.95	99.95	94.55
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.65	8.30	7.65	5.25	5.15	6.60	5.00	8.80	6.40
			$\mathcal{H}_{0,2}$	45.10	70.30	35.75	28.50	33.75	52.70	27.30	59.15	32.10
			$\mathcal{H}_{0,3}$	96.40	99.65	78.05	67.95	80.70	97.00	71.90	97.95	78.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	10.10	14.50	12.75	9.60	9.25	14.20	9.10	15.90	10.40
			$\mathcal{H}_{0,2}$	47.65	81.85	53.25	45.15	54.95	79.35	45.90	82.60	49.65
			$\mathcal{H}_{0,3}$	93.35	99.70	90.90	87.25	94.85	99.60	91.70	99.60	93.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.15	7.80	9.10	6.65	6.20	8.85	5.70	10.00	6.15
			$\mathcal{H}_{0,2}$	17.65	42.10	38.30	32.05	38.10	57.75	27.95	59.75	29.75
			$\mathcal{H}_{0,3}$	38.90	81.30	69.75	65.35	78.40	94.85	69.10	95.05	70.45
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	4.85	5.40	5.45	3.55	3.85	4.15	4.15	5.25	5.35
			$\mathcal{H}_{0,2}$	28.35	46.40	21.90	15.60	18.35	28.40	16.00	34.30	19.20
			$\mathcal{H}_{0,3}$	82.60	94.75	54.30	42.75	53.65	77.40	45.00	82.95	50.95
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	15.20	28.10	20.25	15.80	17.60	28.20	15.40	31.15	17.60
			$\mathcal{H}_{0,2}$	76.30	97.40	74.75	67.75	81.65	96.15	74.65	96.90	77.70
			$\mathcal{H}_{0,3}$	99.80	100.00	98.70	97.25	99.25	100.00	99.35	100.00	99.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.00	15.25	14.70	10.75	11.20	19.90	10.30	20.85	11.30
			$\mathcal{H}_{0,2}$	34.55	76.80	57.65	51.45	61.40	86.90	52.75	88.30	54.40
			$\mathcal{H}_{0,3}$	71.35	98.95	90.95	87.85	95.15	99.75	93.75	99.85	94.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.95	8.60	9.15	6.85	7.30	7.65	5.50	9.75	7.00
			$\mathcal{H}_{0,2}$	46.45	71.70	37.70	30.85	38.00	53.45	29.50	60.40	33.90
			$\mathcal{H}_{0,3}$	96.45	99.20	77.20	66.75	81.30	95.00	74.30	96.55	79.90

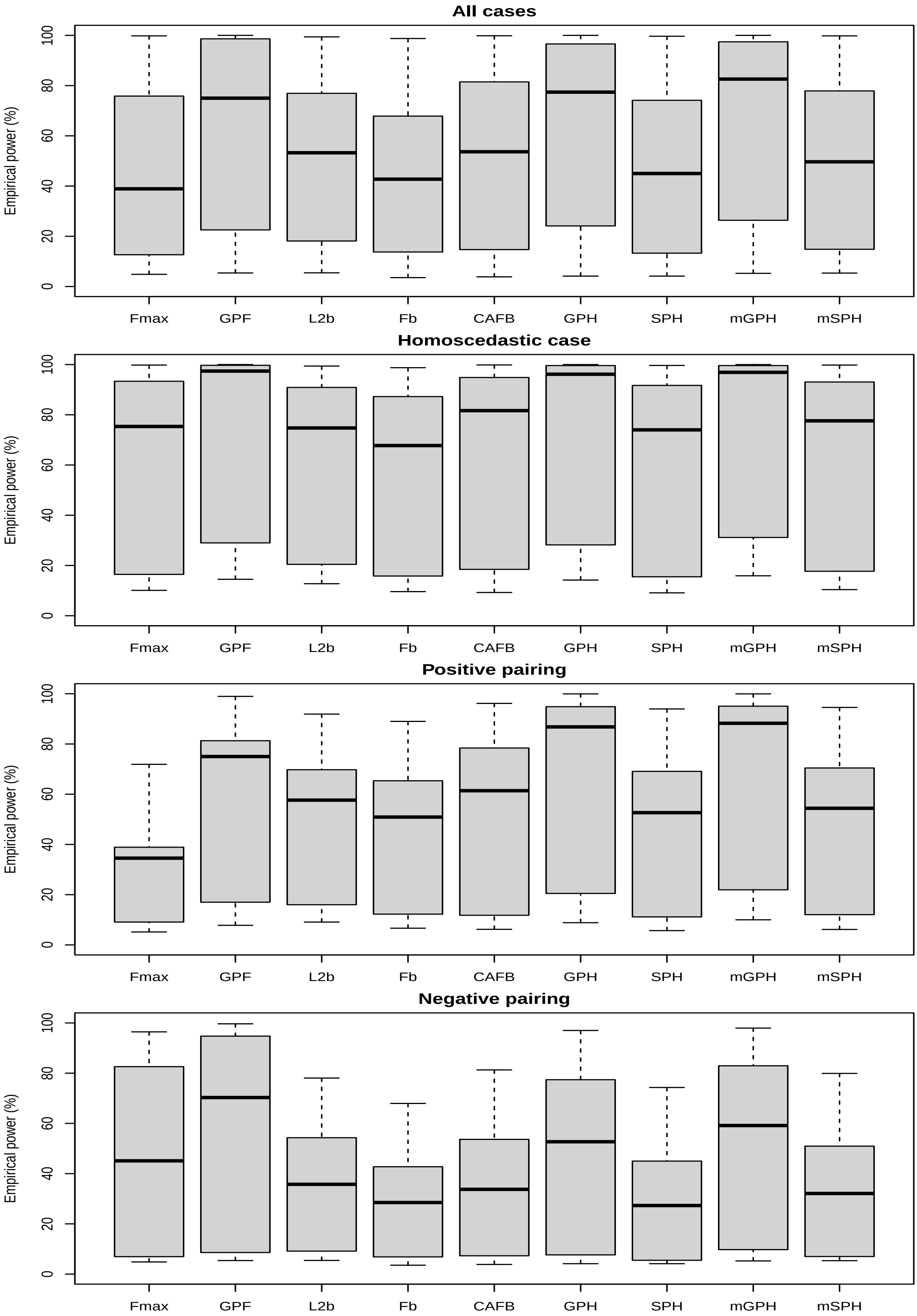


Fig. S46: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

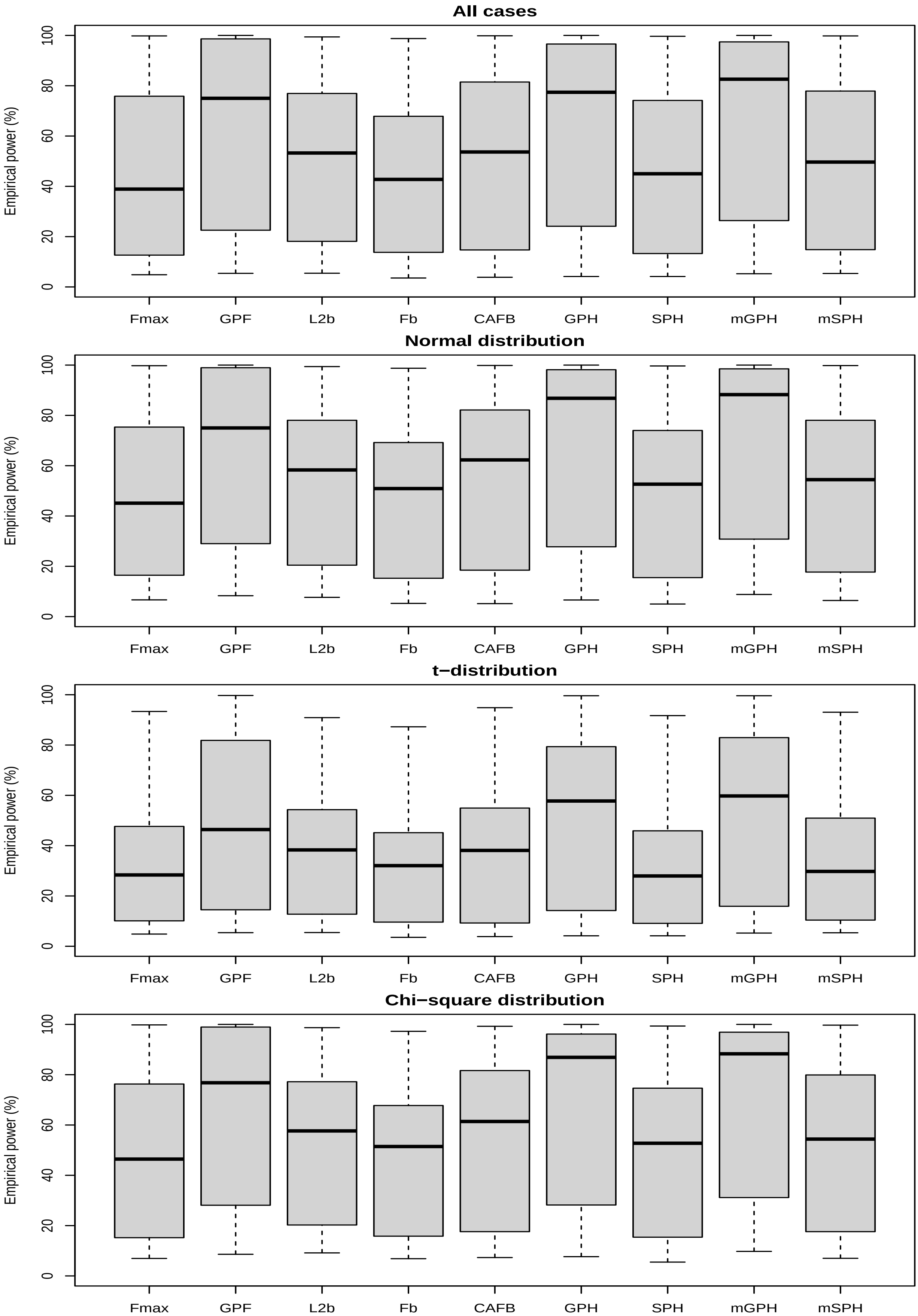


Fig. S47: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Dunnett contrasts with scaling function and different distributions

Table S22: Empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.25	13.05	2.00	1.10	1.20	12.85	12.05	15.65	13.35
			$\mathcal{H}_{0,2}$	77.70	85.05	2.70	1.70	7.20	83.70	75.95	85.85	78.35
			$\mathcal{H}_{0,3}$	99.85	99.95	3.15	2.00	18.65	99.95	99.80	99.95	99.90
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.65	8.40	3.05	1.95	1.30	10.45	9.80	11.15	10.40
			$\mathcal{H}_{0,2}$	36.25	43.75	2.70	1.90	3.05	60.95	51.20	63.10	52.70
			$\mathcal{H}_{0,3}$	77.40	84.00	3.05	2.35	10.00	97.45	94.00	97.75	94.85
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.80	5.80	2.85	2.05	1.85	4.50	5.10	5.95	6.35
			$\mathcal{H}_{0,2}$	43.90	47.50	3.50	2.10	3.25	28.90	26.20	35.30	31.30
			$\mathcal{H}_{0,3}$	95.80	97.00	3.20	2.00	6.50	80.30	72.85	85.35	77.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.20	5.80	2.20	1.80	1.90	5.40	6.50	6.70	7.70
			$\mathcal{H}_{0,2}$	47.10	54.80	2.95	2.00	4.10	51.45	45.35	55.55	48.80
			$\mathcal{H}_{0,3}$	94.05	96.40	3.10	1.80	9.45	94.50	92.45	96.05	93.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.70	4.75	2.75	1.85	1.90	5.90	5.55	6.70	5.80
			$\mathcal{H}_{0,2}$	16.60	19.00	2.50	1.80	2.55	32.35	26.65	34.05	28.15
			$\mathcal{H}_{0,3}$	39.85	47.55	2.50	1.65	5.75	76.80	68.95	79.05	71.35
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.65	3.10	2.30	1.30	1.30	1.90	2.70	3.05	3.40
			$\mathcal{H}_{0,2}$	24.45	27.00	2.75	1.80	2.40	13.15	12.80	17.50	15.55
			$\mathcal{H}_{0,3}$	81.05	82.85	3.15	2.00	3.70	48.50	43.45	56.05	48.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.65	13.60	2.30	1.30	2.00	13.90	12.55	16.00	14.15
			$\mathcal{H}_{0,2}$	76.65	86.25	2.90	2.00	6.00	82.00	74.60	85.60	77.25
			$\mathcal{H}_{0,3}$	99.90	100.00	3.85	2.40	18.85	99.95	99.45	99.95	99.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.05	6.70	2.60	1.55	1.70	9.00	9.35	9.95	9.95
			$\mathcal{H}_{0,2}$	38.85	43.05	3.20	1.80	3.80	61.50	53.40	63.85	55.20
			$\mathcal{H}_{0,3}$	76.60	85.65	3.30	2.35	9.90	96.80	93.70	97.20	94.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	6.05	5.00	2.80	1.80	1.90	4.00	4.60	5.80	6.30
			$\mathcal{H}_{0,2}$	44.30	47.35	2.60	1.55	3.15	30.00	29.20	36.65	33.40
			$\mathcal{H}_{0,3}$	95.15	96.30	3.30	2.30	6.65	79.40	72.60	84.20	77.00

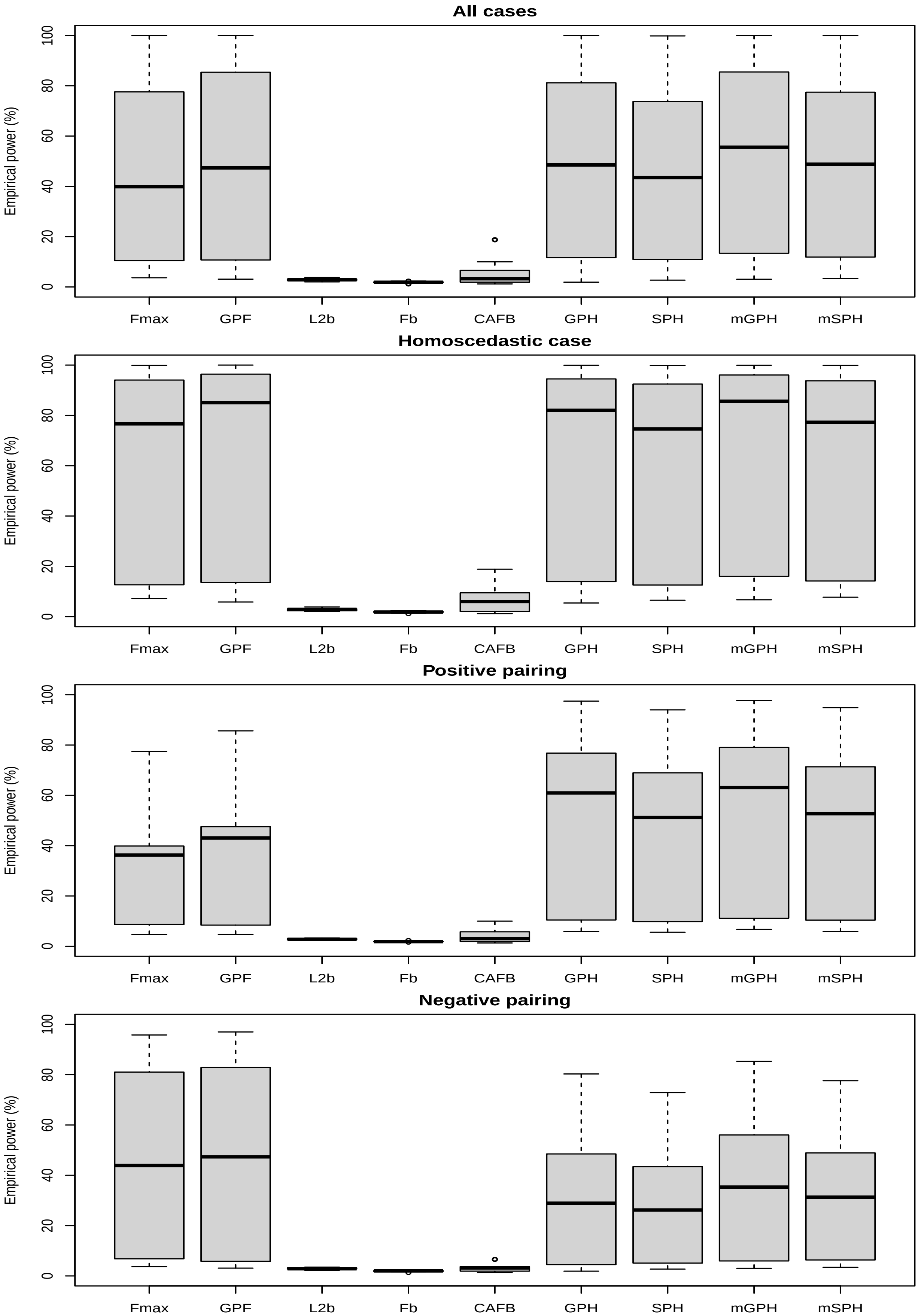


Fig. S48: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

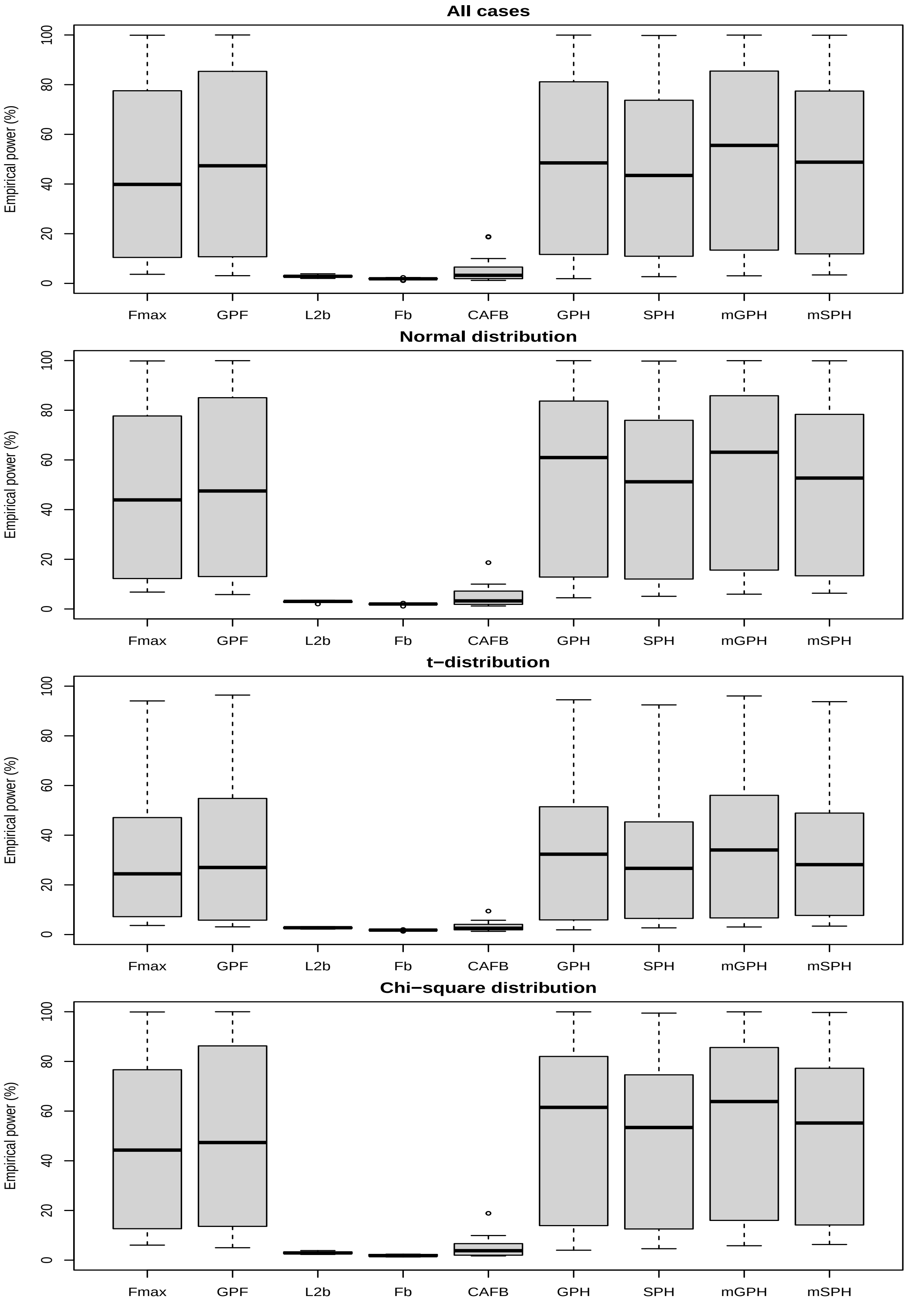


Fig. S49: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Dunnett contrasts with scaling function and different distributions

Table S23: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$) and powers ($\mathcal{H}_{0,3}$) (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.85	1.30	3.00	1.75	1.50	1.40	1.55	1.60	2.20
			$\mathcal{H}_{0,2}$	1.80	1.35	3.05	2.15	1.20	1.25	1.80	1.40	2.05
			$\mathcal{H}_{0,3}$	80.40	86.20	95.15	91.75	94.05	83.40	76.70	86.15	79.15
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.30	0.75	2.70	1.60	1.60	1.10	1.45	1.30	1.70
			$\mathcal{H}_{0,2}$	0.75	0.50	2.80	1.85	1.50	1.35	1.30	1.55	1.40
			$\mathcal{H}_{0,3}$	20.10	23.75	75.35	70.20	74.15	55.05	44.30	56.45	45.85
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.40	2.00	2.40	1.45	1.05	1.55	2.05	2.25	2.50
			$\mathcal{H}_{0,2}$	4.35	3.55	2.75	1.70	0.90	1.30	2.30	1.60	2.75
			$\mathcal{H}_{0,3}$	62.65	68.25	61.25	49.40	49.55	29.80	27.10	36.70	31.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	2.05	0.95	2.45	1.60	1.45	0.80	1.90	1.35	2.20
			$\mathcal{H}_{0,2}$	1.90	1.55	2.50	1.55	1.75	1.60	2.20	1.70	2.60
			$\mathcal{H}_{0,3}$	48.70	54.45	79.40	71.90	73.70	51.40	47.85	56.05	50.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.00	0.70	2.35	1.70	1.30	0.80	1.15	0.90	1.25
			$\mathcal{H}_{0,2}$	0.50	0.25	2.35	1.90	1.45	1.10	1.50	1.30	1.75
			$\mathcal{H}_{0,3}$	9.40	8.30	52.95	47.30	47.85	28.10	25.80	29.60	27.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.20	1.25	2.35	1.15	1.35	0.90	1.70	1.15	2.05
			$\mathcal{H}_{0,2}$	3.75	2.85	2.55	1.15	1.25	0.65	1.25	1.05	2.25
			$\mathcal{H}_{0,3}$	42.75	42.85	39.35	29.55	28.60	13.00	13.25	16.85	16.20
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	2.10	1.45	2.45	1.70	1.35	1.20	2.00	1.65	2.25
			$\mathcal{H}_{0,2}$	1.95	1.35	2.45	1.75	1.80	1.20	2.00	1.60	2.40
			$\mathcal{H}_{0,3}$	79.70	87.35	93.40	90.20	93.55	82.55	76.75	85.55	79.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.35	1.00	2.70	1.85	1.45	1.25	1.65	1.40	1.70
			$\mathcal{H}_{0,2}$	0.65	0.35	3.20	2.30	1.30	1.25	1.40	1.40	1.50
			$\mathcal{H}_{0,3}$	19.00	22.65	78.05	73.30	75.50	55.60	46.15	57.25	47.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.15	1.65	2.80	1.75	1.30	1.30	1.60	1.75	2.15
			$\mathcal{H}_{0,2}$	4.40	3.20	2.90	1.80	1.90	0.95	1.75	1.75	2.45
			$\mathcal{H}_{0,3}$	65.20	66.00	58.90	48.95	54.95	33.70	31.80	38.40	35.95

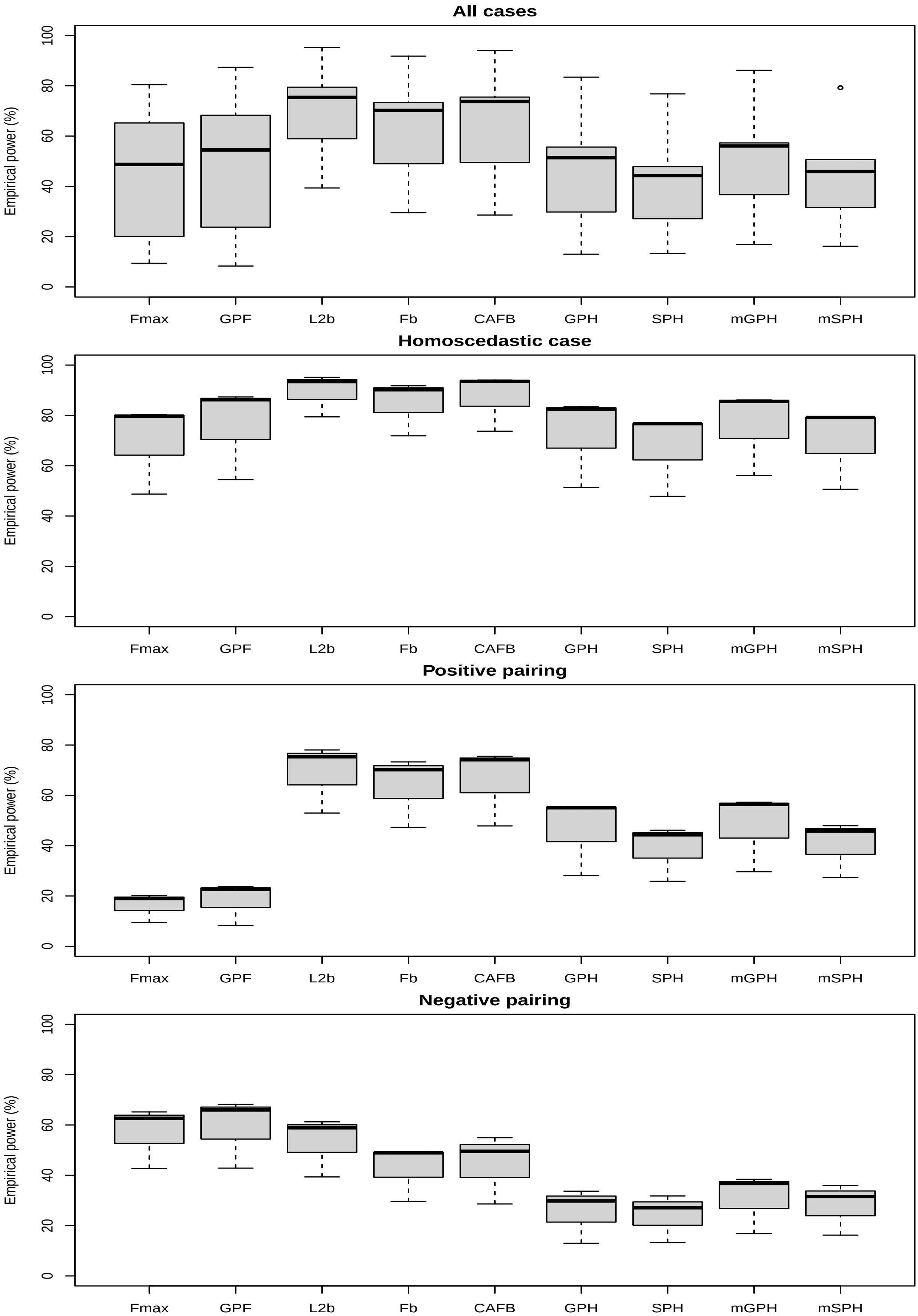


Fig. S50: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

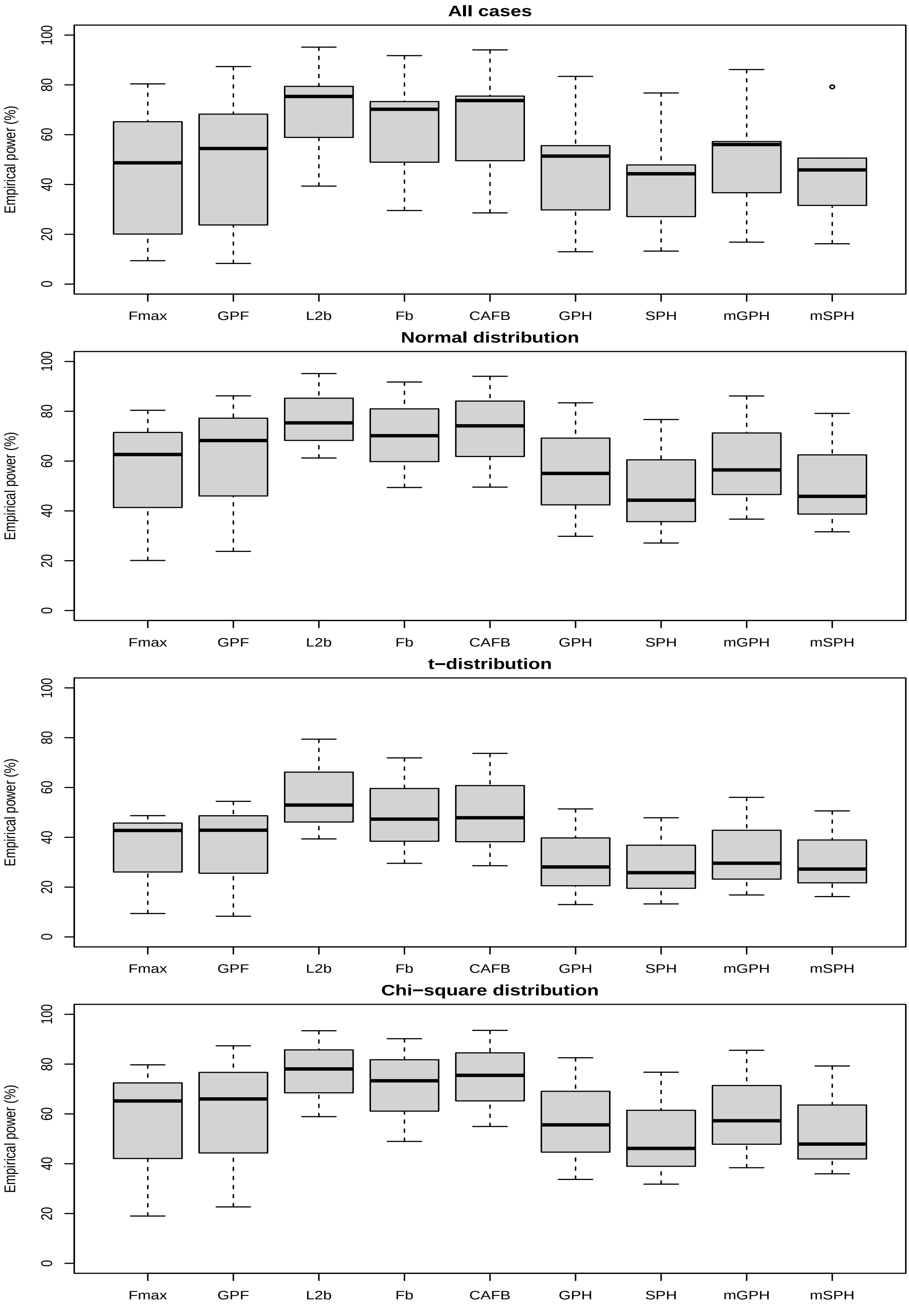


Fig. S51: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Dunnett contrasts with scaling function and different distributions

Table S24: Empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	11.90	13.10	30.10	24.40	22.20	13.15	11.60	15.25	13.55
			$\mathcal{H}_{0,2}$	72.70	81.70	93.25	89.00	90.70	80.10	70.15	83.45	72.95
			$\mathcal{H}_{0,3}$	99.65	99.90	99.95	99.95	99.95	99.95	99.45	99.95	99.55
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	8.95	9.30	25.55	20.75	19.80	10.90	9.95	12.30	10.40
			$\mathcal{H}_{0,2}$	34.00	42.00	81.85	76.65	79.20	60.05	51.20	61.70	52.90
			$\mathcal{H}_{0,3}$	74.15	82.55	98.65	98.10	98.95	96.80	92.75	97.20	93.65
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	5.40	4.50	12.20	8.45	7.25	3.10	4.30	4.70	5.40
			$\mathcal{H}_{0,2}$	38.50	42.95	54.80	45.05	45.20	24.85	22.85	31.25	26.90
			$\mathcal{H}_{0,3}$	94.90	96.85	93.35	87.85	91.15	77.10	67.00	82.50	72.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.20	7.20	19.55	14.90	13.95	6.10	7.70	7.60	8.40
			$\mathcal{H}_{0,2}$	45.45	52.30	75.30	68.60	71.65	48.70	42.40	54.20	46.15
			$\mathcal{H}_{0,3}$	93.10	96.85	98.70	97.50	98.80	95.65	91.55	96.45	93.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.90	3.85	16.30	11.60	9.95	4.80	5.65	5.45	6.05
			$\mathcal{H}_{0,2}$	16.45	18.25	55.20	48.65	50.55	30.90	27.50	32.60	28.80
			$\mathcal{H}_{0,3}$	39.95	45.40	88.85	86.10	90.15	75.35	68.05	78.10	69.35
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.40	3.20	7.35	5.05	3.75	1.80	2.40	2.95	3.70
			$\mathcal{H}_{0,2}$	23.95	24.25	35.10	26.60	26.10	11.45	12.60	14.40	15.75
			$\mathcal{H}_{0,3}$	77.75	80.60	75.55	65.75	68.10	47.00	40.70	55.10	45.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	12.75	12.95	34.10	27.75	25.95	13.00	12.35	14.85	14.10
			$\mathcal{H}_{0,2}$	77.95	84.25	93.15	88.95	92.80	81.25	76.35	84.10	78.65
			$\mathcal{H}_{0,3}$	99.75	100.00	99.90	99.85	99.95	99.80	99.00	99.90	99.10
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	6.50	6.00	26.05	19.75	16.85	8.55	8.75	9.35	9.50
			$\mathcal{H}_{0,2}$	34.55	38.75	81.25	77.20	78.45	56.15	50.90	58.60	52.35
			$\mathcal{H}_{0,3}$	73.60	83.05	99.05	98.25	99.25	95.25	93.10	96.25	93.45
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	7.10	5.80	12.80	9.10	7.60	4.85	5.60	6.60	7.10
			$\mathcal{H}_{0,2}$	43.70	44.70	56.80	47.45	49.90	29.65	28.45	35.65	33.00
			$\mathcal{H}_{0,3}$	95.35	95.50	91.80	84.90	90.20	76.15	70.60	82.10	76.05

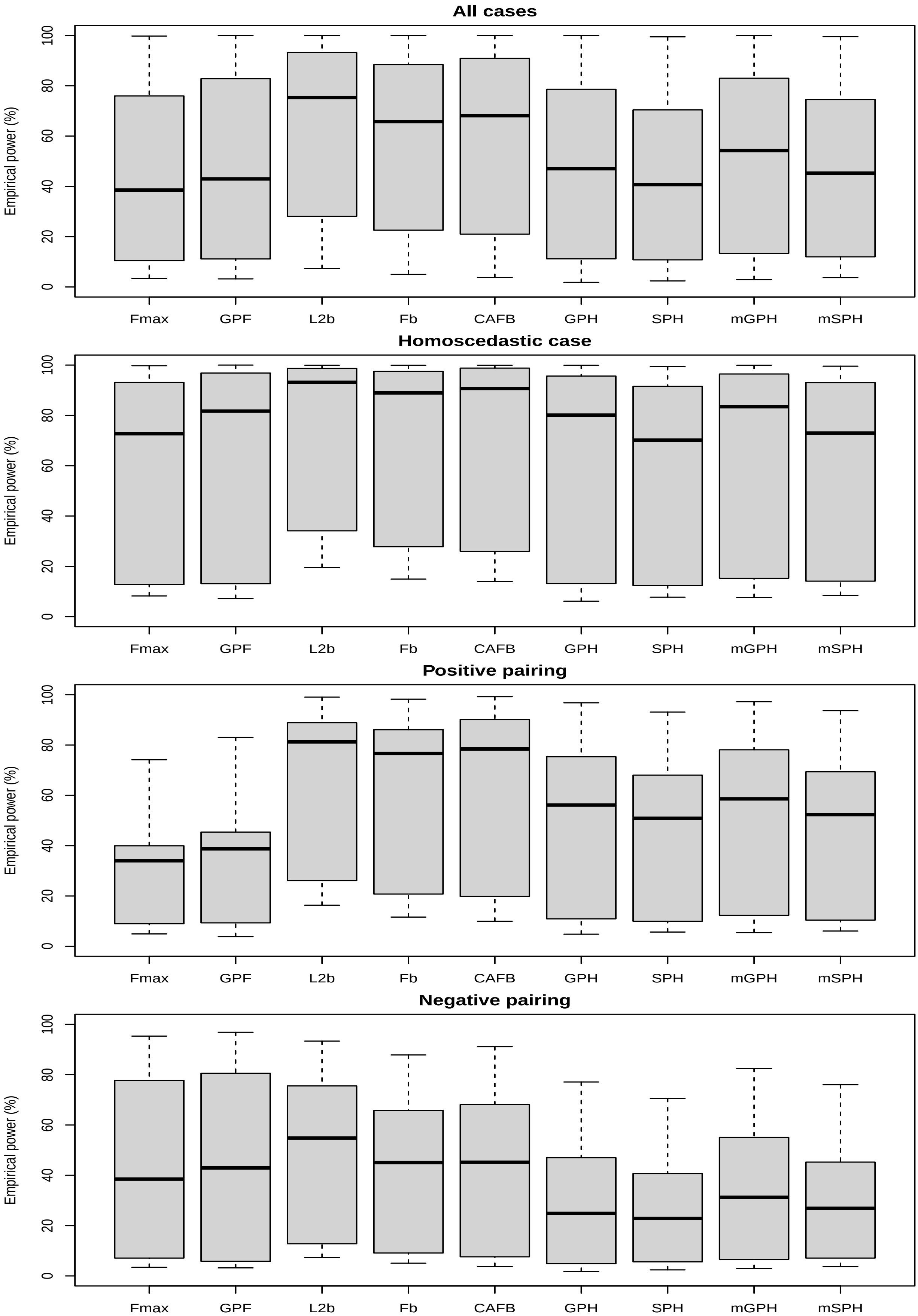


Fig. S52: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts with scaling function and homoscedastic and heteroscedastic cases

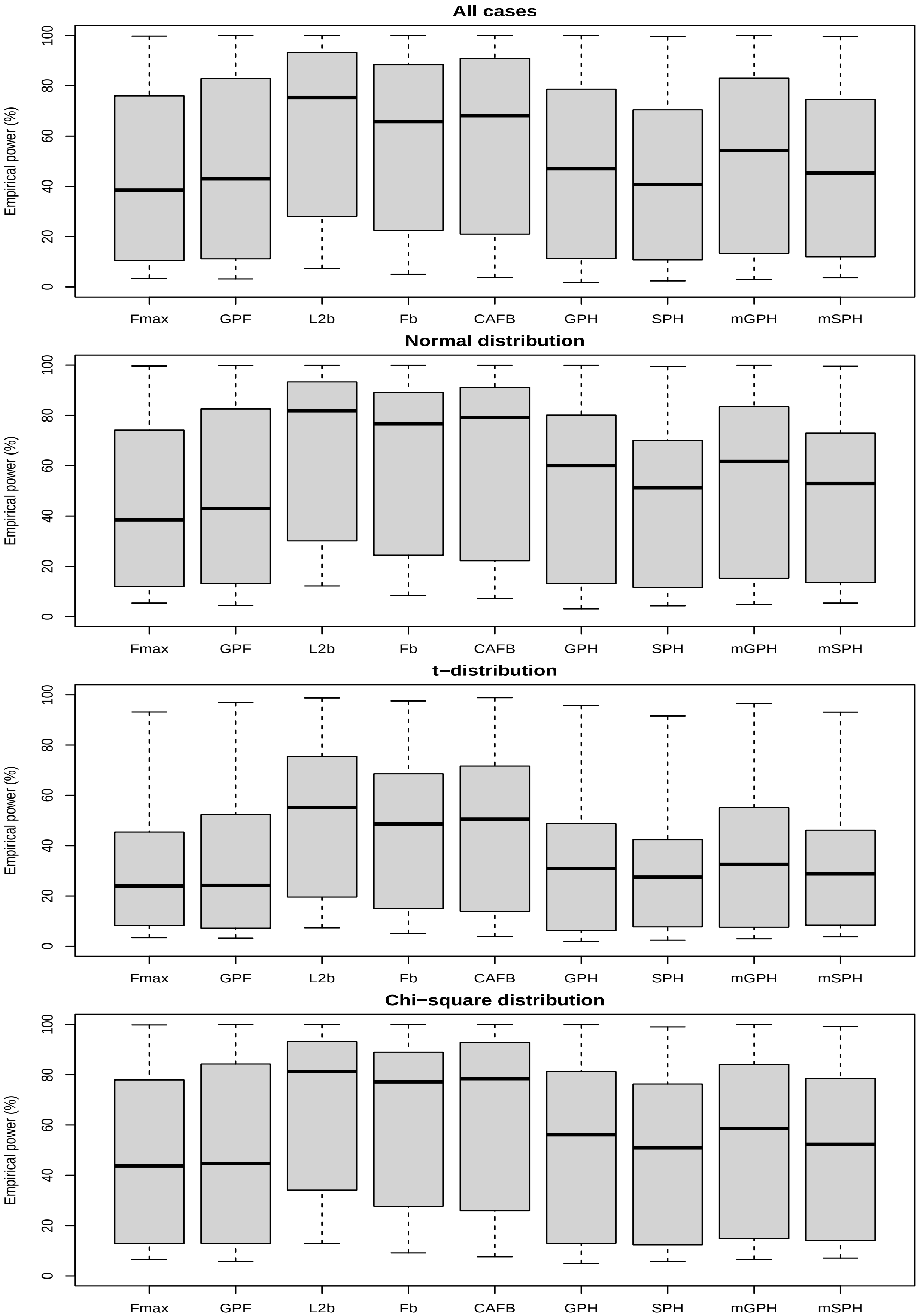


Fig. S53: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Dunnett contrasts with scaling function and different distributions

Table S25: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	4.50	4.55	6.40	4.65	3.85	3.60	4.20	4.55	4.70
			$\mathcal{H}_{0,1}$	1.10	0.95	1.55	0.85	0.65	0.70	0.95	0.90	0.95
			$\mathcal{H}_{0,2}$	0.50	0.75	1.05	0.70	0.55	0.65	0.45	0.65	0.45
			$\mathcal{H}_{0,3}$	0.65	0.65	2.00	1.00	1.05	0.50	0.80	0.60	0.80
			$\mathcal{H}_{0,4}$	0.95	0.65	1.25	0.70	0.35	0.60	0.75	0.70	0.95
			$\mathcal{H}_{0,5}$	0.75	0.85	1.35	0.95	0.60	0.95	0.95	1.10	1.00
			$\mathcal{H}_{0,6}$	1.15	0.80	1.40	1.05	0.75	0.70	1.20	0.95	1.40
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	3.90	1.95	6.20	4.90	4.05	3.45	4.35	3.90	4.95
			$\mathcal{H}_{0,1}$	0.65	0.45	1.45	0.85	0.65	0.70	1.05	0.90	1.10
			$\mathcal{H}_{0,2}$	0.10	0.10	1.50	0.85	0.80	0.35	0.45	0.40	0.65
			$\mathcal{H}_{0,3}$	0.05	0.00	1.25	0.75	0.70	0.50	0.80	0.70	0.85
			$\mathcal{H}_{0,4}$	0.75	0.55	1.35	0.80	0.60	0.85	0.90	1.00	1.05
			$\mathcal{H}_{0,5}$	0.65	0.45	1.20	1.00	0.85	0.90	1.10	1.10	1.25
			$\mathcal{H}_{0,6}$	0.50	0.50	1.40	0.95	0.65	0.55	0.55	0.65	0.70
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	11.05	8.75	6.70	4.35	4.00	2.70	4.50	3.65	5.25
			$\mathcal{H}_{0,1}$	1.15	0.70	1.70	1.30	0.90	0.60	0.70	0.85	0.85
			$\mathcal{H}_{0,2}$	2.30	1.95	1.90	0.95	0.55	0.50	1.15	0.75	1.25
			$\mathcal{H}_{0,3}$	6.10	4.35	1.90	0.85	0.90	0.35	1.05	0.45	1.30
			$\mathcal{H}_{0,4}$	1.05	1.05	1.00	0.70	0.60	0.60	0.90	0.80	1.15
			$\mathcal{H}_{0,5}$	1.95	1.55	1.15	0.80	0.75	0.70	0.60	0.80	0.75
			$\mathcal{H}_{0,6}$	1.10	1.05	1.25	0.95	0.45	0.80	0.90	1.10	1.10
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.95	4.50	5.30	4.15	3.20	3.35	4.45	4.80	5.05
			$\mathcal{H}_{0,1}$	1.00	0.90	1.40	1.05	0.80	0.80	1.05	1.05	1.10
			$\mathcal{H}_{0,2}$	0.40	0.50	1.25	0.90	0.55	0.25	0.25	0.45	0.35
			$\mathcal{H}_{0,3}$	0.65	0.70	0.90	0.70	0.50	0.45	0.65	0.65	0.80
			$\mathcal{H}_{0,4}$	1.25	0.90	0.85	0.70	0.50	0.90	0.95	1.15	1.05
			$\mathcal{H}_{0,5}$	0.95	0.85	0.90	0.80	0.40	0.75	0.80	0.80	1.00
			$\mathcal{H}_{0,6}$	1.30	1.10	0.75	0.65	0.45	1.30	1.25	1.60	1.35
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.85	1.95	5.05	4.45	3.35	3.85	5.00	4.40	5.50
			$\mathcal{H}_{0,1}$	0.65	0.45	1.35	1.15	0.80	0.70	1.00	0.85	1.10
			$\mathcal{H}_{0,2}$	0.10	0.00	1.15	0.85	0.75	0.40	0.60	0.40	0.70
			$\mathcal{H}_{0,3}$	0.10	0.05	1.30	1.10	0.80	1.00	1.25	1.15	1.40
			$\mathcal{H}_{0,4}$	0.65	0.60	0.90	0.70	0.40	0.65	0.90	0.85	0.95
			$\mathcal{H}_{0,5}$	0.55	0.50	0.85	0.85	0.35	0.90	1.00	1.00	1.05
			$\mathcal{H}_{0,6}$	0.40	0.55	0.70	0.65	0.45	0.80	0.75	0.90	0.75
N	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.40	9.45	5.95	5.10	3.60	2.90	4.10	3.20	4.95
			$\mathcal{H}_{0,1}$	1.20	0.60	1.40	1.10	0.55	0.50	0.95	0.60	1.05
			$\mathcal{H}_{0,2}$	2.45	1.95	1.45	1.15	0.80	0.75	0.50	0.80	0.65
			$\mathcal{H}_{0,3}$	6.15	5.30	1.60	1.30	0.70	0.60	0.85	0.75	1.10
			$\mathcal{H}_{0,4}$	1.05	1.00	1.15	0.85	0.65	0.85	0.90	0.95	1.15
			$\mathcal{H}_{0,5}$	2.35	2.00	1.25	1.05	0.55	0.60	0.75	0.75	0.95
			$\mathcal{H}_{0,6}$	0.95	0.90	1.30	1.00	0.60	0.75	0.45	0.85	0.70
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.65	4.60	4.60	3.90	3.30	3.95	5.35	5.00	5.70
			$\mathcal{H}_{0,1}$	1.05	0.55	0.90	0.95	0.75	0.55	1.00	0.75	1.15
			$\mathcal{H}_{0,2}$	1.30	0.75	0.70	0.55	0.40	0.65	1.05	0.80	1.15
			$\mathcal{H}_{0,3}$	0.60	0.65	0.80	0.70	0.45	0.55	0.60	0.65	0.65
			$\mathcal{H}_{0,4}$	1.25	1.05	0.90	0.90	0.60	1.00	1.20	1.10	1.45
			$\mathcal{H}_{0,5}$	1.05	1.40	1.20	1.10	0.75	1.30	1.00	1.55	1.10
			$\mathcal{H}_{0,6}$	1.00	1.10	0.75	0.70	0.45	1.15	1.05	1.40	1.10
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	3.85	2.60	4.95	4.80	3.45	4.80	5.15	5.15	5.50
			$\mathcal{H}_{0,1}$	0.55	0.70	0.95	0.90	0.55	1.05	1.25	1.25	1.35
			$\mathcal{H}_{0,2}$	0.30	0.25	1.00	1.00	0.75	0.75	0.95	0.95	1.10
			$\mathcal{H}_{0,3}$	0.10	0.00	1.20	1.15	0.50	1.10	0.70	1.15	0.80
			$\mathcal{H}_{0,4}$	0.75	0.85	1.15	0.85	0.65	1.00	1.30	1.10	1.30
			$\mathcal{H}_{0,5}$	0.20	0.20	1.20	1.15	0.80	0.85	0.85	0.95	0.85
			$\mathcal{H}_{0,6}$	0.80	0.40	0.90	0.85	0.85	0.60	0.60	0.65	0.75

Table S25: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.05	11.20	5.55	5.20	4.00	4.15	4.80	4.75	5.90
			$\mathcal{H}_{0,1}$	1.35	1.45	1.00	0.85	0.75	1.20	0.95	1.30	1.10
			$\mathcal{H}_{0,2}$	3.15	3.20	0.95	0.80	0.60	1.10	1.40	1.40	1.80
			$\mathcal{H}_{0,3}$	6.15	5.60	1.50	1.35	0.75	0.50	0.75	0.70	1.00
			$\mathcal{H}_{0,4}$	1.30	1.30	1.05	0.90	0.60	0.90	0.95	1.15	1.00
			$\mathcal{H}_{0,5}$	2.05	2.40	1.25	1.10	0.55	0.95	0.80	1.05	1.05
			$\mathcal{H}_{0,6}$	0.80	1.10	1.25	1.20	0.65	0.75	0.65	0.95	0.75
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.20	3.80	6.95	4.85	4.00	2.75	4.65	3.80	5.35
			$\mathcal{H}_{0,1}$	0.95	0.50	1.80	1.40	0.75	0.50	0.80	0.55	0.95
			$\mathcal{H}_{0,2}$	1.00	0.65	2.00	1.00	0.90	0.40	0.95	0.55	1.05
			$\mathcal{H}_{0,3}$	1.10	0.55	1.45	0.75	0.90	0.60	1.15	0.65	1.30
			$\mathcal{H}_{0,4}$	0.90	0.70	1.25	0.80	0.55	0.60	0.80	0.65	0.90
			$\mathcal{H}_{0,5}$	0.90	0.70	1.40	0.75	0.35	0.50	0.80	0.85	0.95
			$\mathcal{H}_{0,6}$	0.90	0.80	1.10	0.90	0.40	0.90	0.85	1.00	1.00
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	3.80	1.70	6.30	4.50	3.10	2.60	4.15	3.20	4.90
			$\mathcal{H}_{0,1}$	0.80	0.45	1.90	1.15	0.65	0.60	0.75	0.85	0.80
			$\mathcal{H}_{0,2}$	0.30	0.20	1.55	0.85	0.75	0.70	1.05	0.75	1.20
			$\mathcal{H}_{0,3}$	0.10	0.00	1.15	0.80	0.30	0.15	0.50	0.25	0.65
			$\mathcal{H}_{0,4}$	0.70	0.50	1.20	0.90	0.55	0.50	0.75	0.60	1.00
			$\mathcal{H}_{0,5}$	0.45	0.15	1.05	0.60	0.30	0.40	0.80	0.45	0.85
			$\mathcal{H}_{0,6}$	0.50	0.55	1.15	0.80	0.45	0.75	0.65	0.85	0.70
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	12.95	8.90	7.25	5.70	5.35	2.90	4.75	4.25	5.55
			$\mathcal{H}_{0,1}$	1.30	0.90	1.90	1.10	0.95	0.45	1.05	0.60	1.20
			$\mathcal{H}_{0,2}$	2.85	1.75	1.65	1.00	1.20	0.55	0.65	0.80	1.15
			$\mathcal{H}_{0,3}$	6.95	4.20	1.55	0.95	1.10	0.25	1.10	0.50	1.40
			$\mathcal{H}_{0,4}$	1.75	1.20	1.70	1.15	1.00	0.95	1.10	1.05	1.30
			$\mathcal{H}_{0,5}$	2.00	1.40	1.60	0.90	0.60	0.45	1.05	0.55	1.05
			$\mathcal{H}_{0,6}$	1.55	1.05	1.80	1.35	0.65	0.80	1.25	0.90	1.40
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	5.30	4.45	6.45	5.85	4.05	3.45	4.50	4.35	5.05
			$\mathcal{H}_{0,1}$	1.15	1.15	1.50	1.25	0.90	0.80	1.00	0.90	1.20
			$\mathcal{H}_{0,2}$	1.10	0.65	1.50	1.20	0.70	0.80	0.70	1.00	0.70
			$\mathcal{H}_{0,3}$	1.10	1.10	1.20	1.05	0.65	0.85	0.75	1.05	0.75
			$\mathcal{H}_{0,4}$	1.05	0.70	1.75	1.25	0.90	0.75	1.05	0.95	1.15
			$\mathcal{H}_{0,5}$	0.65	0.75	1.30	1.05	0.60	0.50	0.65	0.65	0.80
			$\mathcal{H}_{0,6}$	1.10	0.60	1.10	0.85	0.45	0.70	0.75	0.80	1.10
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.15	2.45	5.00	3.85	2.75	3.60	3.90	4.65	4.55
			$\mathcal{H}_{0,1}$	0.35	0.70	1.35	1.00	0.45	0.70	0.90	0.85	0.95
			$\mathcal{H}_{0,2}$	0.30	0.25	0.95	0.85	0.55	0.60	0.80	0.65	0.85
			$\mathcal{H}_{0,3}$	0.10	0.10	1.15	0.90	0.55	0.55	0.85	0.75	0.90
			$\mathcal{H}_{0,4}$	0.65	0.80	0.95	0.70	0.55	0.90	0.55	1.10	0.75
			$\mathcal{H}_{0,5}$	0.45	0.25	0.90	0.65	0.60	0.70	0.70	0.90	0.90
			$\mathcal{H}_{0,6}$	0.60	0.65	0.65	0.65	0.45	0.65	0.80	0.80	1.00
t	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	10.90	9.35	5.50	4.75	3.75	3.60	4.20	4.40	4.75
			$\mathcal{H}_{0,1}$	0.90	0.90	1.25	0.85	0.85	0.70	0.50	0.85	0.55
			$\mathcal{H}_{0,2}$	2.20	1.95	1.30	0.70	0.60	0.45	0.80	0.80	0.80
			$\mathcal{H}_{0,3}$	5.40	4.90	1.15	1.00	0.35	0.35	0.65	0.80	0.80
			$\mathcal{H}_{0,4}$	1.00	1.10	1.30	0.95	0.70	0.70	0.70	0.90	0.85
			$\mathcal{H}_{0,5}$	2.80	2.45	1.00	0.85	0.55	1.00	1.10	1.10	1.25
			$\mathcal{H}_{0,6}$	0.95	0.95	1.45	1.15	0.85	0.85	0.90	1.00	1.10
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.90	3.45	4.45	4.35	3.50	3.80	4.30	4.40	4.75
			$\mathcal{H}_{0,1}$	0.95	0.85	0.90	0.85	0.65	1.05	0.90	1.15	1.05
			$\mathcal{H}_{0,2}$	0.90	0.60	1.20	1.05	0.70	0.75	1.00	0.80	1.00
			$\mathcal{H}_{0,3}$	0.85	0.40	0.95	0.80	0.40	0.35	0.70	0.45	0.70
			$\mathcal{H}_{0,4}$	1.00	0.75	1.00	0.85	0.75	0.95	0.95	1.20	1.10
			$\mathcal{H}_{0,5}$	0.90	0.95	0.95	0.85	0.75	0.80	0.80	0.95	0.90
			$\mathcal{H}_{0,6}$	0.25	0.50	0.90	0.75	0.40	0.25	0.35	0.35	0.50

Table S25: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	4.00	2.10	4.45	3.70	3.05	3.45	4.10	4.15	4.60
			$\mathcal{H}_{0,1}$	0.55	0.40	0.95	0.85	0.90	0.55	0.80	0.70	0.95
			$\mathcal{H}_{0,2}$	0.35	0.40	0.85	0.75	0.50	0.95	0.90	1.10	0.95
			$\mathcal{H}_{0,3}$	0.05	0.00	0.75	0.65	0.30	0.60	0.90	0.65	0.95
			$\mathcal{H}_{0,4}$	0.60	0.40	0.90	0.85	0.60	0.65	0.60	0.70	0.75
			$\mathcal{H}_{0,5}$	0.35	0.30	0.90	0.75	0.40	0.65	0.70	0.75	0.75
			$\mathcal{H}_{0,6}$	0.70	0.60	0.60	0.60	0.40	0.65	0.85	0.75	0.90
t	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	12.25	10.30	4.35	3.85	2.60	3.10	5.05	4.45	5.70
			$\mathcal{H}_{0,1}$	1.05	1.05	0.45	0.45	0.35	0.65	0.85	0.80	0.90
			$\mathcal{H}_{0,2}$	3.05	2.35	1.00	0.80	0.40	0.70	1.10	0.90	1.40
			$\mathcal{H}_{0,3}$	6.35	4.90	1.05	0.85	0.55	0.55	0.90	0.80	1.00
			$\mathcal{H}_{0,4}$	1.15	0.85	0.75	0.55	0.40	0.60	0.95	0.75	1.05
			$\mathcal{H}_{0,5}$	2.40	2.10	0.95	0.90	0.35	0.75	0.85	1.00	0.95
			$\mathcal{H}_{0,6}$	1.35	1.25	1.15	0.85	0.35	1.00	1.05	1.35	1.20
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	FWER	5.20	4.25	6.35	4.70	4.00	3.35	4.90	4.30	5.65
			$\mathcal{H}_{0,1}$	1.25	0.75	1.35	0.70	0.70	0.65	1.10	0.70	1.30
			$\mathcal{H}_{0,2}$	1.05	0.85	1.45	0.80	0.55	0.55	0.80	0.85	1.15
			$\mathcal{H}_{0,3}$	0.85	0.95	1.60	0.90	0.80	0.70	0.65	0.95	0.70
			$\mathcal{H}_{0,4}$	0.90	0.60	0.90	0.75	0.20	0.50	0.85	0.55	1.00
			$\mathcal{H}_{0,5}$	1.15	0.85	1.55	1.05	1.00	0.75	1.30	1.00	1.35
			$\mathcal{H}_{0,6}$	0.85	0.90	1.50	1.10	0.75	0.75	0.85	1.00	1.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	FWER	4.55	1.95	6.25	4.35	3.95	3.45	4.55	3.90	5.05
			$\mathcal{H}_{0,1}$	1.00	0.50	1.25	0.60	0.85	0.75	1.05	0.85	1.10
			$\mathcal{H}_{0,2}$	0.45	0.10	1.25	0.80	0.30	0.40	0.80	0.60	0.95
			$\mathcal{H}_{0,3}$	0.35	0.10	1.90	1.10	0.95	0.70	0.80	0.80	0.85
			$\mathcal{H}_{0,4}$	0.75	0.40	0.95	0.55	0.60	0.50	0.85	0.60	1.00
			$\mathcal{H}_{0,5}$	0.20	0.10	1.55	1.00	1.00	0.50	0.75	0.60	0.85
			$\mathcal{H}_{0,6}$	0.95	0.50	1.25	0.85	0.75	0.85	0.85	0.85	0.95
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	FWER	10.25	7.75	6.40	4.85	4.40	2.70	3.75	3.35	4.75
			$\mathcal{H}_{0,1}$	0.85	0.45	1.50	1.00	1.20	0.50	0.80	0.70	0.90
			$\mathcal{H}_{0,2}$	2.45	1.55	2.05	0.95	0.85	0.45	0.75	0.50	1.05
			$\mathcal{H}_{0,3}$	5.25	4.20	1.75	0.95	0.85	0.40	0.65	0.55	0.90
			$\mathcal{H}_{0,4}$	1.40	0.95	1.40	0.90	0.60	0.80	0.90	0.85	1.10
			$\mathcal{H}_{0,5}$	2.00	1.95	1.30	0.70	0.65	0.70	0.85	0.85	1.20
			$\mathcal{H}_{0,6}$	0.75	0.25	1.20	0.75	0.50	0.25	0.65	0.30	0.70
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_1$	FWER	4.40	4.70	5.20	3.65	3.05	4.40	5.85	5.25	5.65
			$\mathcal{H}_{0,1}$	1.40	1.00	0.95	0.70	0.45	0.70	1.45	0.85	1.50
			$\mathcal{H}_{0,2}$	1.15	1.15	1.10	0.80	0.60	0.85	1.05	1.00	1.30
			$\mathcal{H}_{0,3}$	1.05	0.80	1.05	0.75	0.65	0.95	1.30	1.00	1.35
			$\mathcal{H}_{0,4}$	0.80	0.70	0.95	0.80	0.65	0.75	0.80	1.00	0.85
			$\mathcal{H}_{0,5}$	0.90	0.65	1.00	0.45	0.50	0.70	1.00	1.10	1.00
			$\mathcal{H}_{0,6}$	0.90	1.00	0.80	0.60	0.75	1.05	0.80	1.20	0.95
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_2$	FWER	4.45	2.70	4.90	3.85	2.70	3.90	4.50	4.95	5.10
			$\mathcal{H}_{0,1}$	0.65	0.75	0.70	0.55	0.60	0.70	0.85	0.95	1.00
			$\mathcal{H}_{0,2}$	0.25	0.20	0.90	0.70	0.60	0.85	0.80	1.15	1.00
			$\mathcal{H}_{0,3}$	0.35	0.10	1.10	0.90	0.55	0.60	1.00	0.85	1.05
			$\mathcal{H}_{0,4}$	0.50	0.65	0.70	0.55	0.35	0.75	0.65	0.90	0.75
			$\mathcal{H}_{0,5}$	0.40	0.35	1.20	1.20	0.60	0.75	0.85	0.80	0.95
			$\mathcal{H}_{0,6}$	0.70	1.15	0.70	0.45	0.55	1.05	0.70	1.20	0.85
χ^2	$\mathbf{n}_2$	$\boldsymbol{\lambda}_3$	FWER	11.80	10.50	5.15	4.10	3.35	3.45	4.45	4.35	5.15
			$\mathcal{H}_{0,1}$	1.05	0.80	1.20	0.95	0.50	0.45	0.80	0.60	0.80
			$\mathcal{H}_{0,2}$	2.75	2.40	0.95	0.70	0.80	0.95	1.25	1.00	1.70
			$\mathcal{H}_{0,3}$	6.55	5.50	1.40	0.90	0.60	0.55	0.95	0.80	1.20
			$\mathcal{H}_{0,4}$	1.15	1.15	1.05	0.65	0.70	1.00	1.00	1.10	1.05
			$\mathcal{H}_{0,5}$	1.95	2.20	0.85	0.70	0.45	0.65	0.65	0.80	0.90
			$\mathcal{H}_{0,6}$	1.15	0.90	1.15	0.85	0.45	0.80	0.95	0.95	1.05

Table S25: Empirical FWER and sizes (as percentages) of all tests obtained for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing)) *(continued)*

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_1$	FWER	4.35	4.75	5.40	4.45	3.10	5.00	4.90	5.50	5.70
			$\mathcal{H}_{0,1}$	0.90	1.10	0.90	0.75	0.50	1.10	0.90	1.30	1.05
			$\mathcal{H}_{0,2}$	1.05	0.85	1.35	1.05	0.20	1.00	1.05	1.15	1.25
			$\mathcal{H}_{0,3}$	1.05	1.35	1.20	1.00	0.70	1.15	1.15	1.20	1.25
			$\mathcal{H}_{0,4}$	1.00	1.15	0.90	0.85	0.60	1.35	1.05	1.45	1.15
			$\mathcal{H}_{0,5}$	1.05	0.40	1.15	1.10	0.75	0.55	0.70	0.65	0.90
			$\mathcal{H}_{0,6}$	0.65	0.95	0.75	0.75	0.50	0.80	0.75	1.00	0.85
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_2$	FWER	4.15	2.25	5.75	5.35	3.30	3.60	4.20	4.65	4.55
			$\mathcal{H}_{0,1}$	0.45	0.35	1.25	1.10	0.65	0.70	0.75	0.75	0.85
			$\mathcal{H}_{0,2}$	0.20	0.15	1.30	1.30	0.30	0.75	0.85	1.00	0.85
			$\mathcal{H}_{0,3}$	0.00	0.00	1.15	0.75	0.50	0.55	0.65	0.75	0.70
			$\mathcal{H}_{0,4}$	1.00	0.70	1.30	1.20	0.85	0.80	1.05	1.05	1.25
			$\mathcal{H}_{0,5}$	0.35	0.45	1.60	1.50	0.50	0.80	0.75	0.85	0.80
			$\mathcal{H}_{0,6}$	0.90	0.45	0.90	0.75	0.40	0.55	0.90	0.60	0.90
χ^2	$\mathbf{n}_3$	$\boldsymbol{\lambda}_3$	FWER	11.00	9.20	5.00	4.40	3.05	3.60	4.65	4.85	5.15
			$\mathcal{H}_{0,1}$	1.10	0.75	0.90	0.80	0.70	0.60	0.90	0.85	1.05
			$\mathcal{H}_{0,2}$	2.15	1.60	1.10	0.90	0.50	0.60	0.75	0.80	0.80
			$\mathcal{H}_{0,3}$	5.40	4.55	1.05	1.05	0.55	0.55	0.80	0.85	0.80
			$\mathcal{H}_{0,4}$	1.20	1.15	1.05	1.05	0.55	0.80	0.90	0.90	1.05
			$\mathcal{H}_{0,5}$	2.55	2.05	1.00	0.95	0.45	0.75	1.00	1.20	1.20
			$\mathcal{H}_{0,6}$	1.15	1.30	0.85	0.70	0.60	1.05	0.80	1.30	1.00

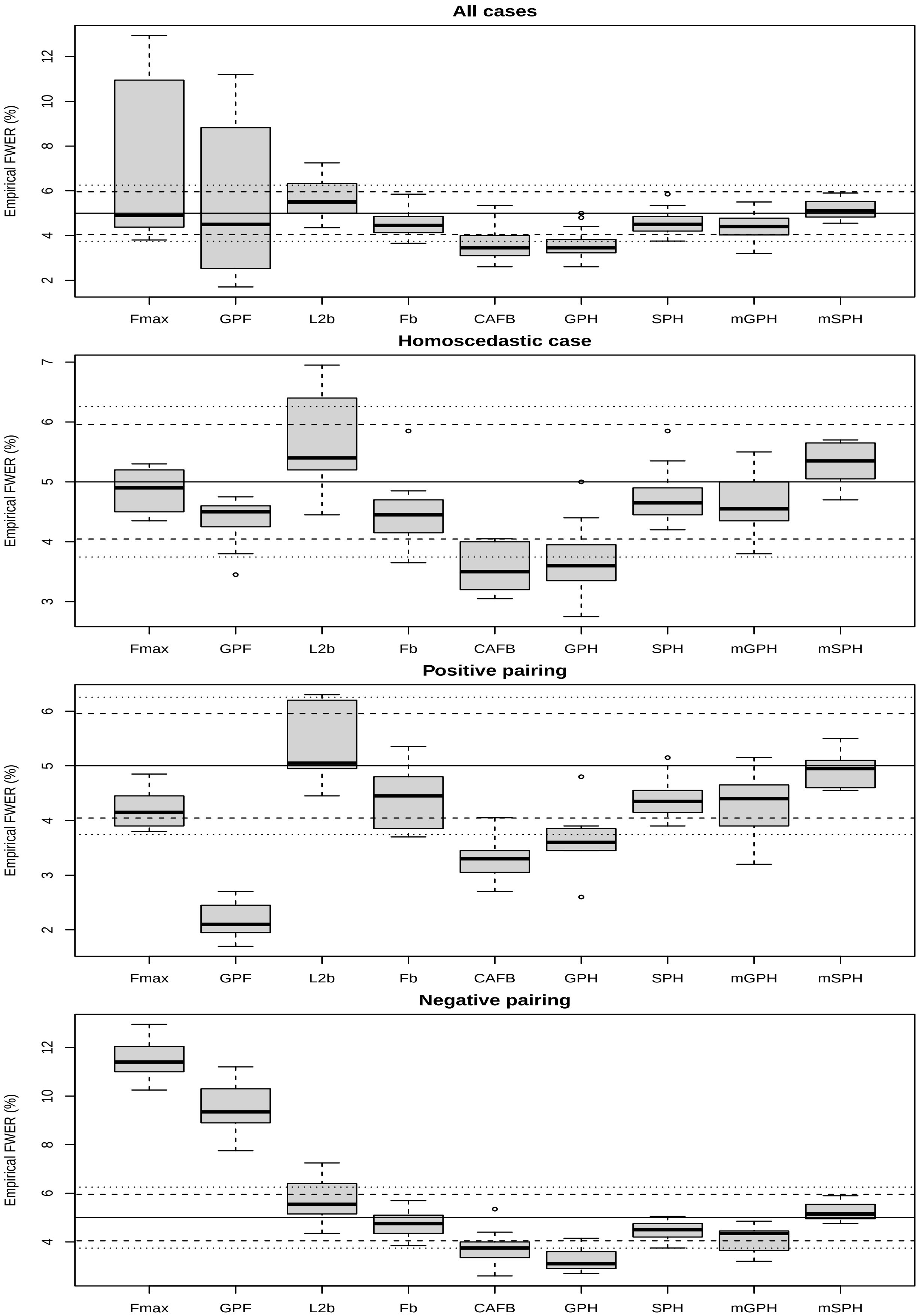


Fig. S54: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

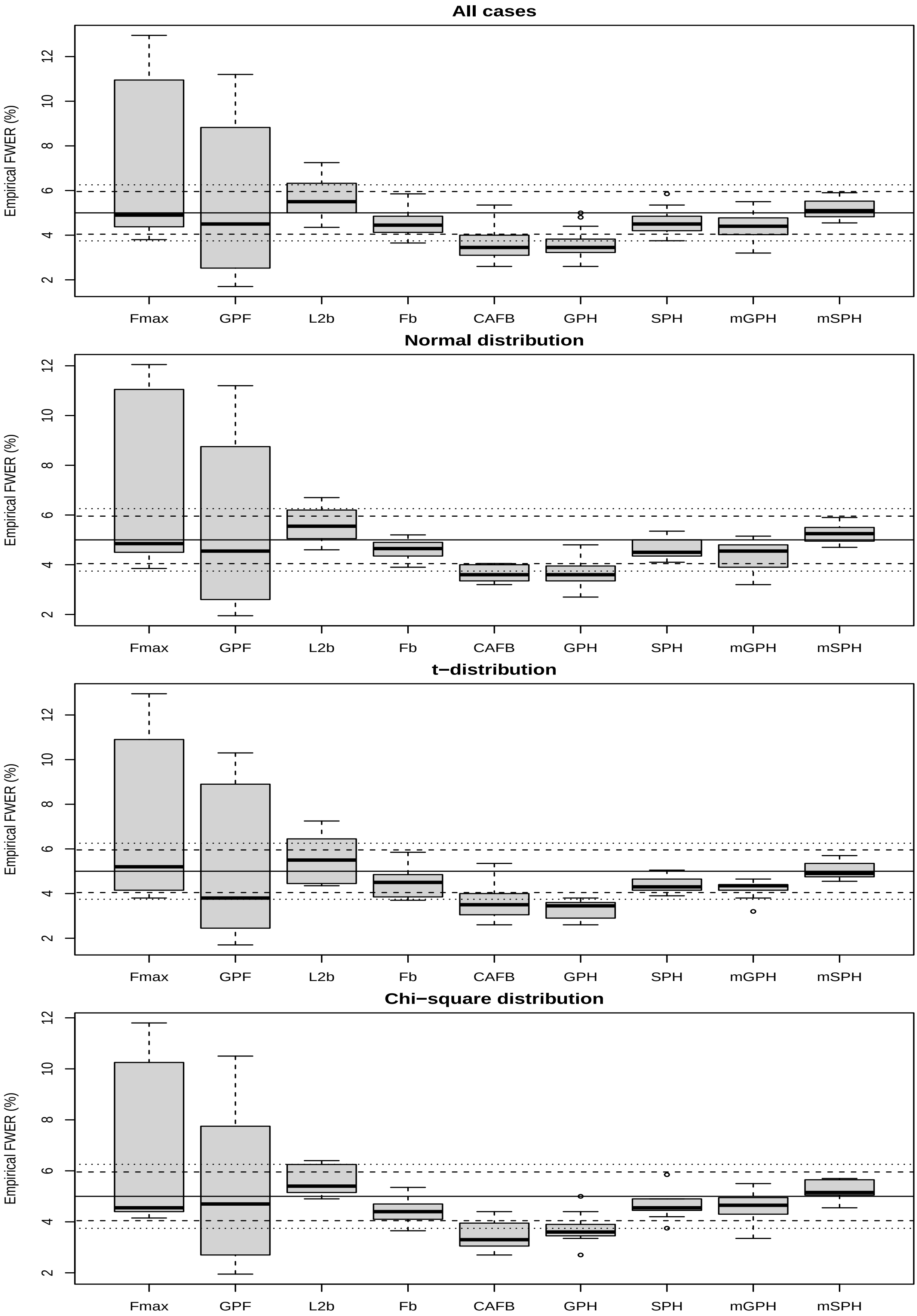


Fig. S55: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts with scaling function and different distributions

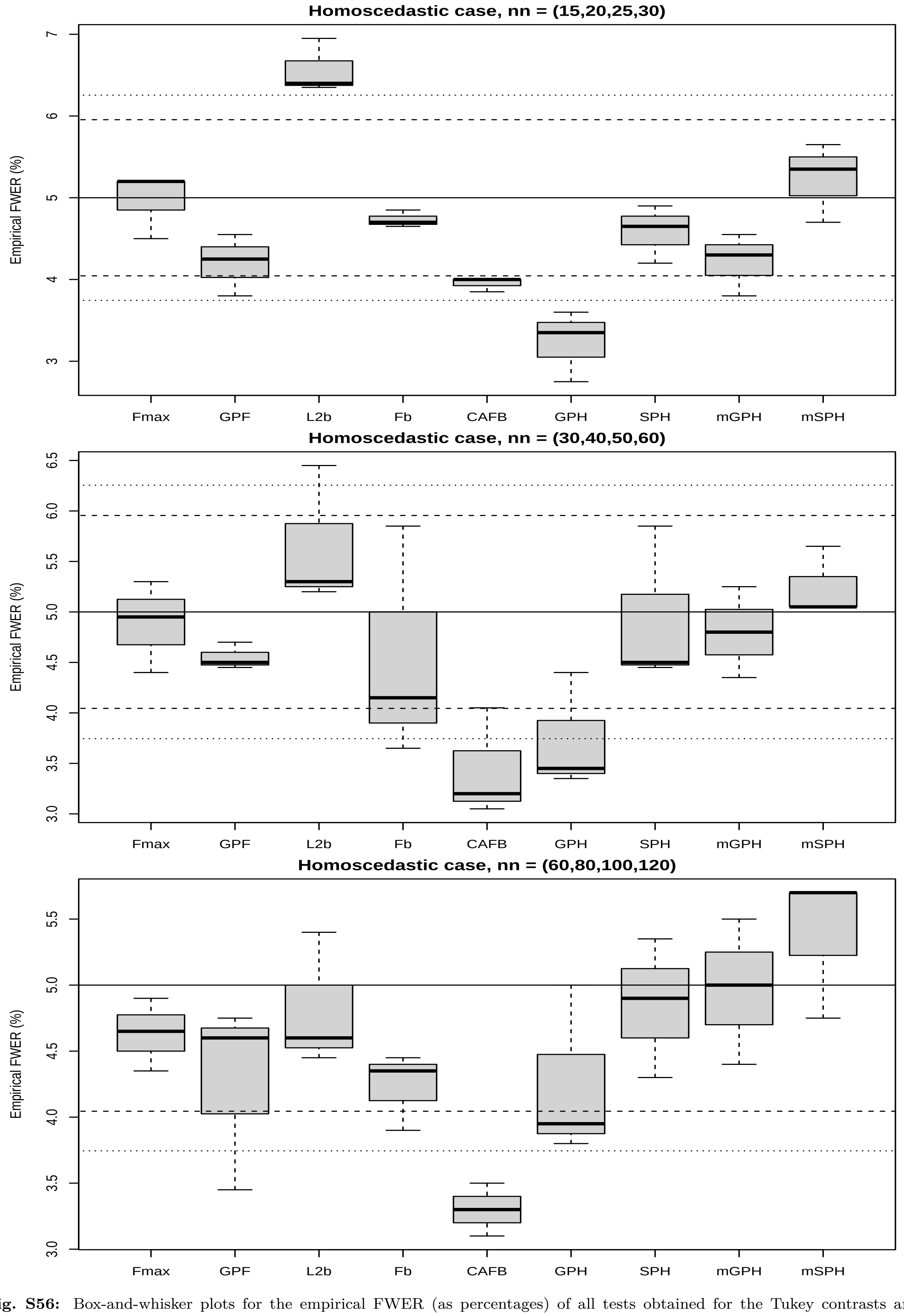


Fig. S56: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes with scaling function and homoscedastic case

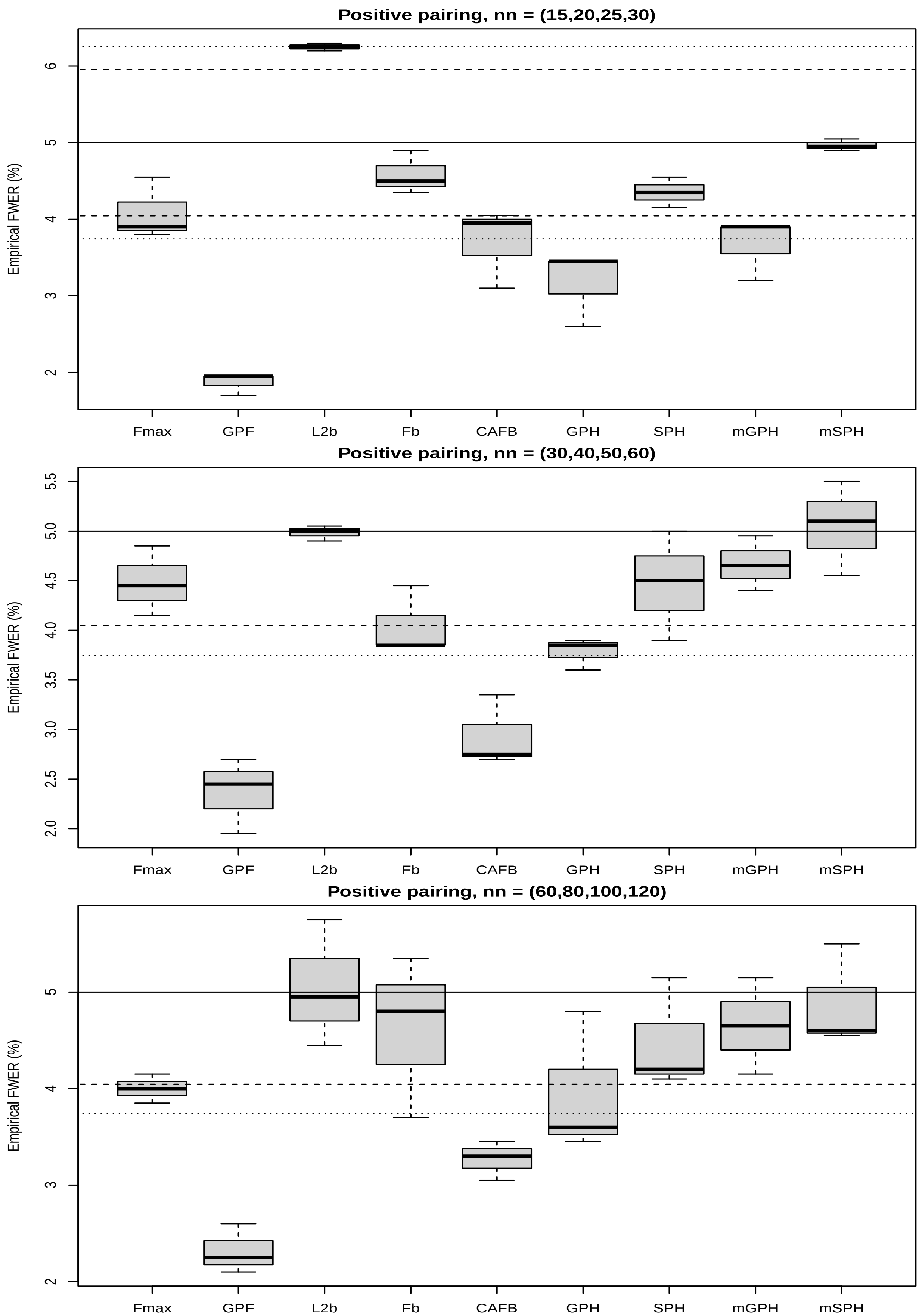


Fig. S57: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes with scaling function and positive pairing

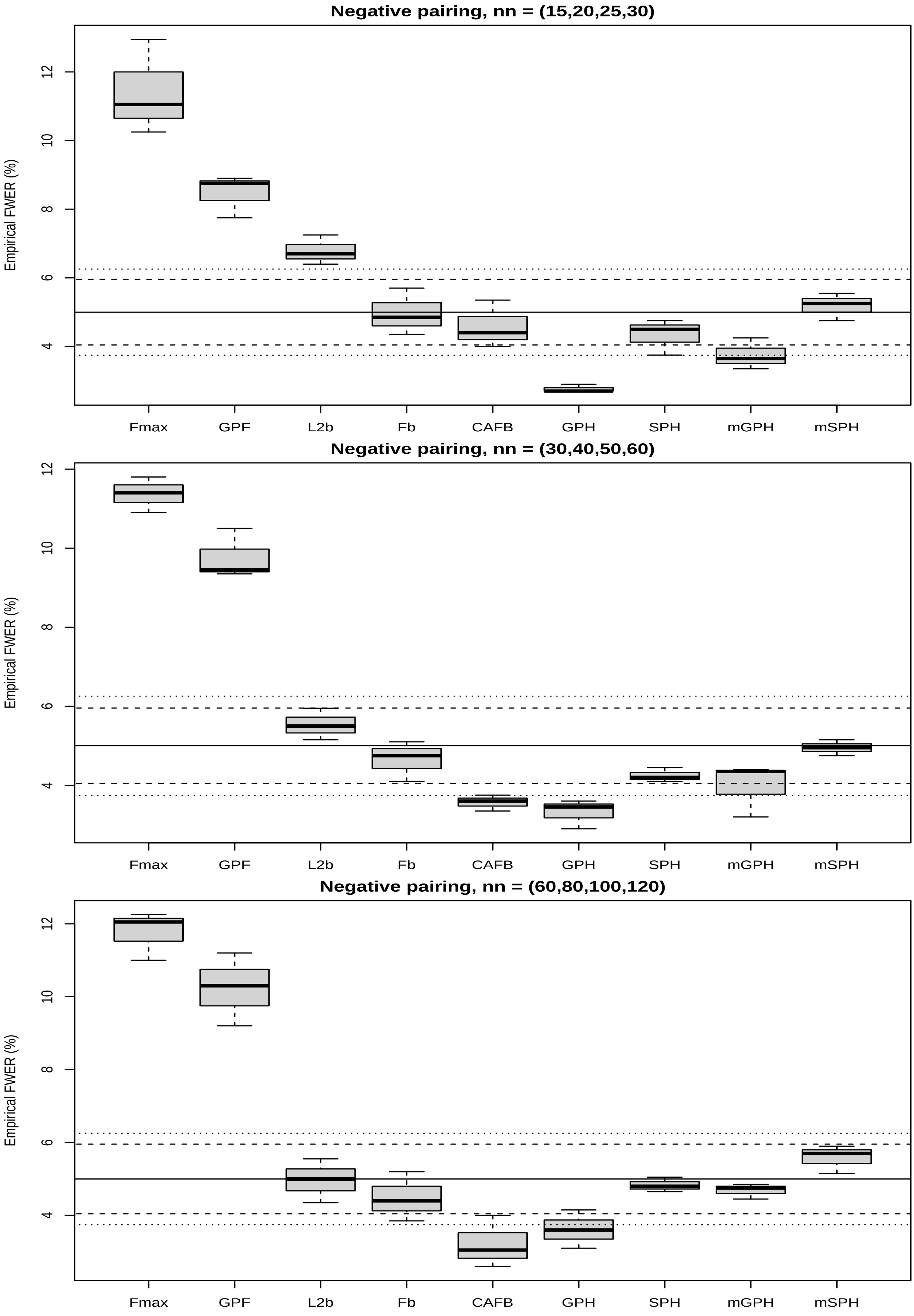


Fig. S58: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and different sample sizes with scaling function and negative pairing

Table S26: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$, $\mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}$, $\mathcal{H}_{0,5}$, $\mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.70	0.60	1.80	0.90	0.65	0.55	0.90	0.70	1.00
			$\mathcal{H}_{0,2}$	0.95	0.50	1.60	1.10	0.80	0.55	0.95	0.55	1.10
			$\mathcal{H}_{0,3}$	74.05	82.10	1.85	1.30	3.90	78.40	68.85	80.55	71.15
			$\mathcal{H}_{0,4}$	0.80	0.80	1.45	0.95	0.80	0.95	0.70	1.10	0.75
			$\mathcal{H}_{0,5}$	84.95	91.90	1.55	1.00	5.30	90.35	83.65	92.40	85.50
			$\mathcal{H}_{0,6}$	90.95	95.80	1.50	1.05	6.45	95.50	90.80	96.25	91.85
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.50	0.40	1.60	1.20	0.95	0.55	0.75	0.55	0.85
			$\mathcal{H}_{0,2}$	0.50	0.40	1.50	1.10	0.90	1.45	1.20	1.50	1.40
			$\mathcal{H}_{0,3}$	14.05	16.50	1.15	1.00	1.85	44.75	38.70	47.90	40.20
			$\mathcal{H}_{0,4}$	0.80	0.60	1.25	0.95	0.75	0.75	0.85	0.90	1.00
			$\mathcal{H}_{0,5}$	27.85	31.05	2.10	1.25	2.35	43.90	37.40	46.50	38.45
			$\mathcal{H}_{0,6}$	32.45	36.20	1.85	1.30	1.60	38.05	34.85	41.40	36.35
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.10	1.00	1.75	1.10	0.80	0.85	1.15	1.05	1.50
			$\mathcal{H}_{0,2}$	2.05	1.75	1.10	0.65	0.70	0.55	0.90	0.70	0.95
			$\mathcal{H}_{0,3}$	54.85	58.75	1.75	1.10	2.00	20.85	19.75	24.10	21.90
			$\mathcal{H}_{0,4}$	1.25	0.80	1.45	0.80	0.65	0.65	1.10	0.80	1.25
			$\mathcal{H}_{0,5}$	65.60	70.80	1.70	1.25	2.40	53.25	46.00	56.65	48.50
			$\mathcal{H}_{0,6}$	80.70	86.55	1.95	1.40	4.55	83.25	75.65	85.95	78.05
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.30	0.65	1.30	0.70	0.70	0.50	1.15	0.70	1.25
			$\mathcal{H}_{0,2}$	1.05	0.55	1.80	1.20	0.75	0.45	1.05	0.70	1.30
			$\mathcal{H}_{0,3}$	40.50	48.85	1.55	0.75	1.90	44.85	37.90	47.10	40.05
			$\mathcal{H}_{0,4}$	1.45	0.75	1.50	1.05	0.60	0.65	1.15	0.80	1.25
			$\mathcal{H}_{0,5}$	54.25	62.45	1.40	0.80	2.50	58.95	52.55	62.40	55.05
			$\mathcal{H}_{0,6}$	62.00	71.15	0.90	0.85	3.30	69.15	60.85	72.40	63.30
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.70	0.40	1.30	0.75	0.60	0.65	0.70	0.75	0.85
			$\mathcal{H}_{0,2}$	0.55	0.15	1.00	0.60	0.55	0.65	1.10	0.75	1.20
			$\mathcal{H}_{0,3}$	4.95	5.10	1.15	0.70	1.05	20.75	17.80	22.70	19.20
			$\mathcal{H}_{0,4}$	0.85	0.50	1.45	0.75	0.70	0.65	1.00	0.80	1.05
			$\mathcal{H}_{0,5}$	12.75	13.05	1.60	1.00	1.15	20.05	17.05	22.65	18.70
			$\mathcal{H}_{0,6}$	15.10	16.20	1.10	0.80	1.00	17.00	15.65	19.40	16.85
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.95	0.65	2.00	1.10	0.75	0.50	0.90	0.70	1.05
			$\mathcal{H}_{0,2}$	2.00	1.40	2.05	1.20	1.15	0.35	0.75	0.45	0.90
			$\mathcal{H}_{0,3}$	36.15	37.65	2.40	1.45	1.85	10.15	9.95	11.95	11.20
			$\mathcal{H}_{0,4}$	1.35	0.85	1.05	0.80	0.65	0.65	1.10	0.70	1.45
			$\mathcal{H}_{0,5}$	38.65	43.80	1.55	0.80	1.55	23.60	23.20	27.00	24.80
			$\mathcal{H}_{0,6}$	49.95	58.60	1.20	0.85	2.85	52.35	44.30	56.05	46.80
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.05	0.50	1.35	0.95	0.40	0.50	0.80	0.65	1.00
			$\mathcal{H}_{0,2}$	0.85	0.90	1.35	0.70	0.85	0.85	0.85	0.95	1.10
			$\mathcal{H}_{0,3}$	73.60	82.70	1.90	1.15	5.55	78.00	70.80	80.40	72.85
			$\mathcal{H}_{0,4}$	0.70	0.70	1.20	0.90	0.65	0.70	0.60	0.90	0.65
			$\mathcal{H}_{0,5}$	84.20	91.50	1.70	1.30	5.90	88.85	81.10	90.10	82.50
			$\mathcal{H}_{0,6}$	91.55	95.60	1.60	1.15	6.10	94.80	90.55	95.75	91.25
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.80	0.65	1.70	0.80	0.60	0.60	0.65	0.65	0.75
			$\mathcal{H}_{0,2}$	0.50	0.10	1.75	1.00	0.90	0.75	1.40	0.85	1.50
			$\mathcal{H}_{0,3}$	13.70	15.60	1.90	1.25	2.30	45.65	37.90	47.85	39.85
			$\mathcal{H}_{0,4}$	0.50	0.35	1.60	0.90	0.45	0.60	0.65	0.65	0.65
			$\mathcal{H}_{0,5}$	25.45	30.05	1.55	1.05	1.90	42.95	34.85	46.20	36.65
			$\mathcal{H}_{0,6}$	32.30	37.70	1.50	1.10	1.75	39.60	34.15	42.45	35.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.70	0.60	1.55	0.85	0.75	0.30	0.60	0.50	0.65
			$\mathcal{H}_{0,2}$	2.70	2.05	2.05	1.25	1.25	0.55	1.25	0.75	1.50
			$\mathcal{H}_{0,3}$	57.55	59.10	2.25	1.10	2.90	26.15	24.05	29.75	26.40
			$\mathcal{H}_{0,4}$	1.40	1.00	1.75	1.35	0.60	0.65	0.90	0.90	1.15
			$\mathcal{H}_{0,5}$	67.05	72.20	1.85	1.15	3.15	53.25	46.95	57.35	49.75
			$\mathcal{H}_{0,6}$	81.95	87.45	1.50	1.30	4.20	84.35	76.70	86.80	79.55

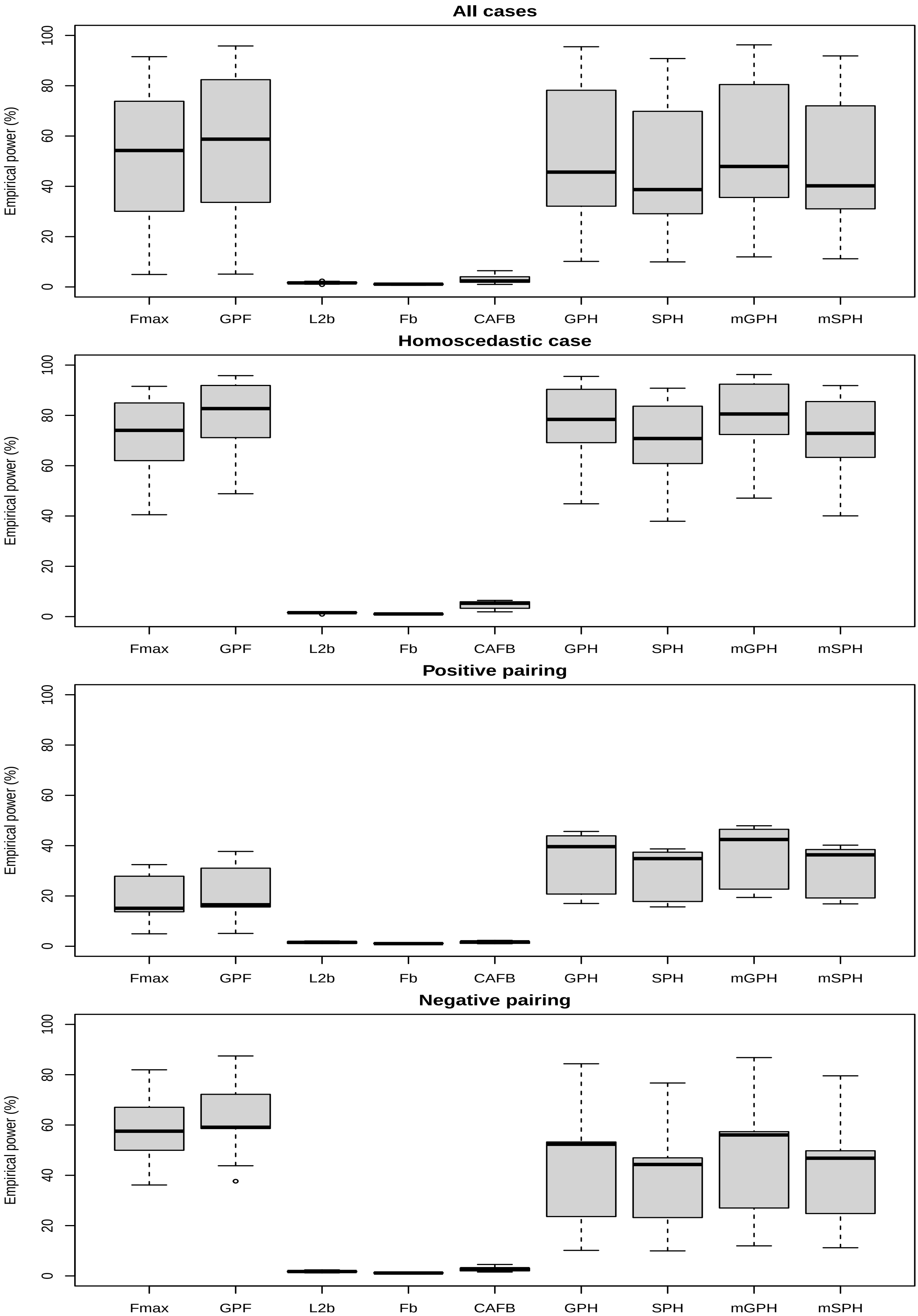


Fig. S59: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

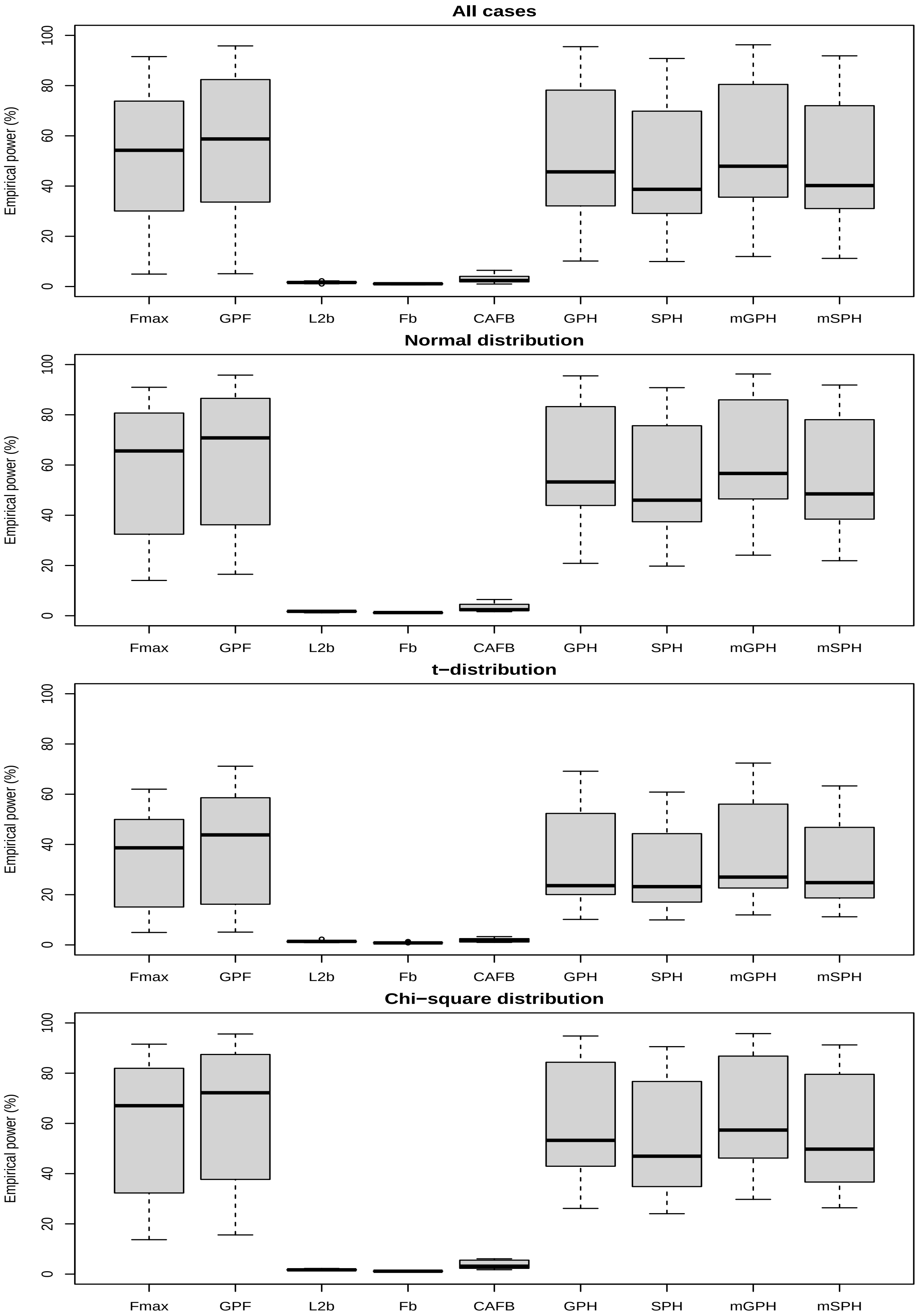


Fig. S60: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts with scaling function and different distributions

Table S27: Empirical sizes ($\mathcal{H}_{0,1}$, $\mathcal{H}_{0,2}$, $\mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}$, $\mathcal{H}_{0,5}$, $\mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.85	0.50	1.55	0.80	0.75	0.50	1.05	0.60	1.20
			$\mathcal{H}_{0,2}$	0.80	0.65	1.45	0.85	0.75	0.60	0.60	0.85	0.75
			$\mathcal{H}_{0,3}$	69.65	97.35	72.85	63.20	78.85	96.15	66.65	96.85	69.10
			$\mathcal{H}_{0,4}$	0.75	0.75	1.40	0.85	0.40	0.55	0.55	0.80	0.80
			$\mathcal{H}_{0,5}$	83.05	99.60	82.45	78.00	89.65	99.45	80.55	99.55	82.40
			$\mathcal{H}_{0,6}$	88.55	99.80	86.95	83.90	94.70	99.85	88.35	99.85	89.60
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	1.00	0.25	1.80	1.20	0.85	0.45	0.90	0.60	1.35
			$\mathcal{H}_{0,2}$	0.60	0.15	1.65	0.95	0.65	0.45	1.15	0.55	1.25
			$\mathcal{H}_{0,3}$	14.85	45.50	44.30	37.45	49.00	75.70	36.45	77.65	38.30
			$\mathcal{H}_{0,4}$	0.85	0.50	1.35	1.05	0.65	0.60	1.10	0.70	1.15
			$\mathcal{H}_{0,5}$	25.60	63.80	39.40	34.20	45.20	74.15	35.45	76.55	37.70
			$\mathcal{H}_{0,6}$	31.05	67.15	38.30	32.60	42.90	69.35	33.30	71.45	35.80
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	0.95	1.00	1.60	0.85	0.80	0.65	0.85	0.75	0.95
			$\mathcal{H}_{0,2}$	2.50	1.75	2.05	1.30	1.20	0.60	0.80	0.70	1.15
			$\mathcal{H}_{0,3}$	59.10	80.70	32.10	23.35	28.00	46.80	21.05	51.50	23.40
			$\mathcal{H}_{0,4}$	1.50	1.05	1.55	1.20	0.65	0.65	1.35	0.95	1.40
			$\mathcal{H}_{0,5}$	64.00	91.90	52.05	44.15	54.95	82.90	43.70	85.45	47.35
			$\mathcal{H}_{0,6}$	76.25	98.05	76.00	70.10	83.05	97.80	71.60	98.15	74.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.00	0.60	1.40	0.80	0.60	0.50	0.85	0.60	0.90
			$\mathcal{H}_{0,2}$	0.80	0.60	1.45	1.20	0.40	0.30	0.80	0.50	0.90
			$\mathcal{H}_{0,3}$	39.15	77.85	45.75	37.85	48.60	74.75	38.05	77.60	39.75
			$\mathcal{H}_{0,4}$	0.85	0.40	1.15	0.75	0.80	0.30	0.65	0.45	0.75
			$\mathcal{H}_{0,5}$	51.75	87.00	53.90	47.40	62.05	85.25	48.70	87.10	51.70
			$\mathcal{H}_{0,6}$	60.90	93.60	61.55	56.70	72.05	92.75	60.60	94.00	63.50
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.55	0.45	1.45	0.85	0.65	0.90	0.55	0.95	0.60
			$\mathcal{H}_{0,2}$	0.25	0.20	2.00	1.25	0.75	0.50	0.60	0.70	0.65
			$\mathcal{H}_{0,3}$	5.30	17.85	26.95	22.50	25.60	44.30	18.15	47.45	19.50
			$\mathcal{H}_{0,4}$	0.65	0.40	2.15	1.35	0.95	0.50	0.70	0.55	0.70
			$\mathcal{H}_{0,5}$	13.15	33.15	23.85	20.00	22.60	42.75	19.00	45.80	20.50
			$\mathcal{H}_{0,6}$	16.45	36.95	22.15	19.25	21.00	38.75	17.35	42.20	18.30
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.25	0.80	2.15	1.35	0.90	0.45	0.75	0.55	1.05
			$\mathcal{H}_{0,2}$	2.20	1.30	1.60	1.05	0.65	0.40	0.70	0.40	0.90
			$\mathcal{H}_{0,3}$	38.65	54.40	18.85	12.30	14.30	21.85	10.70	24.75	12.30
			$\mathcal{H}_{0,4}$	0.85	0.85	1.65	1.20	0.65	0.70	0.60	0.85	0.80
			$\mathcal{H}_{0,5}$	39.05	68.10	31.15	24.80	31.65	50.60	22.70	55.70	25.65
			$\mathcal{H}_{0,6}$	49.10	85.50	48.20	42.15	54.50	82.30	42.90	84.45	46.35
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.95	0.50	1.25	0.65	0.60	0.55	0.95	0.65	0.95
			$\mathcal{H}_{0,2}$	0.90	0.65	1.40	0.85	0.55	0.60	0.90	0.80	0.95
			$\mathcal{H}_{0,3}$	70.65	97.25	72.55	62.70	79.00	94.85	67.40	95.75	70.35
			$\mathcal{H}_{0,4}$	1.10	0.55	1.55	0.90	0.60	0.65	1.15	0.75	1.20
			$\mathcal{H}_{0,5}$	83.30	99.10	80.30	75.40	87.60	98.80	82.20	98.90	83.30
			$\mathcal{H}_{0,6}$	89.90	99.55	86.55	82.90	93.25	99.45	89.00	99.45	90.65
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.75	0.55	1.25	0.75	0.80	0.55	0.95	0.65	1.00
			$\mathcal{H}_{0,2}$	0.30	0.30	1.85	1.05	0.50	0.65	0.75	0.70	0.80
			$\mathcal{H}_{0,3}$	12.20	46.60	43.20	36.55	47.05	76.60	37.20	78.70	39.30
			$\mathcal{H}_{0,4}$	0.50	0.40	1.05	0.65	0.90	0.60	0.75	0.70	0.95
			$\mathcal{H}_{0,5}$	24.00	63.85	40.30	34.35	46.55	72.30	34.10	75.00	35.75
			$\mathcal{H}_{0,6}$	33.40	68.85	38.00	33.25	42.15	69.80	33.90	72.30	36.30
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.00	0.60	1.45	0.90	0.55	0.40	0.85	0.55	0.95
			$\mathcal{H}_{0,2}$	1.95	1.75	1.75	1.00	1.25	0.50	0.85	0.50	1.15
			$\mathcal{H}_{0,3}$	61.25	80.50	36.75	26.90	35.75	50.20	26.30	53.90	28.65
			$\mathcal{H}_{0,4}$	1.10	0.95	1.20	0.90	0.60	0.80	0.90	1.10	1.20
			$\mathcal{H}_{0,5}$	65.70	90.90	51.85	43.75	58.30	79.90	46.25	82.80	49.80
			$\mathcal{H}_{0,6}$	79.20	98.05	76.25	70.50	84.15	96.85	73.35	97.40	76.60

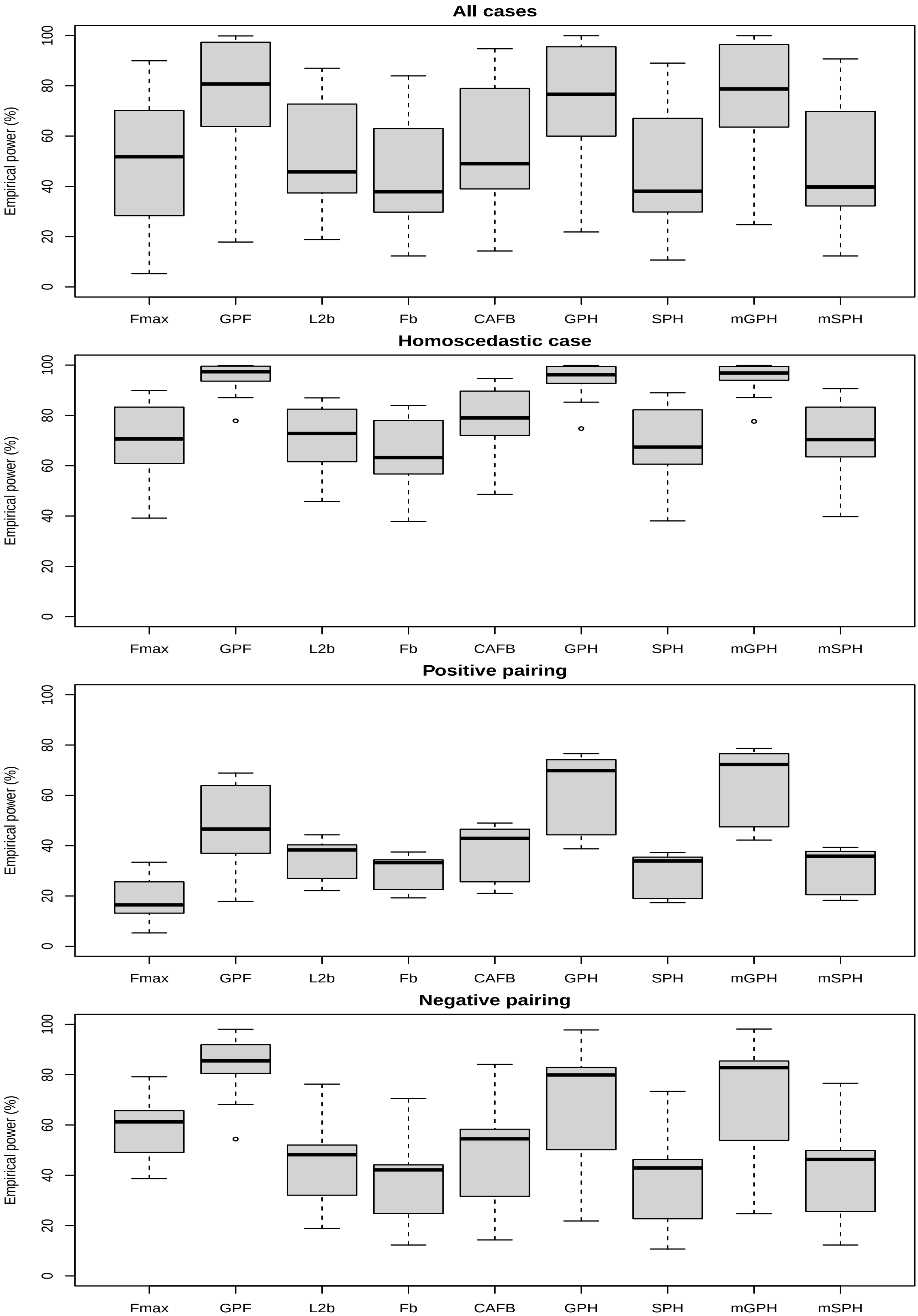


Fig. S61: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

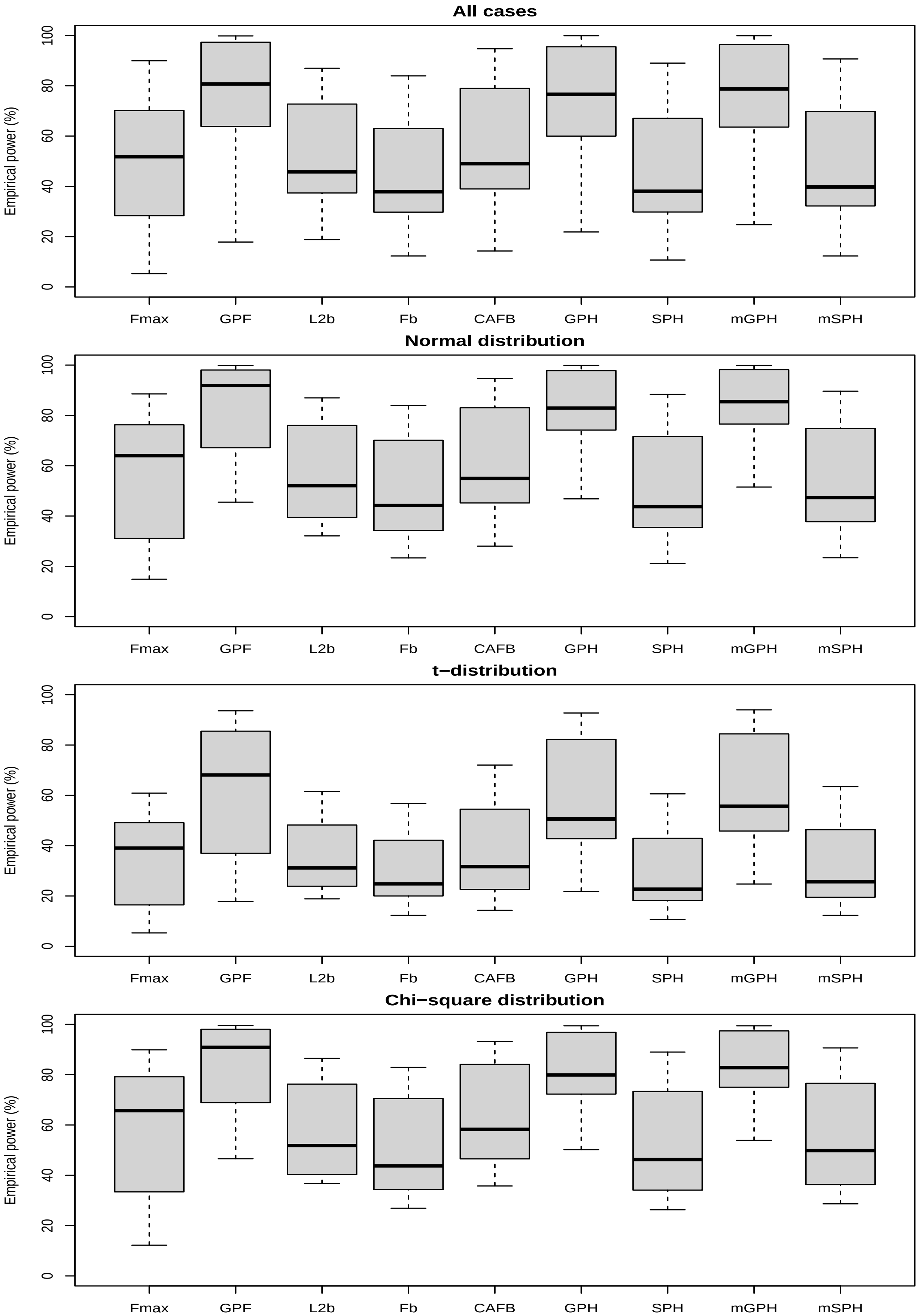


Fig. S62: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A2 for the Tukey contrasts with scaling function and different distributions

Table S28: Empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	10.00	21.05	14.70	9.50	12.75	21.25	9.60	23.25	10.85
			$\mathcal{H}_{0,2}$	62.45	96.65	68.65	58.25	74.45	96.15	60.90	96.85	63.80
			$\mathcal{H}_{0,3}$	99.35	100.00	98.90	97.15	99.60	100.00	98.80	100.00	99.20
			$\mathcal{H}_{0,4}$	12.60	30.00	17.40	13.45	15.65	29.35	12.40	32.20	13.15
			$\mathcal{H}_{0,5}$	79.95	99.15	82.00	77.15	88.75	99.15	79.35	99.30	81.10
			$\mathcal{H}_{0,6}$	18.90	41.40	24.10	20.30	24.05	41.85	18.30	44.75	19.75
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.25	10.90	11.45	7.85	7.00	14.00	6.25	15.35	6.95
			$\mathcal{H}_{0,2}$	25.20	65.35	48.15	40.75	52.25	80.00	39.90	82.10	42.15
			$\mathcal{H}_{0,3}$	57.85	97.45	87.45	83.00	93.10	99.75	87.45	99.80	88.55
			$\mathcal{H}_{0,4}$	5.10	9.85	9.90	7.60	7.30	11.95	6.10	13.65	6.75
			$\mathcal{H}_{0,5}$	25.55	63.15	42.85	37.10	46.20	73.50	35.35	76.05	36.95
			$\mathcal{H}_{0,6}$	5.70	10.25	8.75	6.85	6.65	11.50	6.45	12.60	7.10
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.70	4.85	5.15	3.00	3.05	3.60	2.75	4.75	3.40
			$\mathcal{H}_{0,2}$	34.25	60.85	28.65	20.70	24.15	41.90	16.90	46.00	19.40
			$\mathcal{H}_{0,3}$	92.95	99.35	70.80	58.75	71.20	93.85	58.20	95.50	61.85
			$\mathcal{H}_{0,4}$	6.55	11.95	7.65	5.65	6.10	9.70	5.60	11.60	6.30
			$\mathcal{H}_{0,5}$	65.40	91.80	53.05	43.85	55.20	82.30	44.35	84.75	48.10
			$\mathcal{H}_{0,6}$	15.05	33.55	17.00	14.10	15.55	29.80	12.60	34.15	14.20
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	5.65	9.70	9.45	6.70	6.45	9.20	5.70	10.50	6.45
			$\mathcal{H}_{0,2}$	35.25	73.95	44.10	36.35	45.05	71.75	34.85	74.70	36.90
			$\mathcal{H}_{0,3}$	87.55	99.35	86.65	80.30	91.60	99.15	84.80	99.45	86.25
			$\mathcal{H}_{0,4}$	7.75	14.15	9.95	7.30	7.50	13.80	7.85	16.00	8.50
			$\mathcal{H}_{0,5}$	51.80	88.35	56.95	49.70	64.30	87.35	50.15	88.55	52.50
			$\mathcal{H}_{0,6}$	10.45	21.75	12.95	10.95	11.45	21.55	10.25	23.60	11.15
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	2.95	5.55	6.00	3.60	3.30	5.80	2.95	6.50	3.35
			$\mathcal{H}_{0,2}$	11.65	32.20	30.70	23.70	29.65	46.95	19.45	50.00	21.20
			$\mathcal{H}_{0,3}$	28.10	72.15	62.40	56.20	69.15	91.20	56.40	92.50	58.70
			$\mathcal{H}_{0,4}$	3.65	5.30	6.10	4.60	4.05	6.10	3.75	6.55	4.35
			$\mathcal{H}_{0,5}$	11.95	33.15	25.10	20.05	23.55	42.90	18.85	45.90	19.80
			$\mathcal{H}_{0,6}$	3.75	5.20	4.85	3.70	3.45	5.55	3.75	6.45	4.15
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.55	3.15	3.35	1.70	2.65	2.35	2.05	2.70	2.60
			$\mathcal{H}_{0,2}$	19.95	37.00	15.55	10.95	13.15	20.30	9.75	23.50	11.30
			$\mathcal{H}_{0,3}$	73.75	92.30	45.65	32.15	43.55	68.35	33.80	72.00	36.90
			$\mathcal{H}_{0,4}$	4.10	6.30	5.80	4.20	3.30	4.50	3.45	5.80	3.65
			$\mathcal{H}_{0,5}$	39.05	66.00	30.70	24.10	28.90	47.50	21.75	51.70	24.55
			$\mathcal{H}_{0,6}$	7.80	14.85	9.90	7.65	7.70	12.50	6.25	14.05	7.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	9.90	20.85	14.85	10.70	12.55	21.10	9.90	23.10	11.05
			$\mathcal{H}_{0,2}$	64.90	95.35	67.30	59.30	75.00	93.35	61.45	94.50	64.35
			$\mathcal{H}_{0,3}$	99.40	100.00	97.75	94.70	98.55	100.00	98.50	100.00	98.80
			$\mathcal{H}_{0,4}$	12.90	30.65	17.70	13.30	15.65	29.90	13.25	33.10	14.40
			$\mathcal{H}_{0,5}$	82.65	99.40	78.00	72.80	88.15	98.70	80.70	99.10	82.50
			$\mathcal{H}_{0,6}$	18.20	44.15	23.70	20.05	24.50	43.30	18.25	45.75	20.05
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.95	9.90	10.35	6.85	7.55	12.75	6.15	14.55	6.80
			$\mathcal{H}_{0,2}$	22.90	65.95	49.05	41.00	52.60	79.30	39.40	81.95	41.85
			$\mathcal{H}_{0,3}$	56.50	97.60	86.45	81.85	93.20	99.55	87.75	99.60	89.00
			$\mathcal{H}_{0,4}$	5.45	10.60	9.20	6.85	7.00	12.35	6.70	14.00	6.90
			$\mathcal{H}_{0,5}$	23.60	64.70	41.65	37.05	46.05	74.45	35.60	76.70	37.80
			$\mathcal{H}_{0,6}$	5.15	9.40	7.55	5.35	4.70	9.95	6.00	11.20	6.70
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	4.10	5.55	6.50	4.50	4.65	4.70	3.50	5.70	4.05
			$\mathcal{H}_{0,2}$	36.05	62.60	31.30	23.55	30.35	43.30	22.00	47.80	23.80
			$\mathcal{H}_{0,3}$	93.20	98.85	69.60	56.95	73.35	91.15	62.50	92.50	65.90
			$\mathcal{H}_{0,4}$	7.40	13.65	9.70	7.05	7.25	12.00	6.05	13.55	7.10
			$\mathcal{H}_{0,5}$	66.65	90.80	52.40	44.30	57.80	80.40	45.45	82.85	49.05
			$\mathcal{H}_{0,6}$	15.05	33.80	18.00	14.60	16.95	29.90	12.90	33.20	14.10

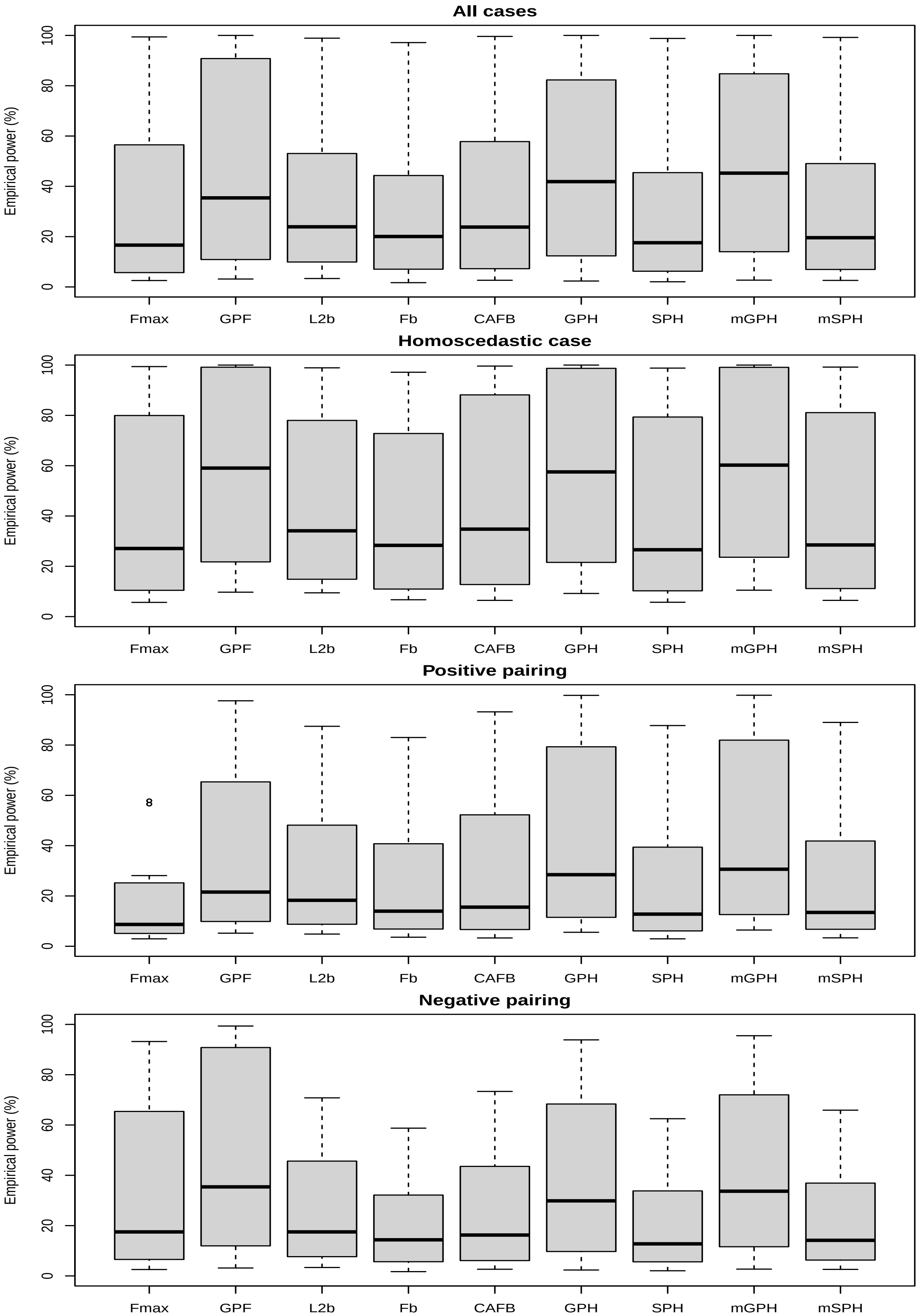


Fig. S63: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

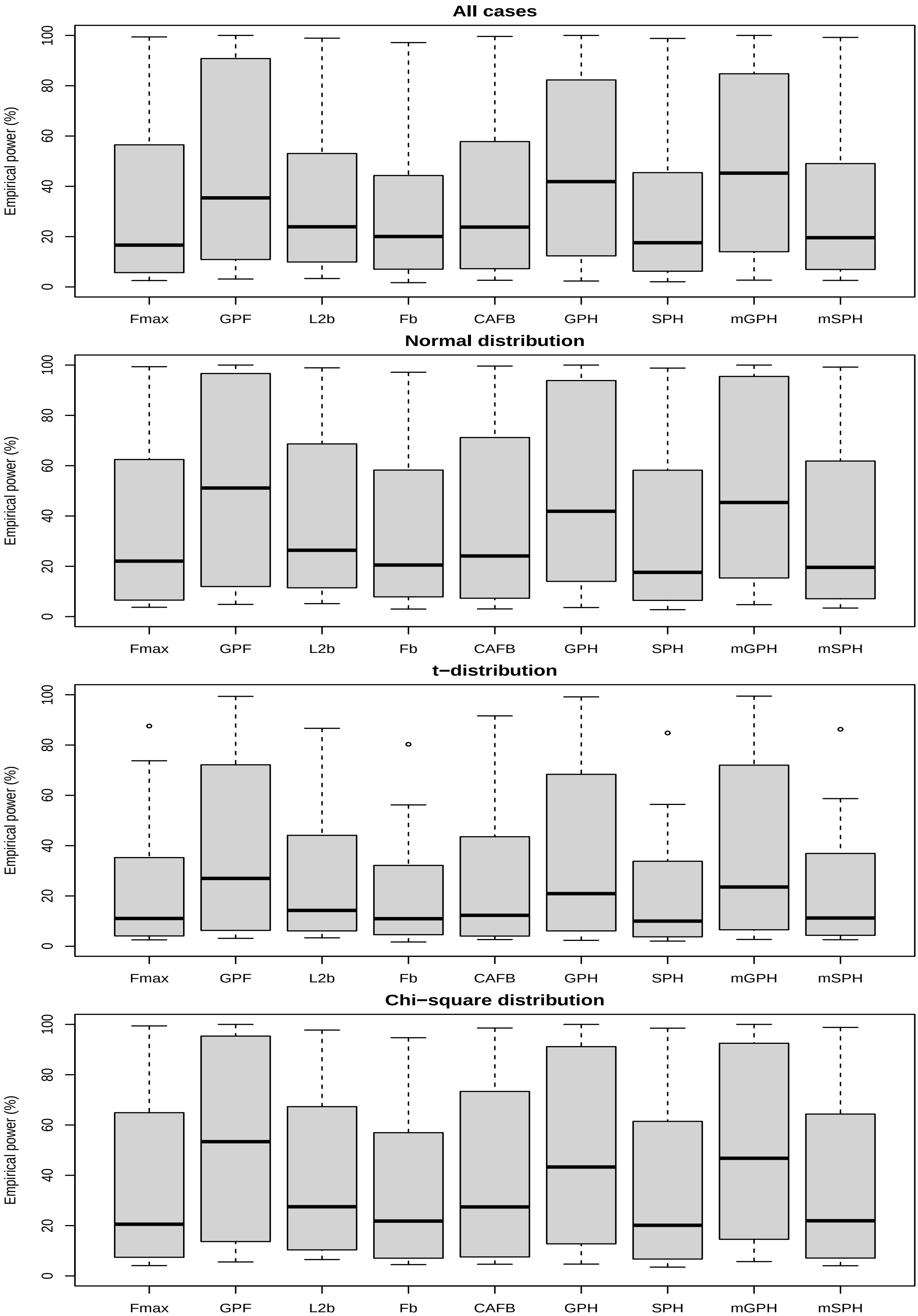


Fig. S64: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A3 for the Tukey contrasts with scaling function and different distributions

Table S29: Empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.70	7.90	1.05	0.55	0.65	7.70	7.55	8.55	8.30
			$\mathcal{H}_{0,2}$	67.15	77.40	1.50	1.00	4.80	75.30	64.90	78.05	67.00
			$\mathcal{H}_{0,3}$	99.35	99.95	1.45	0.90	13.40	99.90	99.10	99.95	99.45
			$\mathcal{H}_{0,4}$	12.35	14.80	1.70	1.20	1.25	14.40	12.05	16.20	13.10
			$\mathcal{H}_{0,5}$	85.45	91.15	1.50	1.00	4.65	90.95	84.90	91.75	86.50
			$\mathcal{H}_{0,6}$	16.70	20.90	1.20	0.80	1.40	20.15	17.15	23.20	18.55
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	4.75	4.60	1.70	0.90	0.75	6.35	5.90	7.15	6.55
			$\mathcal{H}_{0,2}$	25.45	32.95	1.55	1.05	1.55	50.05	39.90	53.05	41.45
			$\mathcal{H}_{0,3}$	65.20	75.05	1.75	1.05	6.65	95.05	88.65	96.05	89.60
			$\mathcal{H}_{0,4}$	3.90	4.90	0.95	0.35	0.75	5.60	4.45	6.60	4.85
			$\mathcal{H}_{0,5}$	26.05	31.65	1.45	1.05	2.35	44.30	35.35	47.00	37.50
			$\mathcal{H}_{0,6}$	4.50	4.65	1.00	0.75	0.85	5.40	4.80	6.40	5.30
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	4.30	3.50	2.00	1.40	1.25	2.50	3.40	3.00	3.75
			$\mathcal{H}_{0,2}$	33.35	37.45	2.20	1.60	2.15	20.00	17.60	22.85	19.60
			$\mathcal{H}_{0,3}$	92.50	94.40	2.00	1.55	4.65	70.70	60.60	74.60	64.00
			$\mathcal{H}_{0,4}$	6.20	5.60	1.75	1.10	1.05	4.55	4.90	5.45	5.65
			$\mathcal{H}_{0,5}$	65.70	70.55	1.70	1.20	2.55	52.45	45.00	56.40	47.95
			$\mathcal{H}_{0,6}$	13.65	15.15	1.30	0.90	1.45	12.40	10.75	14.25	12.30
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	4.20	3.10	1.60	1.10	1.10	2.80	3.55	3.50	4.10
			$\mathcal{H}_{0,2}$	36.80	43.70	1.70	1.05	2.35	40.40	35.35	43.95	37.45
			$\mathcal{H}_{0,3}$	89.10	93.90	1.55	0.80	6.25	91.35	86.65	92.15	88.20
			$\mathcal{H}_{0,4}$	6.50	6.35	1.15	0.80	0.95	6.30	6.50	7.35	7.10
			$\mathcal{H}_{0,5}$	52.55	61.30	1.70	1.30	2.90	59.15	50.00	61.70	52.40
			$\mathcal{H}_{0,6}$	9.30	9.35	1.30	0.80	1.10	8.50	8.25	10.15	9.25
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	2.60	2.90	1.70	1.15	1.10	3.40	3.25	3.85	3.70
			$\mathcal{H}_{0,2}$	11.20	11.65	1.45	1.05	1.30	22.80	18.10	25.05	19.65
			$\mathcal{H}_{0,3}$	29.20	35.40	1.40	1.05	3.65	67.35	58.45	69.65	60.65
			$\mathcal{H}_{0,4}$	2.30	2.15	1.30	0.85	0.85	2.50	2.20	3.15	2.40
			$\mathcal{H}_{0,5}$	10.65	12.90	1.15	0.95	1.15	18.80	16.85	21.05	18.45
			$\mathcal{H}_{0,6}$	2.25	1.95	1.45	1.00	0.75	2.05	2.45	2.60	2.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.90	1.65	1.45	0.75	0.75	1.10	1.55	1.30	1.80
			$\mathcal{H}_{0,2}$	16.95	18.35	1.80	1.35	1.20	8.00	8.45	10.40	9.40
			$\mathcal{H}_{0,3}$	72.65	75.85	2.15	1.10	2.10	37.20	32.55	41.10	35.15
			$\mathcal{H}_{0,4}$	3.20	4.35	1.50	1.05	1.10	2.60	2.60	3.45	3.05
			$\mathcal{H}_{0,5}$	39.40	42.75	1.40	0.85	1.85	25.40	23.80	29.60	25.70
			$\mathcal{H}_{0,6}$	7.85	7.60	1.40	0.80	1.25	5.55	5.80	6.95	6.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.55	8.70	1.25	0.70	1.45	9.45	7.65	10.45	8.25
			$\mathcal{H}_{0,2}$	66.40	77.75	2.05	1.20	3.40	73.75	64.10	76.75	66.65
			$\mathcal{H}_{0,3}$	99.55	99.95	2.30	1.20	12.50	99.70	99.05	99.75	99.20
			$\mathcal{H}_{0,4}$	12.80	13.90	2.05	1.60	1.60	13.70	12.80	15.85	13.95
			$\mathcal{H}_{0,5}$	86.00	91.40	1.75	1.30	5.30	90.40	84.25	92.05	85.50
			$\mathcal{H}_{0,6}$	18.55	21.95	1.65	1.20	1.30	21.70	18.20	23.90	19.45
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.15	3.85	1.30	0.85	0.80	5.80	6.05	6.45	6.45
			$\mathcal{H}_{0,2}$	28.95	31.90	1.50	1.00	2.35	49.80	42.40	53.25	44.45
			$\mathcal{H}_{0,3}$	65.85	76.60	1.95	1.55	6.65	94.00	88.20	94.95	89.80
			$\mathcal{H}_{0,4}$	4.45	4.45	1.20	0.80	1.05	5.30	4.95	5.85	5.25
			$\mathcal{H}_{0,5}$	26.50	30.85	1.65	1.15	1.60	41.50	34.65	44.70	36.85
			$\mathcal{H}_{0,6}$	4.15	3.45	1.50	1.05	0.50	4.10	4.70	4.90	5.35
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.55	3.05	1.55	1.15	1.10	2.35	2.80	2.95	3.40
			$\mathcal{H}_{0,2}$	33.80	36.35	1.65	1.00	1.95	21.25	21.05	24.35	22.90
			$\mathcal{H}_{0,3}$	92.25	93.90	2.40	1.55	4.40	70.50	61.85	74.40	64.35
			$\mathcal{H}_{0,4}$	6.50	6.30	1.50	1.05	0.90	5.15	5.20	6.45	5.95
			$\mathcal{H}_{0,5}$	64.40	71.00	1.60	0.85	3.25	54.35	48.10	58.40	50.65
			$\mathcal{H}_{0,6}$	15.75	17.25	1.50	1.15	0.90	14.00	13.45	16.60	14.85

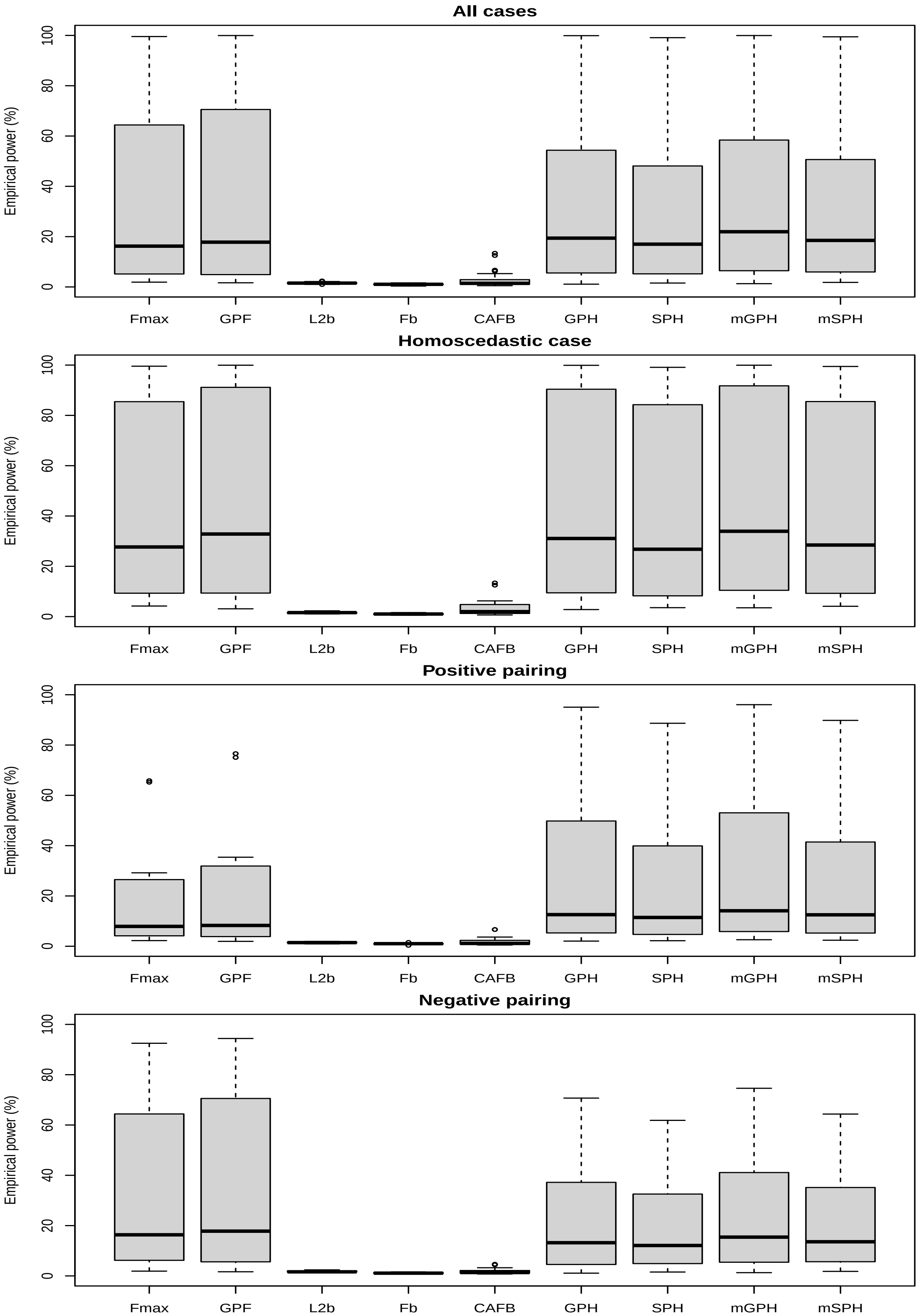


Fig. S65: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

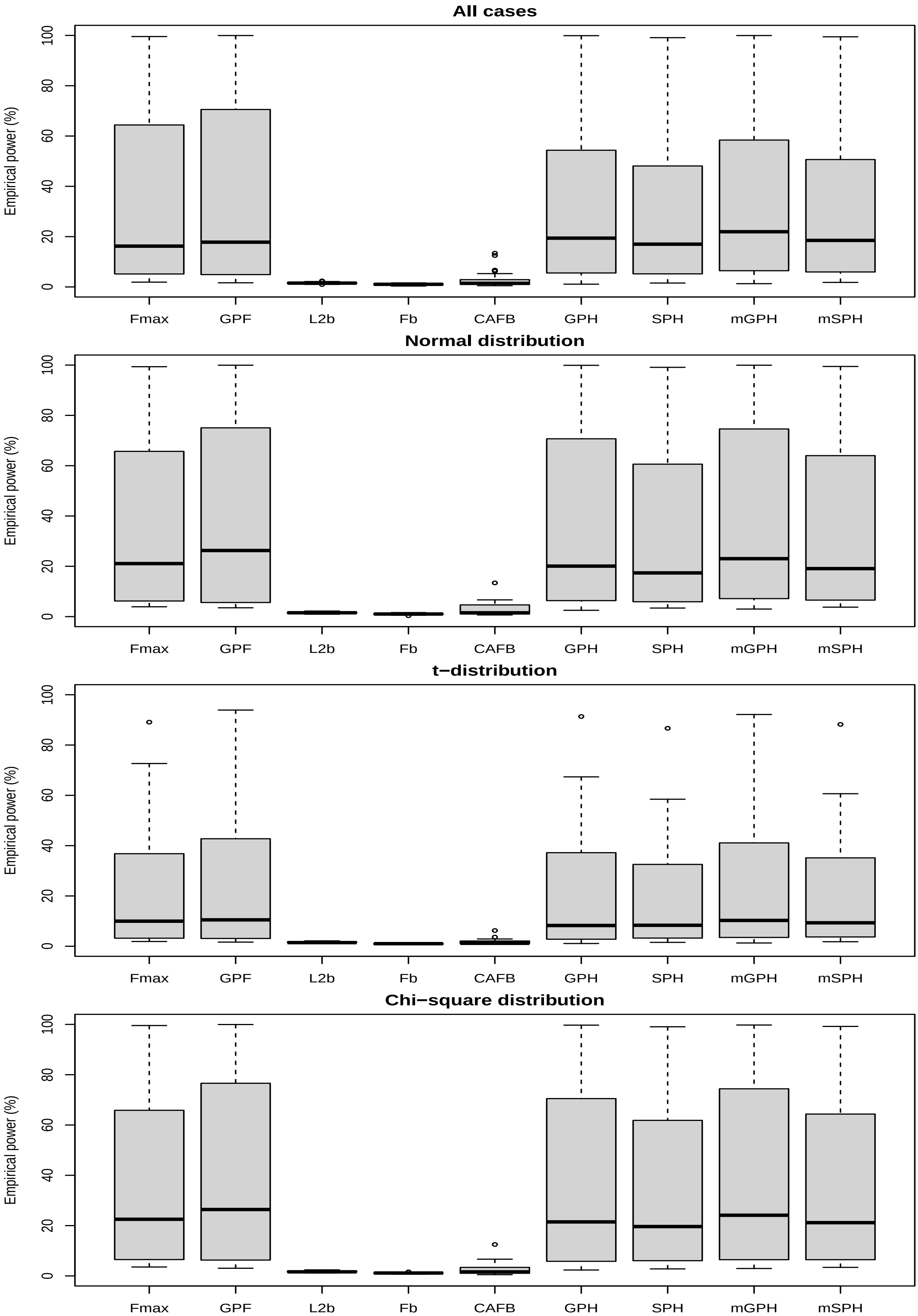


Fig. S66: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A4 for the Tukey contrasts with scaling function and different distributions

Table S30: Empirical sizes ($\mathcal{H}_{0,1}, \mathcal{H}_{0,2}, \mathcal{H}_{0,4}$) and powers ($\mathcal{H}_{0,3}, \mathcal{H}_{0,5}, \mathcal{H}_{0,6}$) (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	0.85	0.75	1.65	1.15	0.70	0.85	0.75	0.95	0.90
			$\mathcal{H}_{0,2}$	0.95	0.60	1.85	0.90	0.50	0.55	0.70	0.70	0.85
			$\mathcal{H}_{0,3}$	69.65	77.85	91.65	87.40	90.35	74.90	65.25	78.10	68.15
			$\mathcal{H}_{0,4}$	0.95	0.60	1.70	1.25	0.90	0.50	0.95	0.60	1.05
			$\mathcal{H}_{0,5}$	83.65	90.55	96.15	94.00	96.80	90.40	81.20	91.60	82.95
			$\mathcal{H}_{0,6}$	89.65	95.30	98.45	97.50	99.25	94.95	89.95	95.85	90.65
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.40	0.30	1.55	1.10	0.80	0.50	0.65	0.55	0.85
			$\mathcal{H}_{0,2}$	0.30	0.20	1.55	1.15	0.75	0.70	0.60	0.75	0.70
			$\mathcal{H}_{0,3}$	13.80	15.80	67.05	60.50	65.05	43.80	33.70	46.50	35.10
			$\mathcal{H}_{0,4}$	0.95	0.50	1.35	1.05	0.75	0.55	0.90	0.85	1.00
			$\mathcal{H}_{0,5}$	24.40	27.55	63.00	57.85	59.50	38.75	34.00	41.90	35.70
			$\mathcal{H}_{0,6}$	27.15	34.90	60.10	55.20	57.65	37.65	29.20	40.20	30.80
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.50	1.15	1.35	0.85	0.65	0.90	1.10	1.00	1.25
			$\mathcal{H}_{0,2}$	2.60	2.20	1.65	0.90	0.50	0.75	1.05	1.00	1.35
			$\mathcal{H}_{0,3}$	53.00	57.95	51.40	39.50	38.90	19.20	17.95	23.05	19.95
			$\mathcal{H}_{0,4}$	0.85	0.80	1.50	0.90	0.85	0.65	0.80	0.85	0.80
			$\mathcal{H}_{0,5}$	62.45	68.95	77.95	70.30	73.70	49.55	42.80	54.55	45.95
			$\mathcal{H}_{0,6}$	79.80	85.95	94.20	91.85	94.60	82.90	76.45	85.15	78.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.40	0.45	1.35	0.85	0.75	0.35	1.25	0.45	1.35
			$\mathcal{H}_{0,2}$	1.10	1.00	1.20	1.00	1.20	0.95	1.35	1.05	1.45
			$\mathcal{H}_{0,3}$	38.50	45.30	71.15	62.75	64.30	40.95	36.60	44.60	39.15
			$\mathcal{H}_{0,4}$	1.15	0.75	1.25	0.80	0.60	0.60	1.15	0.75	1.20
			$\mathcal{H}_{0,5}$	51.80	60.60	77.80	73.15	78.35	58.30	49.45	61.65	52.20
			$\mathcal{H}_{0,6}$	59.20	69.90	84.80	81.00	86.40	68.50	57.80	71.70	59.80
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.15	0.25	1.70	1.10	0.80	0.45	0.60	0.55	0.60
			$\mathcal{H}_{0,2}$	0.15	0.15	1.70	1.00	0.65	0.60	0.85	0.65	0.90
			$\mathcal{H}_{0,3}$	5.55	4.65	44.25	38.50	38.80	19.70	18.55	22.45	20.00
			$\mathcal{H}_{0,4}$	0.80	0.35	0.90	0.75	0.60	0.25	1.00	0.40	1.00
			$\mathcal{H}_{0,5}$	10.40	11.75	40.70	34.40	34.00	18.50	15.55	20.15	16.75
			$\mathcal{H}_{0,6}$	14.65	16.10	37.60	33.15	32.40	17.05	14.75	19.25	16.55
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.25	0.65	1.25	0.60	0.50	0.40	0.75	0.55	0.90
			$\mathcal{H}_{0,2}$	2.30	1.40	1.10	0.50	0.50	0.30	0.75	0.45	0.95
			$\mathcal{H}_{0,3}$	32.80	32.65	30.95	21.55	21.10	8.10	8.05	10.15	9.35
			$\mathcal{H}_{0,4}$	1.00	0.95	1.40	0.95	0.30	0.55	0.85	0.70	0.90
			$\mathcal{H}_{0,5}$	36.35	39.45	47.45	40.50	43.80	23.70	20.30	26.70	22.50
			$\mathcal{H}_{0,6}$	47.30	54.40	72.65	67.05	69.70	48.85	40.95	53.40	43.80
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	1.25	0.60	1.60	1.15	0.35	0.70	1.15	0.80	1.15
			$\mathcal{H}_{0,2}$	1.10	0.75	1.65	1.10	1.00	0.50	1.15	0.65	1.20
			$\mathcal{H}_{0,3}$	71.00	80.50	90.25	85.00	90.00	75.05	67.50	77.15	69.80
			$\mathcal{H}_{0,4}$	1.00	0.85	1.35	0.70	0.55	0.35	0.75	0.60	0.75
			$\mathcal{H}_{0,5}$	81.95	90.35	94.90	92.75	95.85	87.15	80.75	88.65	82.30
			$\mathcal{H}_{0,6}$	89.45	95.25	97.25	96.40	98.55	94.15	88.45	94.95	89.75
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	0.70	0.30	1.75	0.90	0.95	0.50	0.65	0.70	0.85
			$\mathcal{H}_{0,2}$	0.30	0.25	1.85	1.05	0.75	0.55	0.75	0.65	0.80
			$\mathcal{H}_{0,3}$	12.20	14.30	70.95	63.30	67.90	44.00	35.45	47.40	37.00
			$\mathcal{H}_{0,4}$	0.65	0.35	0.95	0.65	0.85	0.35	0.60	0.45	0.70
			$\mathcal{H}_{0,5}$	25.40	29.60	66.30	60.20	62.95	41.40	36.00	44.90	38.30
			$\mathcal{H}_{0,6}$	30.10	34.00	63.05	57.95	58.65	36.30	31.90	39.10	33.60
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.10	0.75	1.70	1.00	0.70	0.55	0.90	0.75	1.05
			$\mathcal{H}_{0,2}$	2.60	1.85	1.70	0.85	0.95	0.35	0.95	0.70	1.20
			$\mathcal{H}_{0,3}$	54.90	56.70	51.25	40.60	46.55	25.05	23.95	28.10	25.85
			$\mathcal{H}_{0,4}$	1.00	0.85	1.05	0.55	0.85	0.50	1.10	0.65	1.20
			$\mathcal{H}_{0,5}$	65.15	70.50	75.35	67.75	74.05	52.95	47.40	57.85	50.90
			$\mathcal{H}_{0,6}$	79.15	86.00	92.35	89.35	93.60	81.80	74.50	84.40	76.70

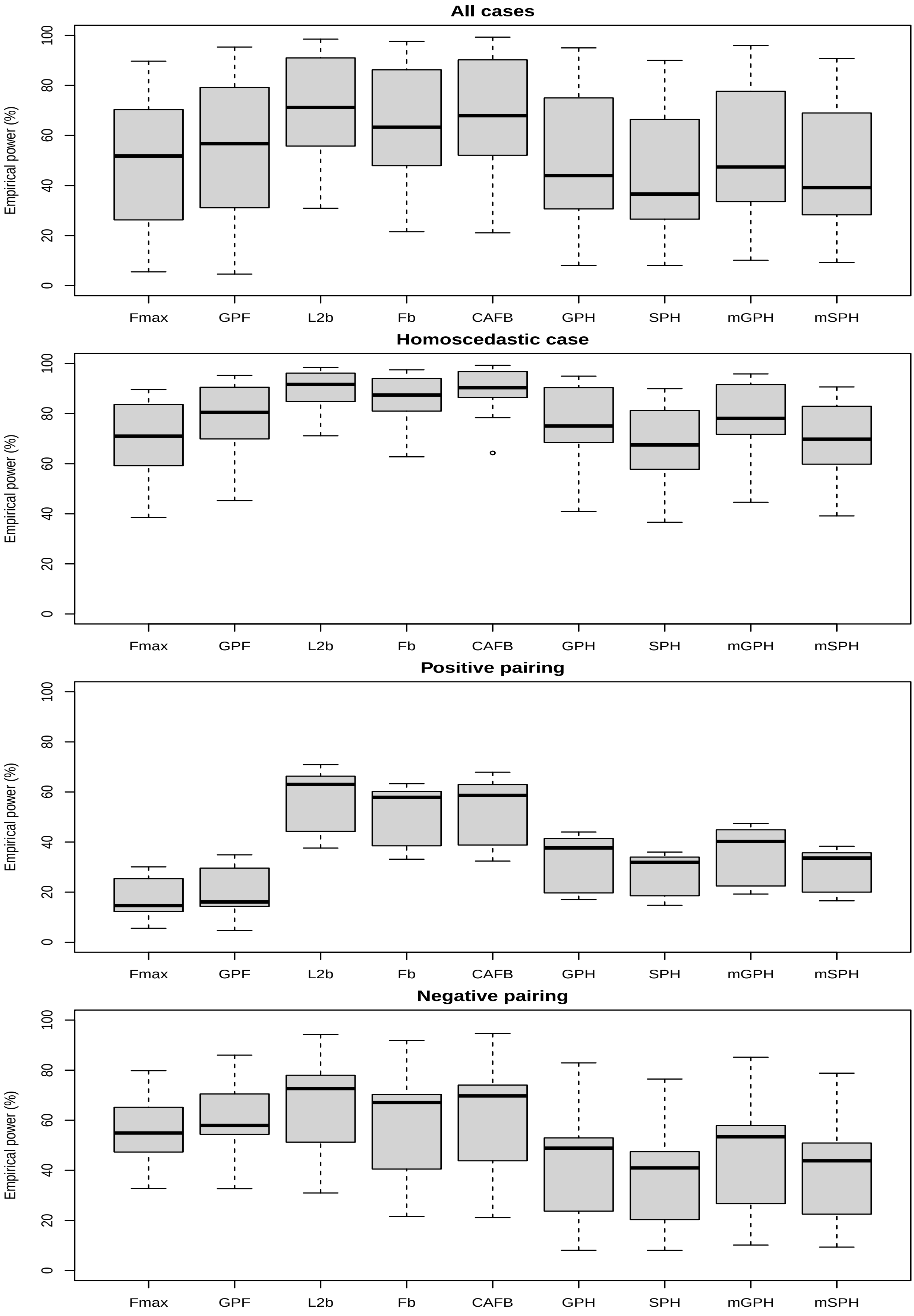


Fig. S67: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

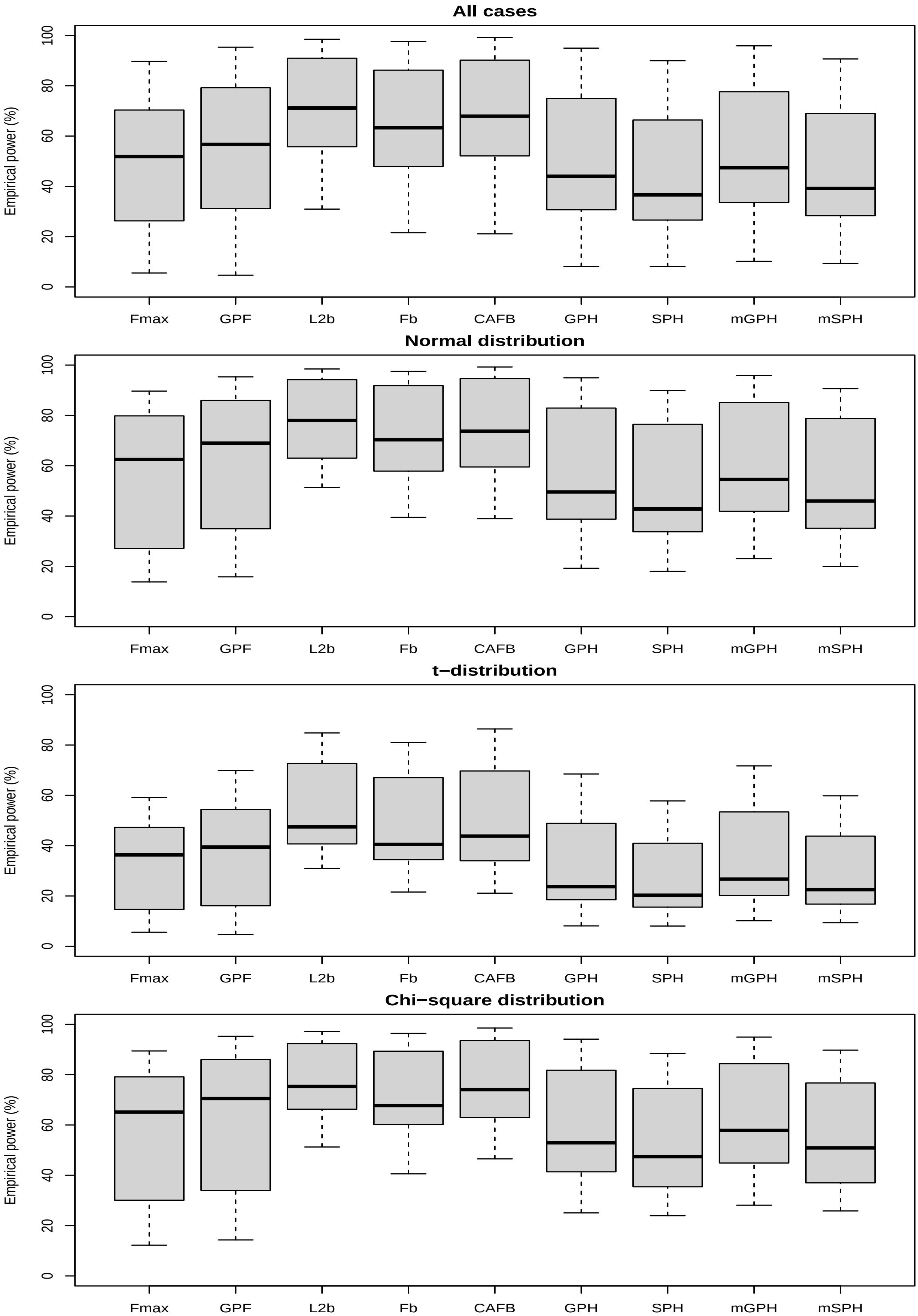


Fig. S68: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A5 for the Tukey contrasts with scaling function and different distributions

Table S31: Empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts with scaling function (D - distribution; $\mathbf{n} = (n_1, n_2, n_3, n_4)$: $\mathbf{n}_1 = (15, 20, 25, 30)$, $\mathbf{n}_2 = (30, 40, 50, 60)$, $\mathbf{n}_3 = (60, 80, 100, 120)$; $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$: $\boldsymbol{\lambda}_1 = (1, 1, 1, 1)$ - homoscedastic case, $\boldsymbol{\lambda}_2 = (1, 1.25, 1.5, 1.75)$ - heteroscedastic case (positive pairing), $\boldsymbol{\lambda}_3 = (1.75, 1.5, 1.25, 1)$ - heteroscedastic case (negative pairing))

D	$\mathbf{n}$	$\boldsymbol{\lambda}$	$\mathcal{H}$	Fmax	GPF	L2b	Fb	CAFB	GPH	SPH	mGPH	mSPH
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	7.75	8.25	24.15	17.15	16.00	8.60	8.00	9.85	8.60
			$\mathcal{H}_{0,2}$	62.25	72.65	88.45	82.90	85.95	70.95	59.20	74.05	60.95
			$\mathcal{H}_{0,3}$	99.10	99.90	99.90	99.75	99.90	99.75	98.30	99.95	98.50
			$\mathcal{H}_{0,4}$	10.75	12.45	32.00	27.10	24.60	12.40	10.30	14.25	11.00
			$\mathcal{H}_{0,5}$	80.30	89.65	97.20	95.30	97.50	88.50	79.40	90.25	81.45
			$\mathcal{H}_{0,6}$	17.05	19.75	40.35	35.45	33.25	19.40	17.05	21.50	18.25
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	5.10	5.60	19.10	14.45	13.65	7.00	6.35	8.05	6.90
			$\mathcal{H}_{0,2}$	24.75	30.10	74.30	66.45	70.50	47.70	39.00	50.60	40.95
			$\mathcal{H}_{0,3}$	61.80	72.60	97.50	96.40	98.15	93.75	87.00	94.80	88.50
			$\mathcal{H}_{0,4}$	5.45	4.60	14.95	11.50	10.95	5.55	6.00	6.45	6.40
			$\mathcal{H}_{0,5}$	24.45	33.05	63.85	58.20	60.70	42.90	34.70	45.80	36.40
			$\mathcal{H}_{0,6}$	4.15	3.80	11.40	9.55	7.80	4.80	4.60	5.50	4.95
N	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	2.95	2.60	8.15	5.30	4.50	1.60	2.15	2.10	2.75
			$\mathcal{H}_{0,2}$	29.55	33.35	45.55	35.25	35.45	16.80	16.05	19.55	18.15
			$\mathcal{H}_{0,3}$	90.95	94.20	89.55	80.80	85.10	66.45	55.10	71.10	58.35
			$\mathcal{H}_{0,4}$	6.50	6.75	16.55	13.15	10.45	5.25	5.15	6.75	5.90
			$\mathcal{H}_{0,5}$	64.95	70.90	77.80	70.25	74.60	54.40	44.70	58.70	47.95
			$\mathcal{H}_{0,6}$	12.75	14.80	30.30	25.35	23.65	12.55	10.55	14.90	11.95
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	5.00	4.00	14.40	9.80	9.15	4.00	4.80	4.55	5.35
			$\mathcal{H}_{0,2}$	33.85	40.50	67.15	59.40	62.35	37.20	31.55	40.60	33.95
			$\mathcal{H}_{0,3}$	88.40	94.50	97.60	95.85	97.75	91.80	85.30	93.30	87.05
			$\mathcal{H}_{0,4}$	4.95	5.95	17.60	14.45	11.55	5.50	5.00	6.50	5.50
			$\mathcal{H}_{0,5}$	49.70	57.85	80.10	75.10	78.45	55.70	47.65	58.95	49.90
			$\mathcal{H}_{0,6}$	8.25	8.65	22.15	18.70	17.50	8.40	7.50	9.40	8.60
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	2.60	2.15	10.85	7.55	6.85	2.90	3.40	3.20	3.75
			$\mathcal{H}_{0,2}$	10.35	11.90	46.10	39.45	40.25	21.40	18.70	24.50	19.80
			$\mathcal{H}_{0,3}$	29.20	33.20	84.05	79.15	83.95	65.80	58.90	68.65	60.55
			$\mathcal{H}_{0,4}$	2.35	2.15	9.00	7.05	5.50	2.80	2.85	3.35	3.05
			$\mathcal{H}_{0,5}$	11.15	12.95	39.80	34.75	35.30	19.10	16.50	21.15	17.70
			$\mathcal{H}_{0,6}$	2.75	2.70	8.35	6.90	5.15	2.60	2.70	3.20	3.15
t	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	1.75	1.35	4.70	2.95	1.85	0.60	1.25	0.90	1.50
			$\mathcal{H}_{0,2}$	16.90	16.25	26.45	18.80	18.70	7.25	8.40	8.70	9.50
			$\mathcal{H}_{0,3}$	68.50	73.25	68.45	55.90	59.50	34.75	29.45	38.70	32.70
			$\mathcal{H}_{0,4}$	2.90	3.00	9.55	6.85	5.00	2.25	2.40	2.85	2.70
			$\mathcal{H}_{0,5}$	34.55	38.60	50.80	41.65	42.55	22.90	20.45	26.00	22.40
			$\mathcal{H}_{0,6}$	6.05	6.25	14.85	11.20	9.85	4.60	5.10	5.80	5.50
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_1$	$\mathcal{H}_{0,1}$	8.25	8.60	27.05	20.90	19.60	9.05	8.05	10.00	8.80
			$\mathcal{H}_{0,2}$	67.10	76.90	88.90	83.00	88.65	73.15	66.80	75.30	68.45
			$\mathcal{H}_{0,3}$	99.15	99.80	99.85	99.50	99.85	99.35	98.15	99.50	98.30
			$\mathcal{H}_{0,4}$	11.85	13.60	35.05	29.30	28.40	13.50	12.30	15.20	13.50
			$\mathcal{H}_{0,5}$	83.10	89.60	95.50	93.10	96.05	87.35	81.05	89.30	82.30
			$\mathcal{H}_{0,6}$	18.10	20.30	40.35	35.45	36.20	20.00	18.50	22.35	19.80
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_2$	$\mathcal{H}_{0,1}$	3.80	2.90	18.55	13.70	11.90	4.50	4.95	5.55	5.30
			$\mathcal{H}_{0,2}$	24.25	26.95	74.60	67.90	71.10	45.95	39.35	48.70	41.60
			$\mathcal{H}_{0,3}$	62.05	74.30	98.10	97.05	98.40	91.50	86.65	93.15	87.95
			$\mathcal{H}_{0,4}$	5.05	4.35	16.25	12.95	10.65	5.35	5.65	6.25	6.25
			$\mathcal{H}_{0,5}$	25.20	31.00	65.00	59.75	63.15	42.45	35.10	45.15	37.15
			$\mathcal{H}_{0,6}$	4.20	4.90	13.70	11.30	9.10	5.45	4.40	6.80	4.90
χ^2	$\mathbf{n}_1$	$\boldsymbol{\lambda}_3$	$\mathcal{H}_{0,1}$	3.60	3.00	8.65	6.10	4.60	2.35	3.00	3.20	3.85
			$\mathcal{H}_{0,2}$	33.45	36.05	48.20	38.00	41.00	21.90	20.55	24.40	22.65
			$\mathcal{H}_{0,3}$	91.85	92.55	87.95	77.60	84.55	67.45	60.30	71.15	62.90
			$\mathcal{H}_{0,4}$	6.50	6.35	17.00	13.35	12.85	5.45	5.15	6.55	6.00
			$\mathcal{H}_{0,5}$	64.95	71.00	74.70	67.25	74.00	53.95	47.80	58.45	51.20
			$\mathcal{H}_{0,6}$	14.95	15.75	30.00	26.05	25.75	13.30	12.70	15.15	14.00

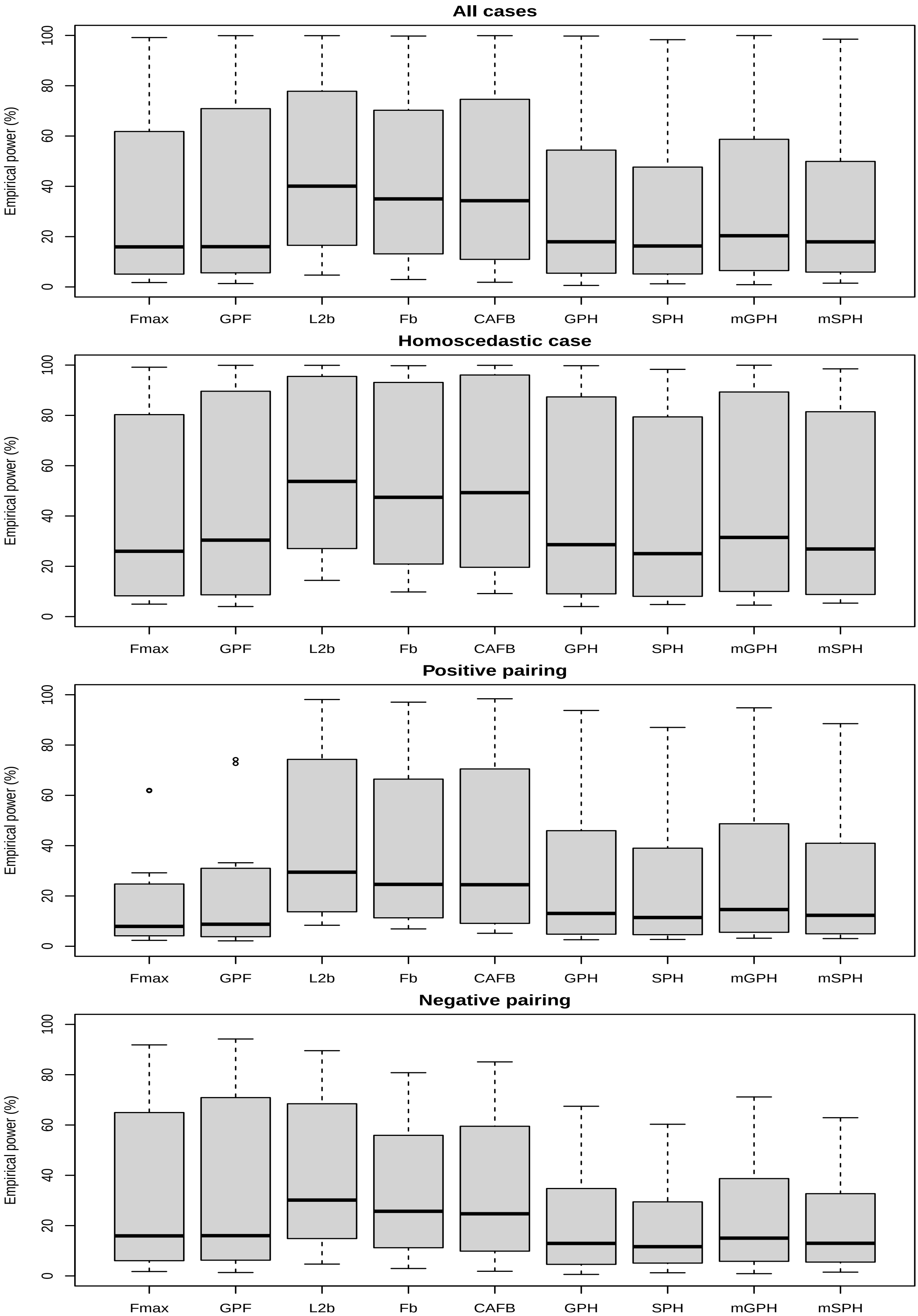


Fig. S69: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts with scaling function and homoscedastic and heteroscedastic cases

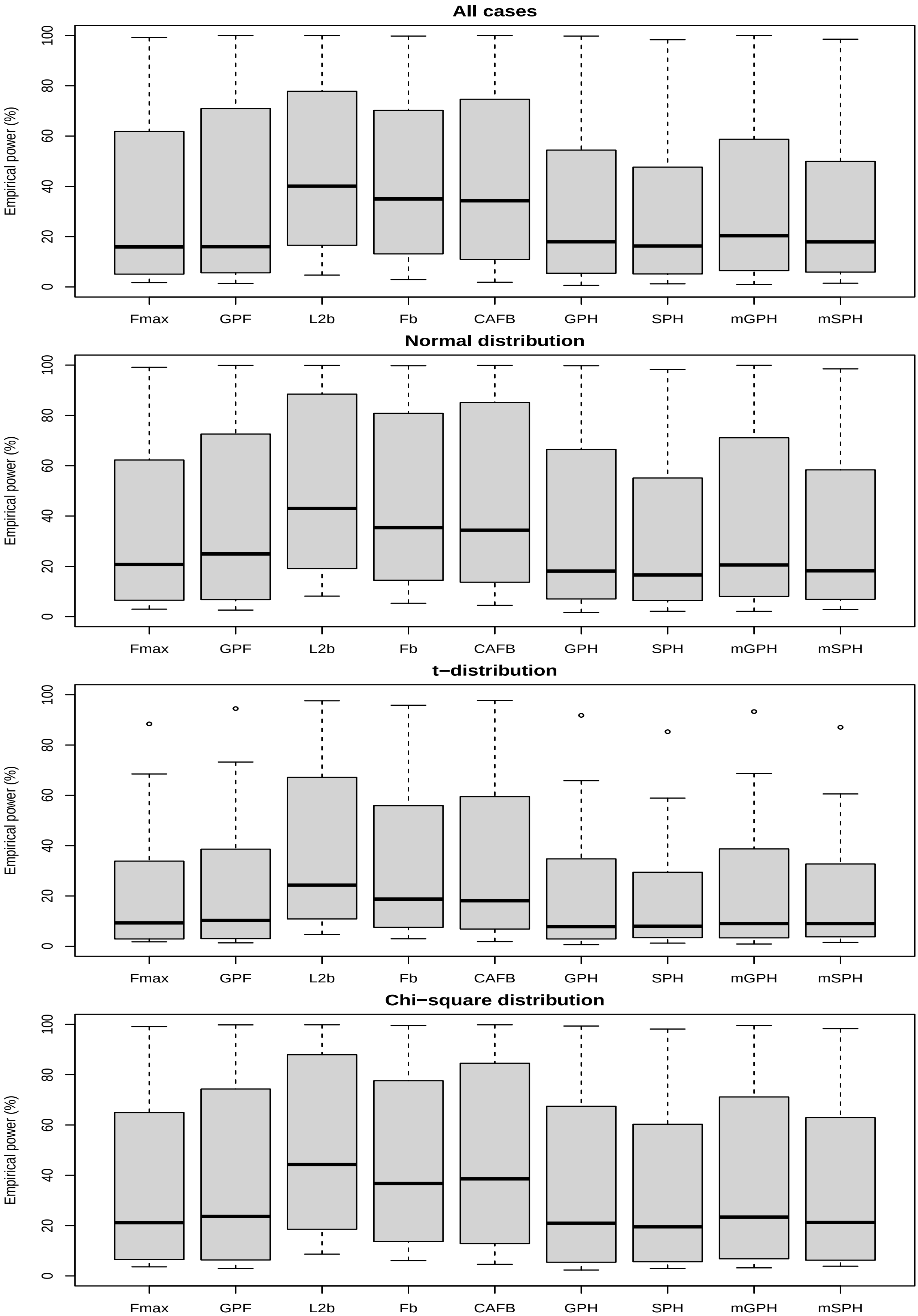


Fig. S70: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A6 for the Tukey contrasts with scaling function and different distributions

S6. Additional Simulation Studies for Heteroscedasticity Level

To investigate the effect of the heteroscedasticity level, we conducted additional simulation studies under the null hypothesis and Alternative A1 for $(n_1, n_2, n_3, n_4) = (15, 20, 25, 30)$. In the setup, we have considered two additional configurations for heteroscedasticity: positive pairing $\lambda_i = 1 + 0.5(i - 1)$ and negative pairing $\lambda_i = 2.5 - 0.5(i - 1)$, where $i = 1, 2, 3, 4$. In comparison to the values of λ_i considered before, the variances are mainly larger, and the differences between them are also larger. Thus, the heteroscedasticity level increases. The results of the comparison are presented in Figures S71-S78. In general, the results for the new λ_i are very similar to the previous ones. There are just two points to mention. First of all, the conservativity or liberality of a test can be even clearer. For example, the empirical sizes of the Fmax- and GPF-tests for a new negative pairing case are much greater than it is for a smaller heteroscedasticity level - liberality of these two tests increases. Second, when the heteroscedasticity level increases, the empirical power decreases, which is natural.

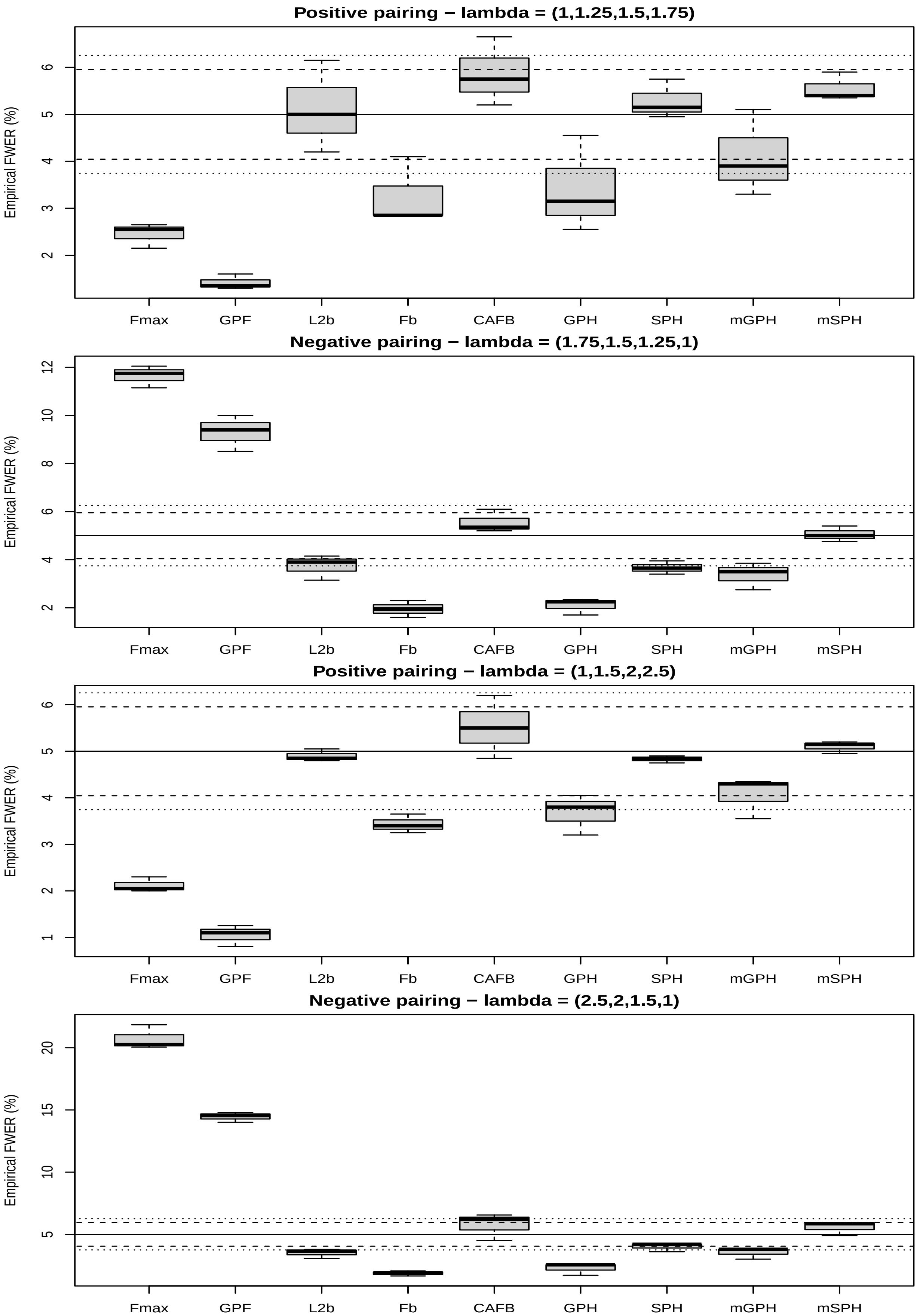


Fig. S71: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts for various heteroscedasticity levels

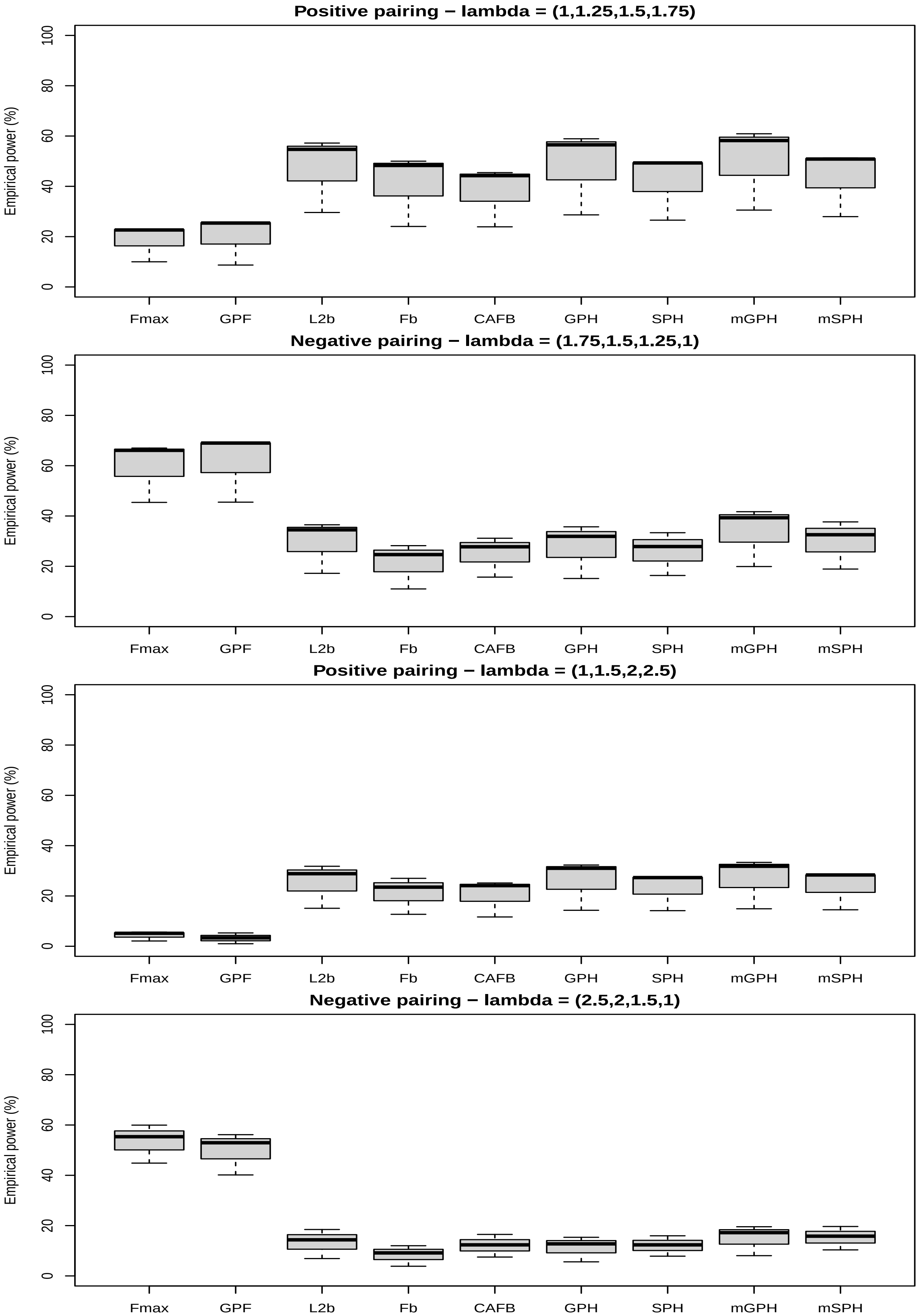


Fig. S72: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and various heteroscedasticity levels

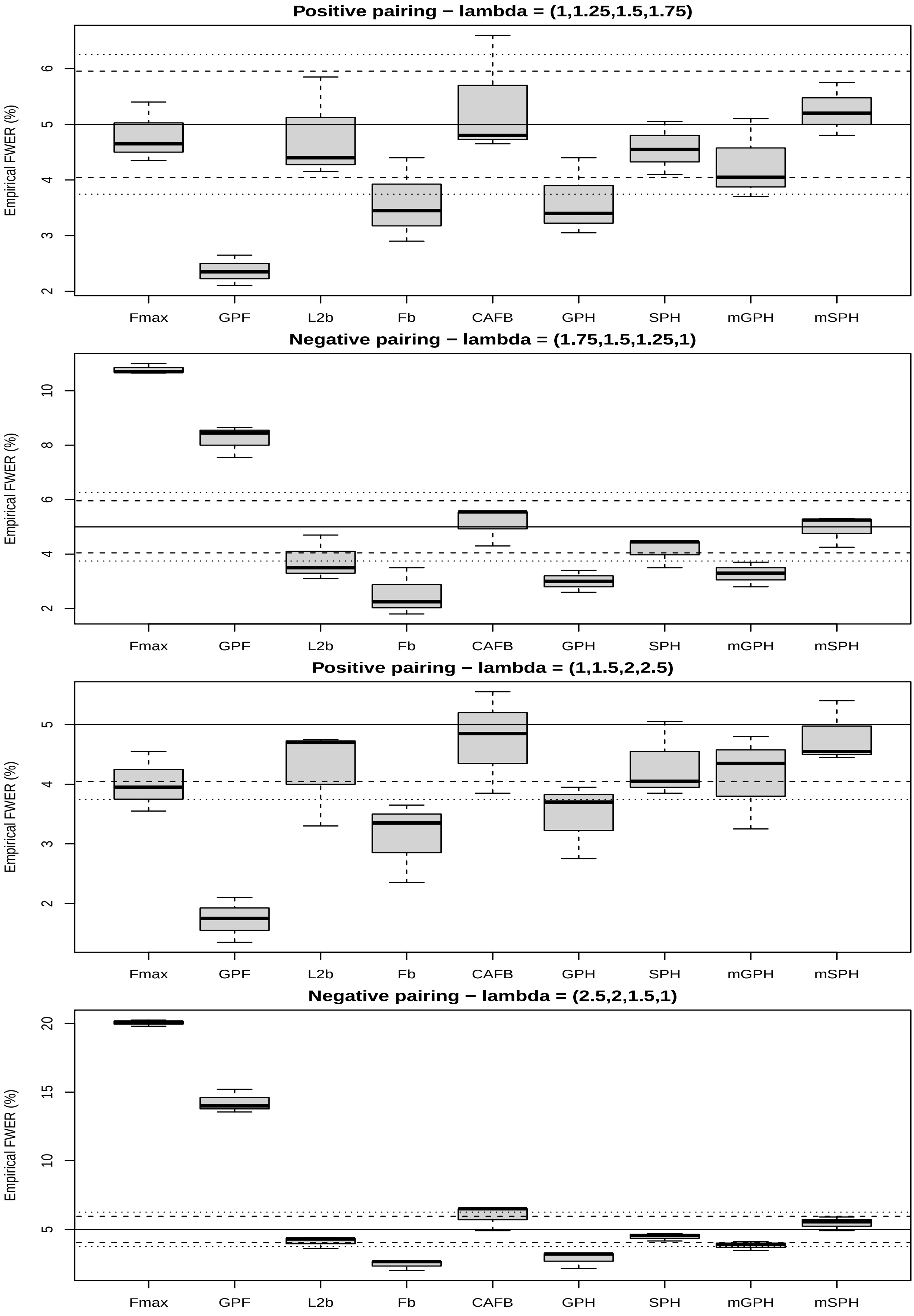


Fig. S73: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts and various heteroscedasticity levels

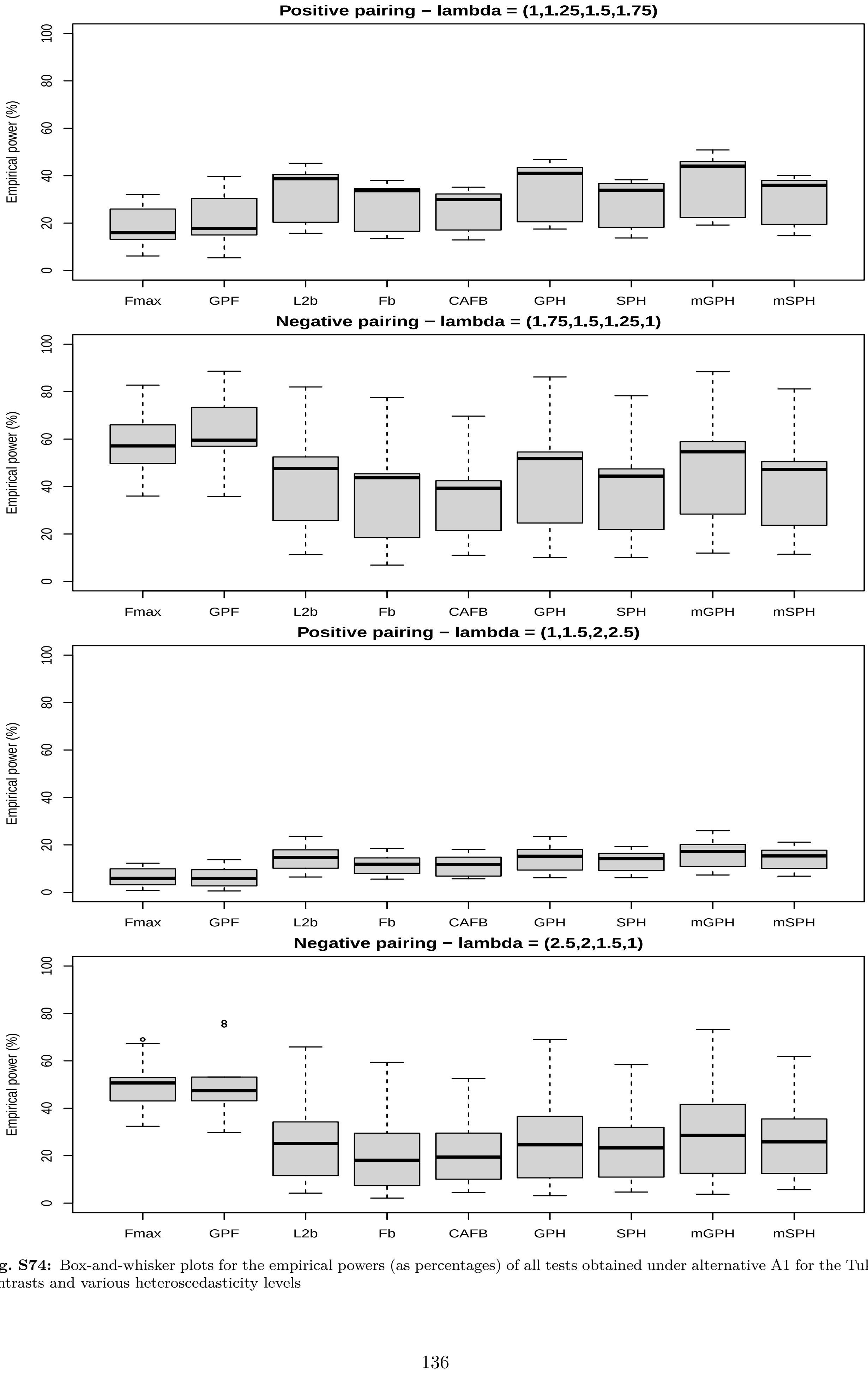


Fig. S74: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and various heteroscedasticity levels

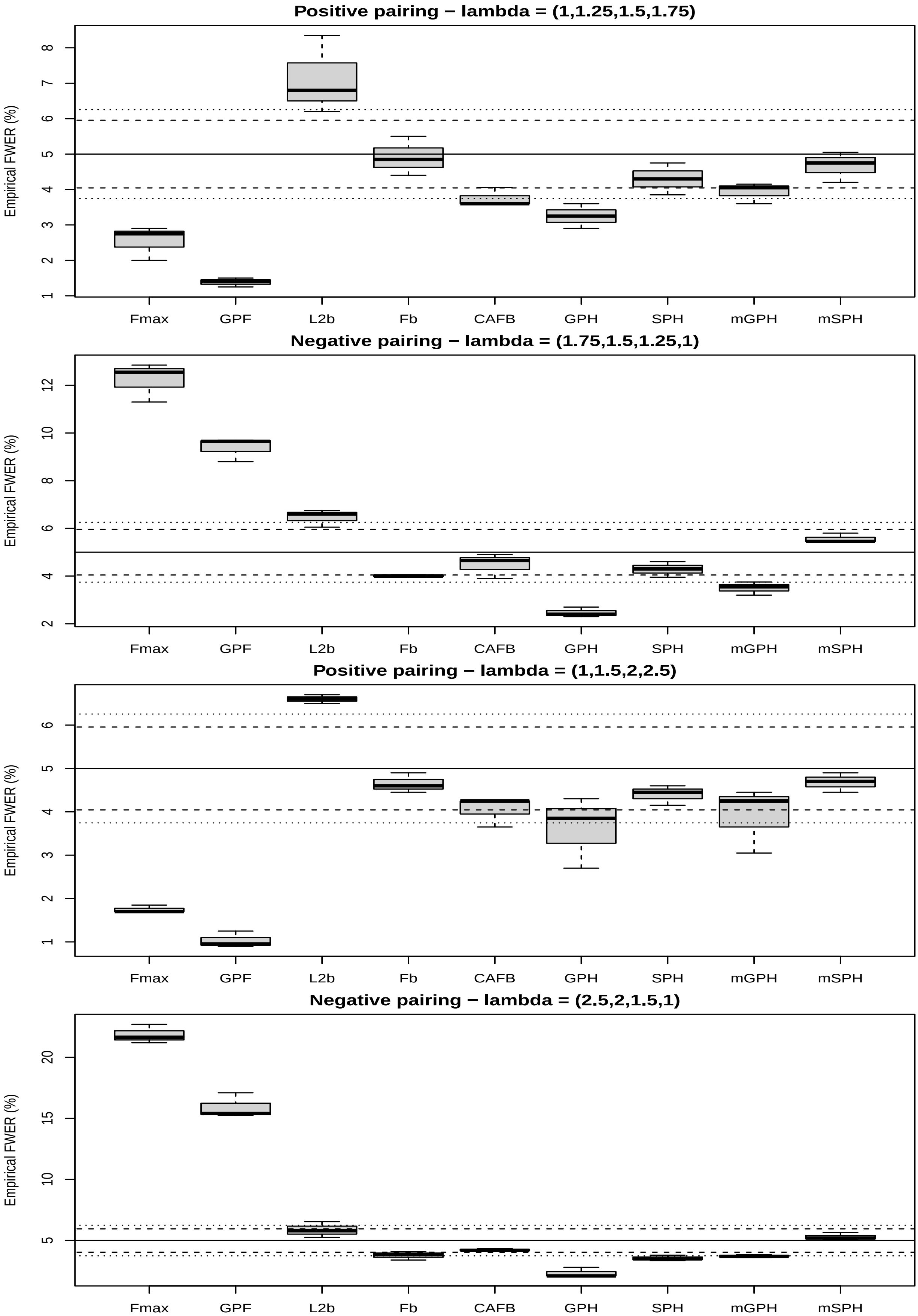


Fig. S75: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts with scaling function for various heteroscedasticity levels

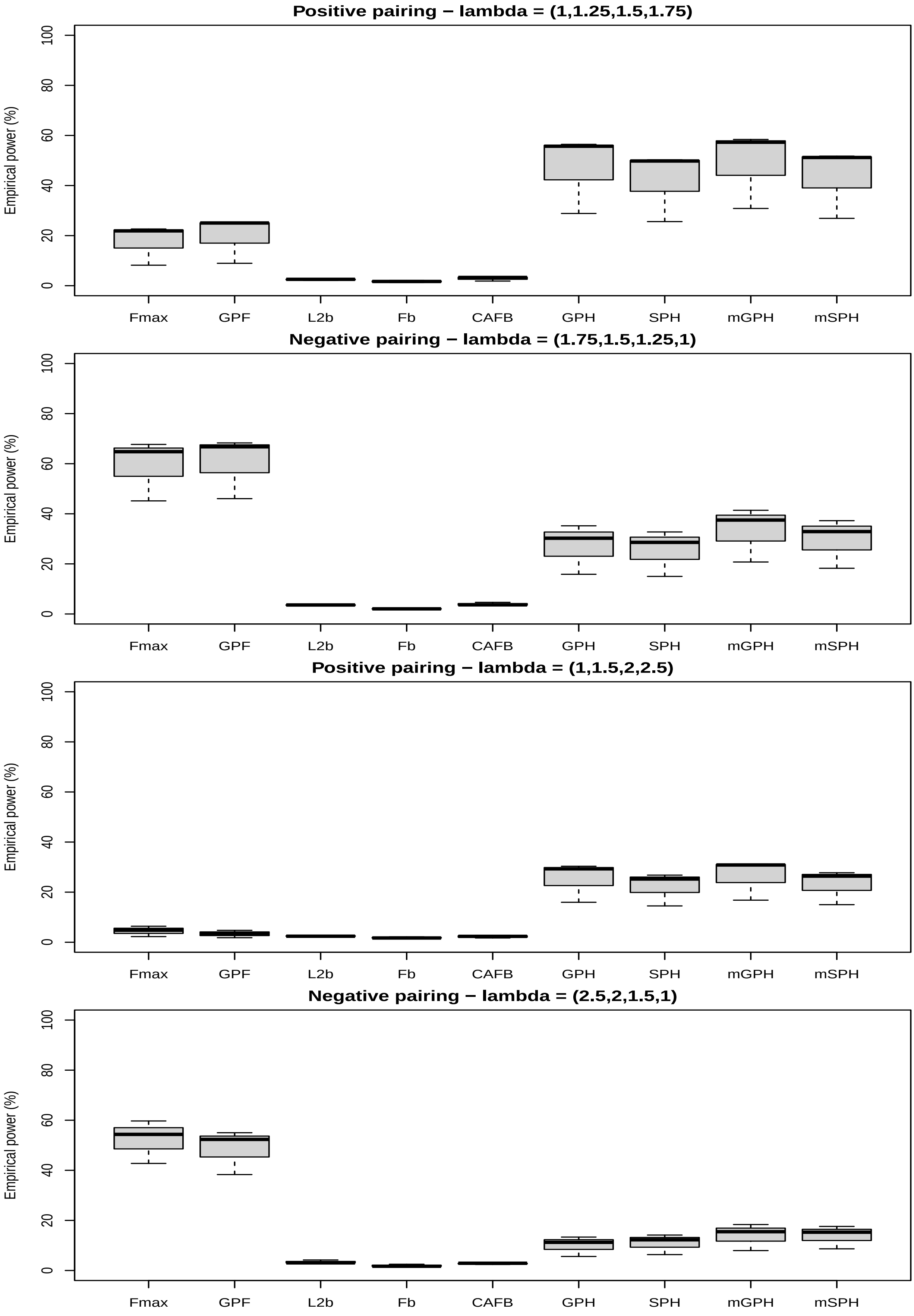


Fig. S76: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts with scaling function and various heteroscedasticity levels

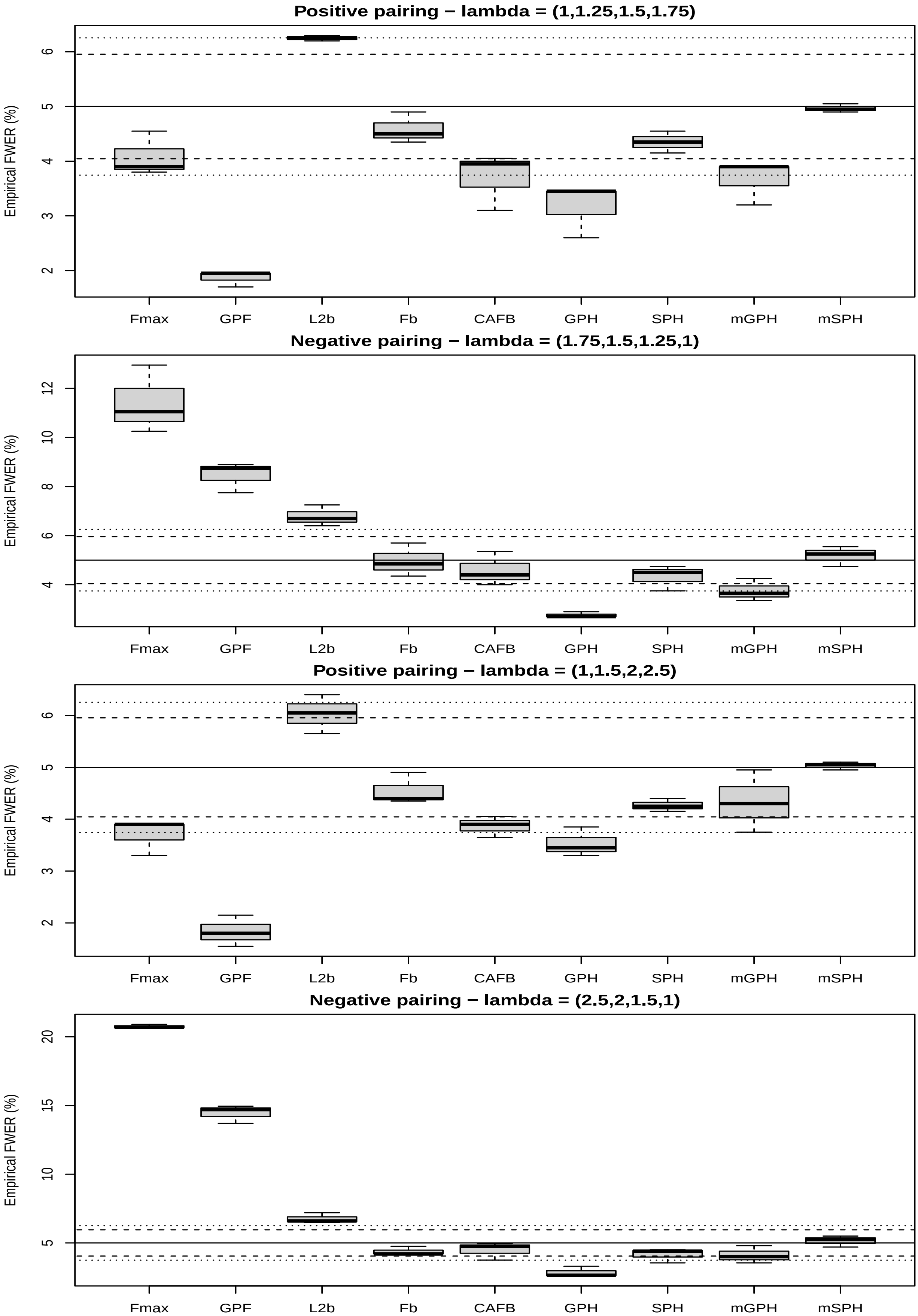


Fig. S77: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts with scaling function and various heteroscedasticity levels

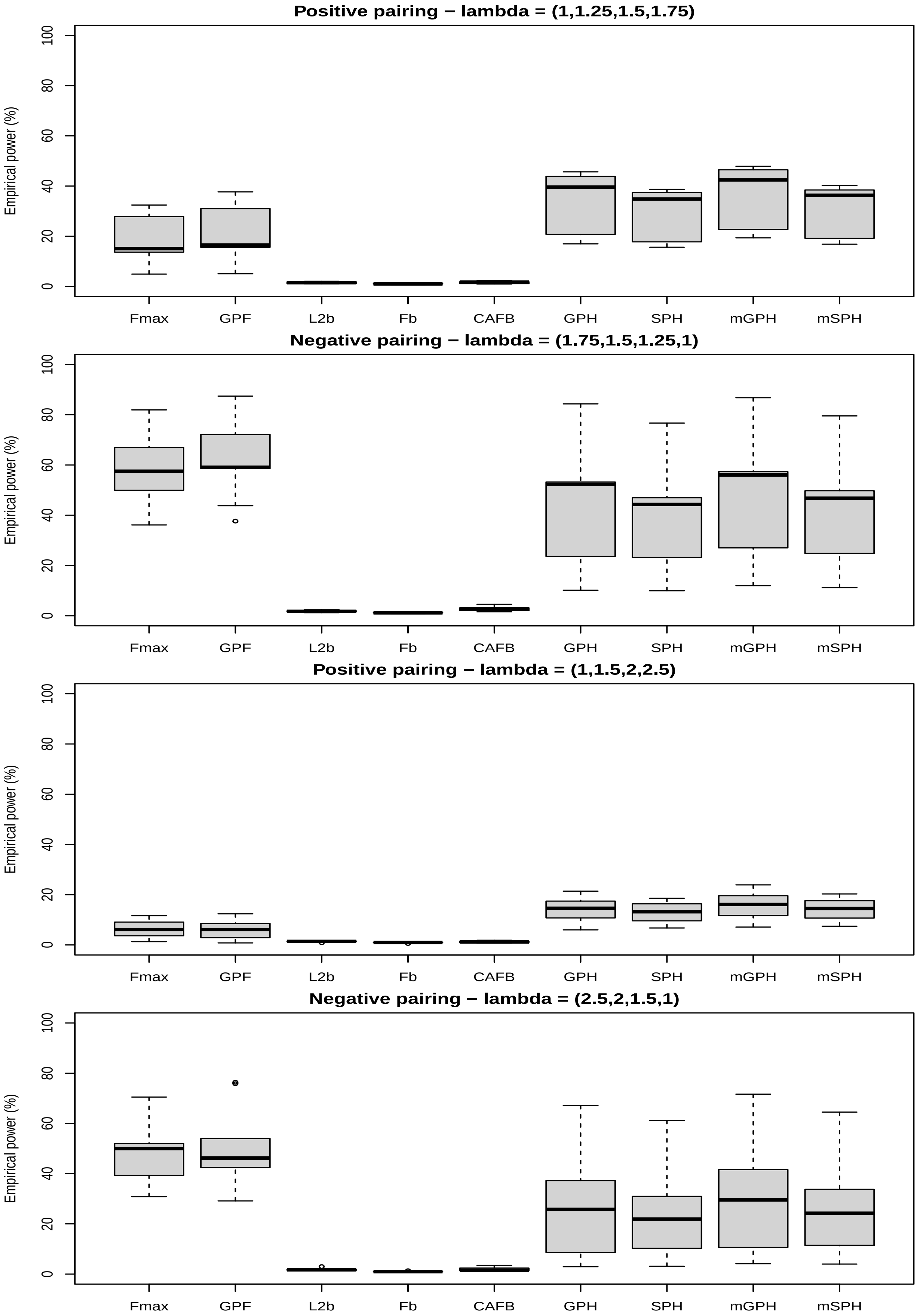


Fig. S78: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts with scaling function and various heteroscedasticity levels

S7. Additional Simulation Studies for Sample Sizes

In this section, we investigate the power behavior of the tests when the sample size increases. We simulate the data in the scenario of alternative A1 with $(n_1, n_2, n_3, n_4) = K \cdot (15, 20, 25, 30)$ for $K = 1, 2, 4$. The resulting empirical powers are summarized in the box-and-whiskers plots in Figures S79-S90. As we expected, the empirical powers increase with the increase in sample size, illustrating the consistency of the tests. The exception is in the scaled functional data, where the L2b- and Fb-tests, which are not scale-invariant, have the trivial power in all cases (see Figures S85-S90).

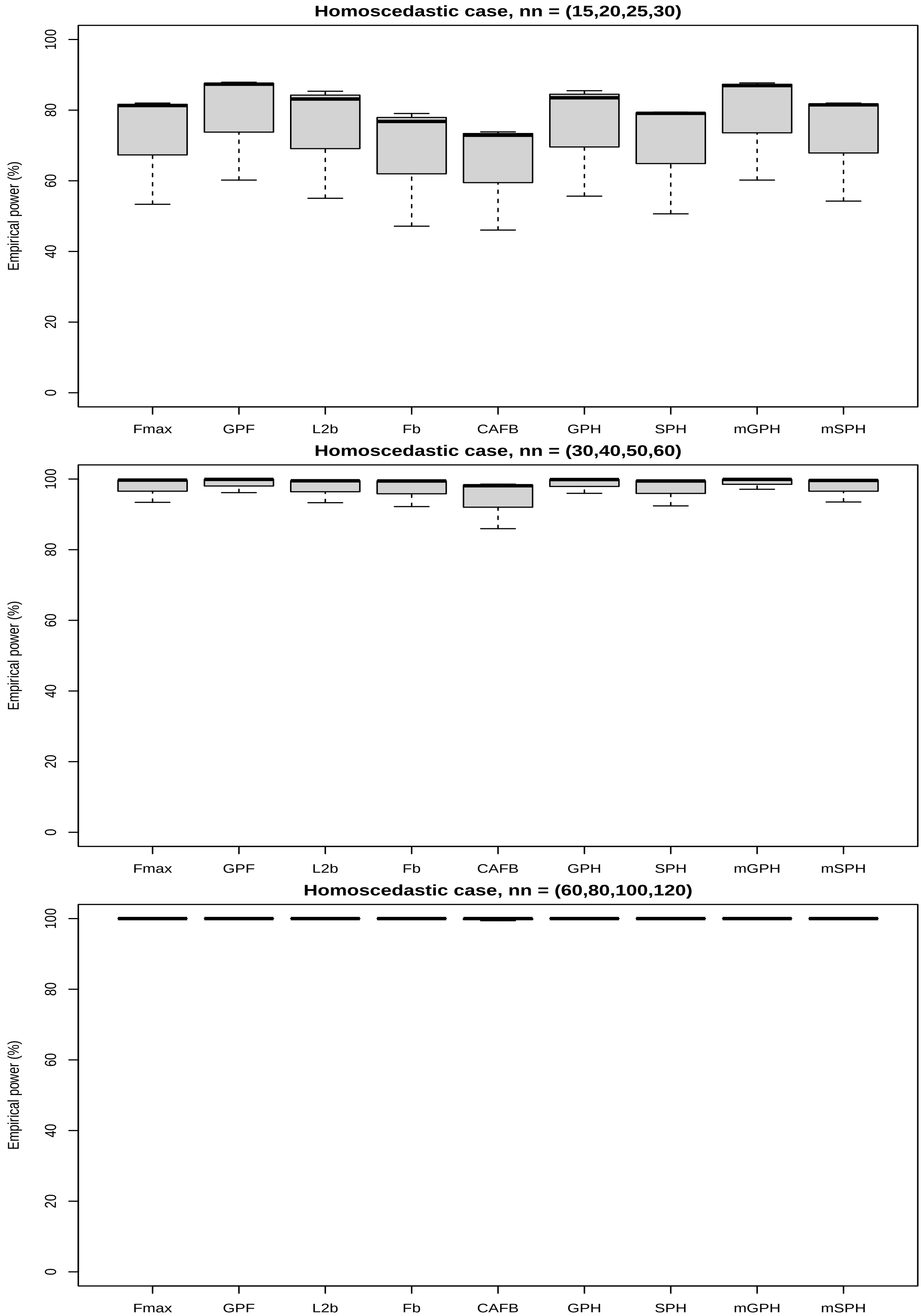


Fig. S79: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes and homoscedastic case

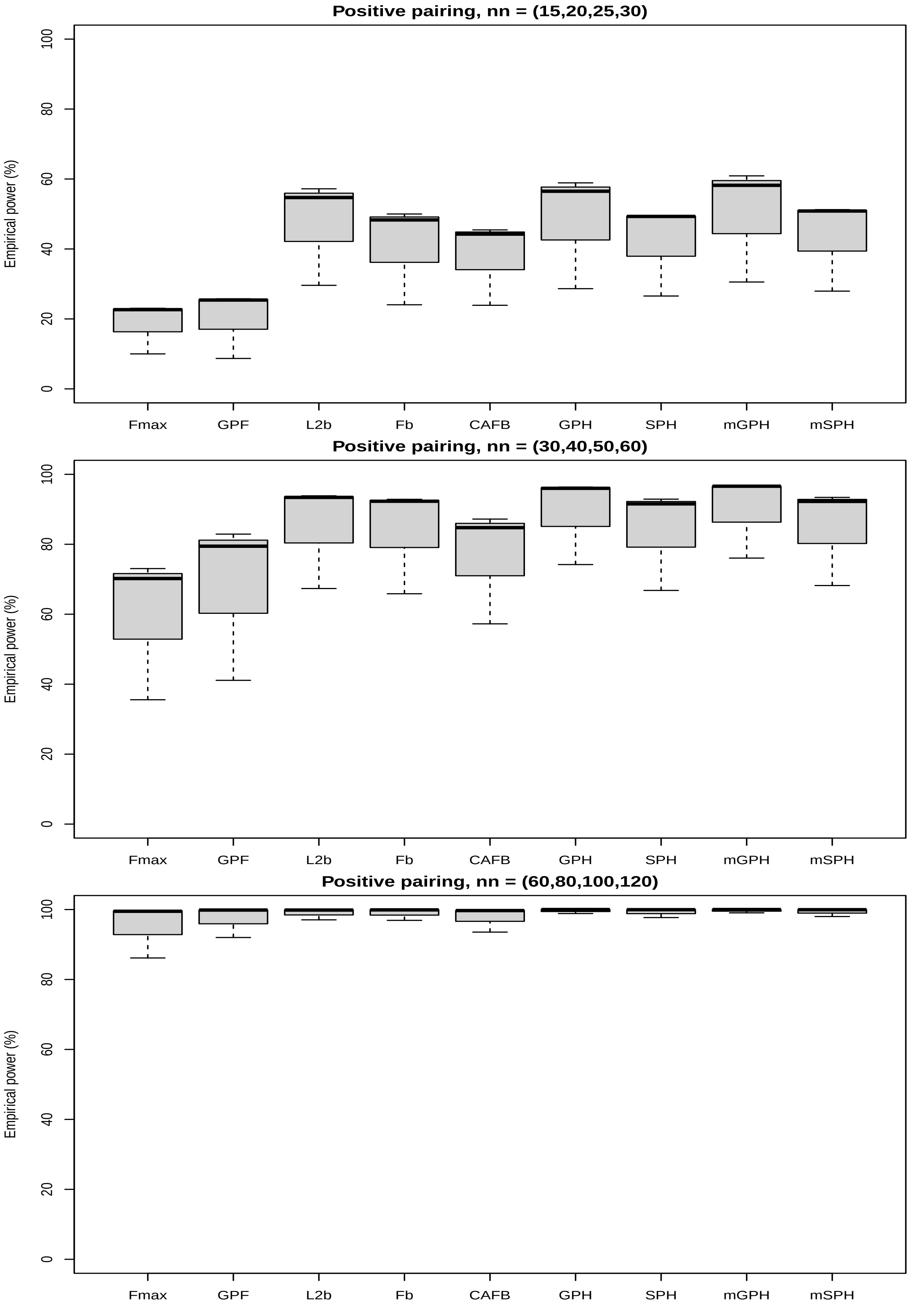


Fig. S80: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes and positive pairing

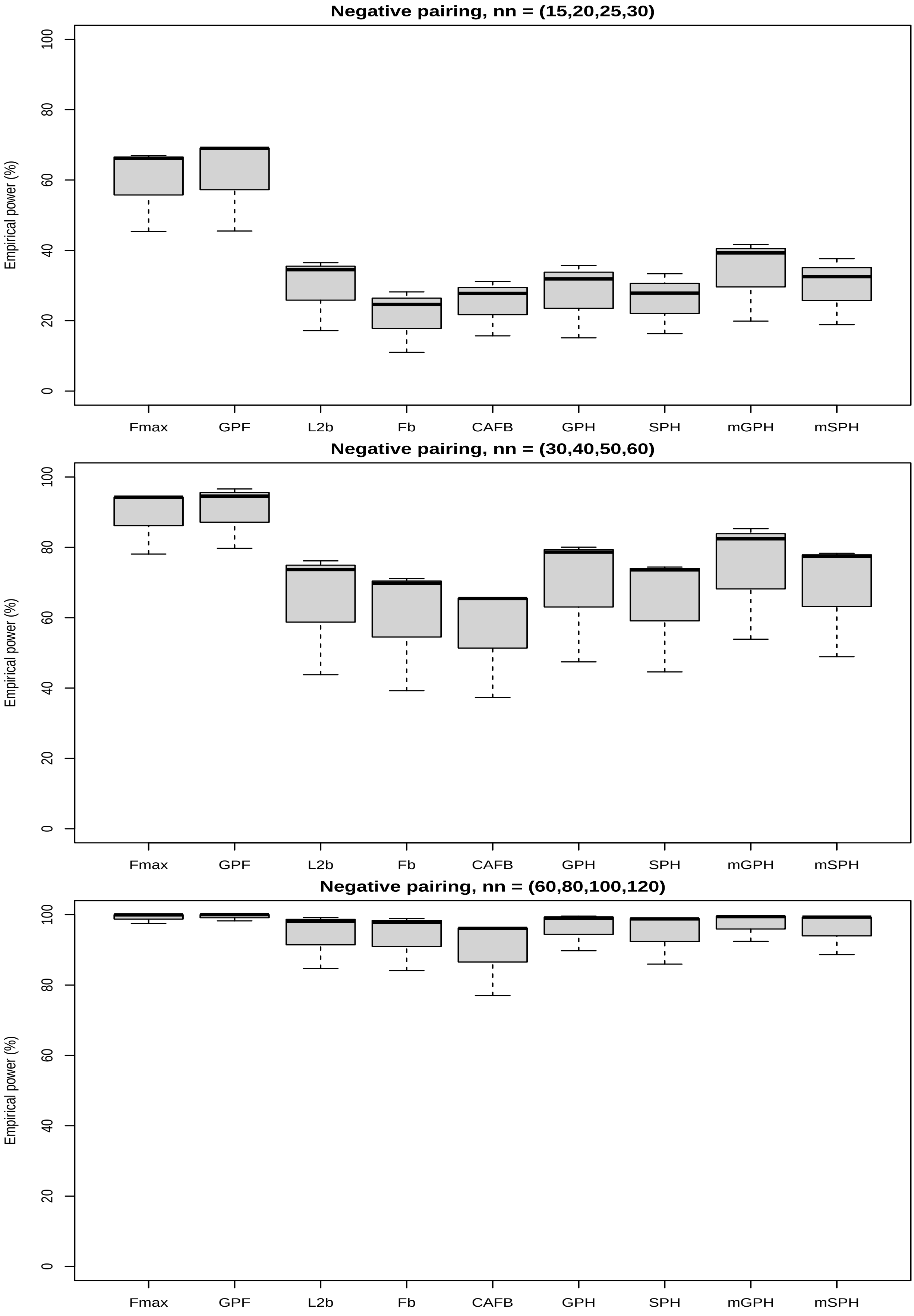


Fig. S81: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes and negative pairing

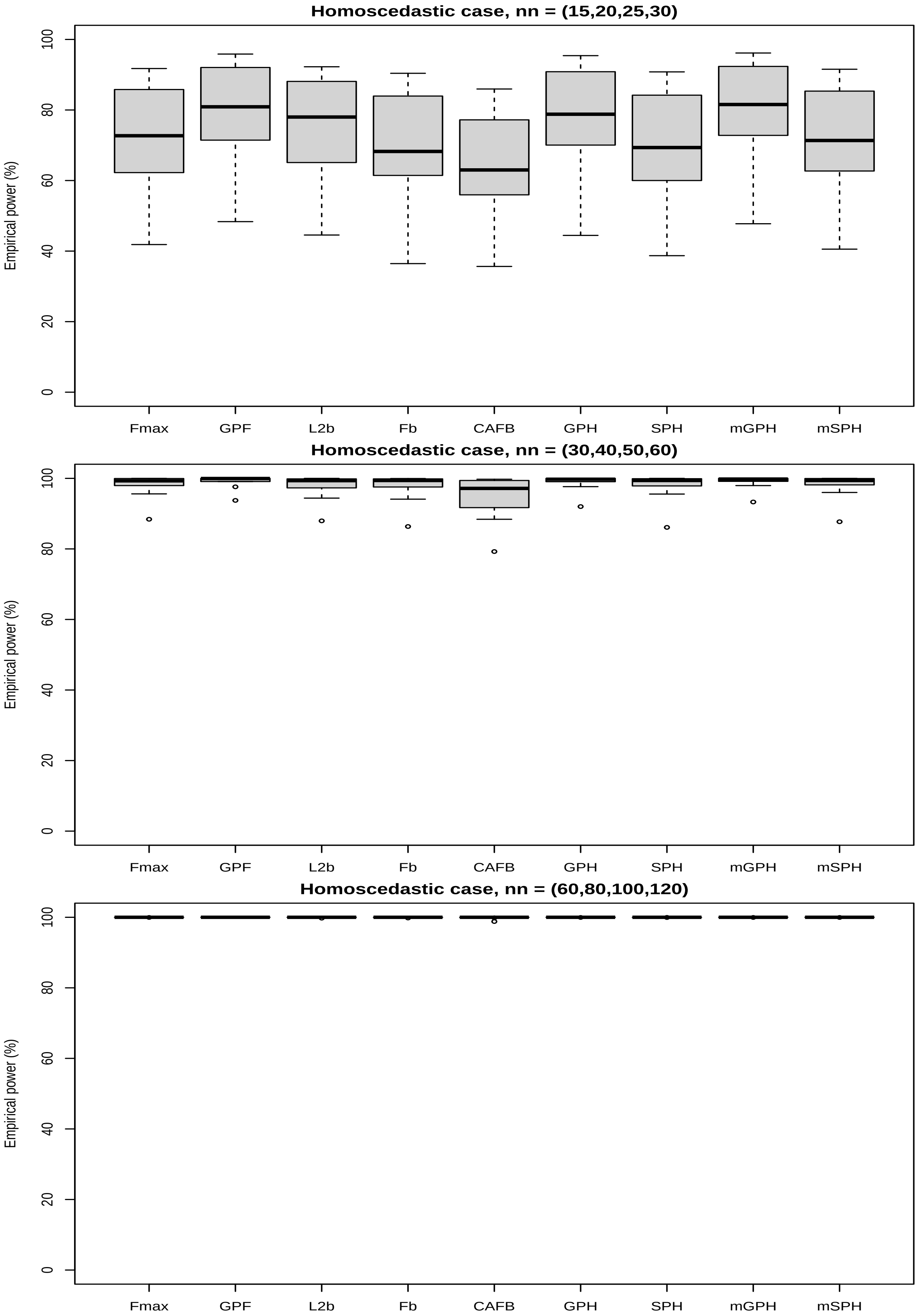
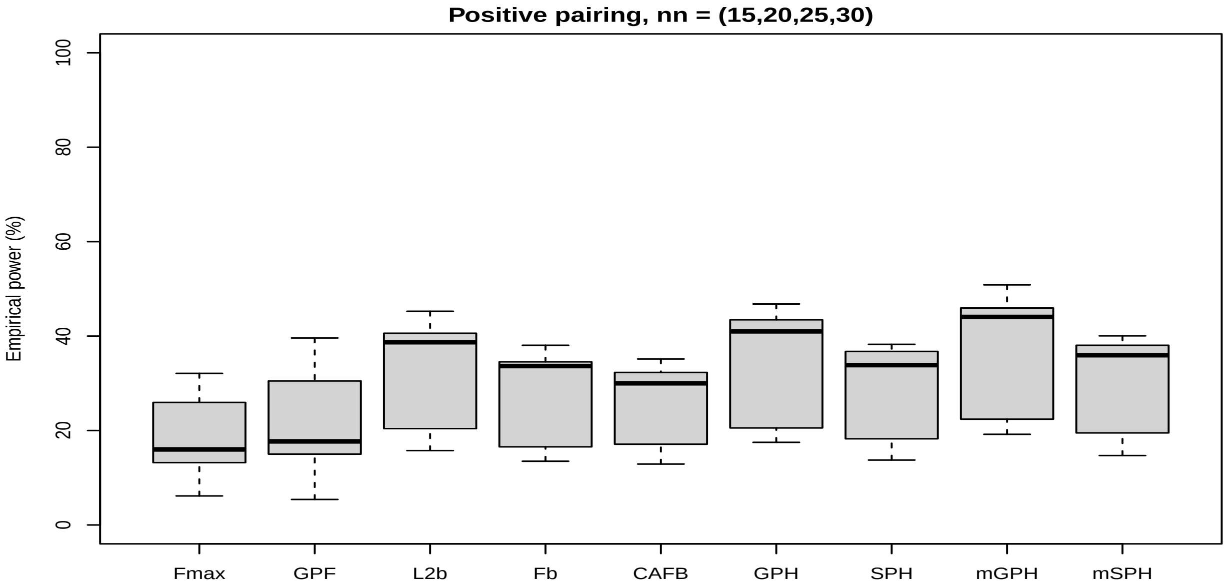
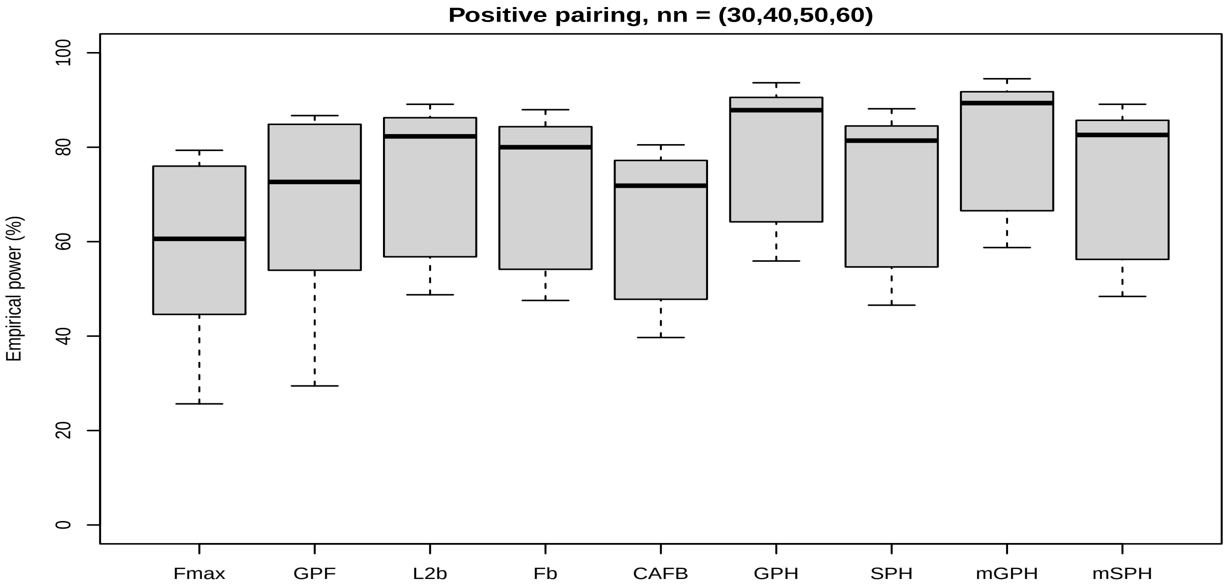


Fig. S82: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes and homoscedastic case





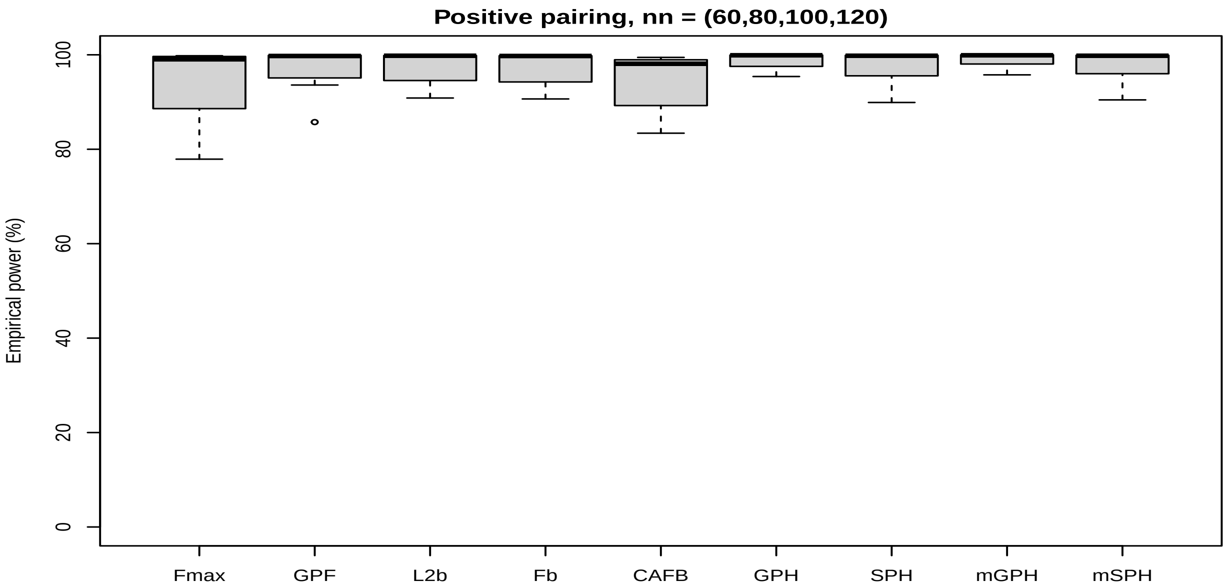


Fig. S83: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes and positive pairing

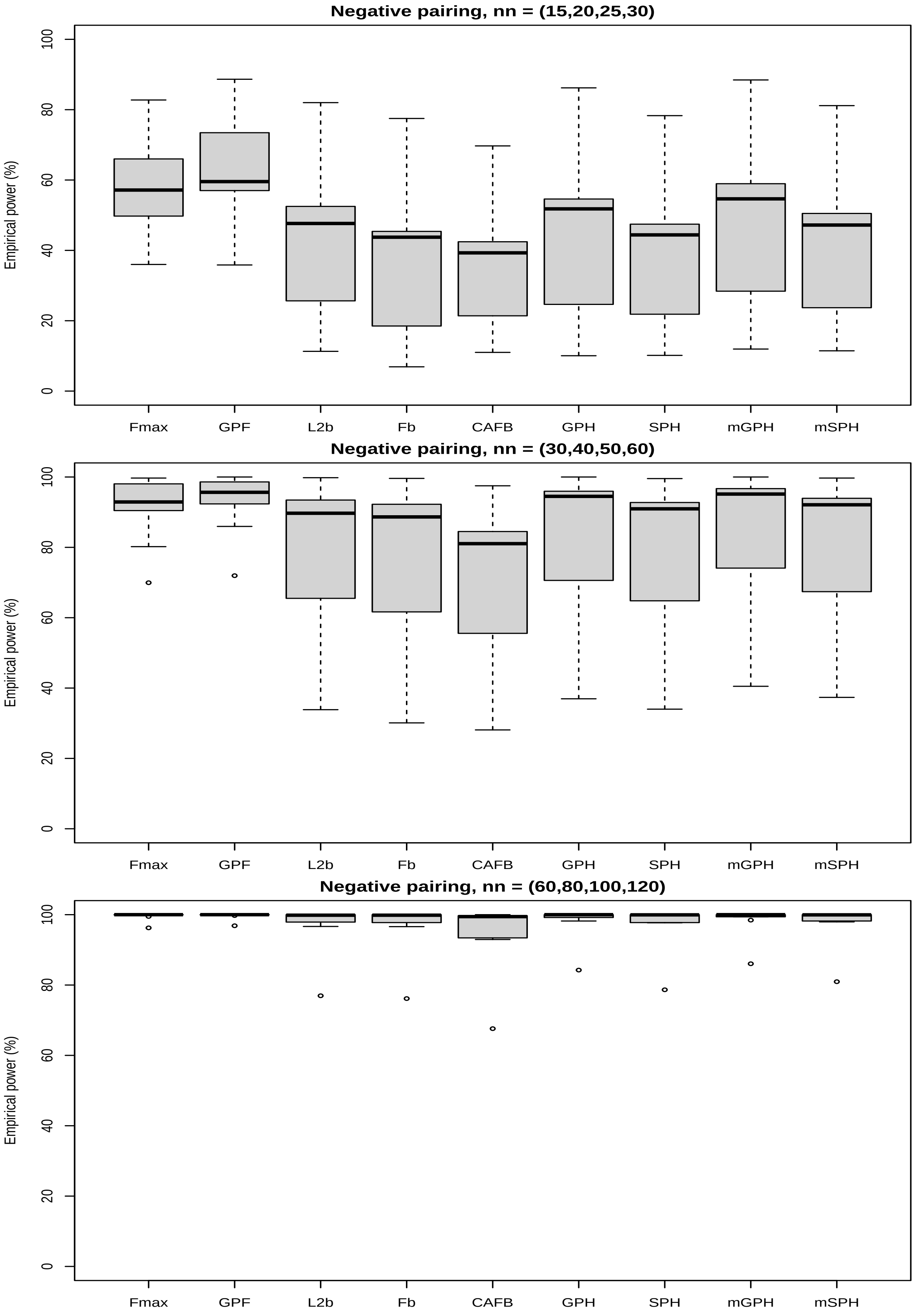


Fig. S84: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes and negative pairing

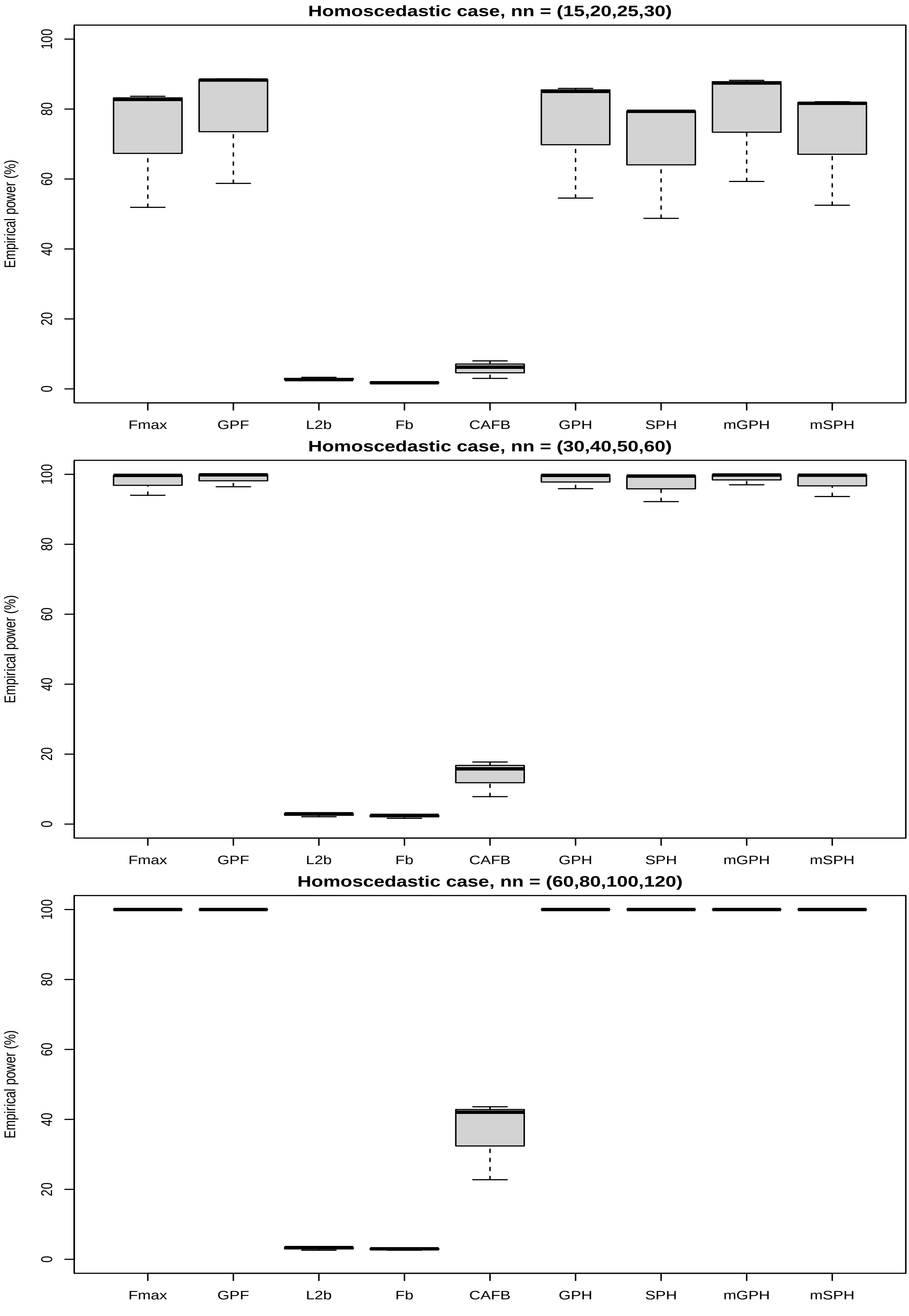


Fig. S85: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes with scaling function and homoscedastic case

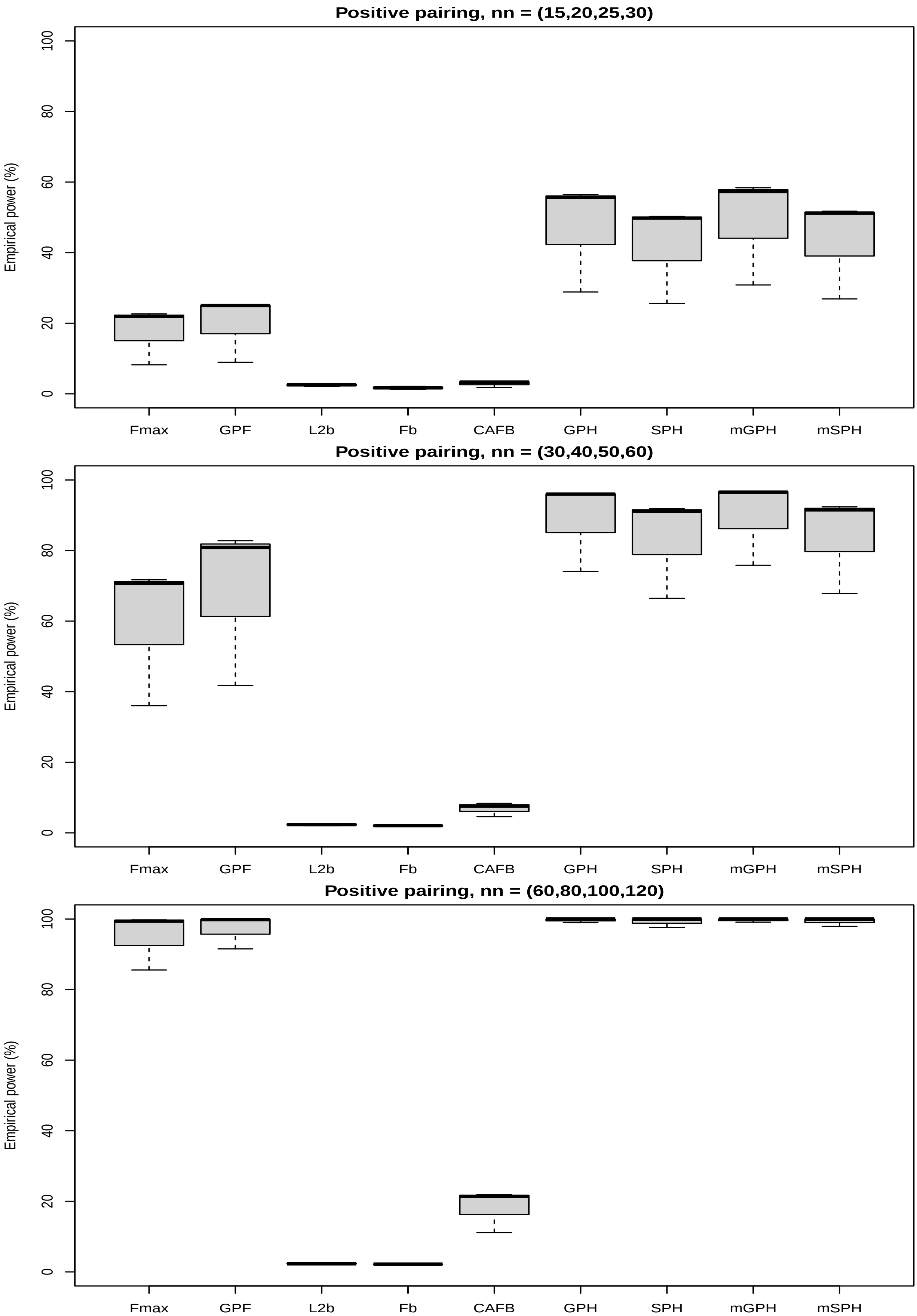


Fig. S86: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes with scaling function and positive pairing

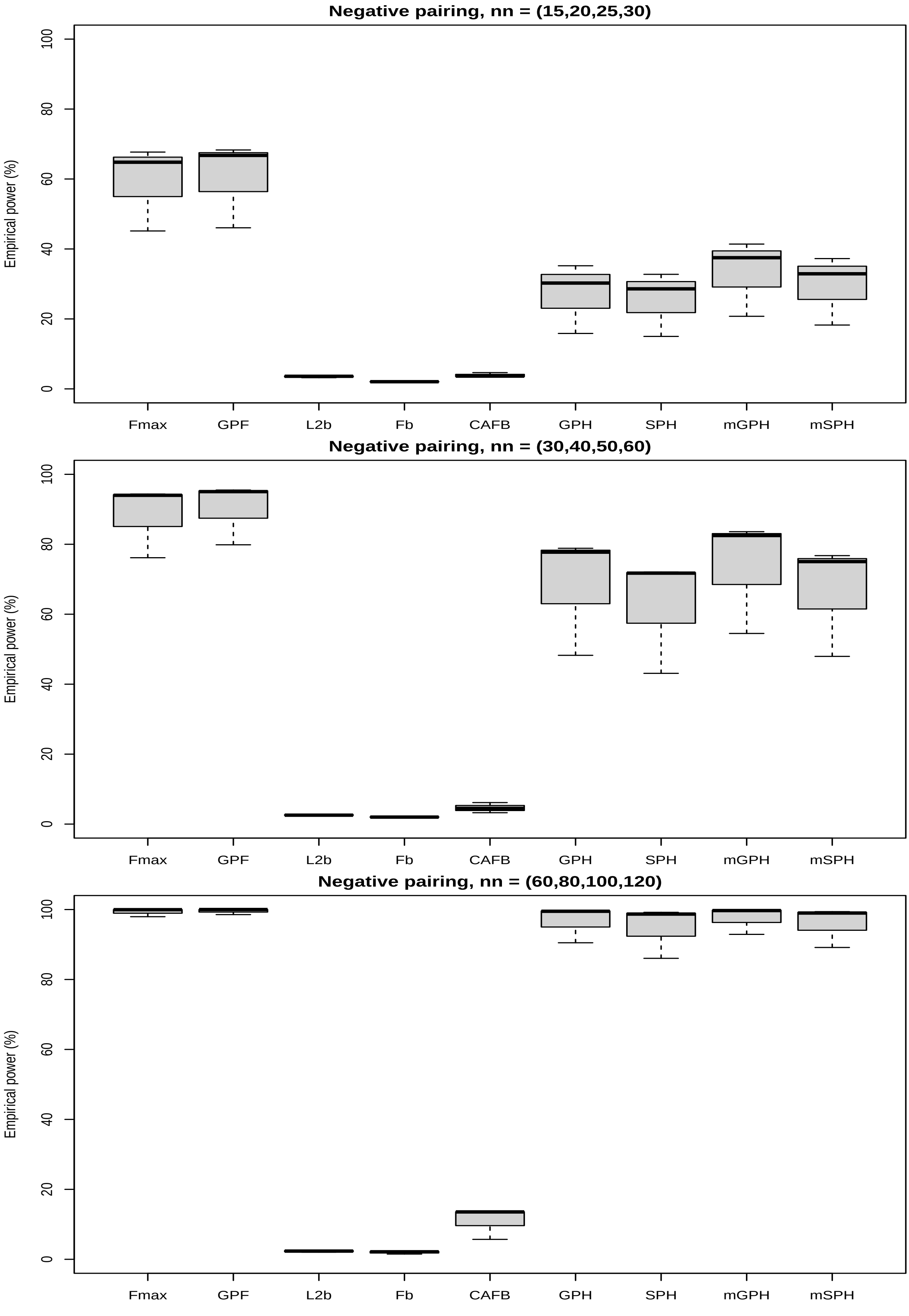


Fig. S87: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts and different sample sizes with scaling function and negative pairing

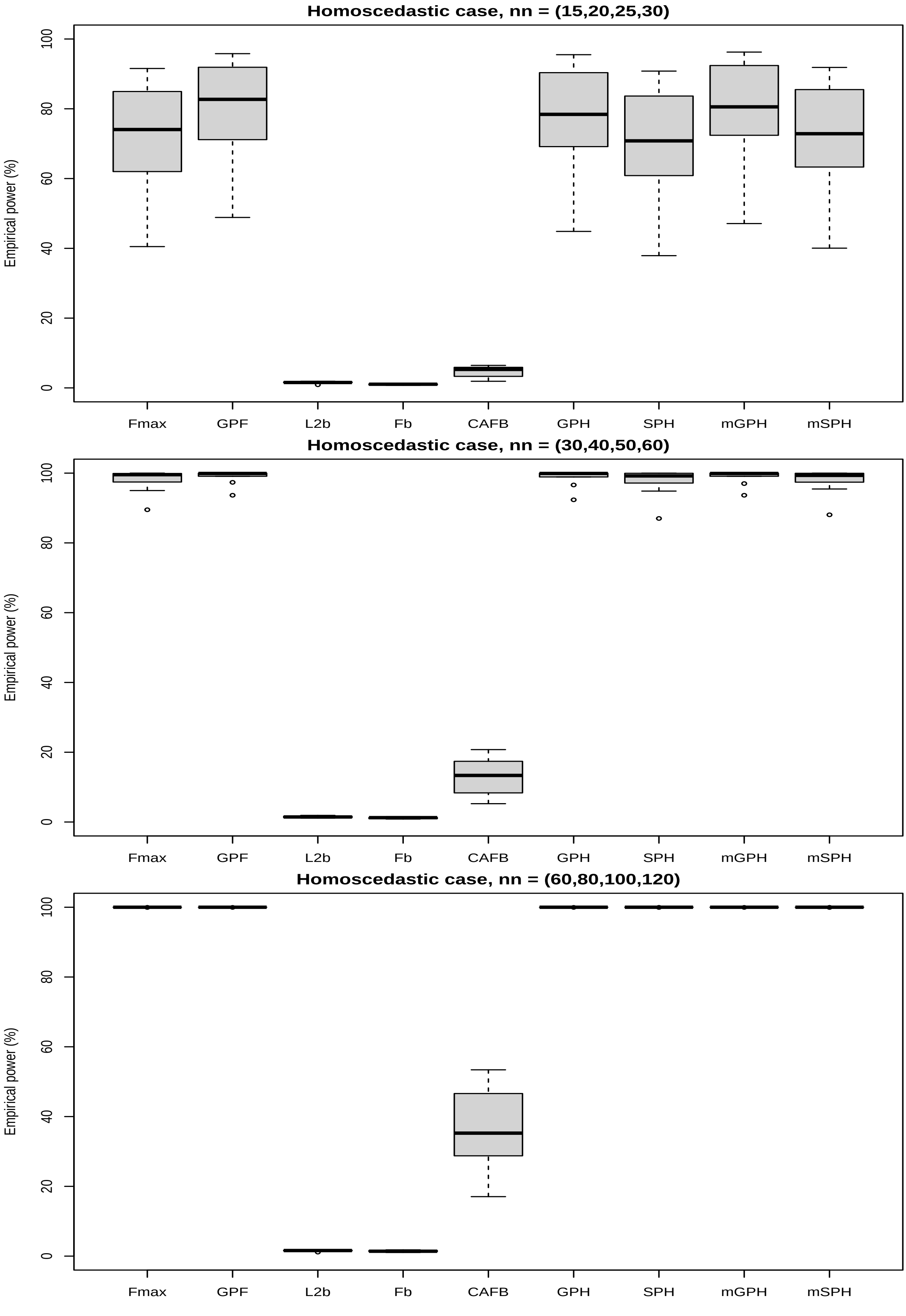


Fig. S88: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes with scaling function and homoscedastic case

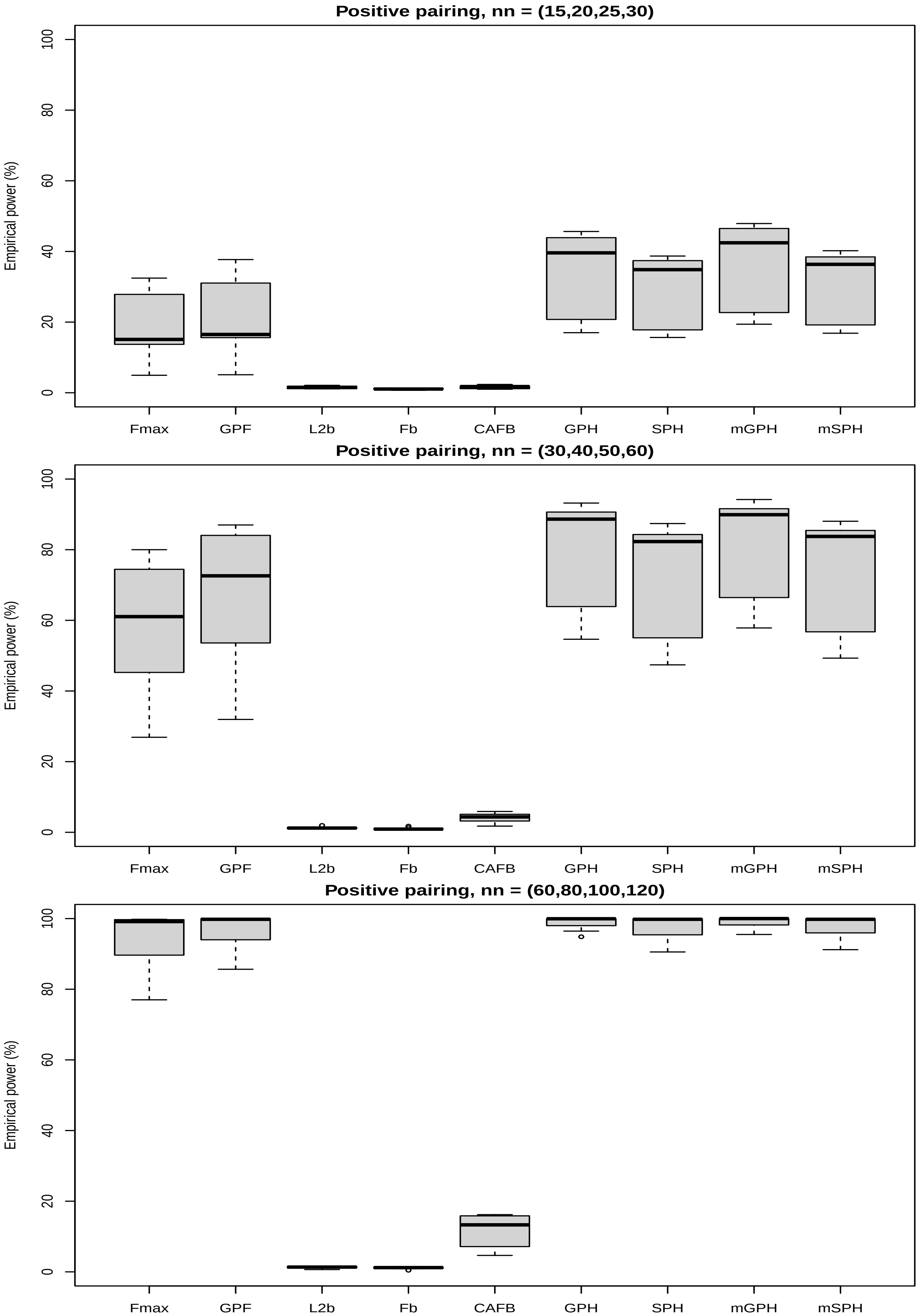


Fig. S89: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes with scaling function and positive pairing

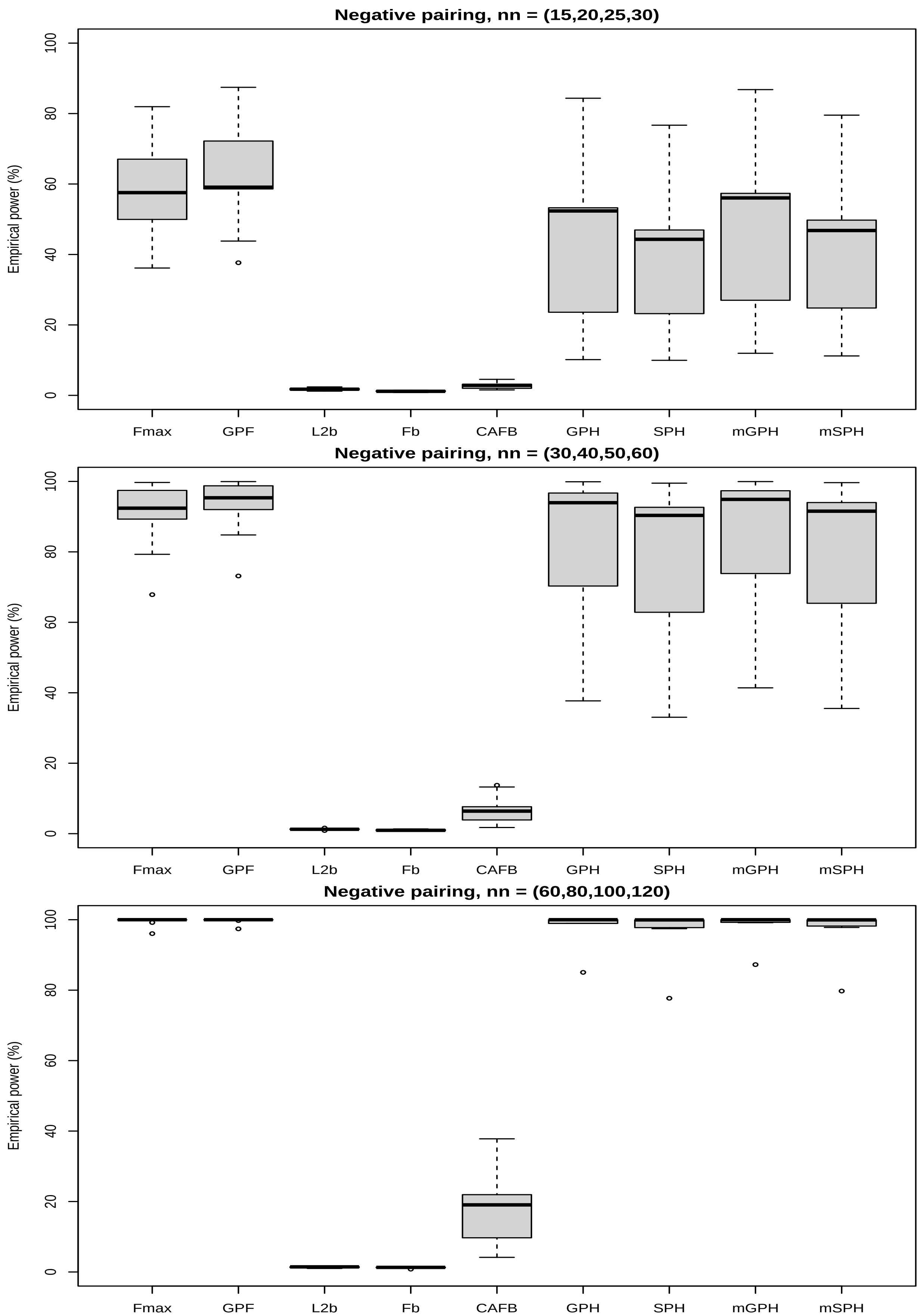


Fig. S90: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts and different sample sizes with scaling function and negative pairing

S8. Additional Simulation Studies for Number of Design Time Points

To investigate the effect of the number of design time points on the properties of the tests, we conducted additional simulation studies under the null hypothesis and alternative A1 for the number of design time points equal to 25, 50, 100, and 200. Note that in the other simulation studies, we generated the data in 50 design time points. The results are presented in Figures S91-S102. We can observe that the increasing number of design time points does not affect the results; they are very stable.

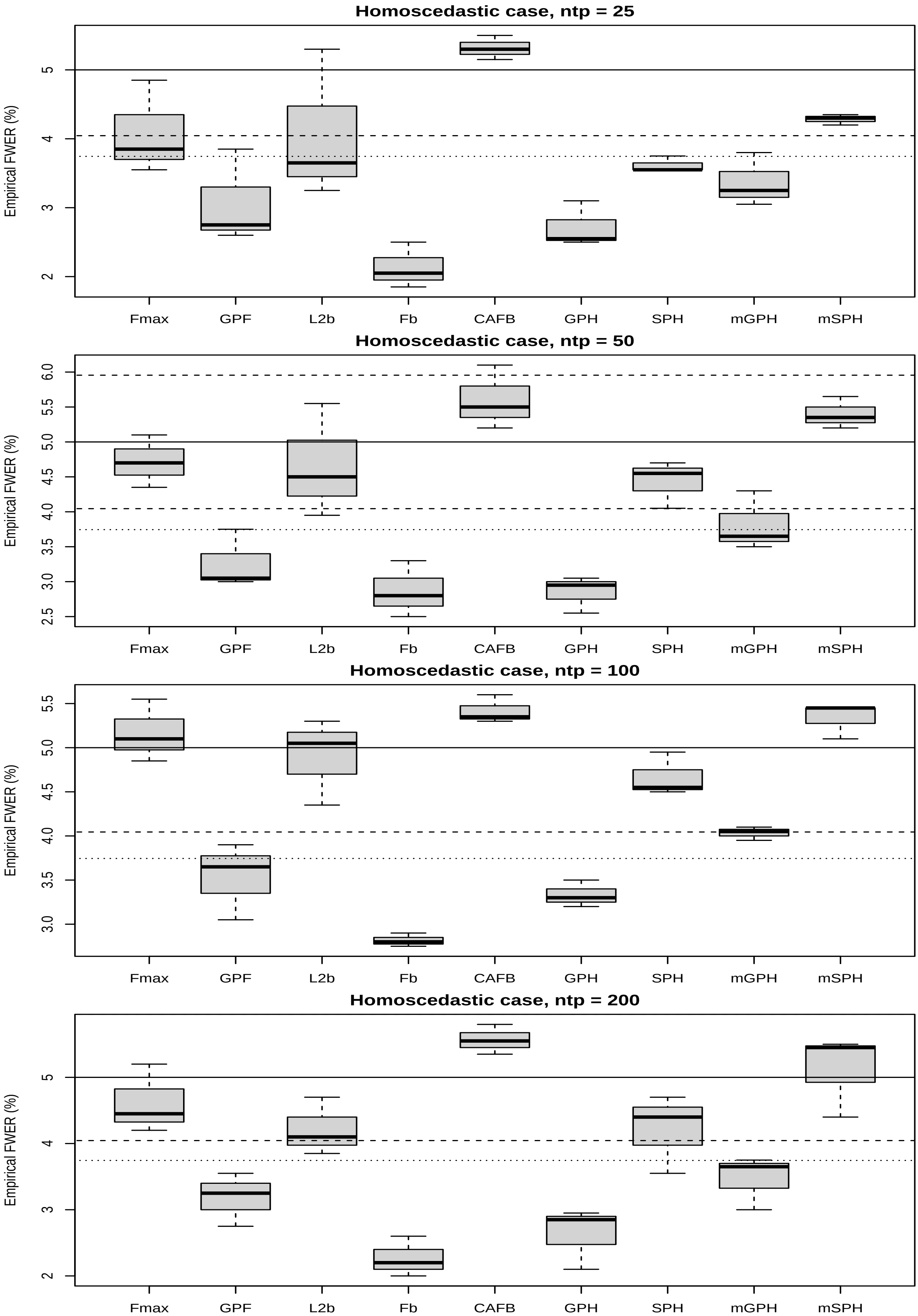


Fig. S91: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts, various numbers of design time points (ntp) for homoscedastic case

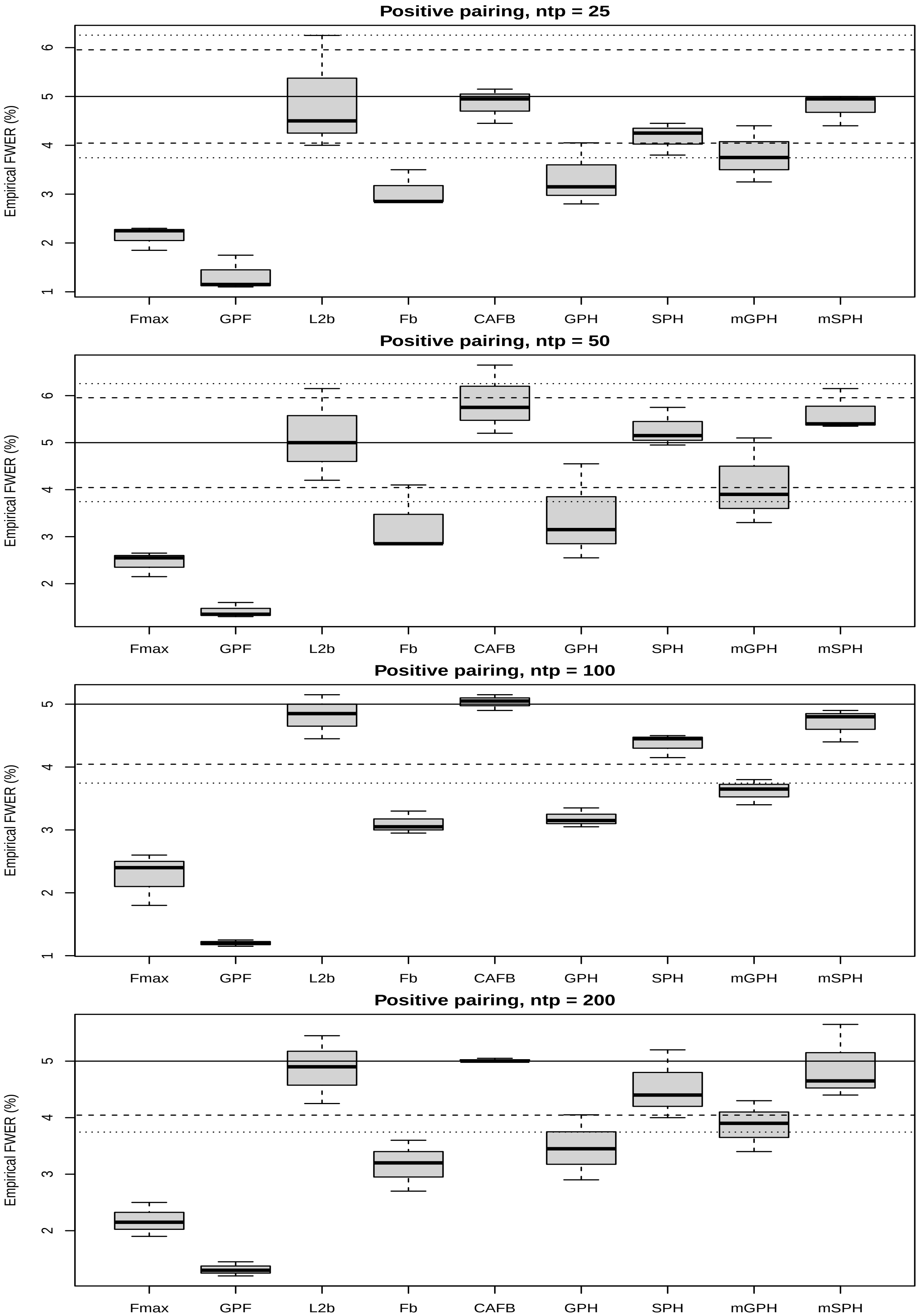


Fig. S92: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts, various numbers of design time points (ntp) for positive pairing

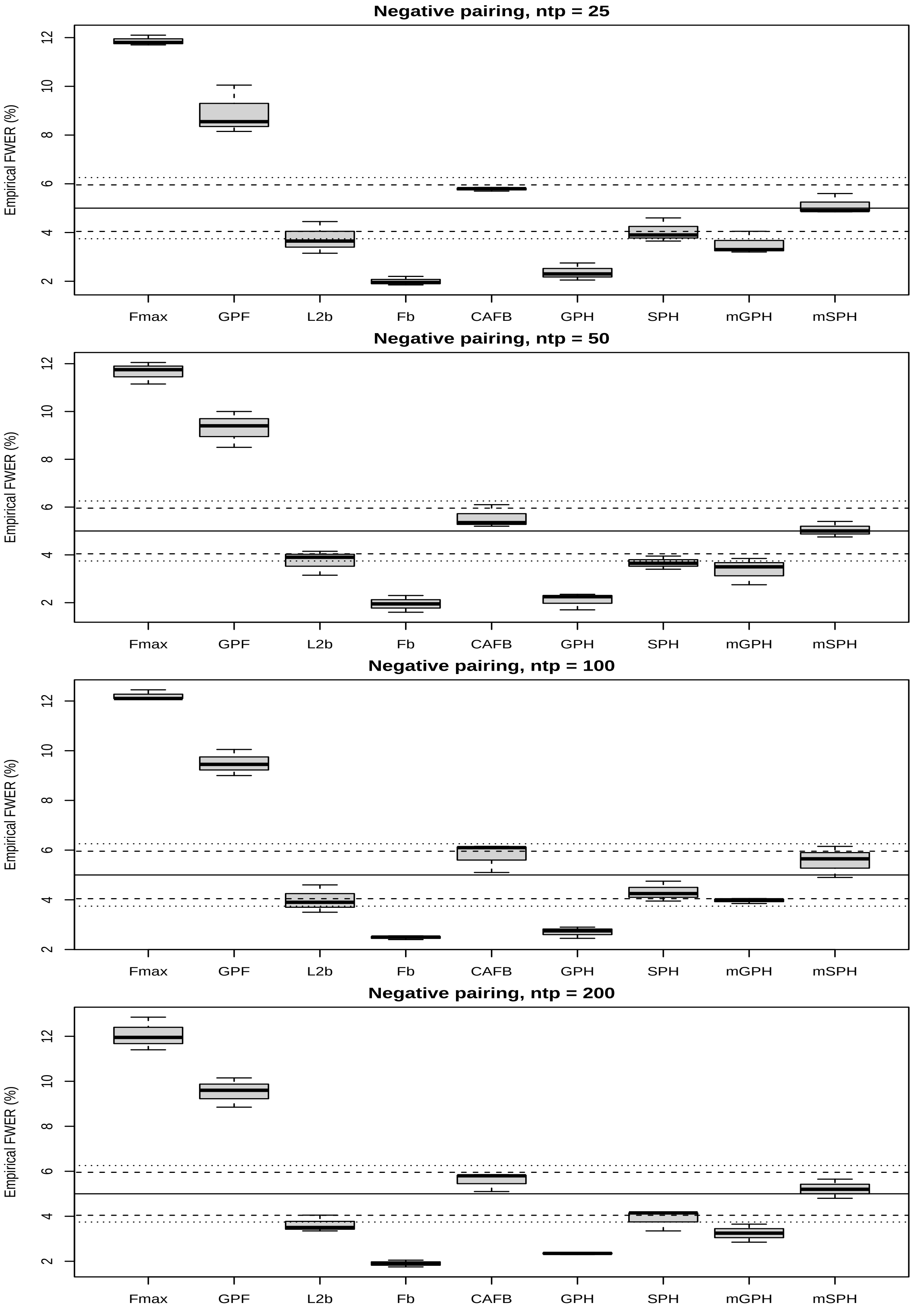


Fig. S93: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Dunnett contrasts, various numbers of design time points (ntp) for negative pairing

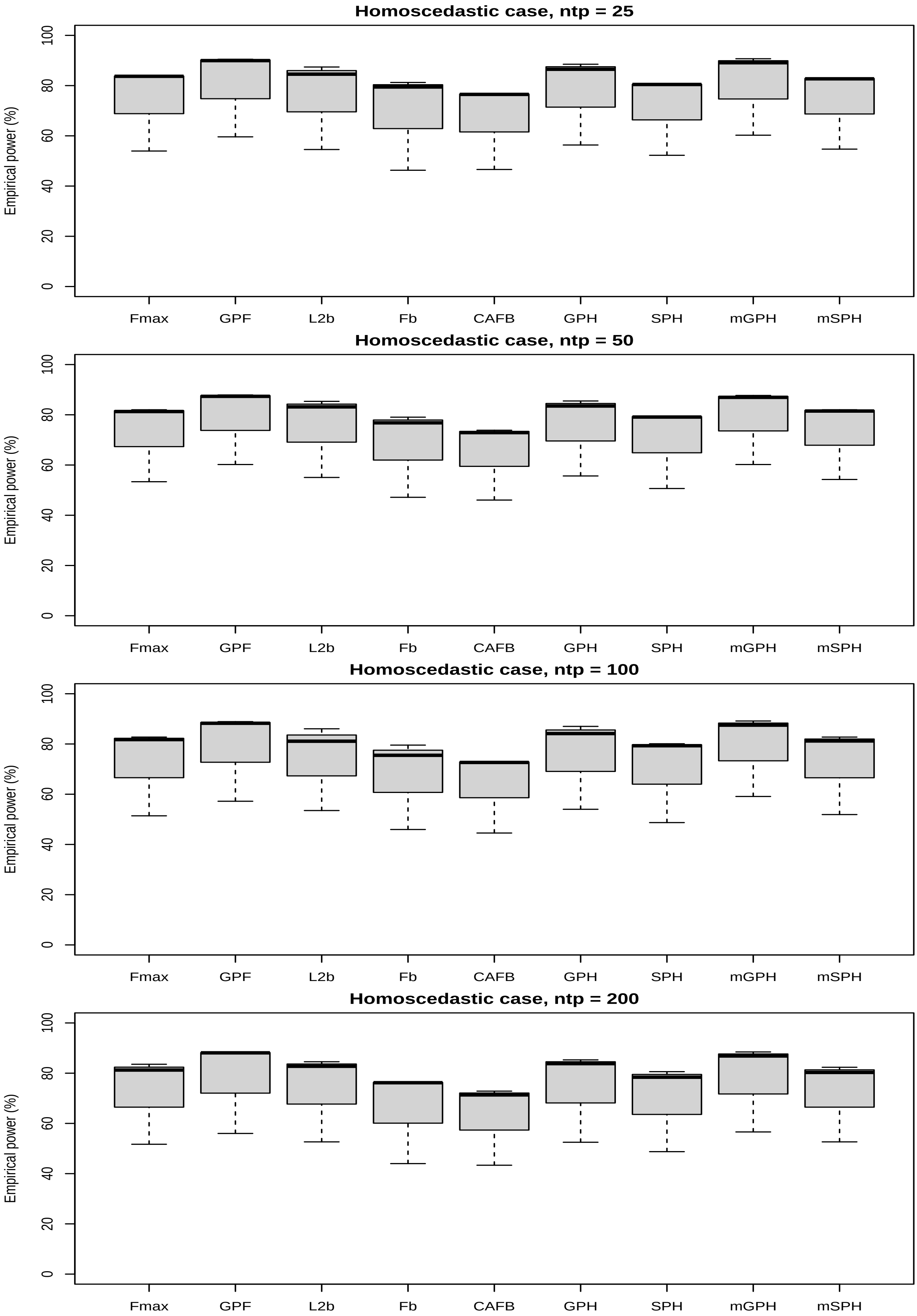


Fig. S94: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts, various numbers of design time points (ntp) and homoscedastic case

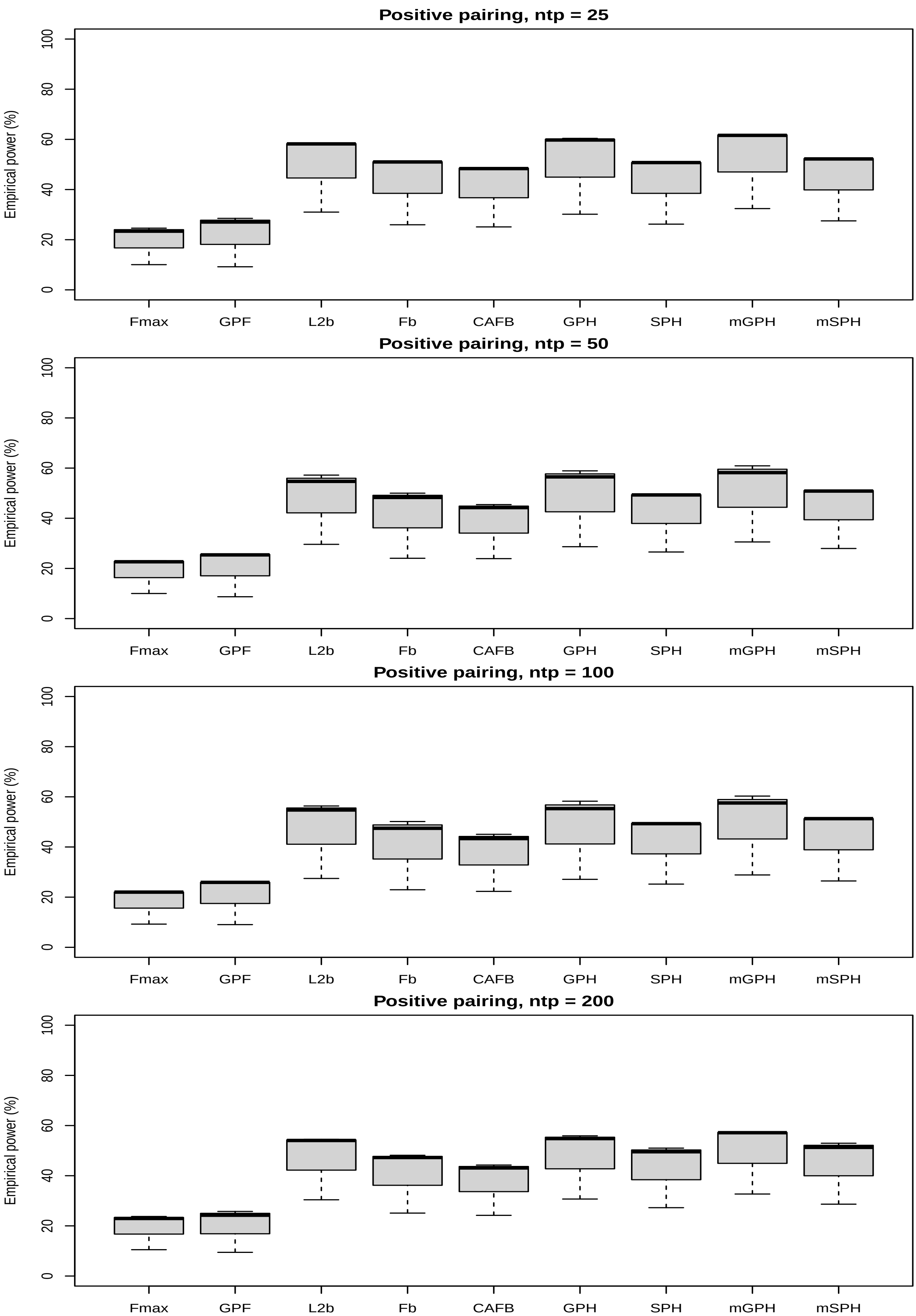


Fig. S95: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts, various numbers of design time points (ntp) and positive pairing

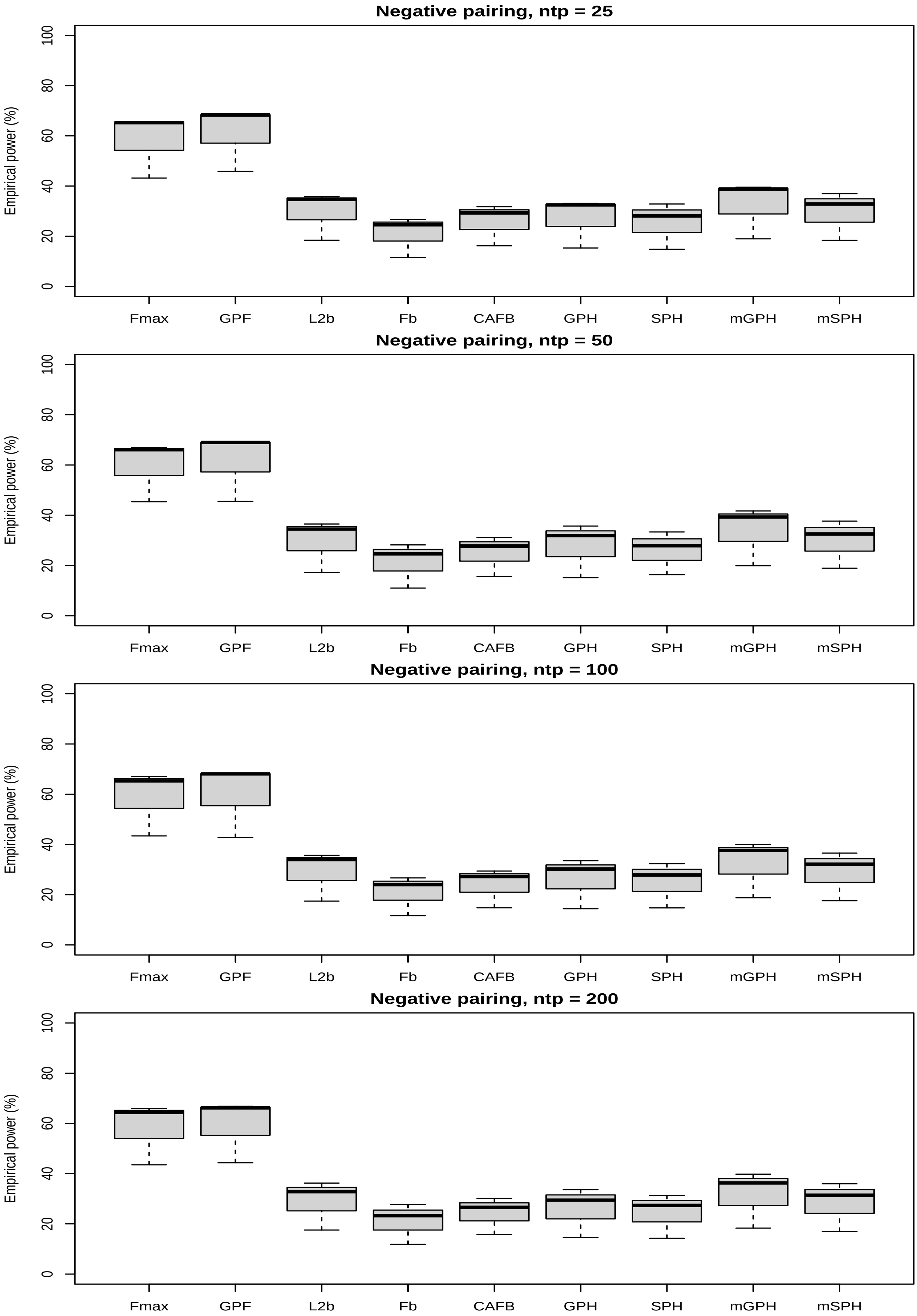


Fig. S96: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Dunnett contrasts, various numbers of design time points (ntp) and negative pairing

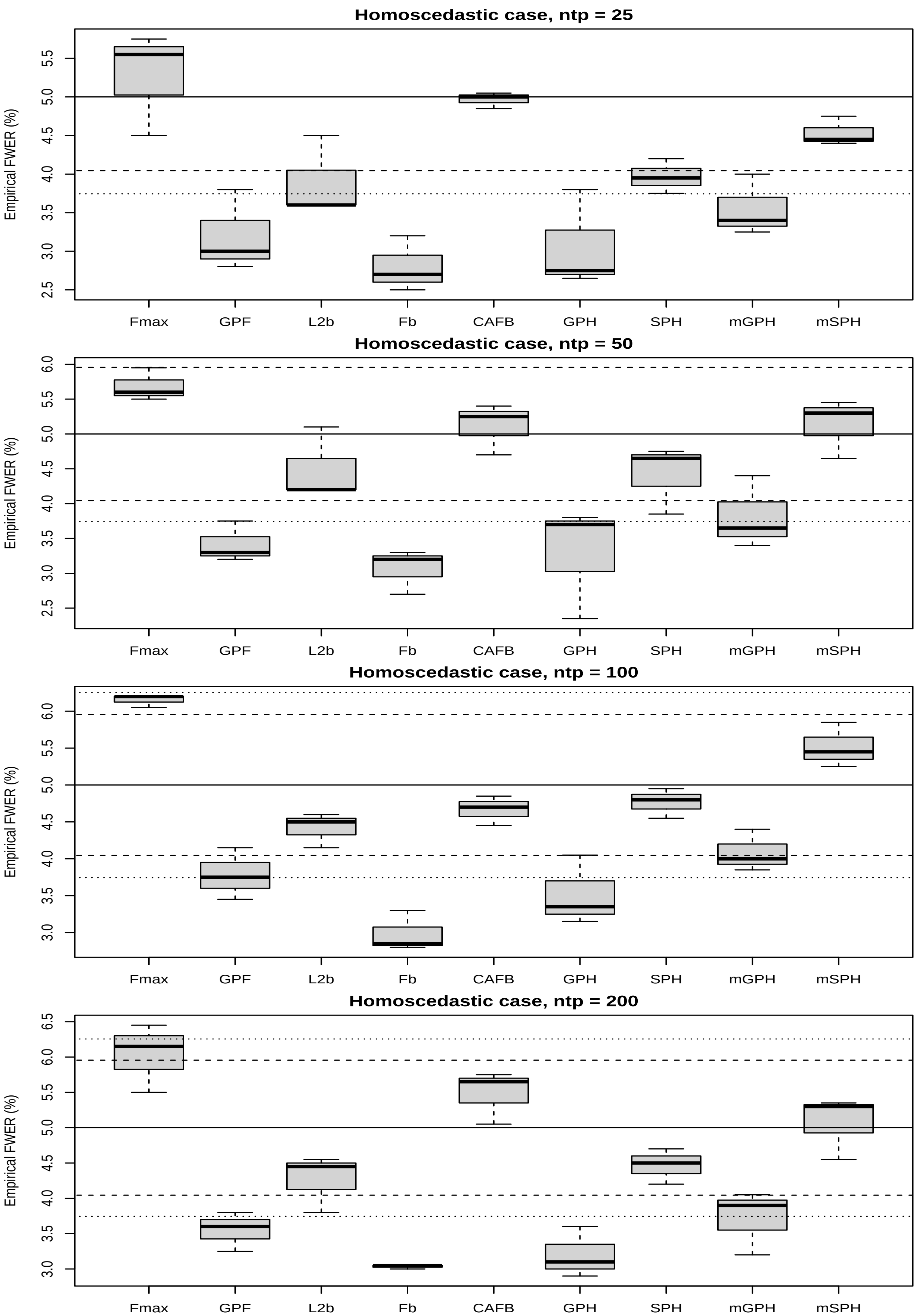


Fig. S97: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts, various numbers of design time points (ntp) and homoscedastic case

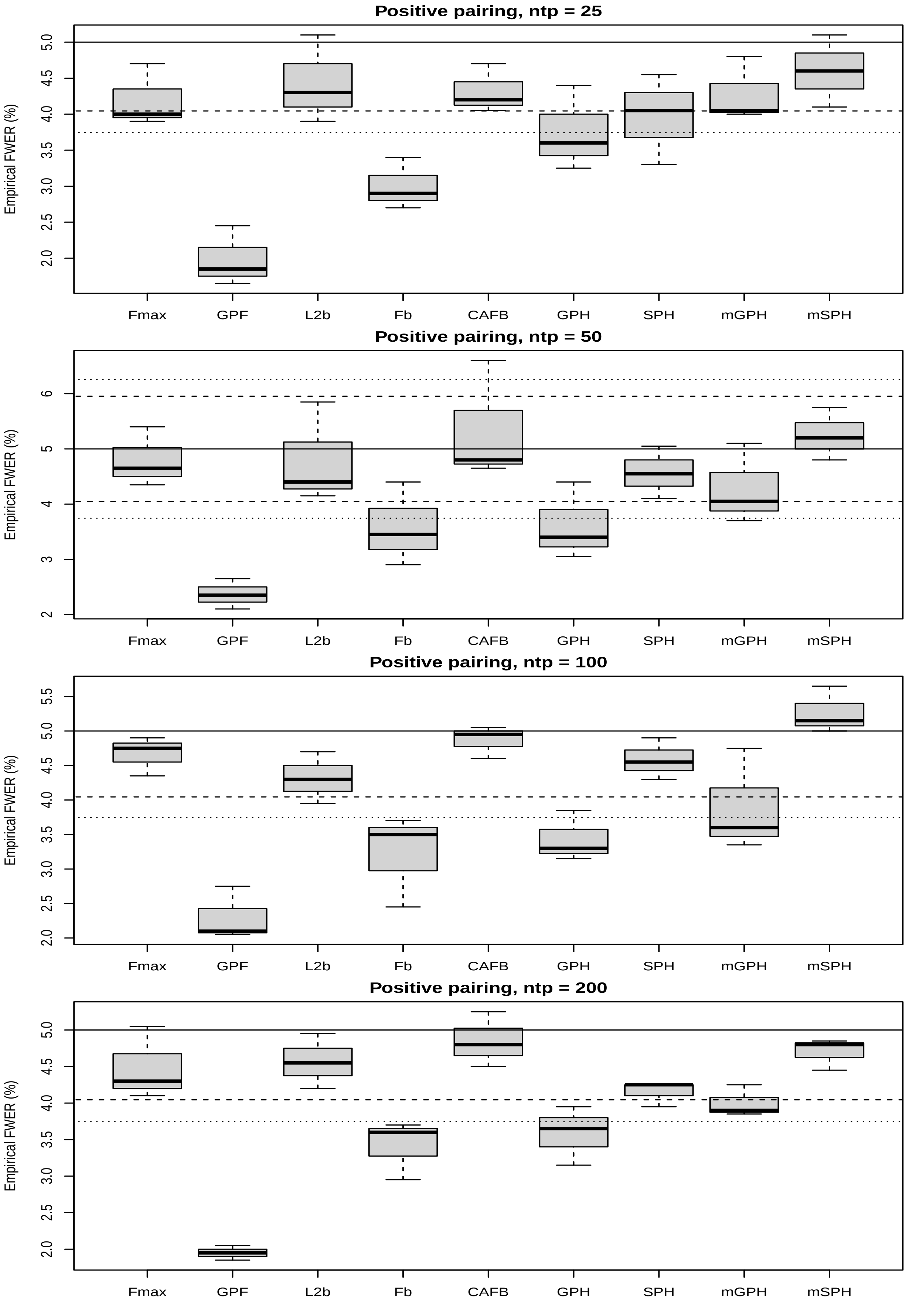


Fig. S98: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts, various numbers of design time points (ntp) and positive pairing

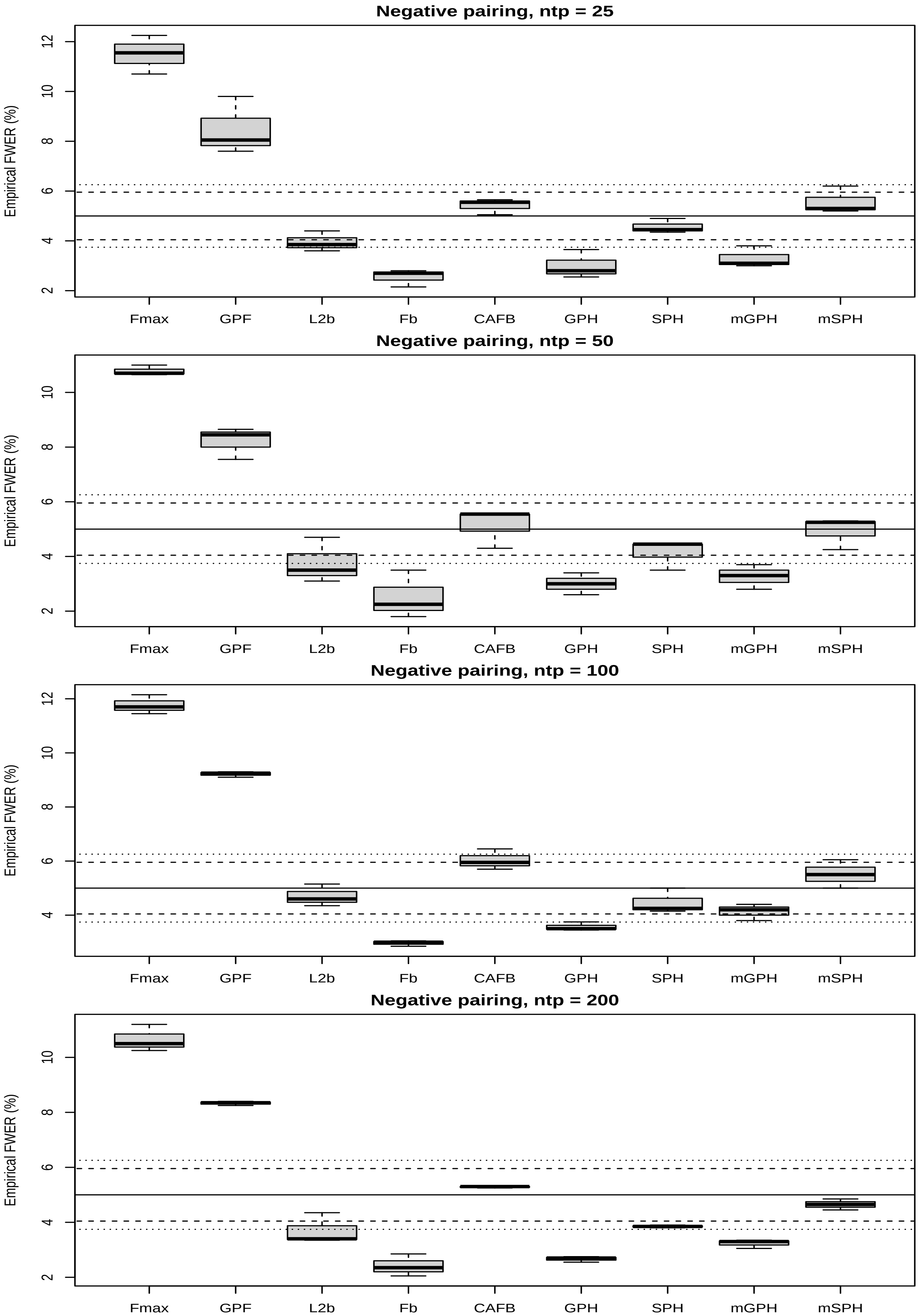


Fig. S99: Box-and-whisker plots for the empirical FWER (as percentages) of all tests obtained for the Tukey contrasts, various numbers of design time points (ntp) and negative pairing

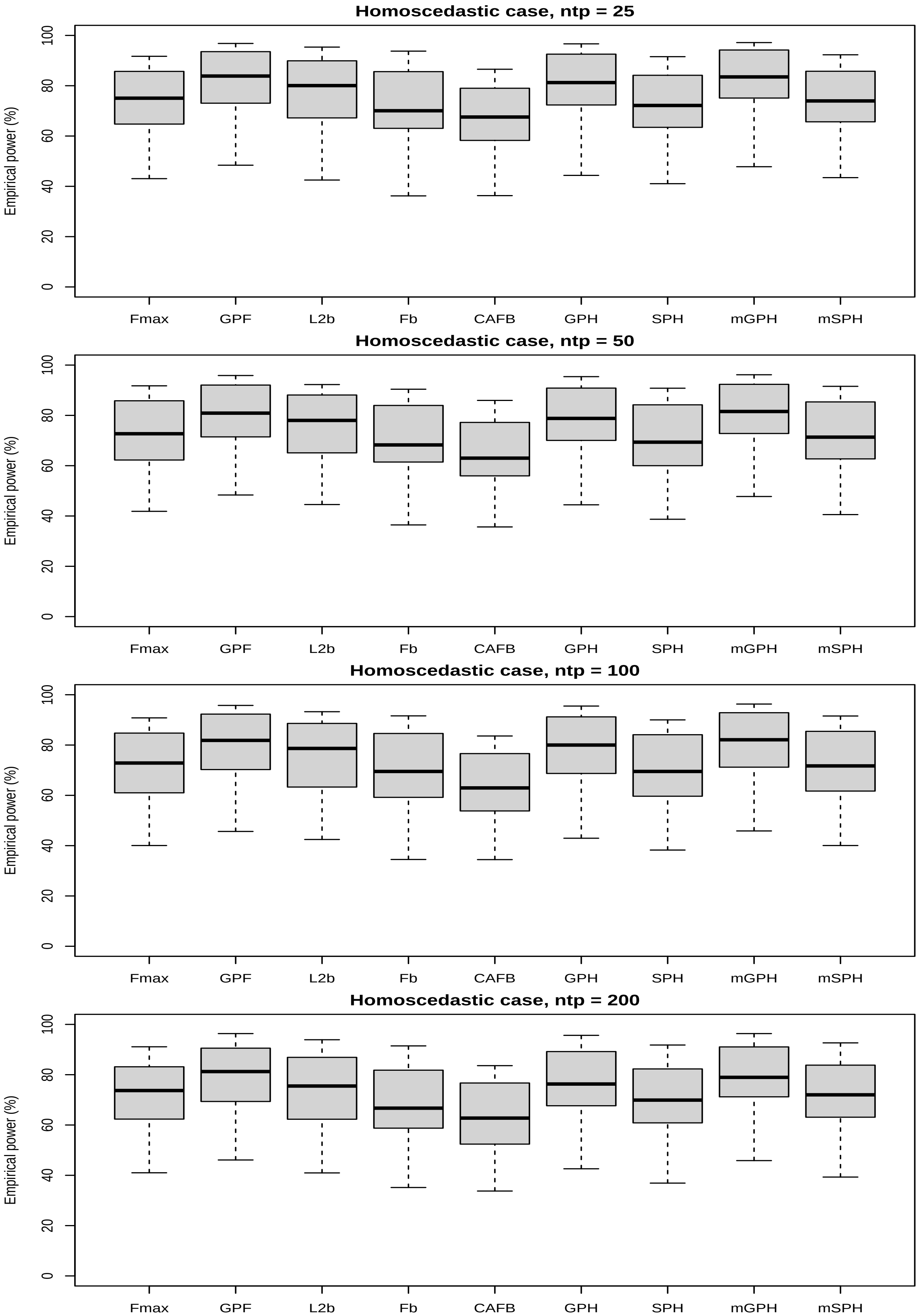


Fig. S100: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts, various numbers of design time points (ntp) and homoscedastic case

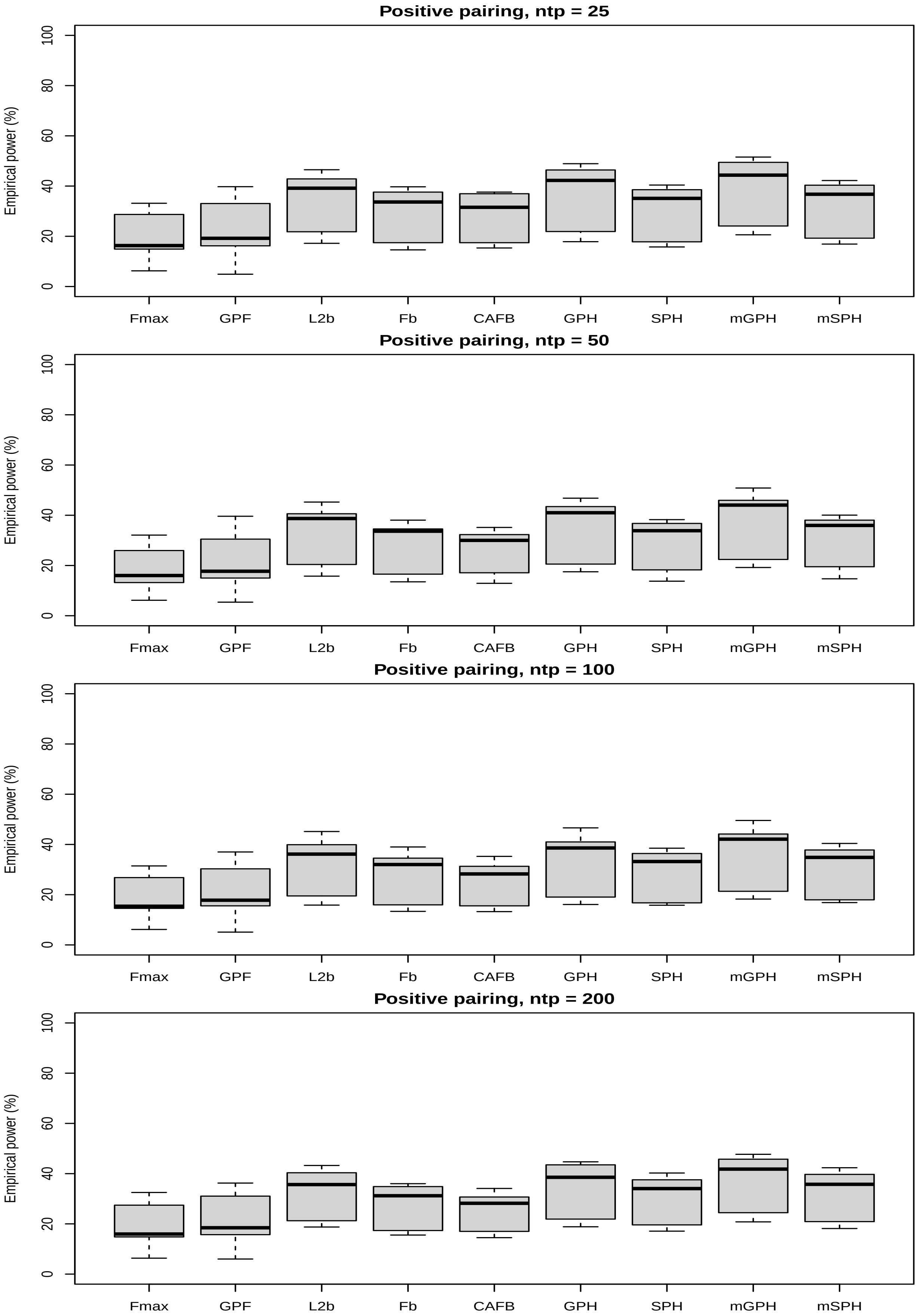


Fig. S101: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts, various numbers of design time points (ntp) and positive pairing

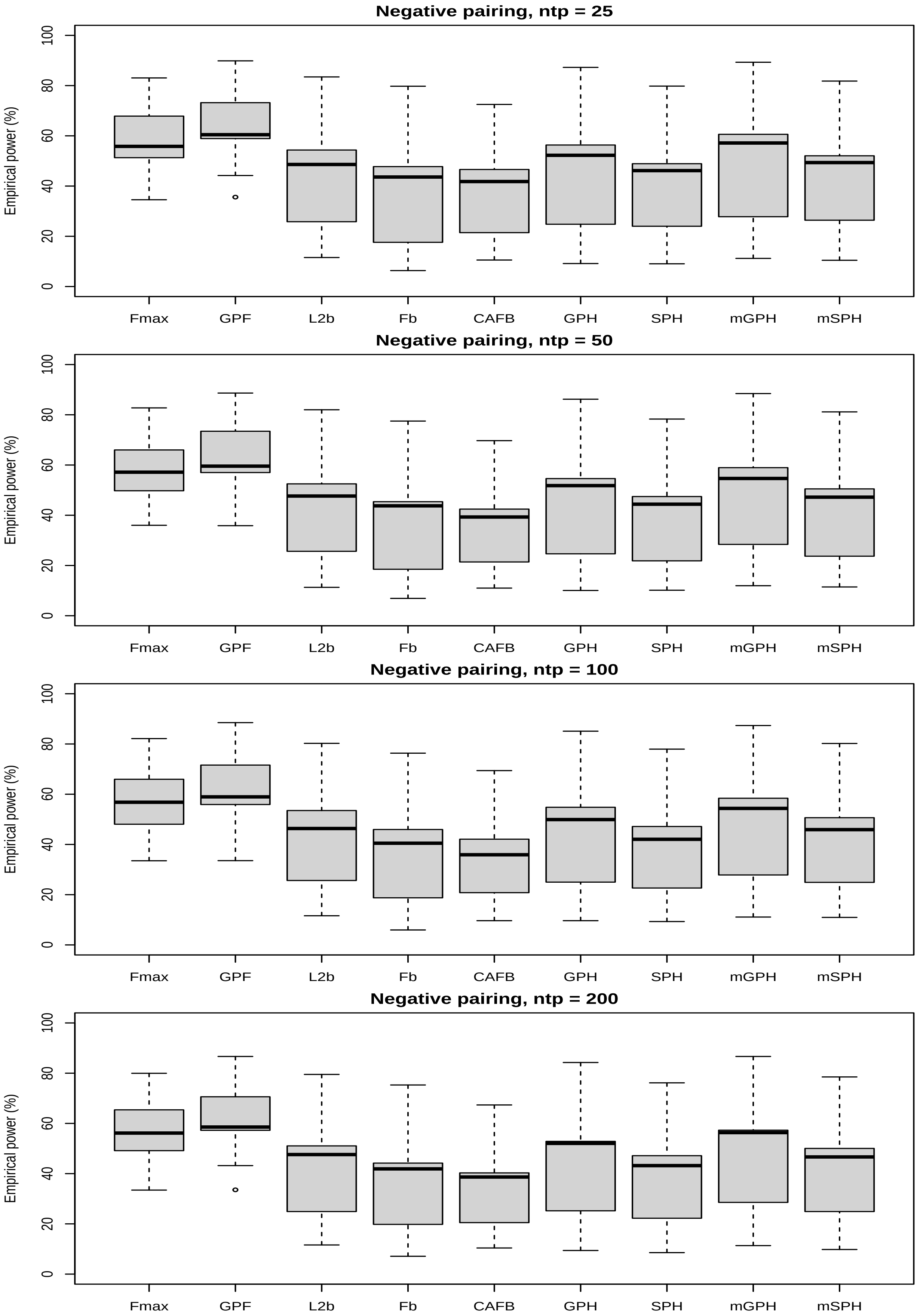


Fig. S102: Box-and-whisker plots for the empirical powers (as percentages) of all tests obtained under alternative A1 for the Tukey contrasts, various numbers of design time points (ntp) and negative pairing

S9. Details on the Competitors for the Simulation Studies

In this section, more details on the competitors for the simulation studies are provided. Here, we use the notation of the main paper.

The Fmax- and GPF-test by Smaga and Zhang [17] are using the test statistics

$$\mathrm{Fmax}_n := \sup_{t\in\mathcal{T}} \left\{ \frac{\mathrm{SSH}_n(t)/r}{\sum_{i=1}^k (n_i-1)\widehat{\gamma}_i(t,t)/(n-k)} \right\} \qquad \text{and} \qquad \mathrm{GPF}_n := \int_{\mathcal{T}} \frac{\mathrm{SSH}_n(t)/r}{\sum_{i=1}^k (n_i-1)\widehat{\gamma}_i(t,t)/(n-k)}\, \mathrm{d}t,$$

where $\mathrm{SSH}_n(t) := (\mathbf{H}\widehat{\boldsymbol{\eta}}(t)-\mathbf{c}(t))^\top(\mathbf{H}\mathbf{D}_n\mathbf{H}^\top)^{-1}(\mathbf{H}\widehat{\boldsymbol{\eta}}(t)-\mathbf{c}(t))$ for all $t\in\mathcal{T}$ and $\mathbf{D}_n := \mathrm{diag}(1/n_1,...,1/n_k)$. The critical values are approximated with a nonparametric pooled bootstrap approach as described in [20, p. 159]. Here, the bootstrap counterparts $v_{ij}^*, j\in\{1,...,n_i\}, i\in\{1,...,k\}$, of the subject-effect functions are randomly generated from $x_{ij}-\widehat{\eta}_i, j\in\{1,...,n_i\}, i\in\{1,...,k\}$. Based on the resulting bootstrap samples $x_{ij}^* = v_{ij}^*+\widehat{\eta}_i, j\in\{1,...,n_i\}, i\in\{1,...,k\}$, the bootstrap counterparts of the Fmax- and GPF-test statistic can be calculated.

The L2b- and Fb-test statistics by Zhang [20] are defined by

$$\mathrm{L2b}_n := \int_{\mathcal{T}} \mathrm{SSH}_n(t)\,\mathrm{d}t \qquad \text{and} \qquad \mathrm{Fb}_n := \frac{\mathrm{L2b}_n}{\sum_{i=1}^k a_{n,ii}\int_{\mathcal{T}}\widehat{\gamma}_i(t,t)\,\mathrm{d}t}$$

with $a_{n,ii}$ being the ith diagonal element of $\mathbf{A}_n = \mathbf{D}_n^{1/2}\mathbf{H}^\top(\mathbf{H}\mathbf{D}_n\mathbf{H}^\top)^{-1}\mathbf{H}\mathbf{D}_n^{1/2}$ for all $i\in\{1,...,k\}$. The critical values are approximated by using a nonparametric groupwise bootstrap approach as described in [20, p. 329]. In contrast to the nonparametric pooled bootstrap, the ith bootstrap sample $x_{ij}^*, j\in\{1,...,n_i\}$, is randomly generated from the ith sample $x_{ij}, j\in\{1,...,n_i\}$, for each $i\in\{1,...,k\}$.

For the CAFB-test by Cuesta-Albertos and Febrero-Bande [2], the null hypothesis

$$\mathcal{H}_0^\nu : \mathbf{H}\int_{\mathcal{T}}\nu(t)\boldsymbol{\eta}(t)\,\mathrm{d}t = \int_{\mathcal{T}}\nu(t)\mathbf{c}(t)\,\mathrm{d}t \tag{S31}$$

is examined with the robust ANOVA test of Brunner et al. [1] based on the projected data $\int_{\mathcal{T}}\nu(t)x_{ij}(t)\,\mathrm{d}t, j\in\{1,...,n_i\}, i\in\{1,...,k\}$, where ν is randomly chosen from a Gaussian distribution such that each of its one-dimensional projections is non-degenerate. This leads to the CAFB-test statistic

$$\mathrm{CAFB}_n(\nu) := \frac{\left(\int_{\mathcal{T}}\nu(t)(\mathbf{H}\widehat{\boldsymbol{\eta}}(t)-\mathbf{c}(t))\,\mathrm{d}t\right)^\top(\mathbf{H}\mathbf{H}^\top)^{-1}\left(\int_{\mathcal{T}}\nu(t)(\mathbf{H}\widehat{\boldsymbol{\eta}}(t)-\mathbf{c}(t))\,\mathrm{d}t\right)}{\sum_{i=1}^k m_{ii}\widehat{\sigma}_i^2(\nu)/n_i},$$

where m_{ii} denotes the ith diagonal element of $\mathbf{M} := \mathbf{H}^\top(\mathbf{H}\mathbf{H}^\top)^{-1}\mathbf{H}$ and $\widehat{\sigma}_i^2(\nu)$ denotes the empirical variance of $\int_{\mathcal{T}}\nu(t)x_{ij}(t)\,\mathrm{d}t, j\in\{1,...,n_i\}$, for all $i\in\{1,...,k\}$. The p-value can be determined from the F-distribution, see [1]. For stabilization, this is done for 30 independent Gaussian white noise processes $\nu_1,...,\nu_{30}$ and, then, the resulting p-values are adjusted with the Bonferroni correction. The global null hypothesis (2) is rejected whenever at least one of the local hypotheses (S31) is.

Finally, the SPH-test by Munko et al. [13] is using the test statistic $\mathrm{SPH}_n := \sup_{t\in\mathcal{T}}\{\mathrm{TF}_{n,\boldsymbol{H},\mathbf{c}}(t)\}$ and approximates the critical values with the same parametric bootstrap approach as described in Section 4. Moreover, for the multiple tests (mSPH), the same procedure as in Section 5 is applied.

S10. Empirical Illustration for the Limiting Distribution of the GPH-based Test Statistic

Let us empirically discuss the convergence in Theorems 1 and 2 of the main paper. Theorem 1 presents the asymptotic null distribution of the GPH-based test statistic, while Theorem 2 shows the same asymptotic distribution for the parametric bootstrap version of this test statistic. In Figure S103, the empirical distribution functions of the GPH-based test statistic under the null hypothesis, its parametric bootstrap, and asymptotic distributions are given. They were obtained based on the simulation study inspired by Example 1 in Section S4. We used the same sample sizes for the three groups: $n_1 = n_2 = n_3 = 10, 20, 30, 40$. We can observe that the empirical distribution functions of the null distribution of the test statistic and its parametric bootstrap version are very similar, which resulted in good finite sample results in the simulation studies. On the other hand, the asymptotic null distribution is quite far away from them. However, as the sample size increases, the null distribution becomes increasingly close to the asymptotic null distribution, which illustrates the convergence in Theorems 1 and 2 in the main paper.

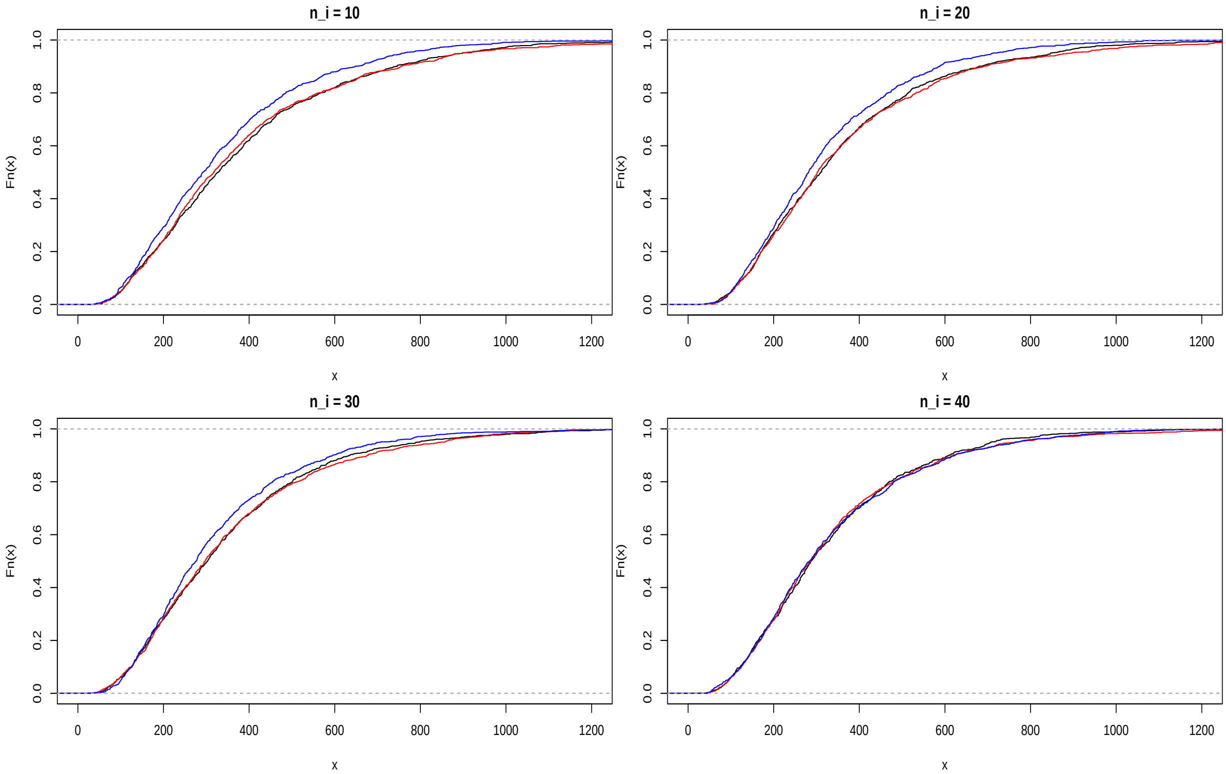


Fig. S103: Empirical distribution functions of the GPH-based test statistic under the null hypothesis (black), its parametric bootstrap distribution (red), and asymptotic distribution (blue) given in Theorem 1 in the main paper.

S11. Computational Time and Speedup of Tests Performing

Let us take into consideration the computational time needed to perform the tests considered. For this purpose, we have calculated the computational time and speedup for all tests considered in simulation studies for data generated under the null hypothesis. The experiments were performed for one physical computer with 16 cores, processor 11th Gen Intel Core i7-11800H @ 2.30GHz, 16 GB RAM, Ubuntu 22.04 64-bit, R 4.5.1.

To check how the sample size affects the computation time, we generated the data with $(n_1, n_2, n_3, n_4) = K \cdot (15, 20, 25, 30)$ for $K = 1, \ldots, 10$. The results are presented in Figure S104. We can observe that the L2b- and CAFB-tests are the fastest ones. This is natural as they need fewer computations than the others. The next group of slower tests consists of the Fmax-, GPF-, and Fb-tests. In this group, we have to calculate the denominator, which distinguishes their test statistics from that of the L2b-test. Interestingly, this group of tests also characterizes the fastest speedup. On the other hand, the four tests based on GPH and SPH spend the most time performing, as they use a lot of bootstrap computations, but their speedup is the smallest.

Now, let us check how the increasing number of design time points affects the computational time and speedup of the tests. For this purpose, we have generated the data with $(n_1, n_2, n_3, n_4) = (15, 20, 25, 30)$ and the number of design time points equal to $K \cdot 50$ for $K = 1, \ldots, 5$. Figure S105 presents the results. We can observe a similar behavior as for increasing sample sizes, but there is an important exception. Namely, an increasing number of design time points results in much greater computational time and faster speedup for the four tests based on GPH and SPH.

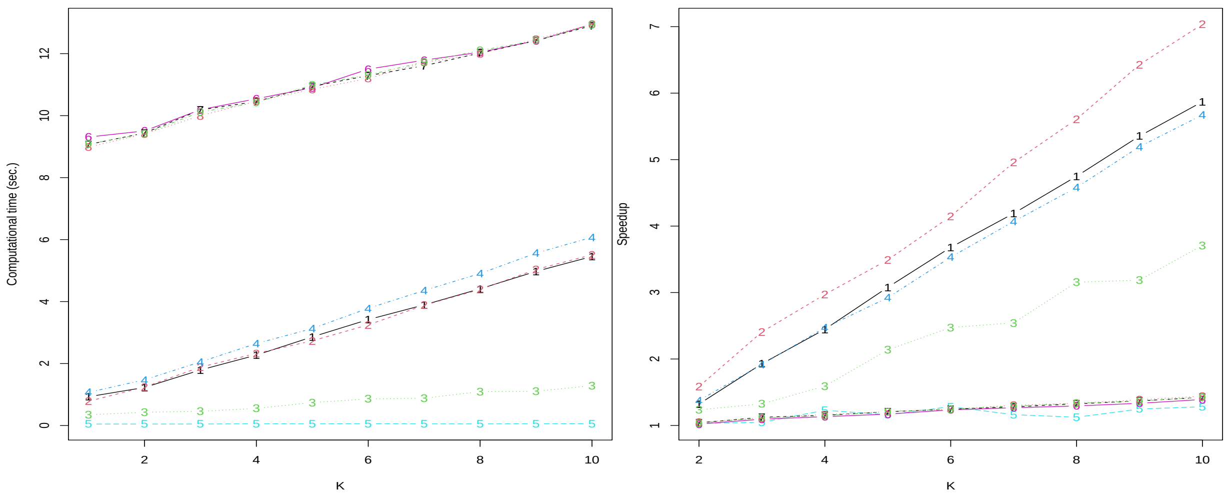


Fig. S104: Computational time and speedup versus the number of observations $(n_1, n_2, n_3, n_4) = K \cdot (15, 20, 25, 30)$ for $K = 1, \ldots, 10$ for 1-Fmax, 2-GPF, 3-L2b, 4-Fb, 5-CAFB, 6-GPH, 7-SPH, 8-mGPH, and 9-mSPH.

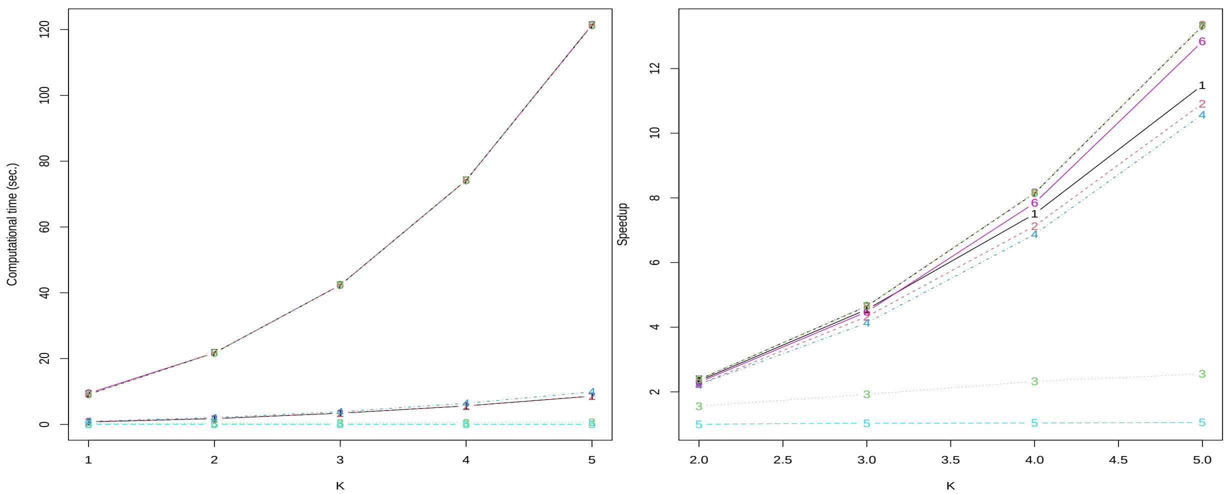


Fig. S105: Computational time and speedup versus the number of design time points $K \cdot 50$ for $K = 1, \ldots, 5$ for 1-Fmax, 2-GPF, 3-L2b, 4-Fb, 5-CAFB, 6-GPH, 7-SPH, 8-mGPH, and 9-mSPH.

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