

Appendix to

Pre-service mathematics teachers' professional modeling competencies: a comparative study between Germany, Mainland China, and Hong Kong

(in: Educational Studies in Mathematics)

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Methodical considerations

An important feature of Qualitative Content Analysis is “to develop techniques of systematic, qualitatively oriented text analysis” (Mayring, 2014, p. 39). Therefore amongst others, a “systematic, rule-bound procedure” (ibid.) and “categories in the focus of analysis” (ibid., p. 40) as well as a “theory-guided character of the analysis” (ibid., p. 41) and the possibility of “integrating quantitative steps of analysis” (ibid.) are characteristic for the method of qualitative content analysis. Against the background of its basic principles qualitative content analysis thus summarizes several forms of respective data analysis, namely the “three fundamental forms of interpreting: summary, explication, and structuring” (ibid., p. 63), which can be further differentiated into sub-concepts.

Following the research question, the presented study uses qualitative content analysis to analyze pre-service teachers' (PSTs') written answers in questionnaires (from Schwarz, 2013) in order to find out commonalities and differences between the PSTs' groups in Germany, Hong Kong and Mainland China with regard to the PSTs' MCK and MPCK of mathematical modeling. Therefore, the concept of structuring the data is nearby, which allows “to assess the material according to certain criteria” (ibid., p. 64). Doing so allows evaluating the different answers on MCK and MPCK related questions with regard to the adequateness. It thereby is further possible

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to not only distinguish between answers which are right or wrong but also use an ordinal scale according to which the PSTs' answers are assessed. In the present study the different grades of adequateness though are predefined basing on the belonging theory. In summary, this leads to an "ordinal deductive category assignment" (ibid., p. 65) as the concrete form of data evaluation used for this study.

In the following to more concretely illustrate the methodical aspects first the questions used in the study are described before the actual coding process is demonstrated by means of one of the items.

Starting point for the items for the PSTs is a modeling task for lower secondary students, which is depicted in figure 1:

Ice-cream shop task

There are four ice-cream shops in Leo's home city, Springfield. Leo is standing in front of his favourite ice-cream shop "Sorrento" as he does often in summertime. One scoop of ice-cream costs \$6.00. He asks himself how much money the owner of the ice-cream shop gets by selling ice-cream on one hot summer Sunday.

To solve the problem Leo does the following: On the next day he asks his best three friends how many scoops of ice-cream they bought last Sunday and gets the following answers:

Marcus: 3 scoops

Peter: 5 scoops

Tom: 4 scoops

Leo calculates on average $(3+4+5) \div 3 = 4$ scoops per day. He multiplies the result by the number of citizens in Springfield (30,000) and divides the result by 4 because there are four ice-cream shops in Springfield.

So 30,000 scoops of ice-cream are sold in the ice-cream shop Sorrento per day.

Income: $30,000 \times \$6.00 = \$180,000$

What do you think about this?

Figure 1: Modeling task as starting point for the items (slightly modified from Maaß, 2004, p. 136, translated)

The first item in the questionnaire for the PSTs then is

- How would you deal with this question yourself as a student teacher: How much money does the owner of the ice-cream shop get by selling ice-cream on one hot summer Sunday? Please outline all the individual steps of your solution.

(item assigned to MCK)

Afterwards, four students' interviews from students working on the task are presented to the PSTs. The interviews are depicted in the following figure 2:

Interview 1

- I: Here you can see one solution of a task. Would you solve it in the same way? First read it and then answer.*
- S: (Pause) I would do it in a similar way. I would divide this, the citizens of Springfield, by two because probably not all of them bought ice-cream on that Sunday, but only round about half.*
- I: Hhm.*
- S: Because of the babies and old grandfathers and so on.*
- I: Exactly. You have to subtract them. And something else? Do you have any other comments?*
- S: Besides that probably I would have done this like that. In addition you can count the people, who are going into the shop, by standing in front of one of the ice-cream shops. But if I really had to calculate that now, I would have done this like that.*

Interview 2

- I: Would you do the task in the same way like the boy in the example? Or do you have an alternative suggestion now?*
- S: I have an alternative suggestion because this is a little bit inexact. For example an old woman wouldn't buy five scoops of ice-cream.*
- I: Exactly.*
- S: And besides that I would place myself somewhere, for example in the morning, and count, when they come out, how many scoops they have since you can see that approximately.*
- I: Mhm.*
- S: Then I would do this in the morning 'round about nine o'clock and then during lunch time, after lunch and then 'round about four or five o'clock. And then I would add this.*
- I: Mhm.*

S: And then, depending on the number of people, I would divide and then I have the result.

Interview 3

I: Would you do this task in the same way as Leo did or would you do it in a different way?

S: So, I would just, okay yes; I just like the thing about the scoops of ice-cream. But the thing about the ice-cream shops, that you divide it by three, I would see where the shops are and such things. And how many people there and how many ice-creams they buy. And perhaps the ice-cream is cheaper in another place so that more people go there. And not all 30,000 go to the ice-cream shop every day, do they?

I: [smiling] Exactly! Do you have another suggestion, how you perhaps could solve this in a better way?

S: Somehow I would ask the people: Where are you going to buy your ice-cream? Or I would have a look, where the shop is, in the centre of the city or so. And then you say if our survey comes out that 50% are going there, 25% are going there and so on. And if you calculate this in proportions. And about the scoops of ice-cream I would ask the owner of the ice-cream shop as well.

I: Mhm.

S: How much do they take by average round about?

Interview 4

I: Would you do this in the same way as the boy did in the example?

S: Well, actually not because asking three are too few.

I: Hhm.

S: You really need more, I would say.

I: How would you do this now?

S: I just would ask more people, so, the ones who bought there for example.

I: Mhm.

S: 100 or something like that. And then just in the same way like he did.

Figure 2: Interviews presented to the PSTs (from the not-published supplement to Maaß, 2004, p. 66ff, translated)

Starting from the interviews the PSTs then are asked to answer the following questions focusing

on the learning process perspective:

- Analyze the statements from a teaching perspective whether the suggested modeling approaches are appropriate. Refer to the text of the interviews.

(item assigned to MPCK - learning process perspective - evaluating students' answers)

- How would you deal with the students' answers? How would you respond to each of the students?

(item assigned to MPCK - learning process perspective -providing feedback to students' answers)

After these tasks a third group of items focuses on the teaching perspective. Precisely the PSTs were asked the following questions:

- Is such a task appropriate for secondary school level? If yes, why? If no, why not?

(item assigned to MPCK - teaching perspective - explaining the appropriateness of the task)

- Imagine you work on this task with Grade 8 students in their mathematics lesson. Which teaching approach (for example forms of social interaction) would you choose? Please explain your position.

(item assigned to MPCK - teaching perspective - proposing an appropriate teaching approach)

Basis for the coding of each item is a coding manual, which is developed upon theoretical considerations about the respective item. In order to illustrate the concrete procedure subsequently the coding process for the item "How would you deal with the students' answers? How would you respond to each of the students?" (task 1c) is presented following the coding manual (from Schwarz, 2013).

Coding Process for task 1c

The future teachers were given four interview-sequences of four students and the future teachers were asked to formulate a response to each of the students. Thus each future teacher formulated four responses, one for every student's answer. The coding of the task was then done in two steps.

Step 1:

Every response to a student's answer is coded separately. That is, as each future teacher formulated four responses, for every future teacher four codes were given in a first step.

Technically if a future teacher formulated an adequate feedback to a student's response, the feedback to this respective student's answer is coded with one assessment unit “(+)” (AU+). If an inadequate but not wrong feedback is formulated an assessment unit “(0)” (AU0) is given. If the feedback is wrong, the assessment unit “(-)” (AU-) is given.

The following table (modified from the coding manual) summarizes the requirements a feedback must contain in order to be assigned to AU+, AU0 or AU-.

Assessment unit	Requirements
AU+	An individual learning aid for the student according to his/her answer, giving the student a meaningful perspective to carry on, is formulated. One aspect demanding them to carry on is sufficient. As the interviews only depict sections, the feedback has to tie in with the students' answers. As theoretical reference for distinguishing forms of feedback a taxonomy of learning aids (compare Zech, 2002, p.315 ff.) is used. It includes the steps of content-related learning aids such as <i>strategic aids focusing on the content</i> and <i>content aids</i> as well as partly <i>general strategic aids</i>. These aids form the general expectations for an adequate feedback because they invite the students to continue working. For each interview against this general background different aspects are accepted as meaningful perspectives to carry on. The criteria for each interview are displayed separately in the tables below.

	Feedback aids in the sense of a simply positive reinforcement without the perspective for carrying on is not regarded as adequate. If the feedback with the perspective for carrying on includes a positive reinforcement this does not harm the coding but is coded as neutral. The same is true if besides a reasonable request for carrying on further unreasonable - but not wrong aspects - are named.
AU0	A feedback which is not wrong but does not fulfil the criteria for “+”. This includes feedback like <i>motivational aids</i> and <i>feedback aids</i> as well as partly <i>general strategic aids</i> if they do not explicitly refer to the students’ answers and therefore do not completely fulfil the criterion of enabling the students to carry on. An inadequate but not wrong feedback also consists of only letting other students correct another student’s answer.
AU-	A wrong/missing feedback. A wrong feedback also includes if the student’s answer is not understood correctly and still unconsidered feedback is given or in consequence no feedback is given at all (missing feedback). If at least one part of the feedback is wrong, the whole part referring to this student is coded with “-“.

Precision of the general coding scheme for the different interviews:

Interview 1: Here are two possibilities of releasing a pulse for continuing work: Request to improve the real model: By asking them questions, the student can be encouraged to modify Leo’s model (on the level of the real model, that is competition or quality/location or average number of scoops per person). Encouragement to carry on working on the empirical level: The student can be asked to specify his/her former elementary considerations about the empirical research.	
AU	Anchor example
+	The student from interview 1 already realized that not every inhabitant eats ice-cream. Together with him I would have tried to find out if every second inhabitant is an infant or a senior citizen. Furthermore I would ask him about his own criteria when choosing an ice cream parlour.

	----- Good approach but it is too general to only divide the number of inhabitants by two. It is good that you realized that it is unrealistic to include all inhabitants into the calculation. Try to work on your idea about finding out the people who go into the ice cream parlour. ----- For 1 I would give the hint that the half is already very approximative and that the approach of counting is good and developable.
0	For 1) -Which day would you chose for the research? -Would you only count the people? ----- I would advice student 1 to interview as many people as possible for a representative poll. Then he/she can evaluate the collected data and present the results to the class. ----- For student from interview 1: “Why do you not count the people coming to the ice cream parlour if you are to calculate that? Wouldn’t that be more realistic?”
-	Interview 1: But the aim is not only to calculate but also to make a statement about the real situation.

Interview 2:

Here are two possibilities of releasing a pulse for continuing work:

Request to develop a real model as a basis of a mathematical modeling:

The student can be encouraged to develop a mathematical modeling or in more general terms to develop an alternative to the empirical research (that is making considerations independent of the empirical data). Or alternatively the student can be encouraged to reflect about Leo’s model in a critical way.

Request to carry on working on the empirical level:

Here the student can be encouraged to develop his former considerations about the empirical research (The student could be asked what other data he could collect or how he could modify his plans). Alternatively the student can be asked to specify or reflect upon his considerations for the empirical poll as these are not definite in some parts and might miss the target.

AU	Anchor example
+	Interview 2 -asking if there is another way to find out about the number of people apart from collecting data -I would definitely tell the students that his approach with the different samples is very sensible. ----- for 2) -How long would you watch and on which days? -how could your observations influence the calculation? ----- Interview 2 -Do you want to wait in front of the ice cream parlour for a whole day? Think about an easier way to find out the average! ----- -Question: "What do you think: How do the sales differ at different times?" -“Can you explain what you mean by saying "divided by" and then the result” -“What do you get when you divide the number of sold scoops by the number of inhabitants?"
0	2: approval! ----- 2: That is a good solution ----- 2 very good idea, but not thought to the end. Think about what you want to calculate. ----- Students 2 and 3: I would tell them that it is a very good idea to do polls and counting because this way they can sum up the situation.
-	Interview 2: -And how do you know that all four parlours have the same amount of customers? ----- 2: I would ask him if he really believes that all inhabitants have ice cream on one day.

Interview 3:

Here are two possibilities of releasing a pulse for continuing work:

Suggestion to improve the real model/the real situation:

Possible suitable suggestions to carry on working on the real model/ the real situation are:

- Further improvements of the assumptions for the real model
- Reasonable integration of the results of the indicated empirical observations into the real model (simply requesting an empirical observation does not make sense and is coded with 0 as the student only mentions the empirical observation in the context of improving the assumptions of the real model)

Suggestions for the transition to the mathematical model:

The current process step, working on the real model/ the real situation, is adequate and in principle sufficient for a student in eighths grade. Alternatively, the student could be asked to transfer their considerations into a mathematical model.

AU	Anchor example
+	Interview 3: Great! Write down your considerations and start a calculation. ----- Interview 3: Delighted about the willingness to deal with the circumstances this intensively, support way of calculation, correct results, praise, if necessary go over the model circle with a simple model, use open questions to make him think about the size of the sample to calculate the number of scoops. ----- I would make this student work on his solution on his own and transform it into a mathematical model to analyse and validate it.
0	Ad 3) Nice. Collect data without relying on percentage ratios. ----- Ad 3) Just try it.
-	n.n.

Interview 4:

Request to improve the real model: By asking her/him questions, the student can be encouraged to modify Leo's model. For example on the level of the real model, that is competition or quality/location or average number of customers per day (simply requesting an empirical observation does not make sense and is coded with 0 as the student only mentions the empirical observation in the context of improving the assumptions of the real model).

AU	Anchor example
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+	I4: Can you find other weaknesses in Leo's solution? Do you believe that exactly $\frac{1}{4}$ of the inhabitants goes to Sorrento, that is <u>all</u> inhabitants eat an average of 4 scoops? ----- IV) You are right, it is not enough to ask three people. Look at Leo's solution, would you go on as he did? Why is Leo's solution not adequate? ----- 4. Ok, and do you believe that on a hot day all people eat ice cream or are you at the parlour every day or do people sometimes not go there? What do you believe has to be taken into account?
0	3: accepted! 4: see 3 ----- student 4: his opinion that three people are too few for a representative study is good. ----- interview 4: This students does not say a lot so that more questions have to be asked.
-	4) Are you up for the task? Consider the remarks above (all missing).

Step 2:

After the separate analysis of the feedback to each of the four student's answer for every future teacher his or her four assessment units are summed up in the following way: One assessment unit AU- compensates for an assessment unit AU+ and the assessment unit AU0 does not have any influence on the coding. The final code then results from the number of remaining AU+ according to the following table:

Category	Definition	Coding manual
+2	Very high competencies regarding feedback can be derived from an adequate feedback for each of the students' answers.	For a coding with +2 the assessment unit + has to remain four times.
+1	High competencies regarding feedback can be derived from an adequate feedback for almost each of the students' answers.	For a coding with +1 the assessment unit + has to remain three times.

0	Average competencies regarding feedback can be derived from an adequate feedback for some of the students' answers.	For a coding with 0 the assessment unit + has to remain twice.
-1	Low competencies regarding feedback can be derived from an adequate feedback for only a few of the students' answers.	For a coding with -1 the assessment unit + has to remain once.
-2	Very low competencies regarding feedback can be derived from an inadequate feedback for all of the students' answers.	A coding with -2 is given for 0 remaining assessment units +. Or A coding with -2 is given for feedback that does not differentiate between the different students' answers. (Anchor items see above)

Anchor example for the rule: "A coding with -2 is given for feedback that does not differentiate between the different students' answers." (Coding -2):

-“Each student recognized something that could improve the calculation. That is praiseworthy. One could ask the students if they can find other aspects that have to be considered. They could also sit down together and discuss and then find a better way together.”

-“I would encourage them in their basic thoughts. But I would ask them which other factors have to be taken into account. If they find out about them then they have to continue thinking about their own ideas.”

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