Supplementary material for **Mathematical induction in education research: A systematic review** (*Educational Studies in Mathematics*) by Kristin Krogh Arnesen and Øystein Ingmar Skartsæterhagen

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Complete data set

This document lists the 85 studies that were reviewed. The studies are alphabetically ordered. The asterisk * marks studies with mathematical induction as a main focus (see the Methods for further information).

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