Online Appendix

A Parametric performance measures

To compare the out-of-sample performance of the optimal portfolios over the entire period we compute a number of commonly used parametric performance measures of the benchmark and augmented portfolios,.

We use the Sharpe ratio, the downside Sharpe ratio (Ziemba, 2005), and the upside potential (UP) ratio (Sortino and Van Der Meer, 1991).

Let R_t be the realized weekly return of portfolio P for $t = 1, ..., T$, of the tth rolling window, with $T = 202$, and let R_f be the return of the risk-free rate for the same period. The Sharpe ratio is given by:

$$SR_p = \frac{E[R_p - R_f]}{(Var[R_p - R_f])^{1/2}}, \tag{19}$$

The downside Sharpe ratio is given by:

$$S_{P_-} = \frac{\bar{R}_p - R_f}{\sqrt{2}\sigma_{P-}}, \tag{20}$$

where the downside risk σ_{P-} is calculated by $\sigma^2_{P_-} = \frac{\sum_{t=1}^{T}(x_t - \bar{x})^2_-}{T-1}$, where the benchmark $\bar{x}$ is set to zero, and the numerator is positive for the quarters t for which the portfolio P has losses (x_t is negative). The UP ratio is given by:

$$\mathrm{UP} = \frac{\frac{1}{K}\sum_{t=1}^{K}\max[0, R_t - \rho_t]}{\sqrt{\frac{1}{K}\sum_{t=1}^{K}(\max[0, \rho_t - R_t])^2}}. \tag{21}$$

where ρ_t is the return of the riskless asset for the same period.

We also use the opportunity cost (Simaan, 1993) to assess the economic significance of the performance difference of the two optimal portfolios. Let R_A and R_B be the realized returns of the portfolio for the augmented and the benchmark asset classes, respectively. The opportunity cost θ is defined as the return that needs to be added to (or subtracted from) the benchmark portfolio return R_B so that the investor is indifferent (in utility terms) between the optimal portfolio strategies for the two investment opportunity sets, i.e.,

$$E[U(1 + R_B + \theta)] = E[U(1 + R_A)]. \tag{22}$$

A positive (negative) θ implies that the investor is better (worse) off if the investment opportunity set includes green bond indices. The opportunity cost takes into account the entire probability density function of asset returns and evaluates strategies even when the asset return distribution is not normal. For the calculation of the opportunity cost, we use exponential and power utility functions, consistent with second-degree stochastic dominance, for different levels of risk aversion.

Next, we compute the portfolio turnover (PT), defined as the average of the absolute change of weights over the T rebalancing points for the N assets.

$$PT = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}(|w_{i,t+1} - w_{i,t}|), \tag{23}$$

where $w_{i,t+1}$ and $w_{i,t}$ are the optimal weights of asset i at time t and $t+1$, respectively. PT indicates the amount of rebalancing required to implement the strategy.

Finally, we evaluate the performance of the two portfolios under the risk-adjusted (net of transaction costs) returns measure (DeMiguel et al., 2009), to account for proportional transaction costs, generated by portfolio turnover, in portfolio returns. Let tc_i be the proportional transaction cost for asset i, and R_{t+1} the realized return of the portfolio at time $t+1$. The portfolio wealth net of transaction costs, NW, is given by:

$$NW_{t+1} = NW_t(1 + R_{t+1})[1 - \sum_{i=1}^{N}(tc_i \times |w_{i,t+1} - w_{i,t}|)], \tag{24}$$

and the return, net of transaction costs, is defined as

$$RTC_{t+1} = \frac{NW_{t+1}}{NW_t} - 1. \tag{25}$$

Let μ_B and μ_A be the out-of-sample mean of quarterly RTC with the benchmark and the augmented opportunity set, respectively, and σ_B and σ_A be the corresponding standard deviations. Then, the return-loss measure is

$$RLM = \frac{\mu_A}{\sigma_A} \times \sigma_B - \mu_B, \tag{26}$$

i.e., the additional return needed so that the benchmark portfolio performs as well as the augmented portfolio. We use transaction costs of 15bp for all assets.

B Out-of-sample robustness tests for the US and European markets

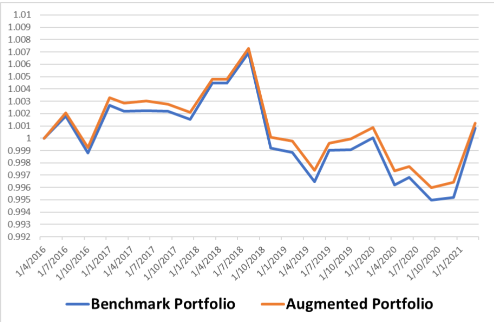
We test the robustness of our findings by performing several robustness tests, changing the estimation windows and rebalancing frequencies.

Figure B1 presents the out-of-sample cumulative returns of the optimal benchmark and augmented in the US and the European markets for three different cases. Panels A and B show the case of the full sample with 600-day estimation windows and 60-day rebalancing frequency. Panels C and D show the case of the full sample with 1200-day estimation windows and 60-day rebalancing frequency. Finally, Panels E and F indicate the case for the full sample with 1000-day estimation windows and 30-day rebalancing frequency.

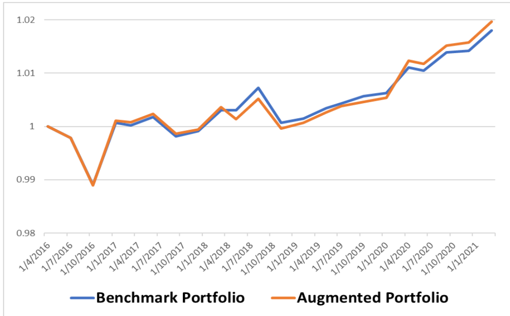
The main result, confirmed by the performance measures of Tables B1 and B2, is that the augmented portfolio with green bonds performs better than the benchmark portfolio in all the cases for both US and European markets.

Figure B1: Out-of-sample robustness tests in the US and European markets

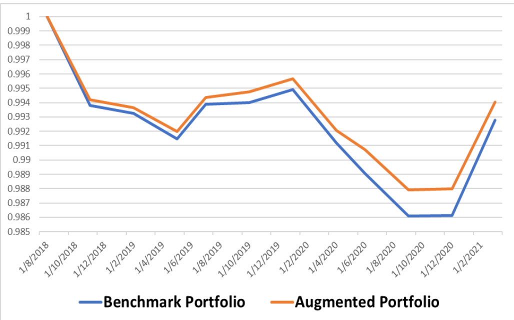
Cumulative performance of the benchmark optimal portfolio and the optimal augmented portfolio with green bonds in the US market and the European markets, respectively. Panels A and B are for the full sample with 600-day estimation windows and 60-day rebalancing frequency, Panels C and D are for the full sample with 1200-day estimation windows and 60-day rebalancing frequency, Panels E and F are for the full sample with 1000-day estimation windows and 30-day rebalancing frequency.



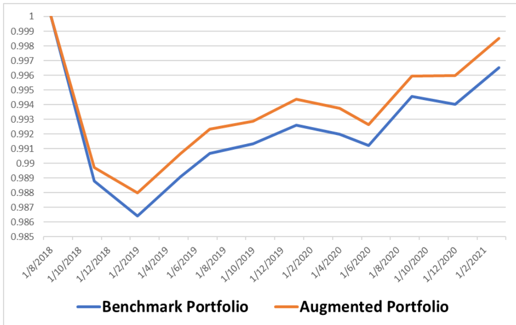
(a) 600-60 for US



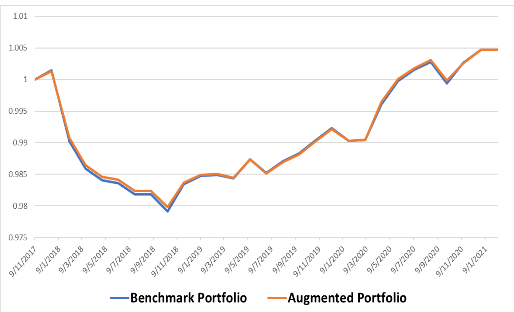
(b) 600-60 for EU



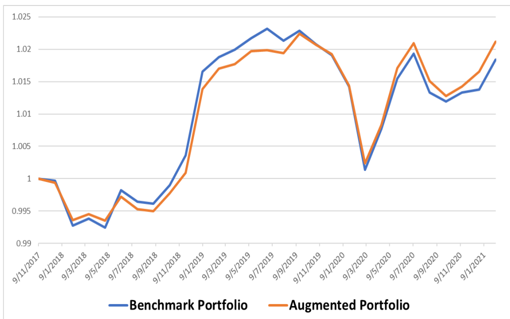
(c) 1200-60 for US



(d) 1200-60 for EU



(e) 1000-30 for US



(f) 1000-30 for EU

Table B1: Out-of-sample robustness portfolio performance in the US market

Entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, and annualized opportunity cost) for the benchmark and the augmented portfolio with green bonds. Panel A is for the full sample with 600-day estimation windows and 60-day rebalancing frequency, Panel B is for the full sample with 1200-day estimation windows and 60-day rebalancing frequency, and Panel C is for the full sample with 1000-day estimation windows and 30-day rebalancing frequency. Mean, volatility, and opportunity cost (θ) are in %. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function.

	(a) 600-60		(b) 1200-60		(c) 1000-30	
Performance measures	Benchmark	Augmented	Benchmark	Augmented	Benchmark	Augmented
Mean	1.087	1.557	-15.21	-12.68	4.469	4.442
Volatility	4.560	4.249	5.427	5.048	5.226	5.029
Sharpe Ratio	-0.005	0.104	-2.974	-2.703	0.583	0.601
Downside_SR	-0.005	0.098	-2.733	-2.491	0.525	0.547
UPratio	0.465	0.483	0.360	0.373	0.455	0.461
Opportunity Cost						
exponential utility						
ARA=2		0.480		3.018		0.000
ARA=4		0.505		3.044		0.025
ARA=6		0.556		3.096		0.025
power utility						
RRA=2		0.480		3.018		0.000
RRA=4		0.505		3.044		0.025
RRA=6		0.556		3.096		0.025

Table B2: Out-of-sample robustness portfolio performance in the European market

Entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, and annualized opportunity cost) for the benchmark and the augmented portfolio with green bonds. Panel A is for the full sample with 600-day estimation windows and 60-day rebalancing frequency, Panel B is for the full sample with 1200-day estimation windows and 60-day rebalancing frequency, and Panel C is for the full sample with 1000-day estimation windows and 30-day rebalancing frequency. Mean, volatility, and opportunity cost (θ) are in %. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function.

	(a) 600-60		(b) 1200-60		(c) 1000-30	
Performance measures	Benchmark	Augmented	Benchmark	Augmented	Benchmark	Augmented
Mean	24.17	26.70	-7.464	-3.191	18.22	21.08
Volatility	6.686	6.959	6.377	5.947	7.806	7.517
Sharpe Ratio	3.409	3.633	-1.331	-0.719	2.131	2.590
Downside_SR	3.856	4.356	-1.043	-0.576	2.282	2.877
UPratio	0.728	0.797	0.309	0.343	0.654	0.699
Opportunity Cost						
exponential utility						
ARA=2		1.985		4.666		2.449
ARA=4		1.959		4.719		2.500
ARA=6		1.934		4.772		2.552
power utility						
RRA=2		1.985		4.666		2.449
RRA=4		1.959		4.719		2.500
RRA=6		1.934		4.772		2.552

C Data Appendix

Table C1 reports summary statistics for the performance of the US and European assets. The Jarque-Berra tests indicate that asset returns are not normally distributed, justifying our choice of stochastic dominance. Except for the commodity index, all return series have positive annualized average daily returns over the sample period. European assets exhibit a less favorable risk-return profile compared to U.S. assets. Table C2 reports the correlation coefficient between US and European assets, respectively. A positive correlation exists between European bonds (green and conventional), stock, commodity, and real estate assets, which is higher than that between US assets.

Table C1: Descriptive statistics for the US and European assets

Entries report the descriptive statistics (annualized mean and standard deviation, skewness, kurtosis and annualized Sharpe ratio) on daily returns for the US and European assets. Mean and standard deviation are in %. The last column presents the p-values of the Jarque-Bera test in order to test the null hypothesis that the data are normally distributed at the 1% significance level. Data spans the period from 31/12/2013 to 11/3/2021 for a total of 1863 daily returns.

	Mean	St.dev.	Skewness	Kurtosis	JB test
			US ASSETS		
BBG Barclays MSCI US Green	3.625	3.326	-0.860	18.02	0.001
BBG Barclays US Agg	3.524	3.364	-0.851	11.55	0.001
MSCI USA Index	12.77	17.57	-0.733	23.52	0.001
S&P GSCI Index	-1.013	21.56	-0.640	12.02	0.001
Dow Jones US Real Estate	6.959	19.72	-1.746	34.76	0.001
			EUROPEAN ASSETS		
BBG Barclays MSCI Euro Green	2.042	8.319	-0.302	8.075	0.001
BBG Barclays Euro Agg	1.690	8.117	-0.136	6.085	0.001
MSCI Europe Index	2.702	17.35	-1.230	20.69	0.001
S&P World Commodity EU Index	-1.667	25.64	-0.362	12.34	0.001
Dow Jones EU Select Real Estate	0.371	20.12	-1.404	22.06	0.001

Table C2: Correlation matrix for the US and European assets

Entries report the correlation coefficients between US and European assets, respectively.

	US ASSETS				
	BBG Barclays MSCI US Green	BBG Barclays US Agg	MSCI USA Index	S&P GSCI Index	Dow Jones US Real Estate
BBG Barclays MSCI US Green	1.000	0.883	-0.160	-0.123	0.072
BBG Barclays US Agg	0.883	1.000	-0.231	-0.160	0.021
MSCI USA Index	-0.160	-0.231	1.000	0.370	0.762
S&P GSCI Index	-0.123	-0.160	0.370	1.000	0.247
Dow Jones US Real Estate	0.072	0.021	0.762	0.247	1.000

	EUROPEAN ASSETS				
	BBG Barclays MSCI Euro Green	BBG Barclays Euro Agg	MSCI Europe Index	S&P World Commodity EU Index	Dow Jones EU Select Real Estate
BBG Barclays MSCI Euro Green	1.000	0.934	0.207	0.020	0.292
BBG Barclays Euro Agg	0.934	1.000	0.241	0.040	0.324
MSCI Europe Index	0.207	0.241	1.000	0.391	0.779
S&P World Commodity EU Index	0.020	0.040	0.391	1.000	0.266
Dow Jones EU Select Real Estate	0.292	0.324	0.779	0.266	1.000

Table C3 reports summary statistics for the performance of the benchmark and green bond indices over the sample period. The Jarque-Berra tests indicate that asset returns are not normally distributed. Except for the commodity index, all return series have positive annualized average daily returns over the sample period. Stock, real estate, and energy indices achieve the highest annualized return, and the green bond indices have the lowest annualized return except for the ICE BofA Green, which has a high annualized return with low annualized volatility. Moreover, the annualized Sharpe ratio of green bonds is lower than that of other financial assets, such as stock, real estate, corporate, and government bonds, with ICE BofA being the sole exception.

Table C4 reports the correlation coefficient between the benchmark assets and green bonds. Green bonds have moderate to high correlations with conventional bonds, ranging from 0.29 to 0.92, and very low to moderate correlations with stock, real estate, commodity, and energy assets, ranging from -0.05 to 0.26, suggesting potential diversification benefits from the introduction of green bonds.

Table C3: Descriptive statistics for the global assets

Entries report the descriptive statistics (annualized mean and standard deviation, skewness, kurtosis and annualized Sharpe ratio) on daily returns for the benchmark assets that proxy the benchmark portfolio and the augmented with green bond indices. Mean and standard deviation are in %. The last column presents the p-values of the Jarque-Bera test in order to test the null hypothesis that the data are normally distributed at the 1% significance level. Data spans the period from 2/12/2014 to 17/9/2021 for a total of 1767 daily returns.

	Mean	**St.dev.**	**Skewness**	**Kurtosis**	**Sharpe ratio**	**JB test**
BENCHMARK ASSETS						
Five markets						
MSCI World	12.47	15.05	-1.161	25.24	0.767	0.001
DJ Global Select RESI	6.962	16.57	-1.853	33.08	0.367	0.001
S&P GSCI Energy Spot	8.512	38.41	-0.437	20.65	0.198	0.001
S&P GSCI Commodity	-2.755	22.27	-0.646	11.14	-0.160	0.001
BBG Barclays Global Agg	2.508	4.619	-0.513	8.285	0.359	0.001
Only bond market						
Corporate						
S&P IG Corp	4.542	4.313	-1.330	17.52	0.853	0.001
S&P INT Corp	2.521	7.912	-0.882	12.73	0.211	0.001
ICE BofA Global Corp	4.038	3.581	-2.343	31.61	0.888	0.001
FTSE WorldBIG Corp	3.981	3.691	-1.824	24.33	0.846	0.001
BBG Barclays Global Corp	3.694	4.589	-1.931	24.74	0.618	0.001
Government						
ICE BofA Global Gvt	2.281	2.928	-0.347	7.871	0.490	0.001
FTSE World Gvt	2.472	3.002	-0.428	8.679	0.542	0.001
BBG Barclays Global Agg Gvt	2.270	5.570	-0.182	6.988	0.256	0.001
US BENCH 5Y Gvt	2.337	3.122	0.159	5.386	0.478	0.001
US BENCH 10Y Gvt	3.629	6.335	-0.119	6.805	0.438	0.001
US BENCH 30Y Gvt	7.490	15.25	0.256	10.86	0.433	0.001
GREEN ASSETS						
S&P Green	2.120	4.950	-0.553	9.063	0.258	0.001
S&P Green Select	2.145	5.645	-0.549	9.239	0.230	0.001
ICE BofA Green	2.711	2.731	-1.400	16.63	0.682	0.001
BBG Barclays MSCI Green	2.117	5.698	-0.558	9.125	0.223	0.001
Solactive Green	1.943	5.452	-0.523	8.203	0.202	0.001

Table C4: Correlation matrix for the global assets

Entries report the correlation coefficients between different cases of the benchmark assets and green assets in daily data.

	MSCI WORLD	DJ GLOBAL SELECT RESI	S&P GSCI Energy Spot	S&P GSCI Commodity	BBG Barclays Global Agg	S&P IG CORP	S&P INT CORP	ICE BofA Global Corp	FTSE WorldBIG Corp	BBG Barclays Global Corp	ICE BofA Global Gvt
MSCI World	1.000	0.754	0.397	0.426	-0.026	-0.063	0.230	0.017	0.025	0.110	-0.242
DJ Global Select RESI	0.754	1.000	0.260	0.283	0.169	0.171	0.345	0.248	0.257	0.308	-0.021
S&P GSCI Energy Spot	0.397	0.260	1.000	0.961	-0.049	-0.088	0.127	-0.043	-0.041	0.010	-0.175
S&P GSCI Commodity	0.426	0.283	0.961	1.000	-0.027	-0.099	0.162	-0.056	-0.053	0.024	-0.190
BBG Barclays Global Agg	-0.026	0.169	-0.049	-0.027	1.000	0.613	0.830	0.641	0.615	0.881	0.652
S&P IG Corp	-0.063	0.171	-0.088	-0.099	0.613	1.000	0.398	0.930	0.950	0.821	0.705
S&P INT Corp	0.230	0.345	0.127	0.162	0.830	0.398	1.000	0.451	0.446	0.800	0.322
ICE BofA Global Corp	0.017	0.248	-0.043	-0.056	0.641	0.930	0.451	1.000	0.962	0.855	0.765
FTSE WorldBIG Corp	0.025	0.257	-0.041	-0.053	0.615	0.950	0.446	0.962	1.000	0.831	0.723
BBG Barclays Global Corp	0.110	0.308	0.010	0.024	0.881	0.821	0.800	0.855	0.831	1.000	0.627
ICE BofA Global Gvt	-0.242	-0.021	-0.175	-0.190	0.652	0.705	0.322	0.765	0.723	0.627	1.000
FTSE World Gvt	-0.210	0.005	-0.157	-0.172	0.647	0.695	0.332	0.764	0.726	0.625	0.993
BBG Barclays Global Gvt	-0.064	0.121	-0.061	-0.038	0.983	0.511	0.807	0.540	0.513	0.801	0.623
US Bench 5Y Gvt	-0.326	-0.085	-0.193	-0.210	0.593	0.691	0.250	0.672	0.634	0.598	0.760
US Bench 10Y Gvt	-0.359	-0.124	-0.232	-0.246	0.604	0.756	0.245	0.748	0.701	0.635	0.848
US Bench 30Y Gvt	-0.371	-0.172	-0.284	-0.293	0.542	0.735	0.188	0.716	0.669	0.587	0.829
S&P Green	0.132	0.257	0.037	0.074	0.900	0.473	0.912	0.498	0.485	0.838	0.418
S&P Green Select	0.110	0.240	0.027	0.060	0.896	0.483	0.913	0.509	0.497	0.841	0.440
ICE BofA Green	-0.053	0.160	-0.071	-0.087	0.587	0.705	0.397	0.812	0.795	0.650	0.845
BBG Barclays MSCI Global Green	0.100	0.237	0.021	0.052	0.917	0.495	0.916	0.534	0.515	0.855	0.486
Solactive Green	0.126	0.242	0.043	0.079	0.881	0.423	0.912	0.463	0.446	0.807	0.401

	FTSE World Gvt	BBG Barclays Global Gvt	US Bench 5Y Gvt	US Bench 10Y Gvt	US Bench 30Y Gvt	S&P Green	S&P Green Select	ICE BofA Green	BBG Barclays MSCI Green	Solactive Green
MSCI World	-0.210	-0.064	-0.326	-0.359	-0.371	0.132	0.110	-0.053	0.100	0.126
DJ Global Select RESI	0.005	0.121	-0.085	-0.124	-0.172	0.257	0.240	0.160	0.237	0.242
S&P GSCI Energy Spot	-0.157	-0.061	-0.193	-0.232	-0.284	0.037	0.027	-0.071	0.021	0.043
S&P GSCI Commodity	-0.172	-0.038	-0.210	-0.246	-0.293	0.074	0.060	-0.087	0.052	0.079
BBG Barclays Global Agg	0.647	0.983	0.593	0.604	0.542	0.900	0.896	0.587	0.917	0.881
S&P IG Corp	0.695	0.511	0.691	0.756	0.735	0.473	0.483	0.705	0.495	0.423
S&P INT Corp	0.332	0.807	0.250	0.245	0.188	0.912	0.913	0.397	0.916	0.912
ICE BofA Global Corp	0.764	0.540	0.672	0.748	0.716	0.498	0.509	0.812	0.534	0.463
FTSE WorldBIG Corp	0.726	0.513	0.634	0.701	0.669	0.485	0.497	0.795	0.515	0.446
BBG Barclays Global Corp	0.625	0.801	0.598	0.635	0.587	0.838	0.841	0.650	0.855	0.807
ICE BofA Global Gvt	0.993	0.623	0.760	0.848	0.829	0.418	0.440	0.845	0.486	0.401
FTSE World Gvt	1.000	0.618	0.736	0.823	0.804	0.421	0.444	0.864	0.491	0.404
BBG Barclays Global Gvt	0.618	1.000	0.544	0.550	0.491	0.877	0.872	0.539	0.892	0.864
US Bench 5Y Gvt	0.736	0.544	1.000	0.938	0.796	0.382	0.392	0.595	0.423	0.347
US Bench 10Y Gvt	0.823	0.550	0.938	1.000	0.932	0.374	0.391	0.666	0.426	0.341
US Bench 30Y Gvt	0.804	0.491	0.796	0.932	1.000	0.320	0.338	0.632	0.372	0.287
S&P Green	0.421	0.877	0.382	0.374	0.320	1.000	0.993	0.446	0.980	0.971
S&P Green Select	0.444	0.872	0.392	0.391	0.338	0.993	1.000	0.474	0.988	0.969
ICE BofA Green	0.864	0.539	0.595	0.666	0.632	0.446	0.474	1.000	0.512	0.434
BBG Barclays MSCI Global Green	0.491	0.892	0.423	0.426	0.372	0.980	0.988	0.512	1.000	0.968
Solactive Green	0.404	0.864	0.347	0.341	0.287	0.971	0.969	0.434	0.968	1.000

D Robustness Check

We additionally test for the non-dominance of the benchmark portfolio over the augmented using the Davidson and Duclos (2013) stochastic non-dominance test, as a robustness check. We do that for two reasons. First, the test allows for correlated samples. Second, the Davidson and Duclos (2013) test has also as null hypothesis that one portfolio does not stochastically dominate another, i.e., the nondominance. We test the null hypothesis using second order stochastic dominance criteria.

We define the empirical versions of the dominance functions D^2_{Tr} and D^2_{Aug} (see Davidson and Duclos (2000) for the definition of dominance functions) as,

$$\hat{D}^2_{Tr} = \frac{1}{T}\sum_{t=1}^{T}(\max(z - y_t, 0)), \tag{27}$$

where T is the number of observations in the distribution sample of benchmark portfolio returns, y_t is the t -th observation, and z is the threshold of interest. Analogously, we define the dominance function for the augmented portfolio (Aug).

Then the augmented portfolio dominates at second order the benchmark portfolio iff $\hat{D}^2_{Tr} > \hat{D}^2_{Aug}$ for all y in the joint support. The null hypothesis of nondominance is not rejected unless there is dominance in the sample.

For each threshold level z, let the standardized difference of the two dominance functions,

$$t(z) = \frac{\hat{D}^2_{Tr} - \hat{D}^2_{Aug}}{(Var(\hat{D}^2_{Tr}) + Var(\hat{D}^2_{Aug}) - 2Cov(\hat{D}^2_{Tr}, \hat{D}^2_{Aug}))^{1/2}}, \tag{28}$$

and the resulting test statistic,

$$t^* = \min_z t(z). \tag{29}$$

In order to simulate the p-values we use the bootstrap methodology described in Davidson and Duclos (2013). Notice that the relevant limiting results of Davidson and Duclos (2013) could be justified in the framework of rebalanced optimal portfolios via results such as Theorem 4.4.2 of Politis et al. (1999). The results entail $T - 1$, i.e., 253 overlapping periods for the in-sample fitting of the two portfolios with corresponding out-of-sample comparisons. The 253 p-values are considered from September 14, 2016 to September 17, 2021, using overlapping periods of 100 weekly returns. Since it is impossible to aggregate the $T-1$ values of the Davidson and Duclos (2013) test statistic to get a unique measure of comparison, we compute quartile p-values from the distribution of the $T-1$ test statistics.