Supplementary material to the paper

# Simulation of an uncertain emission market for greenhouse gases using agent-based methods

by *Zbigniew Nahorski*[1], *Jarosław Stańczak*[1], *Piotr Pałka*[2]

*[1]Systems Research Institute, Polish Academy of Sciences, Newelska 6, 01-447 Warsaw, Poland*
*[2]Warsaw University of Technology, Faculty of Electronics and Information Technology, Institute of Control and Computation Engineering, Nowowiejska 15/19, 00-665, Warsaw, Poland*
*(Corresponding author – Zbigniew Nahorski, email: Zbigniew.Nahorski@ibspan.waw.pl, tel: +48223810275)*

## The central planner view on the optimization of a completely known market

The central planner perceives the use of an emission trading system as a minimization of the social cost function

$$F(\boldsymbol{x}) = \sum_{n=1}^{N} c^{P_n}(x^{P_n}), \qquad \boldsymbol{x} = (x^{P_1}, \dots, x^{P_N}) \tag{s1}$$

where $c^{P_n}(x^{P_n})$ is the abatement cost function of the $n$-th party to reduce the emission to a level of $x^{P_n}$, and subject to

$$\sum_{n=1}^{N} x^{P_n} = K_0$$

where $K_0$ is the total allowable emission of the market participants[1]. The costs typically grow with a raising of reduced quotas, so it can be assumed that $\frac{dc^{P_n}(x)}{dx} < 0$. Using the Lagrange method, an optimal solution satisfies the necessary conditions

$$\lambda = -\frac{dc^{P_n}(x^{P_n})}{dx}, \quad n = 1, \dots N; \qquad \sum_{n=1}^{N} x^{P_n} = K_0. \tag{s3}$$

The solution is optimal globally, if the functions $c^{P_n}$ are convex. Thus, at the optimal emissions the marginal costs of each party are equal to $\lambda$, which is the optimal price. In this manner the market can be optimized centrally, but only if the functions $c^{P_n}(x)$ are known to the central planner.

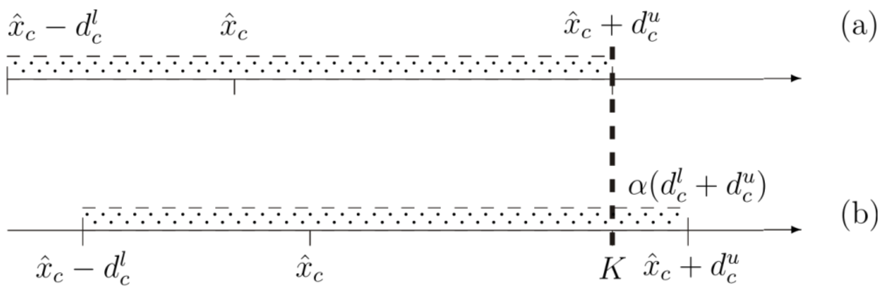


**Figure S1**. An illustration of full compliance and compliance with the risk $\alpha$.

[1] In actual fact, the total emission is bounded from above, i.e. it holds that $\sum_{n=1}^{N} x^{P_n} \le K_0$, but it is easy to notice that it is not optimal to keep the emissions below the boundary.

**Proof of assertion (21)**

To prove property (21)[2], let us first consider a party which only sells its emission permits. From (19), after the $t$-th transaction we have

$$\hat{x}_t^S = \hat{x}_0^S + \frac{\sum_{i=1}^{t} E_{eff,i}^S}{1 - u^S(\alpha)}.$$

Consequently, from (11) we get

$$\hat{x}_t^S = \hat{x}_0^S + \sum_{i=1}^{t} \hat{E}_i^S.$$

As a result, it is simply the sum of the initial emission and any permits that have been sold. All bear an association with the actual seller's uncertainty. Thus, the compliance with a risk $\alpha$ condition, see (9), is

$$\hat{x}_t^S[1 + u^S(\alpha)] \leq K^S.$$

By using now definition (10) we now get (21). This way (21) is true for the seller.

Let us now consider a party which only buys the permits. Similarly, as above, we have

$$\hat{x}_t^B = \hat{x}_0^B - \frac{\sum_{i=1}^{t} E_{eff,i}^B}{1 + u^B(\alpha)} = \frac{[1 + u^B(\alpha)]\hat{x}_0^B - \sum_{i=1}^{t}[1 - u^{S_i}(\alpha)]\hat{E}_i^{S_i}}{1 + u^B(\alpha)}.$$

a situation where $S_i$ are the parties that have sold permits to the buyer $B$. Now, proceeding in an iterative way, analogously to the left hand side of (12),we can see that the numerator on the right hand side is the value used in checking the compliance with the risk $\alpha$. Thus

$$\hat{x}_t^B \leq \frac{K^B}{1 + u^B(\alpha)} = \tilde{K}^B.$$

So, (21) is also true for the buyer.

In general, we can order the selling transactions as the first $j \leq t$ transactions, without loosing any generality. Then, when considering only the first $j$ transactions we realize that (21) is true. Were we to treat the estimated emissions and uncertainty spreads after the first $j$ transactions as a new starting point, and to then consider then the buying transactions, it may be concluded with reference to the first proven stage that (21) is true. This completes the proof of the assertion (21).

[2] All numbers in parentheses pertain to the equations in the main paper.

## Description of the agent system

A widely used standard for description of multi-agent systems is the ODD-protocol, Grimm at al. (2010). Although the presentation below does not strictly follow this protocol, many of its ideas has been applied. **The objective** of the model is to simulate trade of greenhouse gases permissions. An agent represents a trading party. Agents apply different algorithms, depending on a trading mechanism. The interaction protocol is designed according to the FIPA (2010) specifications[3]. A suitable language, known for each agent involved in the trade, is based on the $M^3$ bidding language (Kaleta and Toczyłowski, 2012; Traczyk, 2012), see also Nahorski at al. (2012). Each agent submits a negotiation proposals according to its own preferences, which are a private knowledge of each agent.

**The trade** goes exactly along the scheme presented in subsection 3.4. Auctions can be initiated either by buyers or sellers. A final contract is, however, always concluded bilaterally. After a contract has been reached, the entities exchange the data according to the established rules. Time is continuous, but is used only to record the order of concluded contracts. The trading is consistent with the economy rules, i.e. the cash flow, price/cost equations, etc., are preserved.

There are two kinds of **entities** in the system: the traders (together with the roles of negotiators, operators or bidders) and the morris column (blackboard, pilar column). A trader is characterized by the attributes (the name, the parameters of the cost function, the emission limit (target), the BAU (business as usual) emission, the uncertainty level, the trading mechanism) and the state variables, which are discussed in the sequel. The morris column have no attributes. Its state consists of identifiers of the registered agents.

**The agents** learn about other agents only by communicating with them. They do not observe other agents behavior, nor use predictions. Morris column agent is an intermediary. It keeps the information on the agents that are ready to trade. All results are available to the system operator.

**The initial state** of the simulation is given and constant. The BAU emissions, the emission limits, the cost function parameters, and the trading mechanisms are fixed in all performed simulations. No additional input data are used.

**An offer** is consistent with the $M^3$ standard and contains: the unit offer price (buying or selling, appropriately), the number of permits for trading, and the information if they are for selling or buying. An offer is represented as a Java Class consistent with the $M^3$ standard and serialized before sending the message. Having been received, the message is deserialized, and the offer is recovered. The offer is sent using the PROPOSE performative, and uses the FIPA parameters for addressing and assigning the unique identifier to each offer.

## Data and some results of computer simulations performed

**Table S1**. Data for the case study: parameters of cost reduction function and spreads of emission uncertainties.

| Party | BAU emission $\hat{x}_0^{P_n}$ [MtC/y] | Cost function parameter $a^{P_n}$ [MUSD/(MtC/y)$^2$] | Kyoto limit $K^{P_n}$ [MtC/y] | $d^l = d^u$ |
|---|---|---|---|---|
| USA | 1 820.3 | 0.2755 | 1251 | 0.13 |
| EU | 1 038.0 | 0.9065 | 860 | 0.10 |
| Japan | 350.0 | 2.4665 | 258 | 0.15 |
| CANZ | 312.7 | 1.1080 | 215 | 0.20 |
| EEFSU | 898.6 | 0.7845 | 1314 | 0.30 |

[3] The following FIPA standards are used: FIPA ACL Message Structure Specification (version G) to formulate proper communicative acts, FIPA Agent Management Specification (version K) to apply the Directory Facilitator Agent, and FIPA Propose Interaction Protocol (version H) to develop proper proposals exchange.

**Table S2**. Final results for several values of $\alpha$. Prices in USD/MtC/y, costs in USD.

| Party | Marginal price | Last transaction price | Volume of traded permits | Final emission | Emission reduction cost | Total cost of traded permits |
|---|---|---|---|---|---|---|
| **Sealed bid reverse auctions** | | | | | | |
| $\alpha$**=0.5** | | | | | | |
| USA | 143.3 | 143.5 | 309.3 | 1560.3 | 18 623.80 | 54 831.46 |
| EU | 141.4 | 141.1 | 100.0 | 960.0 | 5 515.15 | 16 216.40 |
| Japan | 144.0 | 141.1 | 62.8 | 320.8 | 2 103.04 | 11 307.24 |
| CANZ | 144.0 | 146.2 | 32.7 | 247.7 | 4 681.30 | 2 275.47 |
| EEFSU | 140.3 | 139.6 | -504.8 | 809.2 | 6 270.01 | -84 630.57 |
| $\alpha$**=0.3** | | | | | | |
| USA | 245.3 | 246.2 | 124.0 | 1375.0 | 60 755.86 | 41 027.40 |
| EU | 241.8 | 241.6 | 44.6 | 904.6 | 14 894.66 | 15 735.65 |
| Japan | 244.1 | 246.2 | 42.5 | 300.5 | 5 353.36 | 12 456.14 |
| CANZ | 240.9 | 242.9 | -11.0 | 204.0 | 11 197.25 | 1 280.45 |
| EEFSU | 238.5 | 241.6 | -567.4 | 746.6 | 23 316.02 | -70 499.64 |
| $\alpha$**=0.1** | | | | | | |
| USA | 319.9 | 318.6 | -11.4 | 1239.6 | 115 585.61 | 29 660.62 |
| EU | 318.7 | 323.4 | 2.2 | 862.2 | 33 038.58 | 15 596.07 |
| Japan | 315.9 | 321.6 | 28.0 | 286.0 | 8 025.75 | 15 365.88 |
| CANZ | 315.3 | 321.6 | -44.6 | 170.4 | 31 670.74 | -2 274.78 |
| EEFSU | 308.2 | 310.8 | -611.8 | 702.2 | 52 389.28 | -58 347.79 |
| **Bilateral transactions** | | | | | | |
| $\alpha$**=0.5** | | | | | | |
| USA | 142.6 | 142.6 | 310.5 | 1561.5 | 18 452.61 | 66 364.93 |
| EU | 142.6 | 142.6 | 99.3 | 959.3 | 5 608.05 | 17 667.54 |
| Japan | 142.6 | 142.6 | 63.1 | 321.1 | 2 061.10 | 12 581.13 |
| CANZ | 142.6 | 142.6 | 33.3 | 248.3 | 4 588.17 | -2 400.45 |
| EEFSU | 142.6 | 142.6 | -506.3 | 807.7 | 6 480.17 | -94 213.16 |
| $\alpha$**=0.3** | | | | | | |
| USA | 250.8 | 250.8 | 114.2 | 1365.2 | 63 502.10 | 51 723.90 |
| EU | 250.8 | 250.8 | 39.7 | 899.7 | 18 819.85 | 19 925.29 |
| Japan | 250.8 | 250.8 | 41.2 | 299.2 | 7 214.22 | 20 466.53 |
| CANZ | 250.8 | 250.8 | -15.5 | 199.5 | 16 765.29 | 1 697.77 |
| EEFSU | 250.8 | 250.8 | -575.2 | 738.8 | 25 880.23 | -93 813.48 |
| $\alpha$**=0.1** | | | | | | |
| USA | 336.4 | 336.4 | -41.2 | 1209.8 | 127 907.68 | 29 874.72 |
| EU | 336.4 | 336.4 | -7.5 | 852.5 | 36 872.98 | 21 193.41 |
| Japan | 336.4 | 336.4 | 23.8 | 281.8 | 14 811.72 | 29 369.90 |
| CANZ | 336.4 | 336.4 | -54.1 | 160.9 | 36 187.08 | 2 209.59 |
| EEFSU | 336.4 | 336.4 | -629.8 | 684.2 | 62 435.51 | -82 647.62 |

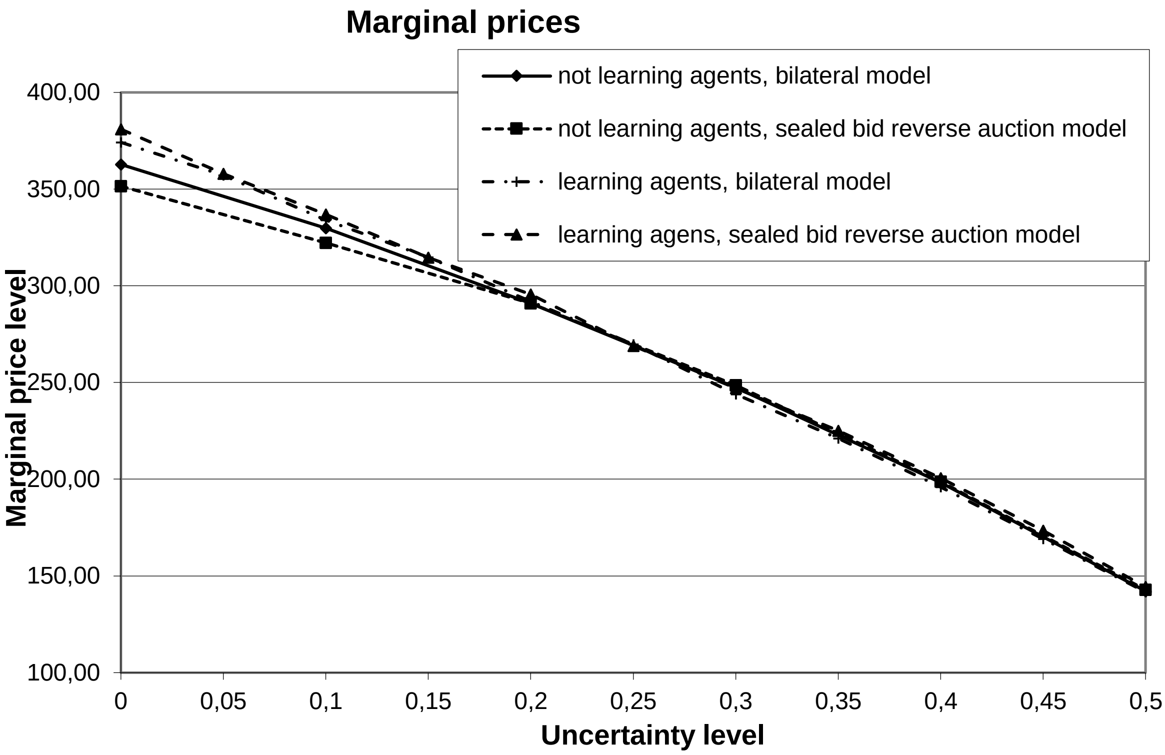


**Figure S2**. Dependence of marginal price, in USD/MtC/y, on the uncertainty level.

## Additional references (not listed in the main paper)


FIPA (2010) Foundation for Intelligent Physical Agents. http://www.fipa.org/

Kaleta M., Toczyłowski E. (2012) $M^3$ – motivations and formal model, In: M. Kaleta, T. Traczyk (Eds.) Modeling Multi-Commodity Trade: Information Exchange Methods. Springer, 3-19

Traczyk T. (2012) M3 Data Structures, In: M. Kaleta, T. Traczyk (eds.) Modeling Multi-Commodity Trade: Information Exchange Methods. Springer, 21-46