APPENDIX 2: MATING FREQUENCIES AND RECURSIONS

A. Mating frequencies in a three-trait cultural model

Mating (♀×♂)	Frequency	Mating (♀×♂)	Frequency
TSM×*TSM*	$\mu_{1,1} = (1-\varepsilon)\left((1-\alpha)x_1^2 + \frac{\alpha x_1^2}{x_1+x_2+x_3+x_4} \right)$	*tSM*×*TSM*	$\mu_{5,1} = (1-\varepsilon)(1-\alpha)x_5x_1$
TSM×*TSm*	$\mu_{1,2} = (1-\varepsilon)\left((1-\alpha)x_1x_2 + \frac{\alpha x_1x_2}{x_1+x_2+x_3+x_4} \right)$	*tSM*×*TSm*	$\mu_{5,2} = (1-\varepsilon)(1-\alpha)x_5x_2$
TSM×*TsM*	$\mu_{1,3} = (1-\alpha)x_1x_3 + \frac{\alpha x_1x_3}{x_1+x_2+x_3+x_4}$	*tSM*×*TsM*	$\mu_{5,3} = (1-\alpha)x_5x_3$
TSM×*Tsm*	$\mu_{1,4} = (1-\alpha)x_1x_4 + \frac{\alpha x_1x_4}{x_1+x_2+x_3+x_4}$	*tSM*×*Tsm*	$\mu_{5,4} = (1-\alpha)x_5x_4$
TSM×*tSM*	$\mu_{1,5} = (1-\varepsilon)(1-\alpha)x_1x_5$	*tSM*×*tSM*	$\mu_{5,5} = (1-\varepsilon)\left((1-\alpha)x_5^2 + \frac{\alpha x_5^2}{x_5+x_6+x_7+x_8} \right)$
TSM×*tSm*	$\mu_{1,6} = (1-\varepsilon)(1-\alpha)x_1x_6$	*tSM*×*tSm*	$\mu_{5,6} = (1-\varepsilon)\left((1-\alpha)x_5x_6 + \frac{\alpha x_5x_6}{x_5+x_6+x_7+x_8} \right)$
TSM×*tsM*	$\mu_{1,7} = (1-\alpha)x_1x_7$	*tSM*×*tsM*	$\mu_{5,7} = (1-\alpha)x_5x_7 + \frac{\alpha x_5x_7}{x_5+x_6+x_7+x_8}$
TSM×*tsm*	$\mu_{1,8} = (1-\alpha)x_1x_8$	*tSM*×*tsm*	$\mu_{5,8} = (1-\alpha)x_5x_8 + \frac{\alpha x_5x_8}{x_5+x_6+x_7+x_8}$
TSm×*TSM*	$\mu_{2,1} = (1-\varepsilon)x_2x_1$	*tSm*×*TSM*	$\mu_{6,1} = (1-\varepsilon)x_6x_1$
TSm×*TSm*	$\mu_{2,2} = (1-\varepsilon)x_2^2$	*tSm*×*TSm*	$\mu_{6,2} = (1-\varepsilon)x_6x_2$
TSm×*TsM*	$\mu_{2,3} = x_2x_3$	*tSm*×*TsM*	$\mu_{6,3} = x_6x_3$
TSm×*Tsm*	$\mu_{2,4} = x_2x_4$	*tSm*×*Tsm*	$\mu_{6,4} = x_6x_4$
TSm×*tSM*	$\mu_{2,5} = (1-\varepsilon)x_2x_5$	*tSm*×*tSM*	$\mu_{6,5} = (1-\varepsilon)x_6x_5$
TSm×*tSm*	$\mu_{2,6} = (1-\varepsilon)x_2x_6$	*tSm*×*tSm*	$\mu_{6,6} = (1-\varepsilon)x_6^2$
TSm×*tsM*	$\mu_{2,7} = x_2x_7$	*tSm*×*tsM*	$\mu_{6,7} = x_6x_7$
TSm×*tsm*	$\mu_{2,8} = x_2x_8$	*tSm*×*tsm*	$\mu_{6,8} = x_6x_8$

TsM×*TSM*	$\mu_{3,1} = (1-\varepsilon)\left((1-\alpha)x_3x_1 + \frac{\alpha x_3x_1}{x_1+x_2+x_3+x_4} \right)$	*tsM*×*TSM*	$\mu_{7,1} = (1-\varepsilon)(1-\alpha)x_7x_1$
TsM×*TSm*	$\mu_{3,2} = (1-\varepsilon)\left((1-\alpha)x_3x_2 + \frac{\alpha x_3x_2}{x_1+x_2+x_3+x_4} \right)$	*tsM*×*TSm*	$\mu_{7,2} = (1-\varepsilon)(1-\alpha)x_7x_2$
TsM×*TsM*	$\mu_{3,3} = (1-\alpha){x_3}^2 + \frac{\alpha {x_3}^2}{x_1+x_2+x_3+x_4}$	*tsM*×*TsM*	$\mu_{7,3} = (1-\alpha)x_7x_3$
TsM×*Tsm*	$\mu_{3,4} = (1-\alpha)x_3x_4 + \frac{\alpha x_3x_4}{x_1+x_2+x_3+x_4}$	*tsM*×*Tsm*	$\mu_{7,4} = (1-\alpha)x_7x_4$
TsM×*tSM*	$\mu_{3,5} = (1-\varepsilon)(1-\alpha)x_3x_5$	*tsM*×*tSM*	$\mu_{7,5} = (1-\varepsilon)\left((1-\alpha)x_7x_5 + \frac{\alpha x_7x_5}{x_5+x_6+x_7+x_8} \right)$
TsM×*tSm*	$\mu_{3,6} = (1-\varepsilon)(1-\alpha)x_3x_6$	*tsM*×*tSm*	$\mu_{7,6} = (1-\varepsilon)\left((1-\alpha)x_7x_6 + \frac{\alpha x_7x_6}{x_5+x_6+x_7+x_8} \right)$
TsM×*tsM*	$\mu_{3,7} = (1-\alpha)x_3x_7$	*tsM*×*tsM*	$\mu_{7,7} = (1-\alpha){x_7}^2 + \frac{\alpha {x_7}^2}{x_5+x_6+x_7+x_8}$
TsM×*tsm*	$\mu_{3,8} = (1-\alpha)x_3x_8$	*tsM*×*tsm*	$\mu_{7,8} = (1-\alpha)x_7x_8 + \frac{\alpha x_7x_8}{x_5+x_6+x_7+x_8}$
Tsm×*TSM*	$\mu_{4,1} = (1-\varepsilon)x_4x_1$	*tsm*×*TSM*	$\mu_{8,1} = (1-\varepsilon)x_8x_1$
Tsm×*TSm*	$\mu_{4,2} = (1-\varepsilon)x_4x_2$	*tsm*×*TSm*	$\mu_{8,2} = (1-\varepsilon)x_8x_2$
Tsm×*TsM*	$\mu_{4,3} = x_4x_3$	*tsm*×*TsM*	$\mu_{8,3} = x_8x_3$
Tsm×*Tsm*	$\mu_{4,4} = {x_4}^2$	*tsm*×*Tsm*	$\mu_{8,4} = x_8x_4$
Tsm×*tSM*	$\mu_{4,5} = (1-\varepsilon)x_4x_5$	*tsm*×*tSM*	$\mu_{8,5} = (1-\varepsilon)x_8x_5$
Tsm×*tSm*	$\mu_{4,6} = (1-\varepsilon)x_4x_6$	*tsm*×*tSm*	$\mu_{8,6} = (1-\varepsilon)x_8x_6$
Tsm×*tsM*	$\mu_{4,7} = x_4x_7$	*tsm*×*tsM*	$\mu_{8,7} = x_8x_7$
Tsm×*tsm*	$\mu_{4,8} = x_4x_8$	*tsm*×*tsm*	$\mu_{8,8} = {x_8}^2$

B. Generalized transmission probabilities for the recursions for *T*, *S,* and *M*.

(a)

	General form		**Current model**	
♀×♂	*T*	*t*	*T*	*t*
T×*T*	b_3	$1-b_3$	$b_{T\to T}$	$1-b_{T\to T}$
T×*t*	b_2	$1-b_2$	$1-b_{t\to t}$	$b_{t\to t}$
t×*T*	b_1	$1-b_1$	$b_{T\to T}$	$1-b_{T\to T}$
t×*t*	b_0	$1-b_0$	$1-b_{t\to t}$	$b_{t\to t}$

(b)

	General form		**Current model**	
	S	*s*	*S*	*s*
S×*S*	c_3	$1-c_3$	1	0
S×*s*	c_2	$1-c_2$	0.5	0.5
s×*S*	c_1	$1-c_1$	0.5	0.5
s×*s*	c_0	$1-c_0$	0	1

(c)

	General form		**Current model**	
	M	*m*	*M*	*m*
M×*M*	d_3	d_3	1	0
M×*m*	d_2	d_2	0.5	0.5
m×*M*	d_1	d_1	0.5	0.5
m×*m*	d_0	d_0	0	1

C. Recursions in a three-trait cultural model after vertical transmission

$$\begin{aligned}\bar{w}x_1{}' = {} & \mu_{11}b_3c_3d_3 + \mu_{12}b_3c_3d_2 + \mu_{13}b_3c_2d_3 + \mu_{14}b_3c_2d_2 + \mu_{15}b_2c_3d_3 + \mu_{16}b_2c_3d_2 \\ & + \mu_{17}b_2c_2d_3 + \mu_{18}b_2c_2d_2 + \mu_{21}b_3c_3d_1 + \mu_{22}b_3c_3d_0 + \mu_{23}b_3c_2d_1 + \mu_{24}b_3c_2d_0 \\ & + \mu_{25}b_2c_3d_1 + \mu_{26}b_2c_3d_0 + \mu_{27}b_2c_2d_1 + \mu_{28}b_2c_2d_0 + \mu_{31}b_3c_1d_3 + \mu_{32}b_3c_1d_2 \\ & + \mu_{33}b_3c_0d_3 + \mu_{34}b_3c_0d_2 + \mu_{35}b_2c_1d_3 + \mu_{36}b_2c_1d_2 + \mu_{37}b_2c_0d_3 + \mu_{38}b_2c_0d_2 \\ & + \mu_{41}b_3c_1d_1 + \mu_{42}b_3c_1d_0 + \mu_{43}b_3c_0d_1 + \mu_{44}b_3c_0d_0 + \mu_{45}b_2c_1d_1 + \mu_{46}b_2c_1d_0 \\ & + \mu_{47}b_2c_0d_1 + \mu_{48}b_2c_0d_0 + \mu_{51}b_1c_3d_3 + \mu_{52}b_1c_3d_2 + \mu_{53}b_1c_2d_3 + \mu_{54}b_1c_2d_2 \\ & + \mu_{55}b_0c_3d_3 + \mu_{56}b_0c_3d_2 + \mu_{57}b_0c_2d_3 + \mu_{58}b_0c_2d_2 + \mu_{61}b_1c_3d_1 + \mu_{62}b_1c_3d_0 \\ & + \mu_{63}b_1c_2d_1 + \mu_{64}b_1c_2d_0 + \mu_{65}b_0c_3d_1 + \mu_{66}b_0c_3d_0 + \mu_{67}b_0c_2d_1 + \mu_{68}b_0c_2d_0 \\ & + \mu_{71}b_1c_1d_3 + \mu_{72}b_1c_1d_2 + \mu_{73}b_1c_0d_3 + \mu_{74}b_1c_0d_2 + \mu_{75}b_0c_1d_3 + \mu_{76}b_0c_1d_2 \\ & + \mu_{77}b_0c_0d_3 + \mu_{78}b_0c_0d_2 + \mu_{81}b_1c_1d_1 + \mu_{82}b_1c_1d_0 + \mu_{83}b_1c_0d_1 + \mu_{84}b_1c_0d_0 \\ & + \mu_{85}b_0c_1d_1 + \mu_{86}b_0c_1d_0 + \mu_{87}b_0c_0d_1 + \mu_{88}b_0c_0d_0\end{aligned}$$

$$\begin{aligned}\bar{w}x_2{}' = {} & \mu_{11}b_3c_3(1-d_3) + \mu_{12}b_3c_3(1-d_2) + \mu_{13}b_3c_2(1-d_3) + \mu_{14}b_3c_2(1-d_2) \\ & + \mu_{15}b_2c_3(1-d_3) + \mu_{16}b_2c_3(1-d_2) + \mu_{17}b_2c_2(1-d_3) + \mu_{18}b_2c_2(1-d_2) \\ & + \mu_{21}b_3c_3(1-d_1) + \mu_{22}b_3c_3(1-d_0) + \mu_{23}b_3c_2(1-d_1) + \mu_{24}b_3c_2(1-d_0) \\ & + \mu_{25}b_2c_3(1-d_1) + \mu_{26}b_2c_3(1-d_0) + \mu_{27}b_2c_2(1-d_1) + \mu_{28}b_2c_2(1-d_0) \\ & + \mu_{31}b_3c_1(1-d_3) + \mu_{32}b_3c_1(1-d_2) + \mu_{33}b_3c_0(1-d_3) + \mu_{34}b_3c_0(1-d_2) \\ & + \mu_{35}b_2c_1(1-d_3) + \mu_{36}b_2c_1(1-d_2) + \mu_{37}b_2c_0(1-d_3) + \mu_{38}b_2c_0(1-d_2) \\ & + \mu_{41}b_3c_1(1-d_1) + \mu_{42}b_3c_1(1-d_0) + \mu_{43}b_3c_0(1-d_1) + \mu_{44}b_3c_0(1-d_0) \\ & + \mu_{45}b_2c_1(1-d_1) + \mu_{46}b_2c_1(1-d_0) + \mu_{47}b_2c_0(1-d_1) + \mu_{48}b_2c_0(1-d_0) \\ & + \mu_{51}b_1c_3(1-d_3) + \mu_{52}b_1c_3(1-d_2) + \mu_{53}b_1c_2(1-d_3) + \mu_{54}b_1c_2(1-d_2) \\ & + \mu_{55}b_0c_3(1-d_3) + \mu_{56}b_0c_3(1-d_2) + \mu_{57}b_0c_2(1-d_3) + \mu_{58}b_0c_2(1-d_2) \\ & + \mu_{61}b_1c_3(1-d_1) + \mu_{62}b_1c_3(1-d_0) + \mu_{63}b_1c_2(1-d_1) + \mu_{64}b_1c_2(1-d_0) \\ & + \mu_{65}b_0c_3(1-d_1) + \mu_{66}b_0c_3(1-d_0) + \mu_{67}b_0c_2(1-d_1) + \mu_{68}b_0c_2(1-d_0) \\ & + \mu_{71}b_1c_1(1-d_3) + \mu_{72}b_1c_1(1-d_2) + \mu_{73}b_1c_0(1-d_3) + \mu_{74}b_1c_0(1-d_2) \\ & + \mu_{75}b_0c_1(1-d_3) + \mu_{76}b_0c_1(1-d_2) + \mu_{77}b_0c_0(1-d_3) + \mu_{78}b_0c_0(1-d_2) \\ & + \mu_{81}b_1c_1(1-d_1) + \mu_{82}b_1c_1(1-d_0) + \mu_{83}b_1c_0(1-d_1) + \mu_{84}b_1c_0(1-d_0) \\ & + \mu_{85}b_0c_1(1-d_1) + \mu_{86}b_0c_1(1-d_0) + \mu_{87}b_0c_0(1-d_1) + \mu_{88}b_0c_0(1-d_0)\end{aligned}$$

$$
\begin{aligned}
\bar{w}x_3{}' = {} & \mu_{11}b_3(1-c_3)d_3 + \mu_{12}b_3(1-c_3)d_2 + \mu_{13}b_3(1-c_2)d_3 + \mu_{14}b_3(1-c_2)d_2 \\
& + \mu_{15}b_2(1-c_3)d_3 + \mu_{16}b_2(1-c_3)d_2 + \mu_{17}b_2(1-c_2)d_3 + \mu_{18}b_2(1-c_2)d_2 \\
& + \mu_{21}b_3(1-c_3)d_1 + \mu_{22}b_3(1-c_3)d_0 + \mu_{23}b_3(1-c_2)d_1 + \mu_{24}b_3(1-c_2)d_0 \\
& + \mu_{25}b_2(1-c_3)d_1 + \mu_{26}b_2(1-c_3)d_0 + \mu_{27}b_2(1-c_2)d_1 + \mu_{28}b_2(1-c_2)d_0 \\
& + \mu_{31}b_3(1-c_1)d_3 + \mu_{32}b_3(1-c_1)d_2 + \mu_{33}b_3(1-c_0)d_3 + \mu_{34}b_3(1-c_0)d_2 \\
& + \mu_{35}b_2(1-c_1)d_3 + \mu_{36}b_2(1-c_1)d_2 + \mu_{37}b_2(1-c_0)d_3 + \mu_{38}b_2(1-c_0)d_2 \\
& + \mu_{41}b_3(1-c_1)d_1 + \mu_{42}b_3(1-c_1)d_0 + \mu_{43}b_3(1-c_0)d_1 + \mu_{44}b_3(1-c_0)d_0 \\
& + \mu_{45}b_2(1-c_1)d_1 + \mu_{46}b_2(1-c_1)d_0 + \mu_{47}b_2(1-c_0)d_1 + \mu_{48}b_2(1-c_0)d_0 \\
& + \mu_{51}b_1(1-c_3)d_3 + \mu_{52}b_1(1-c_3)d_2 + \mu_{53}b_1(1-c_2)d_3 + \mu_{54}b_1(1-c_2)d_2 \\
& + \mu_{55}b_0(1-c_3)d_3 + \mu_{56}b_0(1-c_3)d_2 + \mu_{57}b_0(1-c_2)d_3 + \mu_{58}b_0(1-c_2)d_2 \\
& + \mu_{61}b_1(1-c_3)d_1 + \mu_{62}b_1(1-c_3)d_0 + \mu_{63}b_1(1-c_2)d_1 + \mu_{64}b_1(1-c_2)d_0 \\
& + \mu_{65}b_0(1-c_3)d_1 + \mu_{66}b_0(1-c_3)d_0 + \mu_{67}b_0(1-c_2)d_1 + \mu_{68}b_0(1-c_2)d_0 \\
& + \mu_{71}b_1(1-c_1)d_3 + \mu_{72}b_1(1-c_1)d_2 + \mu_{73}b_1(1-c_0)d_3 + \mu_{74}b_1(1-c_0)d_2 \\
& + \mu_{75}b_0(1-c_1)d_3 + \mu_{76}b_0(1-c_1)d_2 + \mu_{77}b_0(1-c_0)d_3 + \mu_{78}b_0(1-c_0)d_2 \\
& + \mu_{81}b_1(1-c_1)d_1 + \mu_{82}b_1(1-c_1)d_0 + \mu_{83}b_1(1-c_0)d_1 + \mu_{84}b_1(1-c_0)d_0 \\
& + \mu_{85}b_0(1-c_1)d_1 + \mu_{86}b_0(1-c_1)d_0 + \mu_{87}b_0(1-c_0)d_1 + \mu_{88}b_0(1-c_0)d_0
\end{aligned}
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\begin{aligned}
\bar{w}{x_4}' = {} & \mu_{11}b_3(1-c_3)(1-d_3) + \mu_{12}b_3(1-c_3)(1-d_2) + \mu_{13}b_3(1-c_2)(1-d_3) \\
& + \mu_{14}b_3(1-c_2)(1-d_2) + \mu_{15}b_2(1-c_3)(1-d_3) + \mu_{16}b_2(1-c_3)(1-d_2) \\
& + \mu_{17}b_2(1-c_2)(1-d_3) + \mu_{18}b_2(1-c_2)(1-d_2) + \mu_{21}b_3(1-c_3)(1-d_1) \\
& + \mu_{22}b_3(1-c_3)(1-d_0) + \mu_{23}b_3(1-c_2)(1-d_1) + \mu_{24}b_3(1-c_2)(1-d_0) \\
& + \mu_{25}b_2(1-c_3)(1-d_1) + \mu_{26}b_2(1-c_3)(1-d_0) + \mu_{27}b_2(1-c_2)(1-d_1) \\
& + \mu_{28}b_2(1-c_2)(1-d_0) + \mu_{31}b_3(1-c_1)(1-d_3) + \mu_{32}b_3(1-c_1)(1-d_2) \\
& + \mu_{33}b_3(1-c_0)(1-d_3) + \mu_{34}b_3(1-c_0)(1-d_2) + \mu_{35}b_2(1-c_1)(1-d_3) \\
& + \mu_{36}b_2(1-c_1)(1-d_2) + \mu_{37}b_2(1-c_0)(1-d_3) + \mu_{38}b_2(1-c_0)(1-d_2) \\
& + \mu_{41}b_3(1-c_1)(1-d_1) + \mu_{42}b_3(1-c_1)(1-d_0) + \mu_{43}b_3(1-c_0)(1-d_1) \\
& + \mu_{44}b_3(1-c_0)(1-d_0) + \mu_{45}b_2(1-c_1)(1-d_1) + \mu_{46}b_2(1-c_1)(1-d_0) \\
& + \mu_{47}b_2(1-c_0)(1-d_1) + \mu_{48}b_2(1-c_0)(1-d_0) + \mu_{51}b_1(1-c_3)(1-d_3) \\
& + \mu_{52}b_1(1-c_3)(1-d_2) + \mu_{53}b_1(1-c_2)(1-d_3) + \mu_{54}b_1(1-c_2)(1-d_2) \\
& + \mu_{55}b_0(1-c_3)(1-d_3) + \mu_{56}b_0(1-c_3)(1-d_2) + \mu_{57}b_0(1-c_2)(1-d_3) \\
& + \mu_{58}b_0(1-c_2)(1-d_2) + \mu_{61}b_1(1-c_3)(1-d_1) + \mu_{62}b_1(1-c_3)(1-d_0) \\
& + \mu_{63}b_1(1-c_2)(1-d_1) + \mu_{64}b_1(1-c_2)(1-d_0) + \mu_{65}b_0(1-c_3)(1-d_1) \\
& + \mu_{66}b_0(1-c_3)(1-d_0) + \mu_{67}b_0(1-c_2)(1-d_1) + \mu_{68}b_0(1-c_2)(1-d_0) \\
& + \mu_{71}b_1(1-c_1)(1-d_3) + \mu_{72}b_1(1-c_1)(1-d_2) + \mu_{73}b_1(1-c_0)(1-d_3) \\
& + \mu_{74}b_1(1-c_0)(1-d_2) + \mu_{75}b_0(1-c_1)(1-d_3) + \mu_{76}b_0(1-c_1)(1-d_2) \\
& + \mu_{77}b_0(1-c_0)(1-d_3) + \mu_{78}b_0(1-c_0)(1-d_2) + \mu_{81}b_1(1-c_1)(1-d_1) \\
& + \mu_{82}b_1(1-c_1)(1-d_0) + \mu_{83}b_1(1-c_0)(1-d_1) + \mu_{84}b_1(1-c_0)(1-d_0) \\
& + \mu_{85}b_0(1-c_1)(1-d_1) + \mu_{86}b_0(1-c_1)(1-d_0) + \mu_{87}b_0(1-c_0)(1-d_1) \\
& + \mu_{88}b_0(1-c_0)(1-d_0)
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}{x_5}' = {} & \mu_{11}(1-b_3)c_3d_3 + \mu_{12}(1-b_3)c_3d_2 + \mu_{13}(1-b_3)c_2d_3 + \mu_{14}(1-b_3)c_2d_2 \\
& + \mu_{15}(1-b_2)c_3d_3 + \mu_{16}(1-b_2)c_3d_2 + \mu_{17}(1-b_2)c_2d_3 + \mu_{18}(1-b_2)c_2d_2 \\
& + \mu_{21}(1-b_3)c_3d_1 + \mu_{22}(1-b_3)c_3d_0 + \mu_{23}(1-b_3)c_2d_1 + \mu_{24}(1-b_3)c_2d_0 \\
& + \mu_{25}(1-b_2)c_3d_1 + \mu_{26}(1-b_2)c_3d_0 + \mu_{27}(1-b_2)c_2d_1 + \mu_{28}(1-b_2)c_2d_0 \\
& + \mu_{31}(1-b_3)c_1d_3 + \mu_{32}(1-b_3)c_1d_2 + \mu_{33}(1-b_3)c_0d_3 + \mu_{34}(1-b_3)c_0d_2 \\
& + \mu_{35}(1-b_2)c_1d_3 + \mu_{36}(1-b_2)c_1d_2 + \mu_{37}(1-b_2)c_0d_3 + \mu_{38}(1-b_2)c_0d_2 \\
& + \mu_{41}(1-b_3)c_1d_1 + \mu_{42}(1-b_3)c_1d_0 + \mu_{43}(1-b_3)c_0d_1 + \mu_{44}(1-b_3)c_0d_0 \\
& + \mu_{45}(1-b_2)c_1d_1 + \mu_{46}(1-b_2)c_1d_0 + \mu_{47}(1-b_2)c_0d_1 + \mu_{48}(1-b_2)c_0d_0 \\
& + \mu_{51}(1-b_1)c_3d_3 + \mu_{52}(1-b_1)c_3d_2 + \mu_{53}(1-b_1)c_2d_3 + \mu_{54}(1-b_1)c_2d_2 \\
& + \mu_{55}(1-b_0)c_3d_3 + \mu_{56}(1-b_0)c_3d_2 + \mu_{57}(1-b_0)c_2d_3 + \mu_{58}(1-b_0)c_2d_2 \\
& + \mu_{61}(1-b_1)c_3d_1 + \mu_{62}(1-b_1)c_3d_0 + \mu_{63}(1-b_1)c_2d_1 + \mu_{64}(1-b_1)c_2d_0 \\
& + \mu_{65}(1-b_0)c_3d_1 + \mu_{66}(1-b_0)c_3d_0 + \mu_{67}(1-b_0)c_2d_1 + \mu_{68}(1-b_0)c_2d_0 \\
& + \mu_{71}(1-b_1)c_1d_3 + \mu_{72}(1-b_1)c_1d_2 + \mu_{73}(1-b_1)c_0d_3 + \mu_{74}(1-b_1)c_0d_2 \\
& + \mu_{75}(1-b_0)c_1d_3 + \mu_{76}(1-b_0)c_1d_2 + \mu_{77}(1-b_0)c_0d_3 + \mu_{78}(1-b_0)c_0d_2 \\
& + \mu_{81}(1-b_1)c_1d_1 + \mu_{82}(1-b_1)c_1d_0 + \mu_{83}(1-b_1)c_0d_1 + \mu_{84}(1-b_1)c_0d_0 \\
& + \mu_{85}(1-b_0)c_1d_1 + \mu_{86}(1-b_0)c_1d_0 + \mu_{87}(1-b_0)c_0d_1 + \mu_{88}(1-b_0)c_0d_0
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}{x_6}' = {} & \mu_{11}(1-b_3)c_3(1-d_3) + \mu_{12}(1-b_3)c_3(1-d_2) + \mu_{13}(1-b_3)c_2(1-d_3) \\
& + \mu_{14}(1-b_3)c_2(1-d_2) + \mu_{15}(1-b_2)c_3(1-d_3) + \mu_{16}(1-b_2)c_3(1-d_2) \\
& + \mu_{17}(1-b_2)c_2(1-d_3) + \mu_{18}(1-b_2)c_2(1-d_2) + \mu_{21}(1-b_3)c_3(1-d_1) \\
& + \mu_{22}(1-b_3)c_3(1-d_0) + \mu_{23}(1-b_3)c_2(1-d_1) + \mu_{24}(1-b_3)c_2(1-d_0) \\
& + \mu_{25}(1-b_2)c_3(1-d_1) + \mu_{26}(1-b_2)c_3(1-d_0) + \mu_{27}(1-b_2)c_2(1-d_1) \\
& + \mu_{28}(1-b_2)c_2(1-d_0) + \mu_{31}(1-b_3)c_1(1-d_3) + \mu_{32}(1-b_3)c_1(1-d_2) \\
& + \mu_{33}(1-b_3)c_0(1-d_3) + \mu_{34}(1-b_3)c_0(1-d_2) + \mu_{35}(1-b_2)c_1(1-d_3) \\
& + \mu_{36}(1-b_2)c_1(1-d_2) + \mu_{37}(1-b_2)c_0(1-d_3) + \mu_{38}(1-b_2)c_0(1-d_2) \\
& + \mu_{41}(1-b_3)c_1(1-d_1) + \mu_{42}(1-b_3)c_1(1-d_0) + \mu_{43}(1-b_3)c_0(1-d_1) \\
& + \mu_{44}(1-b_3)c_0(1-d_0) + \mu_{45}(1-b_2)c_1(1-d_1) + \mu_{46}(1-b_2)c_1(1-d_0) \\
& + \mu_{47}(1-b_2)c_0(1-d_1) + \mu_{48}(1-b_2)c_0(1-d_0) + \mu_{51}(1-b_1)c_3(1-d_3) \\
& + \mu_{52}(1-b_1)c_3(1-d_2) + \mu_{53}(1-b_1)c_2(1-d_3) + \mu_{54}(1-b_1)c_2(1-d_2) \\
& + \mu_{55}(1-b_0)c_3(1-d_3) + \mu_{56}(1-b_0)c_3(1-d_2) + \mu_{57}(1-b_0)c_2(1-d_3) \\
& + \mu_{58}(1-b_0)c_2(1-d_2) + \mu_{61}(1-b_1)c_3(1-d_1) + \mu_{62}(1-b_1)c_3(1-d_0) \\
& + \mu_{63}(1-b_1)c_2(1-d_1) + \mu_{64}(1-b_1)c_2(1-d_0) + \mu_{65}(1-b_0)c_3(1-d_1) \\
& + \mu_{66}(1-b_0)c_3(1-d_0) + \mu_{67}(1-b_0)c_2(1-d_1) + \mu_{68}(1-b_0)c_2(1-d_0) \\
& + \mu_{71}(1-b_1)c_1(1-d_3) + \mu_{72}(1-b_1)c_1(1-d_2) + \mu_{73}(1-b_1)c_0(1-d_3) \\
& + \mu_{74}(1-b_1)c_0(1-d_2) + \mu_{75}(1-b_0)c_1(1-d_3) + \mu_{76}(1-b_0)c_1(1-d_2) \\
& + \mu_{77}(1-b_0)c_0(1-d_3) + \mu_{78}(1-b_0)c_0(1-d_2) + \mu_{81}(1-b_1)c_1(1-d_1) \\
& + \mu_{82}(1-b_1)c_1(1-d_0) + \mu_{83}(1-b_1)c_0(1-d_1) + \mu_{84}(1-b_1)c_0(1-d_0) \\
& + \mu_{85}(1-b_0)c_1(1-d_1) + \mu_{86}(1-b_0)c_1(1-d_0) + \mu_{87}(1-b_0)c_0(1-d_1) \\
& + \mu_{88}(1-b_0)c_0(1-d_0)
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}x_7{}' = {} & \mu_{11}(1-b_3)(1-c_3)d_3 + \mu_{12}(1-b_3)(1-c_3)d_2 + \mu_{13}(1-b_3)(1-c_2)d_3 \\
& + \mu_{14}(1-b_3)(1-c_2)d_2 + \mu_{15}(1-b_2)(1-c_3)d_3 + \mu_{16}(1-b_2)(1-c_3)d_2 \\
& + \mu_{17}(1-b_2)(1-c_2)d_3 + \mu_{18}(1-b_2)(1-c_2)d_2 + \mu_{21}(1-b_3)(1-c_3)d_1 \\
& + \mu_{22}(1-b_3)(1-c_3)d_0 + \mu_{23}(1-b_3)(1-c_2)d_1 + \mu_{24}(1-b_3)(1-c_2)d_0 \\
& + \mu_{25}(1-b_2)(1-c_3)d_1 + \mu_{26}(1-b_2)(1-c_3)d_0 + \mu_{27}(1-b_2)(1-c_2)d_1 \\
& + \mu_{28}(1-b_2)(1-c_2)d_0 + \mu_{31}(1-b_3)(1-c_1)d_3 + \mu_{32}(1-b_3)(1-c_1)d_2 \\
& + \mu_{33}(1-b_3)(1-c_0)d_3 + \mu_{34}(1-b_3)(1-c_0)d_2 + \mu_{35}(1-b_2)(1-c_1)d_3 \\
& + \mu_{36}(1-b_2)(1-c_1)d_2 + \mu_{37}(1-b_2)(1-c_0)d_3 + \mu_{38}(1-b_2)(1-c_0)d_2 \\
& + \mu_{41}(1-b_3)(1-c_1)d_1 + \mu_{42}(1-b_3)(1-c_1)d_0 + \mu_{43}(1-b_3)(1-c_0)d_1 \\
& + \mu_{44}(1-b_3)(1-c_0)d_0 + \mu_{45}(1-b_2)(1-c_1)d_1 + \mu_{46}(1-b_2)(1-c_1)d_0 \\
& + \mu_{47}(1-b_2)(1-c_0)d_1 + \mu_{48}(1-b_2)(1-c_0)d_0 + \mu_{51}(1-b_1)(1-c_3)d_3 \\
& + \mu_{52}(1-b_1)(1-c_3)d_2 + \mu_{53}(1-b_1)(1-c_2)d_3 + \mu_{54}(1-b_1)(1-c_2)d_2 \\
& + \mu_{55}(1-b_0)(1-c_3)d_3 + \mu_{56}(1-b_0)(1-c_3)d_2 + \mu_{57}(1-b_0)(1-c_2)d_3 \\
& + \mu_{58}(1-b_0)(1-c_2)d_2 + \mu_{61}(1-b_1)(1-c_3)d_1 + \mu_{62}(1-b_1)(1-c_3)d_0 \\
& + \mu_{63}(1-b_1)(1-c_2)d_1 + \mu_{64}(1-b_1)(1-c_2)d_0 + \mu_{65}(1-b_0)(1-c_3)d_1 \\
& + \mu_{66}(1-b_0)(1-c_3)d_0 + \mu_{67}(1-b_0)(1-c_2)d_1 + \mu_{68}(1-b_0)(1-c_2)d_0 \\
& + \mu_{71}(1-b_1)(1-c_1)d_3 + \mu_{72}(1-b_1)(1-c_1)d_2 + \mu_{73}(1-b_1)(1-c_0)d_3 \\
& + \mu_{74}(1-b_1)(1-c_0)d_2 + \mu_{75}(1-b_0)(1-c_1)d_3 + \mu_{76}(1-b_0)(1-c_1)d_2 \\
& + \mu_{77}(1-b_0)(1-c_0)d_3 + \mu_{78}(1-b_0)(1-c_0)d_2 + \mu_{81}(1-b_1)(1-c_1)d_1 \\
& + \mu_{82}(1-b_1)(1-c_1)d_0 + \mu_{83}(1-b_1)(1-c_0)d_1 + \mu_{84}(1-b_1)(1-c_0)d_0 \\
& + \mu_{85}(1-b_0)(1-c_1)d_1 + \mu_{86}(1-b_0)(1-c_1)d_0 + \mu_{87}(1-b_0)(1-c_0)d_1 \\
& + \mu_{88}(1-b_0)(1-c_0)d_0
\end{aligned}
$$

$$
\begin{aligned}
\bar{w}x_8{}' = {} & \mu_{11}(1-b_3)(1-c_3)(1-d_3) + \mu_{12}(1-b_3)(1-c_3)(1-d_2) \\
& + \mu_{13}(1-b_3)(1-c_2)(1-d_3) + \mu_{14}(1-b_3)(1-c_2)(1-d_2) \\
& + \mu_{15}(1-b_2)(1-c_3)(1-d_3) + \mu_{16}(1-b_2)(1-c_3)(1-d_2) \\
& + \mu_{17}(1-b_2)(1-c_2)(1-d_3) + \mu_{18}(1-b_2)(1-c_2)(1-d_2) \\
& + \mu_{21}(1-b_3)(1-c_3)(1-d_1) + \mu_{22}(1-b_3)(1-c_3)(1-d_0) \\
& + \mu_{23}(1-b_3)(1-c_2)(1-d_1) + \mu_{24}(1-b_3)(1-c_2)(1-d_0) \\
& + \mu_{25}(1-b_2)(1-c_3)(1-d_1) + \mu_{26}(1-b_2)(1-c_3)(1-d_0) \\
& + \mu_{27}(1-b_2)(1-c_2)(1-d_1) + \mu_{28}(1-b_2)(1-c_2)(1-d_0) \\
& + \mu_{31}(1-b_3)(1-c_1)(1-d_3) + \mu_{32}(1-b_3)(1-c_1)(1-d_2) \\
& + \mu_{33}(1-b_3)(1-c_0)(1-d_3) + \mu_{34}(1-b_3)(1-c_0)(1-d_2) \\
& + \mu_{35}(1-b_2)(1-c_1)(1-d_3) + \mu_{36}(1-b_2)(1-c_1)(1-d_2) \\
& + \mu_{37}(1-b_2)(1-c_0)(1-d_3) + \mu_{38}(1-b_2)(1-c_0)(1-d_2) \\
& + \mu_{41}(1-b_3)(1-c_1)(1-d_1) + \mu_{42}(1-b_3)(1-c_1)(1-d_0) \\
& + \mu_{43}(1-b_3)(1-c_0)(1-d_1) + \mu_{44}(1-b_3)(1-c_0)(1-d_0) \\
& + \mu_{45}(1-b_2)(1-c_1)(1-d_1) + \mu_{46}(1-b_2)(1-c_1)(1-d_0) \\
& + \mu_{47}(1-b_2)(1-c_0)(1-d_1) + \mu_{48}(1-b_2)(1-c_0)(1-d_0) \\
& + \mu_{51}(1-b_1)(1-c_3)(1-d_3) + \mu_{52}(1-b_1)(1-c_3)(1-d_2) \\
& + \mu_{53}(1-b_1)(1-c_2)(1-d_3) + \mu_{54}(1-b_1)(1-c_2)(1-d_2) \\
& + \mu_{55}(1-b_0)(1-c_3)(1-d_3) + \mu_{56}(1-b_0)(1-c_3)(1-d_2) \\
& + \mu_{57}(1-b_0)(1-c_2)(1-d_3) + \mu_{58}(1-b_0)(1-c_2)(1-d_2) \\
& + \mu_{61}(1-b_1)(1-c_3)(1-d_1) + \mu_{62}(1-b_1)(1-c_3)(1-d_0) \\
& + \mu_{63}(1-b_1)(1-c_2)(1-d_1) + \mu_{64}(1-b_1)(1-c_2)(1-d_0) \\
& + \mu_{65}(1-b_0)(1-c_3)(1-d_1) + \mu_{66}(1-b_0)(1-c_3)(1-d_0) \\
& + \mu_{67}(1-b_0)(1-c_2)(1-d_1) + \mu_{68}(1-b_0)(1-c_2)(1-d_0) \\
& + \mu_{71}(1-b_1)(1-c_1)(1-d_3) + \mu_{72}(1-b_1)(1-c_1)(1-d_2) \\
& + \mu_{73}(1-b_1)(1-c_0)(1-d_3) + \mu_{74}(1-b_1)(1-c_0)(1-d_2) \\
& + \mu_{75}(1-b_0)(1-c_1)(1-d_3) + \mu_{76}(1-b_0)(1-c_1)(1-d_2) \\
& + \mu_{77}(1-b_0)(1-c_0)(1-d_3) + \mu_{78}(1-b_0)(1-c_0)(1-d_2) \\
& + \mu_{81}(1-b_1)(1-c_1)(1-d_1) + \mu_{82}(1-b_1)(1-c_1)(1-d_0) \\
& + \mu_{83}(1-b_1)(1-c_0)(1-d_1) + \mu_{84}(1-b_1)(1-c_0)(1-d_0) \\
& + \mu_{85}(1-b_0)(1-c_1)(1-d_1) + \mu_{86}(1-b_0)(1-c_1)(1-d_0) \\
& + \mu_{87}(1-b_0)(1-c_0)(1-d_1) + \mu_{88}(1-b_0)(1-c_0)(1-d_0)
\end{aligned}
$$