

# Appendix of “Draw procedures for balanced group rounds in sports competitions”

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## A Example of the backtracking algorithm

Suppose  $E = \{1, \dots, 12\}$ , meaning we have to form 4 groups of size 3. Take  $B_2 = \{5, 6, 7, 8\}$  and  $B_3 = \{9, 10, 11, 12\}$ , with  $s_1 = s_2 = 19$ ,  $s_3 = s_4 = 20$ . Additionally, suppose there is a geographical constraint forbidding the group  $\{3, 5, 12\}$ . The corresponding array is:

$$\begin{array}{c} \begin{array}{cccc} & 5 & 6 & 7 & 8 \\ 1 & \left( \begin{array}{cccc} 0 & 12 & 11 & 10 \\ 12 & 11 & 10 & 9 \\ 0 & 11 & 10 & 9 \\ 11 & 10 & 9 & 0 \end{array} \right) \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

The backtracking algorithm proceeds as follows. Select the first row and place a queen on the first available square (1, 2). Move to the second row and place a queen on the first available square, which is (2, 3), since (2, 1) is occupied by a 12 (the same value as (1, 2)) and the square (2, 2) is in the same column as (1, 2). Next, move to the third row and place a queen on the first available square, which is (3, 4). Finally, go to the fourth row and place a queen on (4, 1). This yields a perfect solution formed by the groups (1, 6, 12), (2, 7, 10), (3, 8, 9), (4, 5, 11). We now backtrack by removing the queens from the fourth and third rows and try to place the third queen to the right of the previous choice, which is impossible. Therefore, we backtrack again, remove the second queen, and place it on (2, 4). Then, we move to the third row and place the queen on (3, 3), and the queen of the fourth row on (4, 1). This gives the solution  $\{(1, 6, 12), (2, 8, 9), (3, 7, 10), (4, 5, 11)\}$ . We backtrack again and find that there are no more solutions with the queen of the first row on (1, 2). Thus, we place it to its right at (1, 3) and repeat the above procedure. It is straightforward to check that this gives only the solution  $\{(1, 7, 11), (2, 5, 12), (3, 8, 9), (4, 6, 10)\}$ . Finally, placing the first queen on (1, 4) and repeating the procedure yields the solution  $\{(1, 8, 10), (2, 5, 12), (3, 6, 11), (4, 7, 9)\}$ . The list of all perfect solutions in this case is shown in Table A.1.

Table A.1: List of perfect solutions in Example A.

| Solution | Group A | Group B | Group C | Group D |
|----------|---------|---------|---------|---------|
| 1        | 1-6-12  | 2-7-10  | 3-8-9   | 4-5-11  |
| 2        | 1-6-12  | 2-8-9   | 3-7-10  | 4-5-11  |
| 3        | 1-7-11  | 2-5-12  | 3-8-9   | 4-6-10  |
| 4        | 1-8-10  | 2-5-12  | 3-6-11  | 4-7-9   |

Table B.1: National teams taken for the 2026 World Cup in Example 1.1. The first two columns are their absolute and relative rankings (as of 23-Dec-2021). The third column is the score of the teams used for forming the groups.

| Absolute | Relative | Score | Team           | FIFA Code | Confederation |
|----------|----------|-------|----------------|-----------|---------------|
| 1        | 1        | 1     | Belgium        | BEL       | UEFA          |
| 2        | 2        | 2     | Brazil         | BRA       | CONMEBOL      |
| 3        | 3        | 3     | France         | FRA       | UEFA          |
| 4        | 4        | 4     | England        | ENG       | UEFA          |
| 5        | 5        | 5     | Argentina      | ARG       | CONMEBOL      |
| 6        | 6        | 6     | Italy          | ITA       | UEFA          |
| 7        | 7        | 7     | Spain          | ESP       | UEFA          |
| 8        | 8        | 8     | Portugal       | POR       | UEFA          |
| 9        | 9        | 9     | Denmark        | DEN       | UEFA          |
| 10       | 10       | 10    | Netherlands    | NED       | UEFA          |
| 11       | 11       | 11    | USA            | USA       | CONCACAF      |
| 12       | 12       | 12    | Germany        | GER       | UEFA          |
| 13       | 13       | 13    | Switzerland    | SUI       | UEFA          |
| 14       | 14       | 14    | Mexico         | MEX       | CONCACAF      |
| 15       | 15       | 15    | Croatia        | CRO       | UEFA          |
| 40       | 34       | 16    | Canada         | CAN       | CONCACAF      |
| 16       | 16       | 17    | Colombia       | COL       | CONMEBOL      |
| 17       | 17       | 18    | Uruguay        | URU       | CONMEBOL      |
| 18       | 18       | 19    | Sweden         | SWE       | UEFA          |
| 19       | 19       | 20    | Wales          | WAL       | UEFA          |
| 20       | 20       | 21    | Senegal        | SEN       | CAF           |
| 21       | 21       | 22    | Iran           | IRN       | AFC           |
| 22       | 22       | 23    | Peru           | PER       | CONMEBOL      |
| 23       | 23       | 24    | Serbia         | SRB       | UEFA          |
| 24       | 24       | 25    | Chile          | CHI       | CONMEBOL      |
| 25       | 25       | 26    | Ukraine        | UKR       | UEFA          |
| 26       | 26       | 27    | Japan          | JPN       | AFC           |
| 27       | 27       | 28    | Poland         | POL       | UEFA          |
| 28       | 28       | 29    | Morocco        | MAR       | CAF           |
| 29       | 29       | 30    | Algeria        | ALG       | CAF           |
| 30       | 30       | 31    | Tunisia        | TUN       | CAF           |
| 33       | 31       | 32    | Korea Republic | KOR       | AFC           |
| 35       | 32       | 33    | Australia      | AUS       | AFC           |
| 36       | 33       | 34    | Nigeria        | NGA       | CAF           |
| 43       | 35       | 35    | Paraguay       | PAR       | CONMEBOL      |
| 45       | 36       | 36    | Egypt          | EGY       | CAF           |
| 48       | 37       | 37    | Qatar          | QAT       | AFC           |
| 49       | 38       | 38    | Costa Rica     | CRC       | CONCACAF      |
| 50       | 39       | 39    | Cameroon       | CMR       | CAF           |
| 51       | 40       | 40    | Saudi Arabia   | KSA       | AFC           |
| 52       | 41       | 41    | Ghana          | GHA       | CAF           |
| 53       | 42       | 42    | Mali           | MLI       | CAF           |
| 56       | 43       | 43    | Côte d'Ivoire  | CIV       | CAF           |
| 57       | 44       | 44    | Jamaica        | JAM       | CONCACAF      |
| 63       | 45       | 45    | Panama         | PAN       | CONCACAF      |
| 69       | 46       | 46    | UAE            | UAE       | AFC           |
| 74       | 47       | 47    | China          | CHN       | AFC           |
| 110      | 48       | 48    | New Zealand    | NZL       | OFC           |

Table B.2: National teams for the 2026 World Cup using rankings as of 22-Dec-2022.

| Absolute | Relative | Score | Team                 | FIFA Code | Confederation |
|----------|----------|-------|----------------------|-----------|---------------|
| 1        | 1        | 1     | Brazil               | BRA       | CONMEBOL      |
| 2        | 2        | 2     | Argentina            | ARG       | CONMEBOL      |
| 3        | 3        | 3     | France               | FRA       | UEFA          |
| 4        | 4        | 4     | Belgium              | BEL       | UEFA          |
| 5        | 5        | 5     | England              | ENG       | UEFA          |
| 6        | 6        | 6     | Netherlands          | NED       | UEFA          |
| 7        | 7        | 7     | Croatia              | CRO       | UEFA          |
| 8        | 8        | 8     | Italy                | ITA       | UEFA          |
| 9        | 9        | 9     | Portugal             | POR       | UEFA          |
| 10       | 10       | 10    | Spain                | ESP       | UEFA          |
| 11       | 11       | 11    | Morocco              | MAR       | CAF           |
| 12       | 12       | 12    | Switzerland          | SUI       | UEFA          |
| 13       | 13       | 13    | USA                  | USA       | CONCACAF      |
| 14       | 14       | 14    | Germany              | GER       | UEFA          |
| 15       | 15       | 15    | Mexico               | MEX       | CONCACAF      |
| 53       | 42       | 16    | Canada               | CAN       | CONCACAF      |
| 16       | 16       | 17    | Uruguay              | URU       | CONMEBOL      |
| 17       | 17       | 18    | Colombia             | COL       | CONMEBOL      |
| 18       | 18       | 19    | Denmark              | DEN       | UEFA          |
| 19       | 19       | 20    | Senegal              | SEN       | CAF           |
| 20       | 20       | 21    | Japan                | JPN       | AFC           |
| 21       | 21       | 22    | Peru                 | PER       | CONMEBOL      |
| 22       | 22       | 23    | Poland               | POL       | UEFA          |
| 23       | 23       | 24    | Sweden               | SWE       | UEFA          |
| 24       | 24       | 25    | Iran                 | IRN       | AFC           |
| 25       | 25       | 26    | Korea Republic       | KOR       | AFC           |
| 26       | 26       | 27    | Ukraine              | UKR       | UEFA          |
| 27       | 27       | 28    | Australia            | AUS       | AFC           |
| 28       | 28       | 29    | Wales                | WAL       | UEFA          |
| 29       | 29       | 30    | Serbia               | SRB       | UEFA          |
| 30       | 30       | 31    | Tunisia              | TUN       | CAF           |
| 31       | 31       | 32    | Chile                | CHI       | CONMEBOL      |
| 32       | 32       | 33    | Costa Rica           | CRC       | CONCACAF      |
| 33       | 33       | 34    | Cameroon             | CMR       | CAF           |
| 35       | 34       | 35    | Nigeria              | NGA       | CAF           |
| 39       | 35       | 36    | Egypt                | EGY       | CAF           |
| 40       | 36       | 37    | Algeria              | ALG       | CAF           |
| 41       | 37       | 38    | Ecuador              | ECU       | CONMEBOL      |
| 45       | 38       | 39    | Mali                 | MLI       | CAF           |
| 47       | 39       | 40    | Côte d'Ivoire        | CIV       | CAF           |
| 49       | 40       | 41    | Saudi Arabia         | KSA       | AFC           |
| 50       | 41       | 42    | Burkina Faso         | BFA       | CAF           |
| 60       | 42       | 43    | Qatar                | QAT       | AFC           |
| 61       | 43       | 44    | Panama               | PAN       | CONCACAF      |
| 64       | 44       | 45    | Jamaica <sup>4</sup> | JAM       | CONCACAF      |
| 68       | 45       | 46    | Iraq                 | IRQ       | AFC           |
| 70       | 46       | 47    | UAE                  | UAE       | AFC           |
| 105      | 47       | 48    | New Zeland           | NZL       | OFC           |

Table B.3: National teams for the 2026 World Cup using rankings as of 21-Dec-2023.

| Absolute | Relative | Score | Team               | FIFA Code | Confederation |
|----------|----------|-------|--------------------|-----------|---------------|
| 1        | 1        | 1     | Argentina          | ARG       | CONMEBOL      |
| 2        | 2        | 2     | France             | FRA       | UEFA          |
| 3        | 3        | 3     | England            | ENG       | UEFA          |
| 4        | 4        | 4     | Belgium            | BEL       | UEFA          |
| 5        | 5        | 5     | Brazil             | BRA       | CONMEBOL      |
| 6        | 6        | 6     | Netherlands        | NED       | UEFA          |
| 7        | 7        | 7     | Portugal           | POR       | UEFA          |
| 8        | 8        | 8     | Spain              | ESP       | UEFA          |
| 9        | 9        | 9     | Italy              | ITA       | UEFA          |
| 10       | 10       | 10    | Croatia            | CRO       | UEFA          |
| 11       | 11       | 11    | Uruguay            | URU       | CONMEBOL      |
| 12       | 12       | 12    | USA                | USA       | CONCACAF      |
| 13       | 13       | 13    | Morocco            | MAR       | CAF           |
| 14       | 14       | 14    | Colombia           | COL       | CONMEBOL      |
| 15       | 15       | 15    | Mexico             | MEX       | CONCACAF      |
| 48       | 38       | 16    | Canada             | CAN       | CONCACAF      |
| 16       | 16       | 17    | Germany            | GER       | UEFA          |
| 17       | 17       | 18    | Japan              | JPN       | AFC           |
| 18       | 18       | 19    | Switzerland        | SUI       | UEFA          |
| 19       | 19       | 20    | Denmark            | DEN       | UEFA          |
| 20       | 20       | 21    | Senegal            | SEN       | CAF           |
| 21       | 21       | 22    | Iran               | IRN       | AFC           |
| 22       | 22       | 23    | Ukraine            | UKR       | UEFA          |
| 23       | 23       | 24    | Korea Republic     | KOR       | AFC           |
| 24       | 24       | 25    | Austria            | AUT       | UEFA          |
| 25       | 25       | 26    | Australia          | AUS       | AFC           |
| 26       | 26       | 27    | Sweden             | SWE       | UEFA          |
| 27       | 27       | 28    | Hungary            | HUN       | UEFA          |
| 28       | 28       | 29    | Tunisia            | TUN       | CAF           |
| 29       | 29       | 30    | Wales              | WAL       | UEFA          |
| 30       | 30       | 31    | Algeria            | ALG       | CAF           |
| 32       | 31       | 32    | Ecuador            | ECU       | CONMEBOL      |
| 33       | 32       | 33    | Egypt              | EGY       | CAF           |
| 35       | 33       | 34    | Peru               | PER       | CONMEBOL      |
| 40       | 34       | 35    | Chile              | CHI       | CONMEBOL      |
| 41       | 35       | 36    | Panama             | PAN       | CONCACAF      |
| 42       | 36       | 37    | Nigeria            | NGA       | CAF           |
| 46       | 37       | 38    | Cameroon           | CMR       | CAF           |
| 49       | 38       | 39    | Côte d'Ivoire      | CIV       | CAF           |
| 51       | 39       | 40    | Mali               | MLI       | CAF           |
| 52       | 40       | 41    | Costa Rica         | CRC       | CONCACAF      |
| 55       | 41       | 42    | Jamaica            | JAM       | CONCACAF      |
| 56       | 42       | 43    | Saudi Arabia       | KSA       | AFC           |
| 57       | 43       | 44    | Burkina Faso       | BFA       | CAF           |
| 58       | 44       | 45    | Qatar <sup>5</sup> | QAT       | AFC           |
| 63       | 45       | 46    | Iraq               | IRQ       | AFC           |
| 64       | 46       | 47    | UAE                | UAE       | AFC           |
| 104      | 47       | 48    | New Zeland         | NZL       | OFC           |

Table B.4: Number of solutions from each valid configuration with countries in Table B.2. The total number of perfect solutions is 3,284,230,574. Total run time: 29,644 sec (8.23 hours).

| Changes | Perf. sol.    | Changes     | Perf. sol. | Changes           | Perf. sol. |
|---------|---------------|-------------|------------|-------------------|------------|
| -       | 775,076,111   | 29,31,33,34 | 0          | 30,32,33,34       | 0          |
| 29,33   | 53,245,176    | 29,31,33,35 | 0          | 30,32,33,35       | 0          |
| 29,34   | 0             | 29,31,33,36 | 0          | 30,32,33,36       | 0          |
| 29,35   | 0             | 29,31,34,35 | 0          | 30,32,34,35       | 0          |
| 29,36   | 0             | 29,31,34,36 | 0          | 30,32,34,36       | 0          |
| 30,33   | 188,461,885   | 29,32,33,34 | 0          | 31,32,33,34       | 0          |
| 30,34   | 0             | 29,32,33,35 | 0          | 31,32,33,35       | 0          |
| 30,35   | 0             | 29,32,33,36 | 0          | 31,32,33,36       | 0          |
| 30,36   | 0             | 29,32,34,35 | 0          | 31,32,34,35       | 0          |
| 31,33   | 1,245,595,895 | 29,32,34,36 | 0          | 31,32,34,36       | 0          |
| 31,34   | 0             | 30,31,33,34 | 0          | 29,31,32,33,34,35 | 0          |
| 31,35   | 0             | 30,31,33,35 | 0          | 29,31,32,33,34,36 | 0          |
| 31,36   | 0             | 30,31,33,36 | 0          | 30,31,32,33,34,35 | 0          |
| 32,33   | 1,021,851,507 | 30,31,34,35 | 0          | 30,31,32,33,34,36 | 0          |
| 32,34   | 0             | 30,31,34,36 | 0          |                   |            |
| 32,35   | 0             |             |            |                   |            |
| 32,36   | 0             |             |            |                   |            |

Table B.5: Number of solutions from each valid configuration with countries in Table B.3. The total number of perfect solutions is 714,837,429. Total run time: 58,811 sec (16.61 hours).

| Changes | Perf. sol.  | Changes     | Perf. sol. | Changes           | Perf. sol. |
|---------|-------------|-------------|------------|-------------------|------------|
| -       | 159,882,508 | 29,31,33,34 | 736,432    | 30,32,33,34       | 10,059,893 |
| 29,33   | 8,861,835   | 29,31,33,35 | 564,488    | 30,32,33,35       | 5,569,986  |
| 29,34   | 7,924,610   | 29,31,33,36 | 400,969    | 30,32,33,36       | 2,087,596  |
| 29,35   | 4,763,099   | 29,31,34,35 | 364,130    | 30,32,34,35       | 2,344,146  |
| 29,36   | 1,776,573   | 29,31,34,36 | 91,280     | 30,32,34,36       | 620,605    |
| 30,33   | 37,712,856  | 29,32,33,34 | 2,104,146  | 31,32,33,34       | 21,167,103 |
| 30,34   | 27,883,667  | 29,32,33,35 | 1,214,552  | 31,32,33,35       | 12,393,926 |
| 30,35   | 14,322,840  | 29,32,33,36 | 467,633    | 31,32,33,36       | 5,578,885  |
| 30,36   | 6,491,401   | 29,32,34,35 | 547,252    | 31,32,34,35       | 5,193,055  |
| 31,33   | 66,331,586  | 29,32,34,36 | 151,121    | 31,32,34,36       | 1,493,220  |
| 31,34   | 54,676,902  | 30,31,33,34 | 4,651,914  | 29,31,32,33,34,35 | 0          |
| 31,35   | 27,473,595  | 30,31,33,35 | 3,754,896  | 29,31,32,33,34,36 | 17,788     |
| 31,36   | 14,227,986  | 30,31,33,36 | 2,383,735  | 30,31,32,33,34,35 | 0          |
| 32,33   | 80,564,250  | 30,31,34,35 | 2,036,359  | 30,31,32,33,34,36 | 103,781    |
| 32,34   | 69,692,577  | 30,31,34,36 | 440,883    |                   |            |
| 32,35   | 32,548,337  |             |            |                   |            |
| 32,36   | 13,163,033  |             |            |                   |            |

Table C.1: Proportion ( $\times 10^{-3}$ ) of perfect solutions with row team in the same group as column team

|     | COL | URU | SWE | WAL | SEN | IRN | PER | SRB | CHI | UKR | JPN | POL | MAR | ALG | TUN | KOR | AUS | NGA | PAR | EGY | QAT | CRC | CMR | KSA | GHA | MLI | CIV | JAM | PAN | UAE | CHN | NZL |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| BEL |     |     |     |     |     |     |     | 52  | 158 | 121 | 63  | 99  | 133 |     |     |     | 57  | 59  | 71  | 112 | 73  | 130 | 126 | 59  |     | 57  |     |     | 183 | 113 | 62  | 142 | 128 |     |     |     |     |     |     |
| BRA |     |     |     |     |     |     |     | 129 |     | 131 | 148 | 107 |     |     |     | 89  | 78  | 134 | 184 |     |     | 94  | 142 | 160 | 89  |     |     | 54  | 119 | 142 | 70  | 67  | 61  |     |     |     |     |     |     |
| FRA |     |     |     |     |     |     | 78  | 126 | 97  | 116 | 69  | 87  | 70  |     | 72  | 64  | 75  | 29  | 55  | 61  | 61  | 55  | 56  | 48  | 137 |     | 32  | 83  | 72  | 90  | 113 | 114 | 138 |     |     |     |     |     |     |
| ENG |     |     |     |     |     |     | 87  | 76  | 119 | 83  | 87  | 64  | 86  | 61  | 77  | 70  | 56  | 26  | 47  | 61  | 61  | 47  | 82  |     | 146 | 29  | 75  | 78  | 68  | 85  | 108 | 57  | 164 |     |     |     |     |     |     |
| ARG |     |     |     |     |     |     | 68  | 238 |     | 68  |     | 75  | 91  | 53  | 78  | 74  | 142 | 55  | 57  |     |     | 112 | 71  | 145 |     | 98  | 78  | 84  | 38  | 31  | 37  |     | 117 | 188 |     |     |     |     |     |
| ITA |     |     |     |     |     |     | 46  | 97  | 156 | 50  | 71  | 51  | 70  | 48  | 29  | 35  | 50  | 114 | 51  | 66  | 64  | 37  | 54  | 39  | 111 | 105 | 22  | 13  | 52  | 60  | 62  | 57  | 89  | 77  | 131 | 92  |     |     |     |
| ESP |     |     |     |     |     |     | 43  | 93  | 91  | 127 | 89  | 63  | 46  | 59  | 34  | 24  | 32  | 101 | 50  | 53  | 61  | 34  | 69  | 79  |     | 105 | 60  | 44  | 13  | 49  | 53  | 57  | 71  | 104 | 120 | 91  | 87  |     |     |
| POR |     |     |     |     |     |     | 44  | 101 | 105 | 87  | 104 | 90  | 67  | 44  | 49  | 33  | 40  | 27  | 53  | 65  | 58  | 35  |     |     | 67  | 28  | 45  | 33  | 51  | 55  | 82  | 90  | 96  | 102 | 104 | 92  |     |     |     |
| DEN |     |     |     |     |     |     | 108 | 124 | 108 | 79  | 49  | 96  | 67  | 37  | 51  | 35  | 16  | 39  | 75  | 72  | 45  |     | 147 | 24  | 14  | 30  | 52  | 35  | 55  | 82  | 48  | 84  | 91  | 125 | 117 | 50  |     |     |     |
| NED |     |     |     |     |     |     | 130 | 119 | 89  | 76  |     | 95  | 57  | 39  | 51  | 30  | 32  | 48  | 72  | 58  | 104 | 51  | 110 |     | 55  | 37  | 82  | 47  | 39  | 76  | 100 | 139 | 60  |     |     |     |     |     |     |
| USA |     |     |     |     |     |     | 84  |     | 63  | 112 |     | 62  | 76  | 31  | 30  | 56  | 81  | 62  | 87  | 184 | 73  | 163 | 181 | 35  | 61  | 80  | 53  |     | 77  | 30  | 62  | 52  | 123 |     | 84  |     |     |     |     |
| GER |     |     |     |     |     |     | 98  | 100 | 101 | 79  | 36  | 48  | 64  | 36  | 66  | 54  | 66  | 46  | 142 | 64  |     | 135 | 88  | 67  | 53  | 69  | 45  | 42  | 86  | 36  | 37  | 90  | 109 | 92  | 52  |     |     |     |     |
| SUI |     |     |     |     |     |     | 127 | 135 | 115 | 82  | 48  | 95  | 63  | 60  | 49  | 65  | 44  | 40  | 78  |     | 98  | 20  | 80  | 29  | 86  | 55  | 69  | 57  | 82  | 103 | 124 | 139 | 58  |     |     |     |     |     |     |
| MEX |     |     |     |     |     |     | 179 | 172 | 123 | 105 |     | 114 | 91  | 67  | 74  |     | 77  |     |     |     | 52  | 24  | 26  | 114 | 38  | 127 | 40  | 62  | 90  | 157 | 181 | 87  |     |     |     |     |     |     |     |
| CRO |     |     |     |     |     |     | 128 | 103 | 98  | 87  | 100 | 78  | 59  | 70  | 84  | 42  | 56  | 95  |     | 49  | 102 | 20  | 52  | 94  | 30  | 138 | 52  | 81  | 113 | 78  | 126 | 66  |     |     |     |     |     |     |     |
| CAN |     |     |     |     |     |     | 102 | 103 | 61  | 37  | 55  | 144 | 55  | 66  | 62  | 70  | 135 | 110 | 53  | 127 | 112 | 54  | 55  | 97  | 58  | 86  | 92  |     | 122 | 83  | 61  |     |     |     |     |     |     |     |     |
| COL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 41  | 124 | 157 | 156 | 104 | 52  | 155 | 118 | 94  |     |     |     |
| URU |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 61  | 82  | 146 | 154 | 126 | 45  | 50  | 134 | 112 | 91  |     |     |
| SWE |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 120 | 86  | 130 | 108 | 116 | 39  | 98  | 113 | 99  | 91  |     |     |
| WAL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 37  | 34  | 116 | 77  | 86  | 93  | 100 | 73  | 82  | 96  | 94  | 113 |
| SEN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 108 | 47  |     | 84  |     |     | 96  | 112 | 149 | 196 | 208 |     |
| IRN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     | 154 |     |     |     |
| PER |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| SRB |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| CHI |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| UKR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| JPN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| POL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| MAR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ALG |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| TUN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| KOR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| AUS |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| NGA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| PAR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| EGY |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |





Table C.4: Proportion ( $\times 10^{-3}$ ) of simulation runs of Draw 2B with row team in the same group as column team. Green (red) color means that the proportion under this draw is higher (lower) than under the Uniform Draw; darker colors indicate larger differences.

|     | COL | URU | SWE | WAL | SEN | IRN | PER | SRB | CHI | UKR | JPN | POL | MAR | ALG | TUN | KOR | AUS | NGA | PAR | EGY | QAT | CRC | CMR | KSA | GHA | MLI | CIV | JAM | PAN | PAK | CHN | NZL |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| BEL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| BRA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| FRA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ENG |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ARG |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ITA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ESP |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| POR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| DEN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| NED |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| USA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| GER |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| SUI |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| MEX |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| CRO |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| CAN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| COL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| URU |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| SWE |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| WAL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| SEN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| IRN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| PER |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| SRB |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| CHI |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| UKR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| JPN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| POL |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| MAR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| ALG |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| TUN |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| KOR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| AUS |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| NGA |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| PAR |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| EGY |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |