9 Technical appendix: not for publication

9.1 Balance of payments and government budget constraint equations

In this section I prove that the balance of payments equation is redundant in defining the steady-state equilibrium of the benchmark model.

Government I assume that the government needs to finance an exogenously given stream of public goods G as well as transfers to the unemployed:

$$G + uL_m b = \tau_L w_m (1-u) L_m \left(1 + \nu \frac{r+\lambda}{q}\right) + \tau_{e,m} p_E e_m (1-u) L_m. \tag{40}$$

We can rewrite the budget constraint by normalizing it with respect to the number of workers in the urban sector:

$$g + ub = \tau_L w_m (1-u) \left(1 + \nu \frac{r+\lambda}{q}\right) + \tau_{e,m} p_E e_m (1-u) \tag{41}$$

where $g \equiv G/L_m$.

Balance of payments Aggregate consumption in the rural area amounts to $C_a = w_a L_a$ and aggregate production of agricultural goods amounts to $g(k_a)L_a$, so that the excess income in the rural area that exceeds demand is used to rent capital that is fixed in the agricultural sector:

$$g(k_a)L_a - C_a = r_a K_a \tag{42}$$

where K_a is the total stock of capital used in the agricultural sector. Given perfect competition in the agricultural sector, equation (42) holds.

Aggregate private consumption in the city is the sum of consumption of the formally employed and the unemployed:

$$C = w_m(1-u)L_m + \lambda P(1-u)L_m + uL_m b + uL_m z \tag{43}$$

The firm-level capitalized hiring costs amount to:

$$\frac{\lambda + r}{q} c = \frac{\lambda + r}{q} \nu w_m + \frac{\lambda + r}{q} \nu w_m \tau_L \tag{44}$$

where the second term is a payment to the government by a firm. The aggregate hiring costs are then:

$$\frac{\lambda + r}{q} \nu w_m (1-u) L_m \tag{45}$$

and the balance of payments in an economy implies that the excess of the aggregate income in the city over the aggregate demand is used to import both capital and energy at the international markets:

$$\begin{aligned} \left[(1-u)L_m A_m f(k_m, e_m) + uL_m z - \frac{\lambda + r}{q} \nu w_m (1-u) L_m\right] - C - G = \\ p_E e_m (1-u) L_m + r_m k_m (1-u) L_m \end{aligned} \tag{46}$$

By substituting the expression for C from (43) into (46) and dividing by L_m, we obtain:

$$\begin{aligned} (1-u)A_m f(k_m, e_m) - w_m(\lambda + r)\frac{\nu}{q}(1-u) - \frac{G}{L_m} - w_m(1-u) - \lambda P(1-u) - ub = \\ p_E e_m (1-u) + r_m k_m (1-u) \end{aligned} \tag{47}$$

Substituting G/L_m instead into the expression from the budget constraint (41), and dividing by $1-u$, I obtain:

$$A_m f(k_m, e_m) - w_m(\lambda + r)\frac{\nu}{q} - w_m - \lambda P - \tau_L w_m(1 + \nu\frac{r+\lambda}{q}) - \\ -\tau_{e,m} p_E e_m = p_E e_m + r_m k_m \tag{48}$$

which is exactly the wage determination equation derived as part of the steady-state equilibrium in the main text:

$$A_m f(k_m, e_m) - (1 + \tau_{e,m)} p_E e_m - r_m k_m = (1 + \tau_L) w_m \left[1 + (\lambda + r)\frac{\nu}{q}\right] + \lambda P$$