

HIGH RESOLUTION FIGURES FROM WITT AND MALAMUD (2013)

- Fig. 1.** Three geoscientific time series examples exhibiting long-range persistence. (a) Bi-decadal oxygen isotope data set $\delta^{18}\text{O}$ (proxy for palaeotemperature). (b) Discharge of Elkhorn river. (c) Geomagnetic auroral electrojet (AE) index. (d) Differenced AE index.
- Fig. 2.** Lagged scatter plots of the three geophysical time series shown in Fig. 1.
- Fig. 3.** Realisations of short-range persistence autoregressive (AR(1)) processes.
- Fig. 4.** Correlograms of the four AR(1) time series shown in Fig. 3.
- Fig. 5.** Autocorrelation function of the three geophysical time series shown in Fig. 1, given as a function of increasing lag.
- Fig. 6.** Cartoon sketch of power spectral densities of self-affine and four other long-range persistent processes.
- Fig. 7.** Realizations of uncorrelated time series ($N = 1024$) with the following one-point probability distributions: (a) Gaussian, (b) Log-normal (constructed with Box-Cox transform), $c_v = 0.5$ (c) Levy, $\alpha = 1.5$. In (d) is shown the aggregation of these three uncorrelated time series.
- Fig. 8.** Ensembles of 20 realisations of the running sums of the three uncorrelated noises shown in Fig. 7.
- Fig. 9.** Three one-point probability distributions which typically occur in time series (Gaussian, Levy, and Log-normal).
- Fig. 10.** Examples of synthetic fractional Gaussian noises and motions (FGN).
- Fig. 11.** Examples of synthetic fractional Levy noises and motions (FLevyN).
- Fig. 12.** Examples of synthetic fractional log-normal noises and motions (FLNN_{LN}) constructed by Box-Cox transform.
- Fig. 13.** Examples of synthetic fractional log-normal noises and motions (FLNN_{LN}) (constructed by the Schreiber-Schmitz algorithm).
- Fig. 14.** Long-range persistence analysis of a FLNN with persistence strength $\beta_{\text{model}} = 1.0$ (coefficient of variation $c_v = 0.5$, $N = 4096$ data points).
- Fig. 15.** Pre-processing of a time series and the effect of Welch windowing on the power spectral densities of a FLNN ($\beta_{\text{model}} = 2.5$, $c_v = 0.5$).
- Fig. 16.** Whittle estimator and its corresponding maximum likelihood function.
- Fig. 17.** Illustration of systematic and random errors using detrended fluctuation analysis with quadratic trend removed (DFA2).
- Fig. 18.** Distribution of estimated long-range persistence strengths for power spectral analysis applied to fractional noises, $\beta_{\text{model}} = 1.0$.
- Fig. 19.** Distribution of estimated long-range persistence strengths for detrended fluctuation analysis applied to fractional noises, $\beta_{\text{model}} = 1.0$.
- Fig. 20.** Distribution of estimated long-range persistence strengths for Hurst rescaled range (R/S) analysis applied to fractional noises, $\beta_{\text{model}} = 1.0$.
- Fig. 21.** Performance of the Hurst rescaled range analysis (R/S) applied to realisations of fractional noises and motions.
- Fig. 22.** Performance of semivariogram analysis applied to realisations of fractional noises and motions.
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- Fig. 24.** Performance of power spectral analysis (best-fit) applied to realisations of fractional noises and motions.
- Fig. 25.** Performance of power spectral analysis (Whittle) applied to realisations of fractional noises and motions.
- Fig. 26.** Performance of three techniques for evaluating long-range persistence in FLNN created to have $c_v = 0.0$ to 2.0.
- Fig. 27.** Performance of four techniques for evaluating long-range persistence in FLevyN created to have $\alpha = 1.0$ to 2.0.
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- Fig. 29.** Visual overview of the systematic error (bias) of five techniques for evaluating long-range persistence.
- Fig. 30.** Visual overview of the random error of five techniques for evaluating long-range persistence.
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- Fig. 32.** Cartoon illustration of joint probability distributions.
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- Fig. 34.** Illustration of conditional probability distribution construction for FLNN realisations ($0.0 \leq \beta_{\text{model}} \leq 2.2$, $c_v = 0.5$, $N = 1024$) using DFA.
- Fig. 35.** Three screen captures from *Supplementary Material Excel Spreadsheet* for a user to determine conditional probability distributions based on a user-measured β_{measured} for a time series, and the corresponding benchmark performance results discussed in this paper.
- Fig. 36.** Conditional distributions of the strength of long-range persistence of a FLNN ($\beta_{\text{model}} = 1.0$, $c_v = 0.5$, $N = 1024$) for five techniques.
- Fig. 37.** Long-range dependence analysis of 10,000 year GISP2 bi-decadal $\delta^{18}\text{O}$ isotopes (palaeotemperature proxy, Fig. 1A) using four techniques.
- Fig. 38.** Long-range dependence analysis of 1929–2001 daily discharge (Elkhorn river, Fig. 1B) using four techniques.
- Fig. 39.** Long-range dependence analysis of the 1 February 1978 (24 h) geomagnetic auroral electrojet (AE) index (Fig. 1C) using four techniques.
- Fig. 40.** Power-law functions with additive and multiplicative noises.
- Fig. 41.** Distribution of estimated power-law exponents for two different fitting techniques applied to three different power-law models (Fig. 40).

Short figure captions are given above.

Please see Witt and Malamud (2013) for detailed figure captions.

(Please use the following citation)

Witt, AW and Malamud, BD (2013) Quantification of long-range persistence in geophysical time series: conventional and benchmark-based improvement techniques. *Surveys in Geophysics*, doi:10.1007/s10712-012-9217-8.

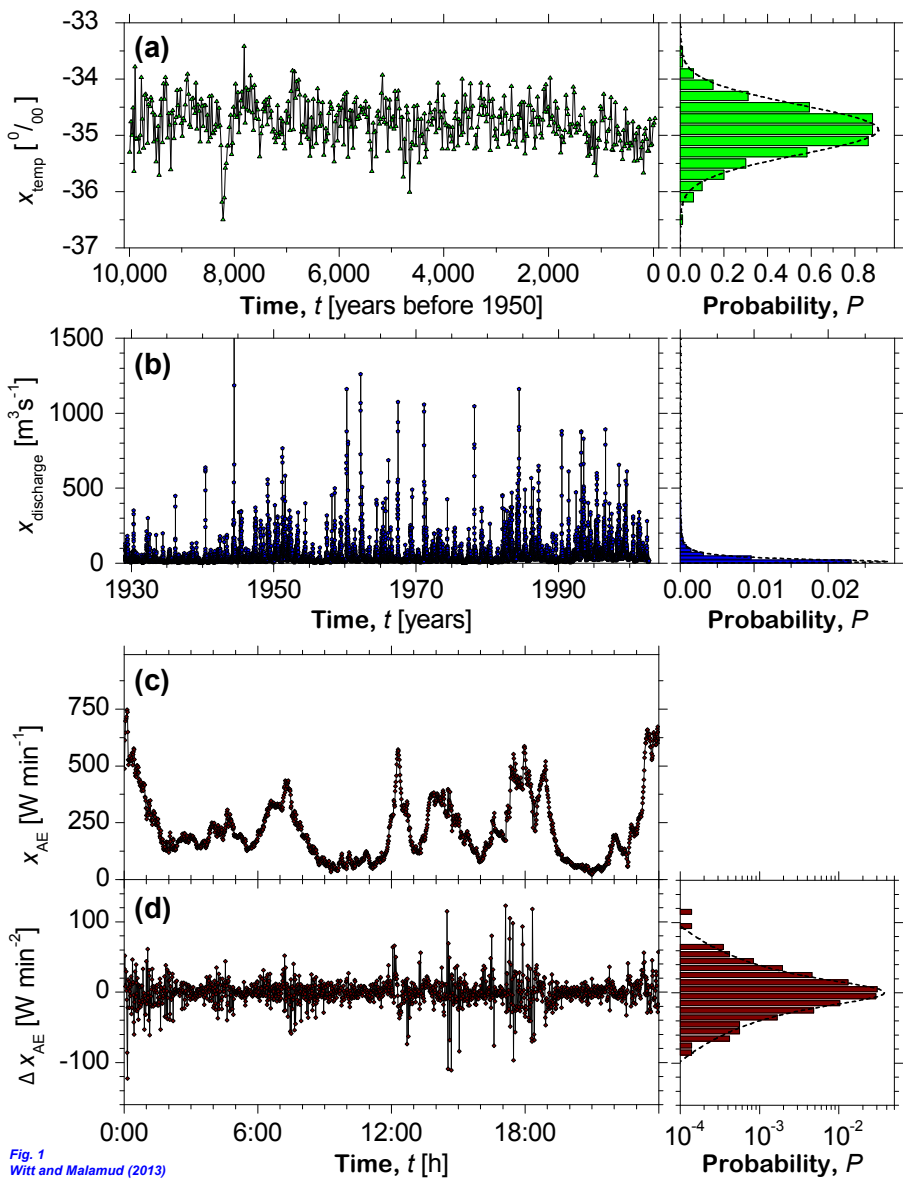


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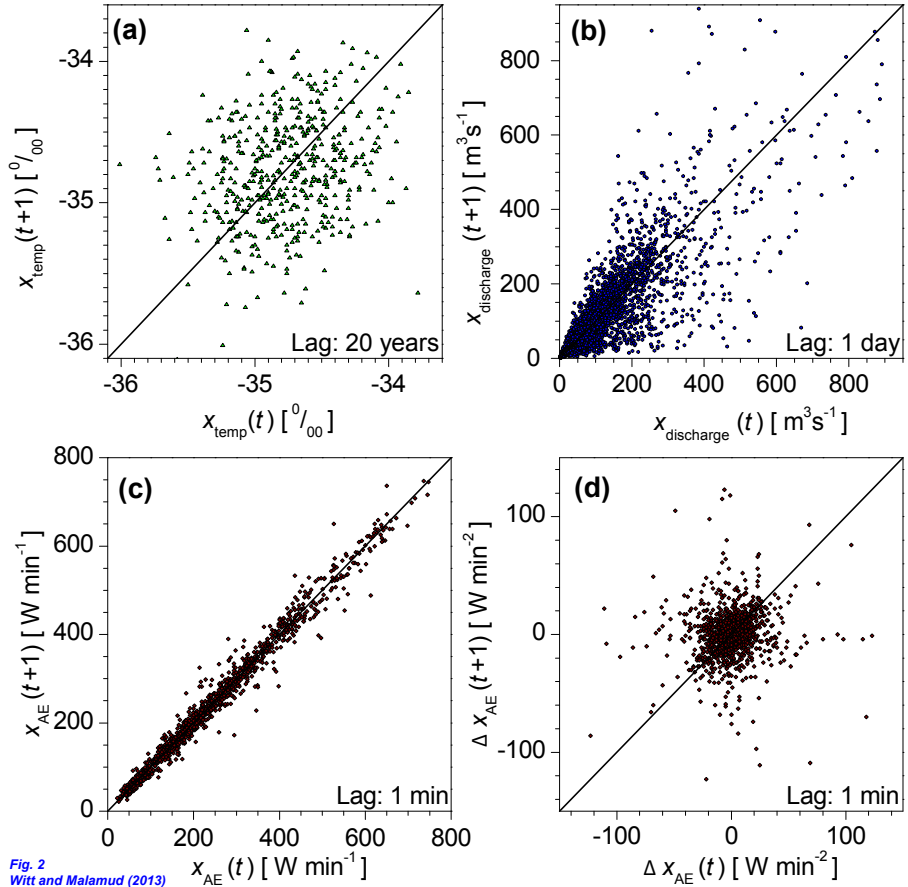


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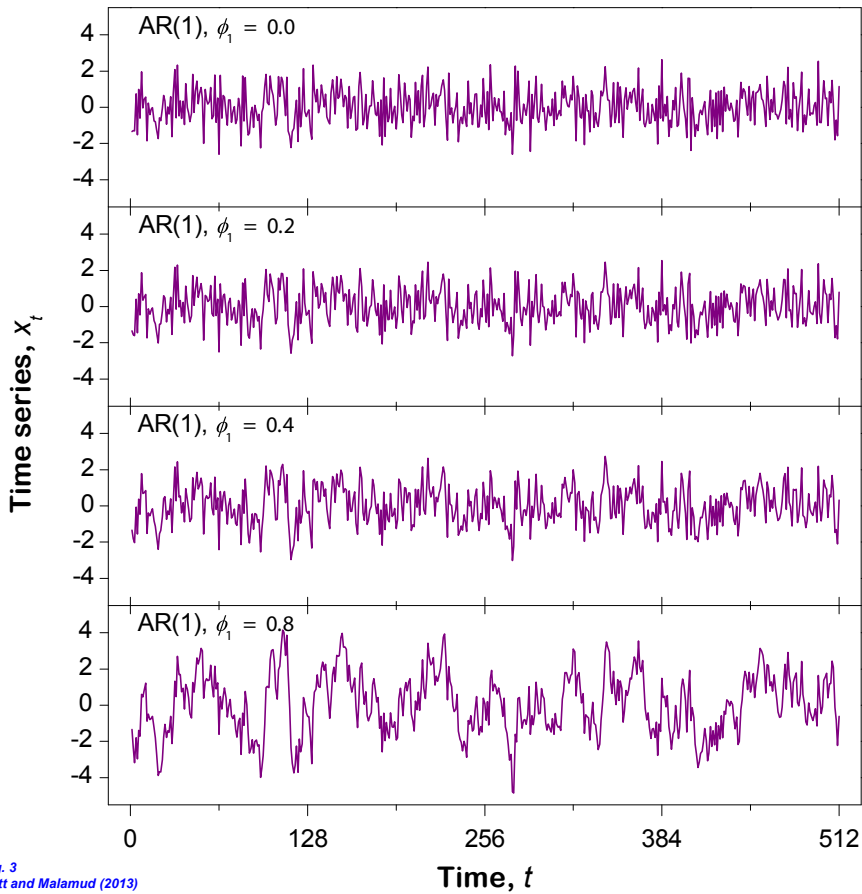


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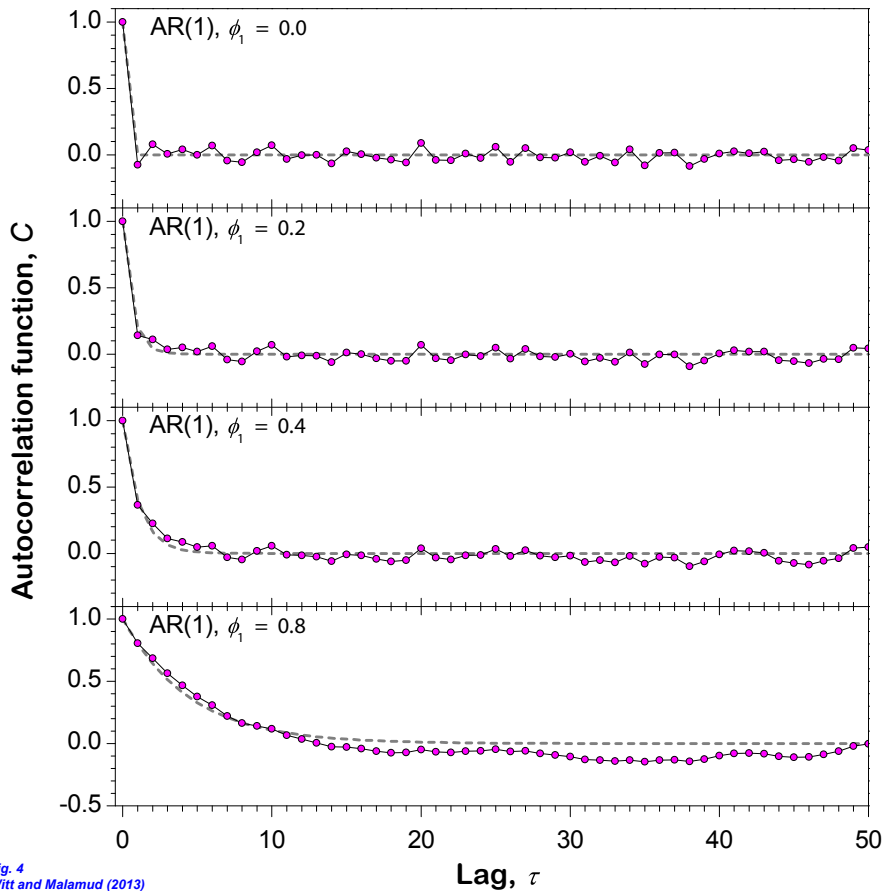


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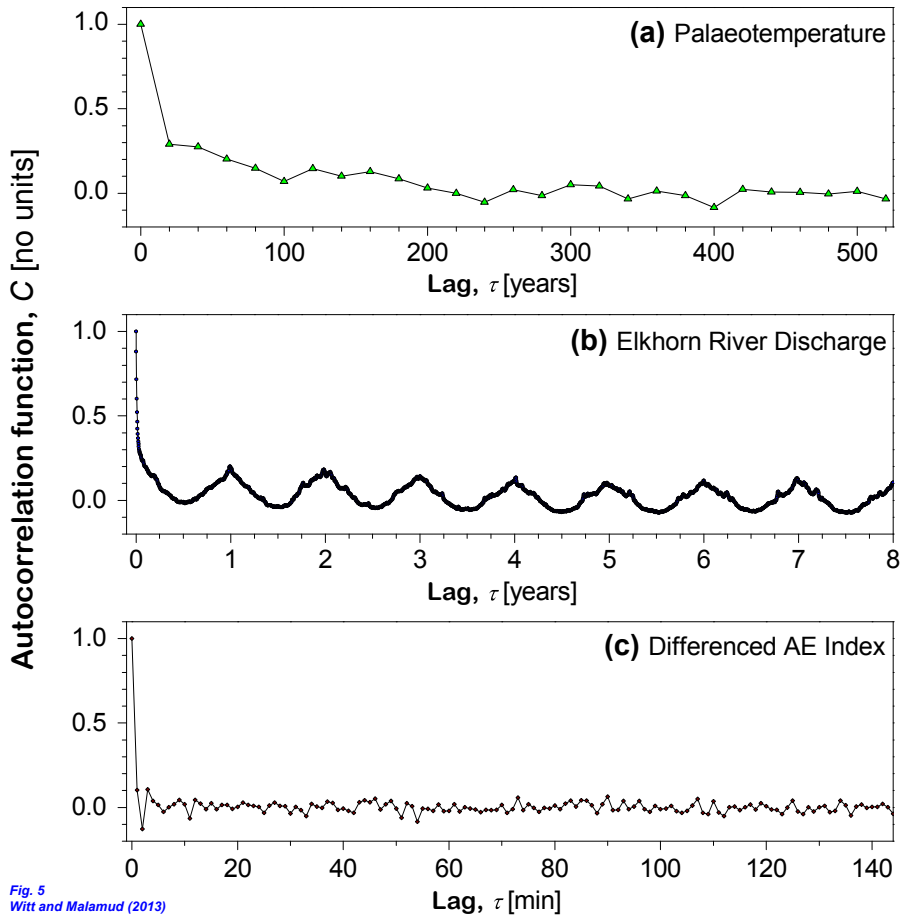


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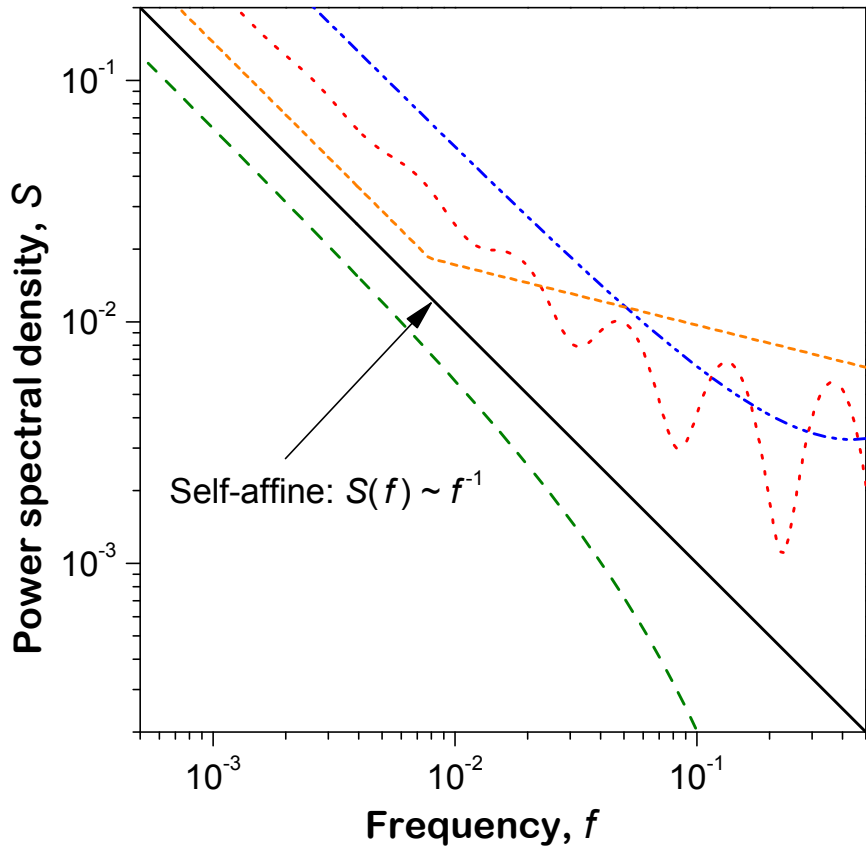


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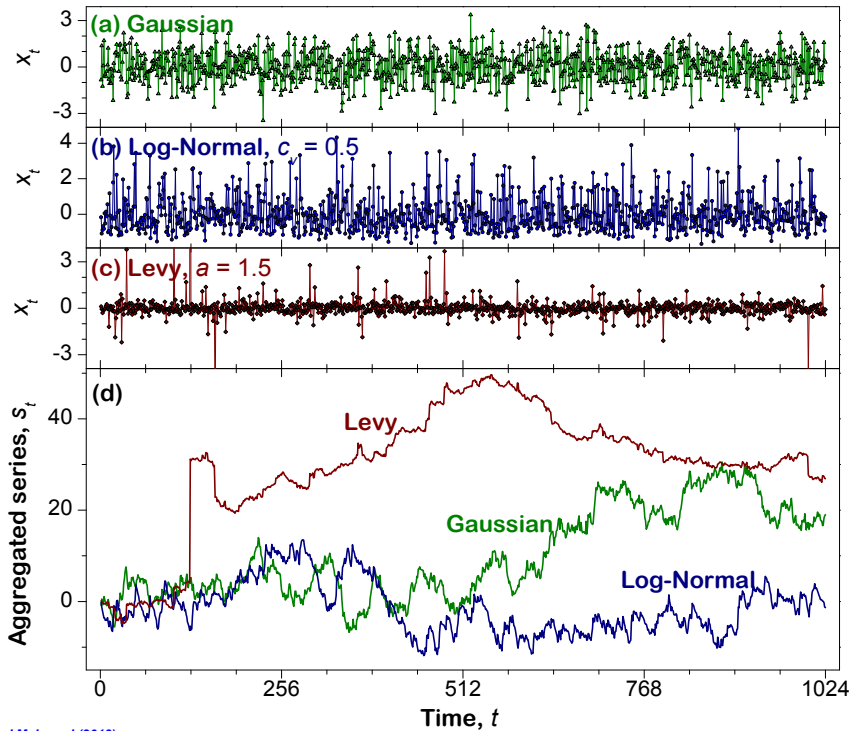


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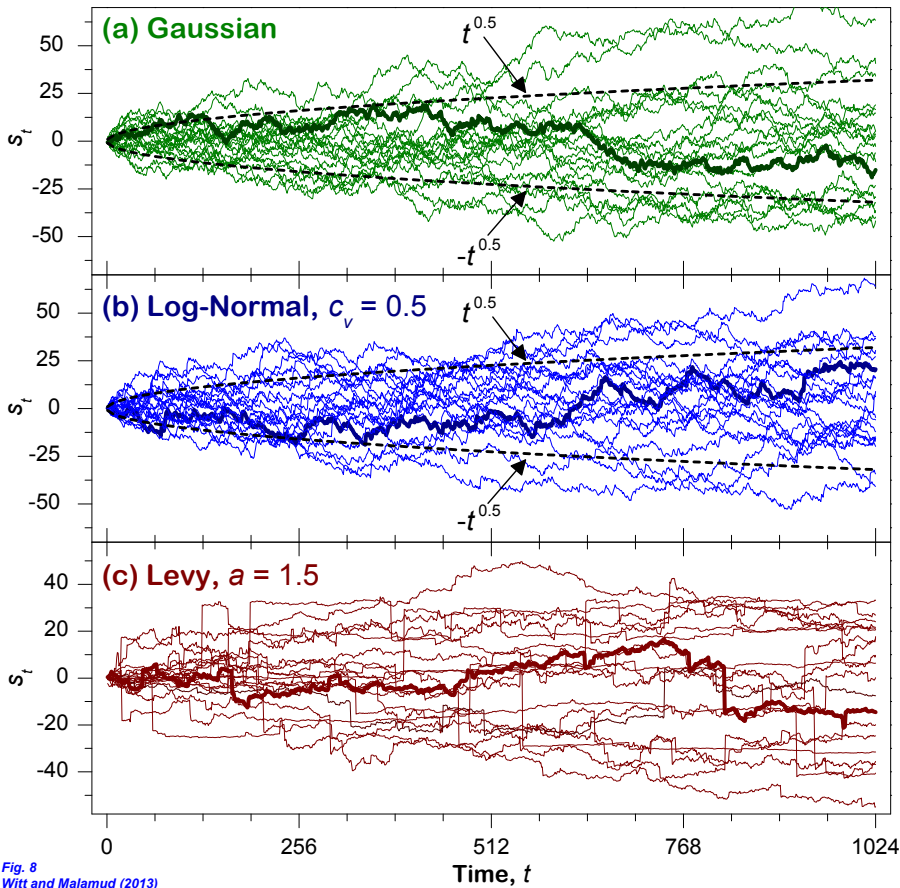


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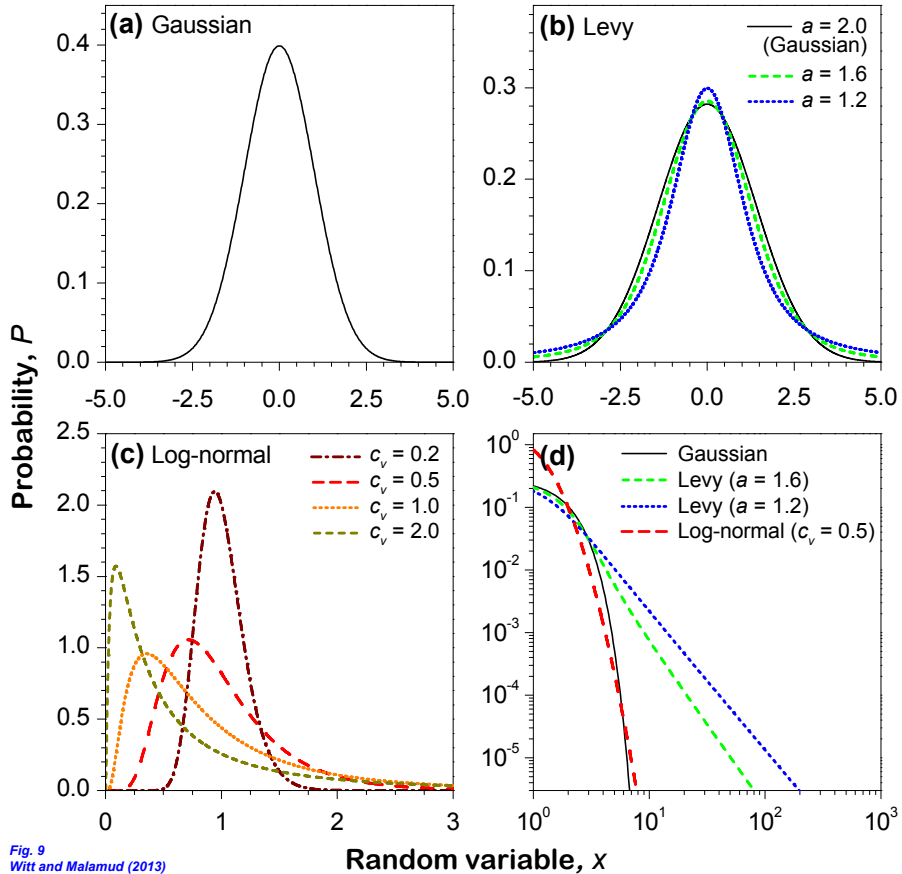


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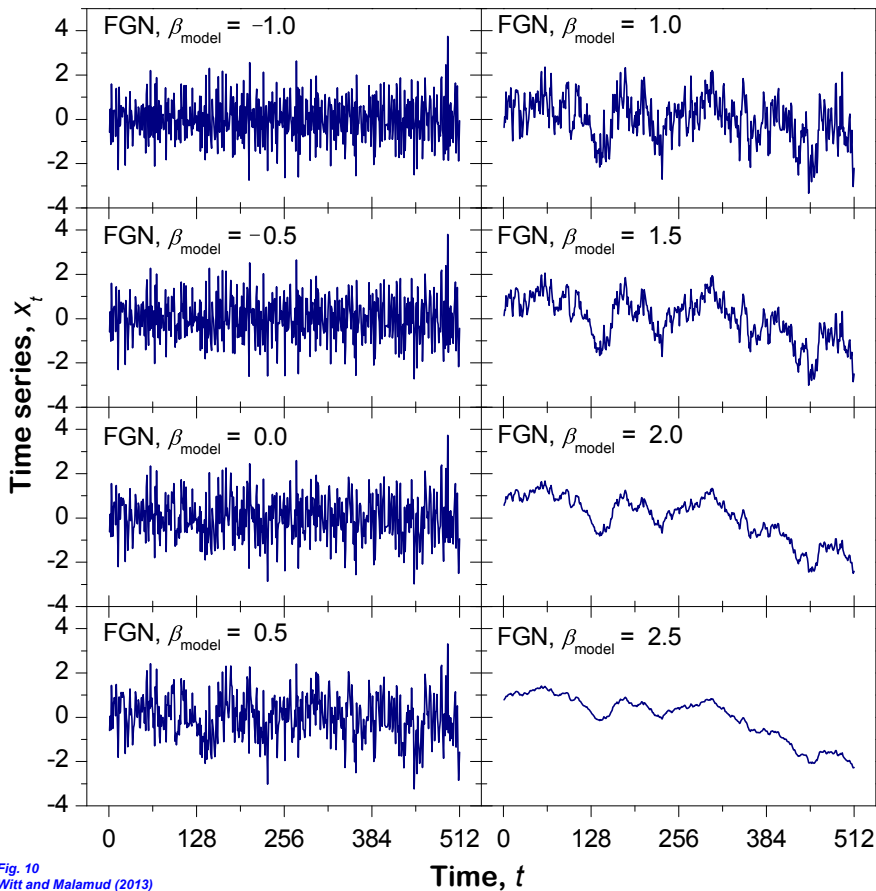


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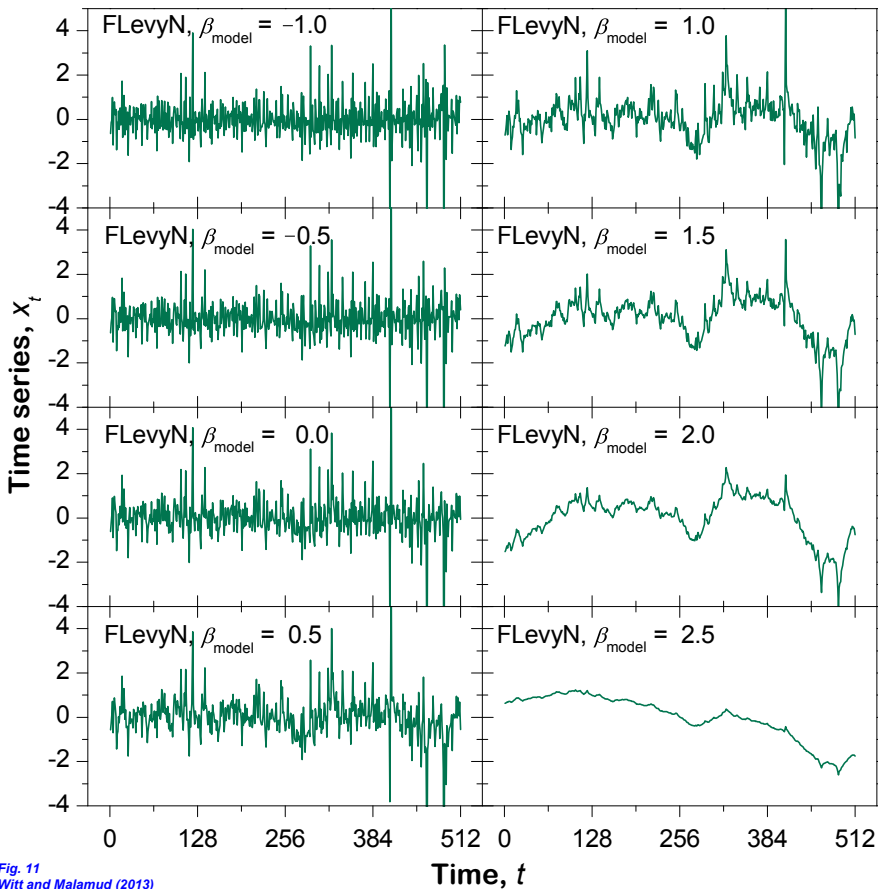


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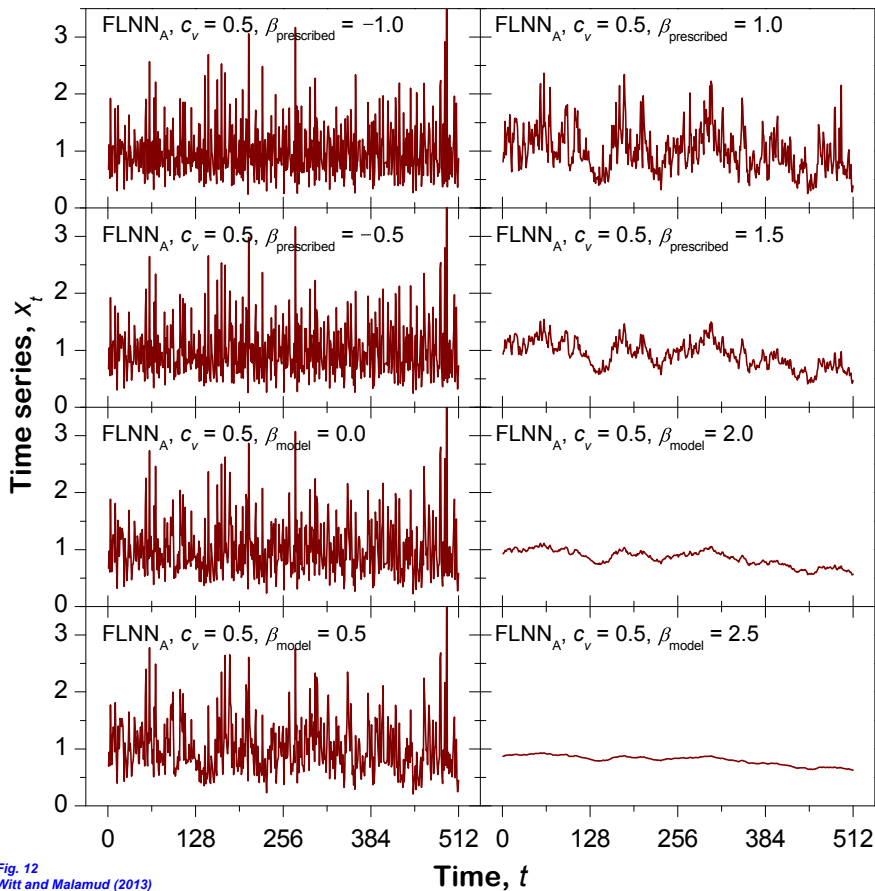


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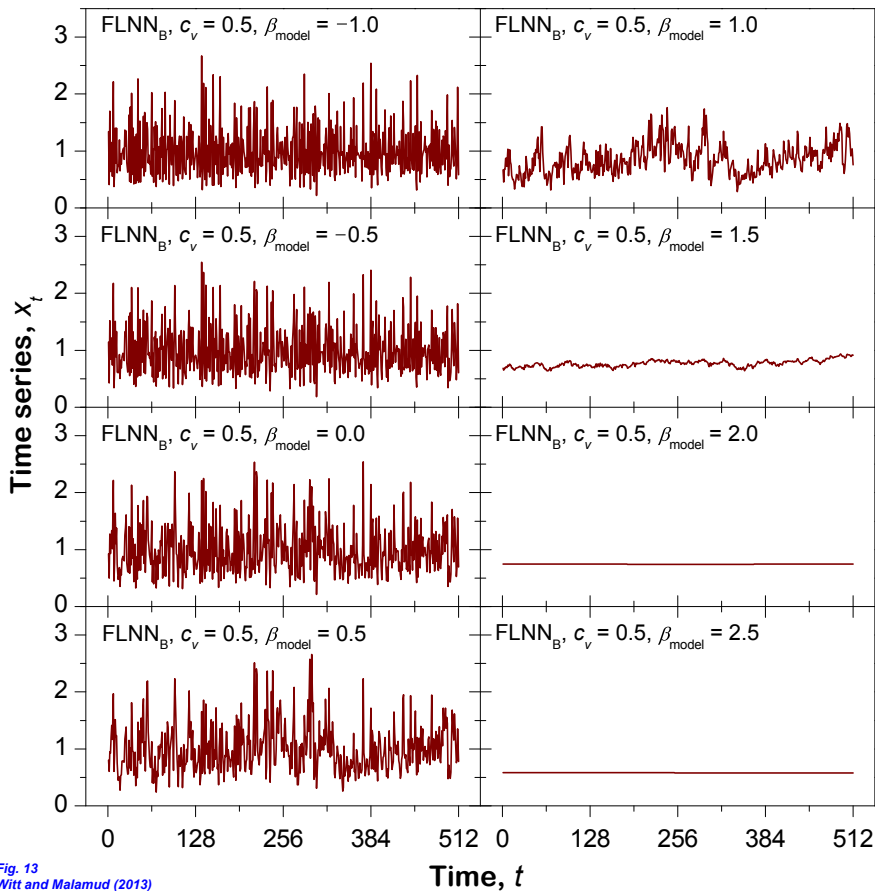


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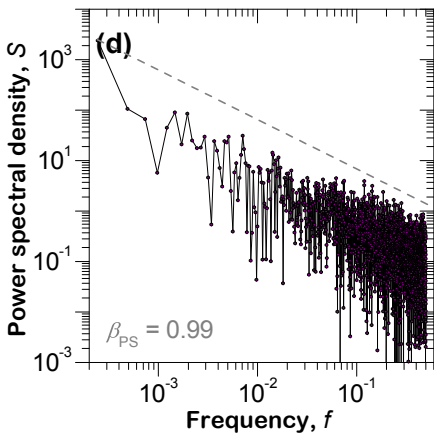
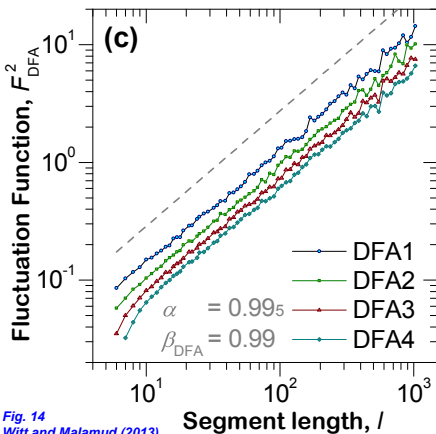
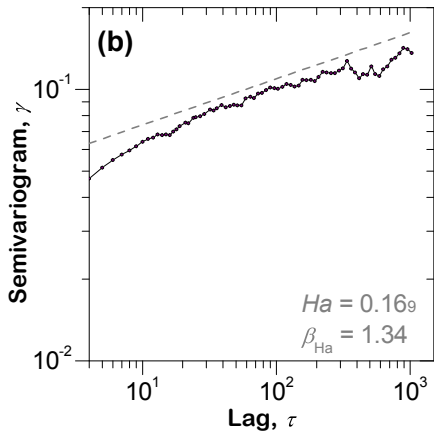
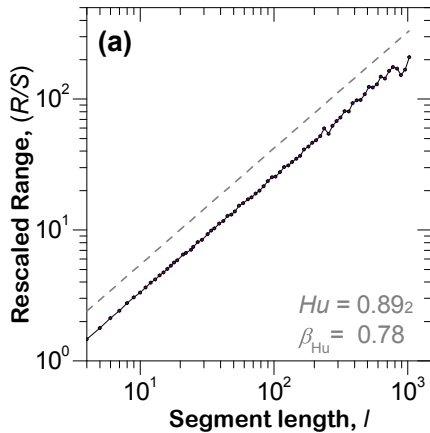


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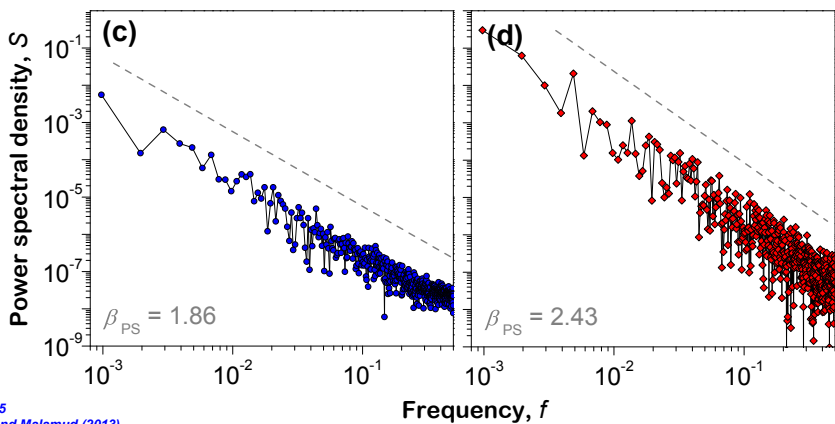
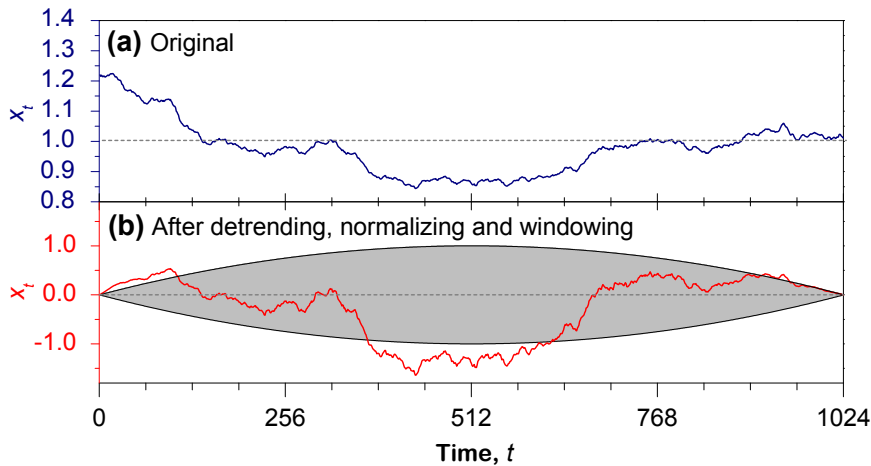


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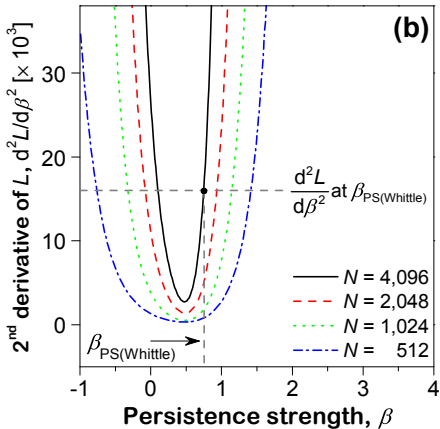
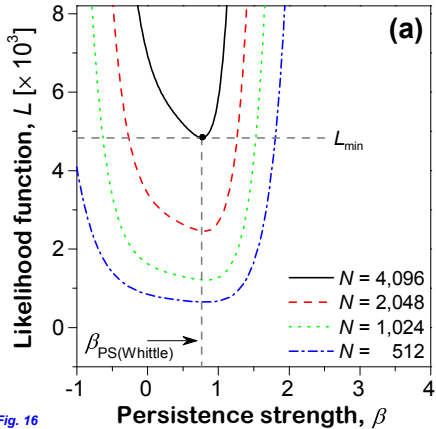


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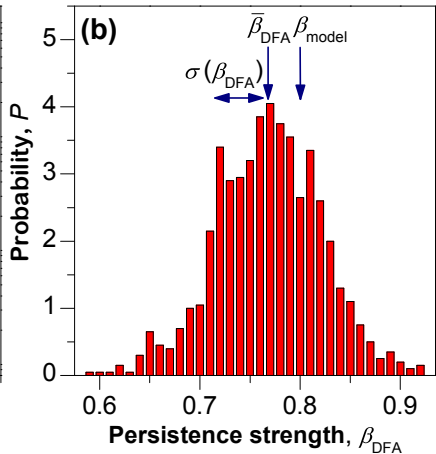
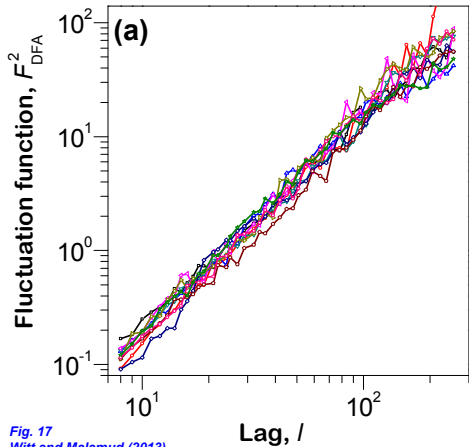


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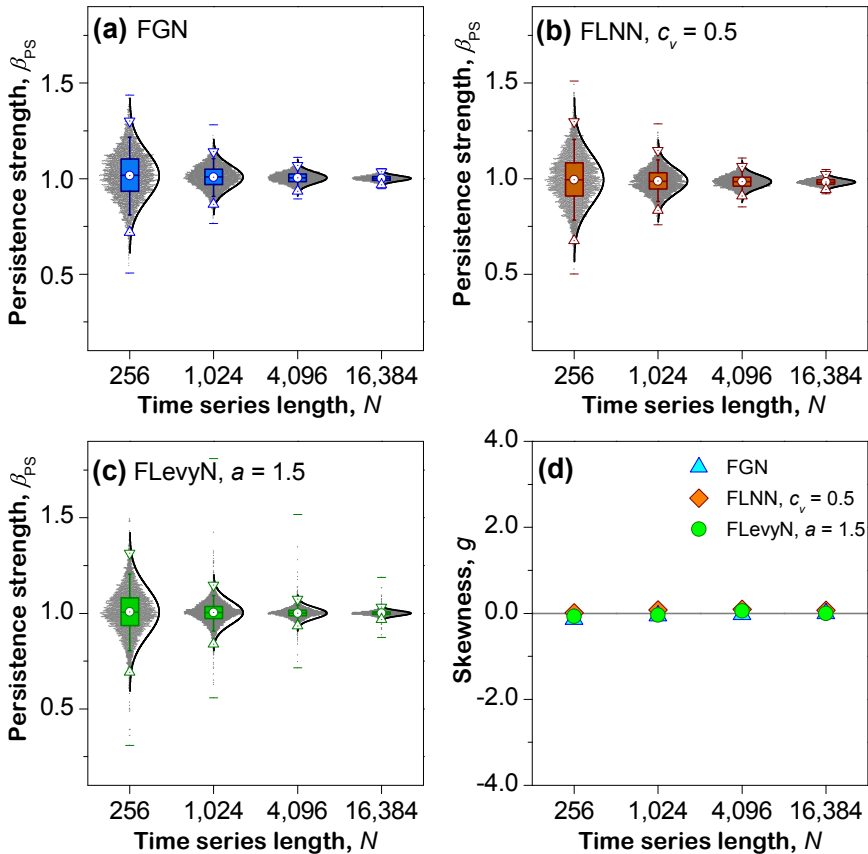


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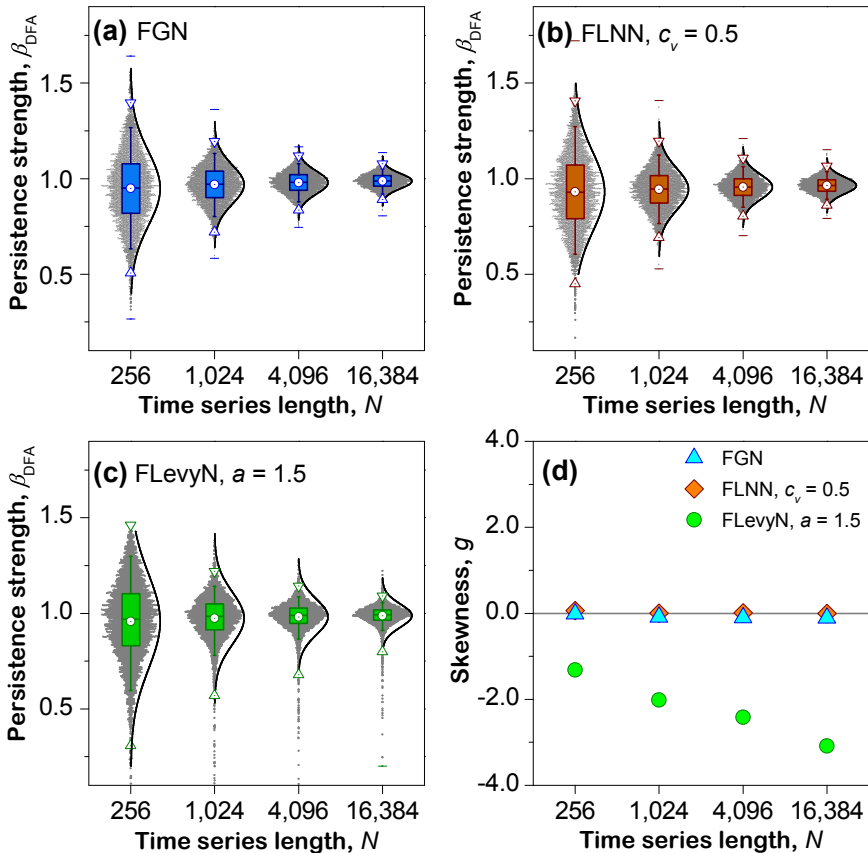


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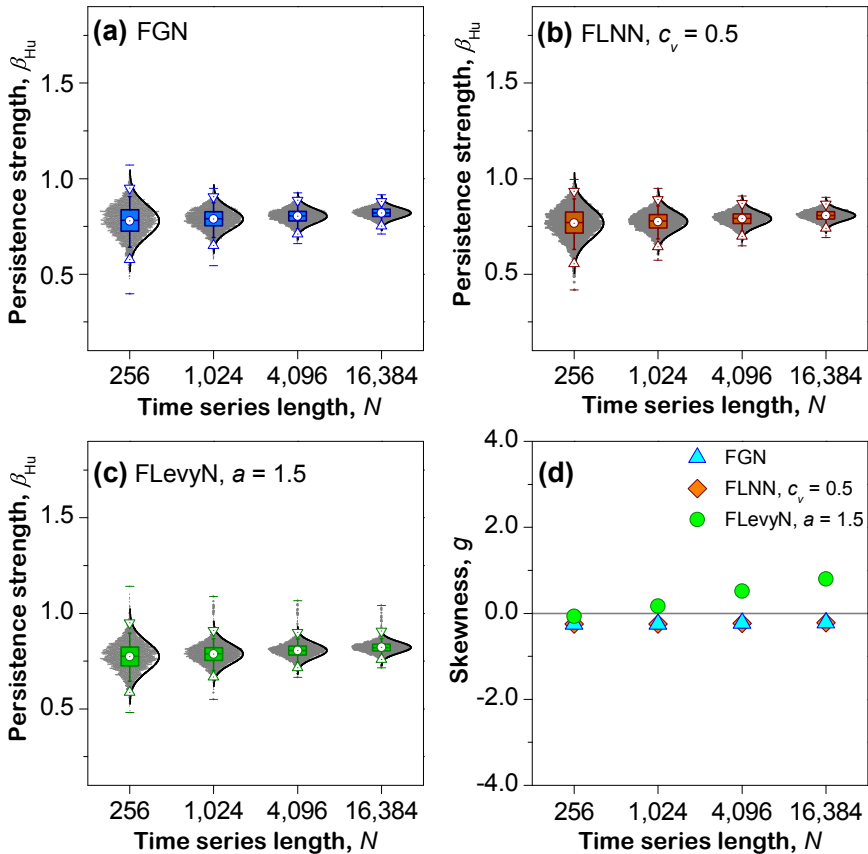


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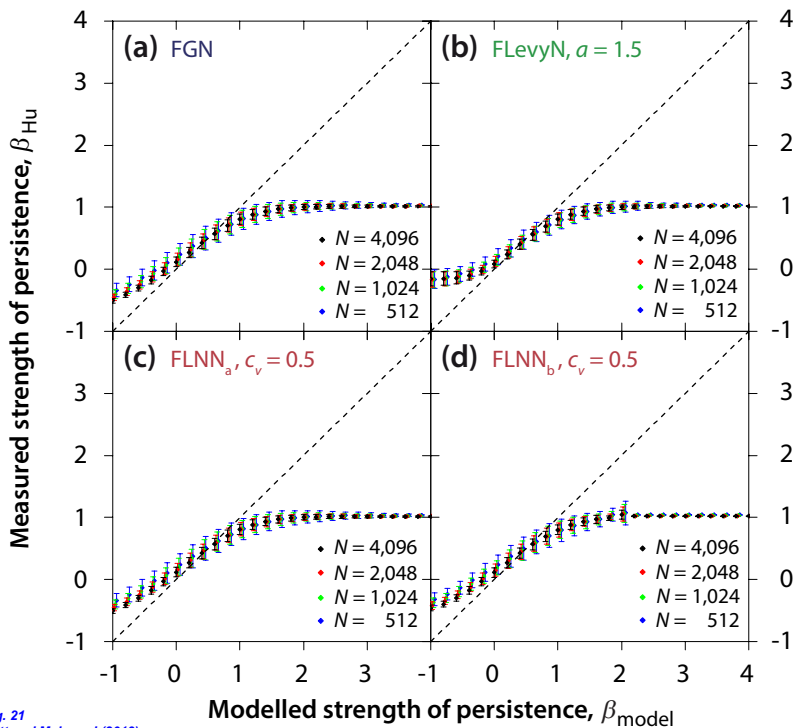


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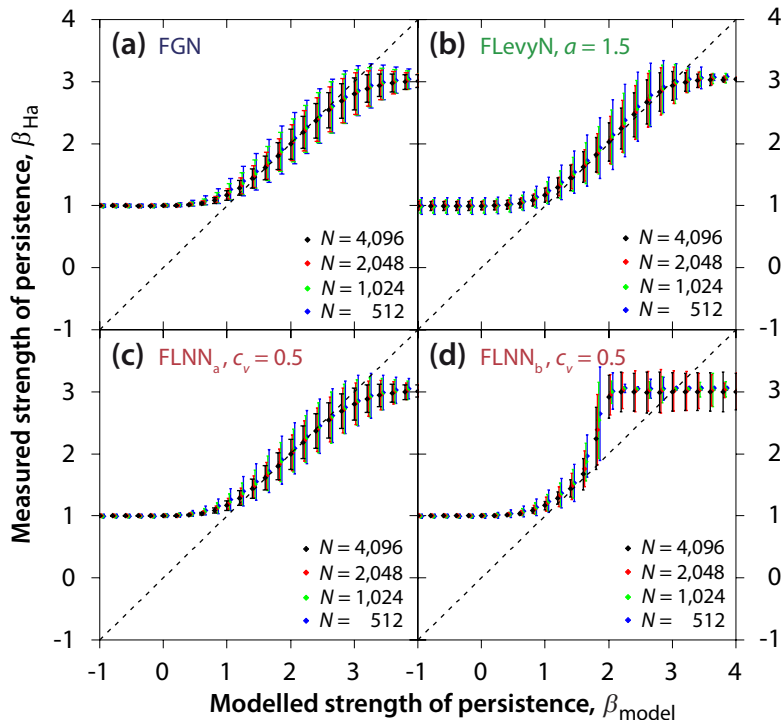


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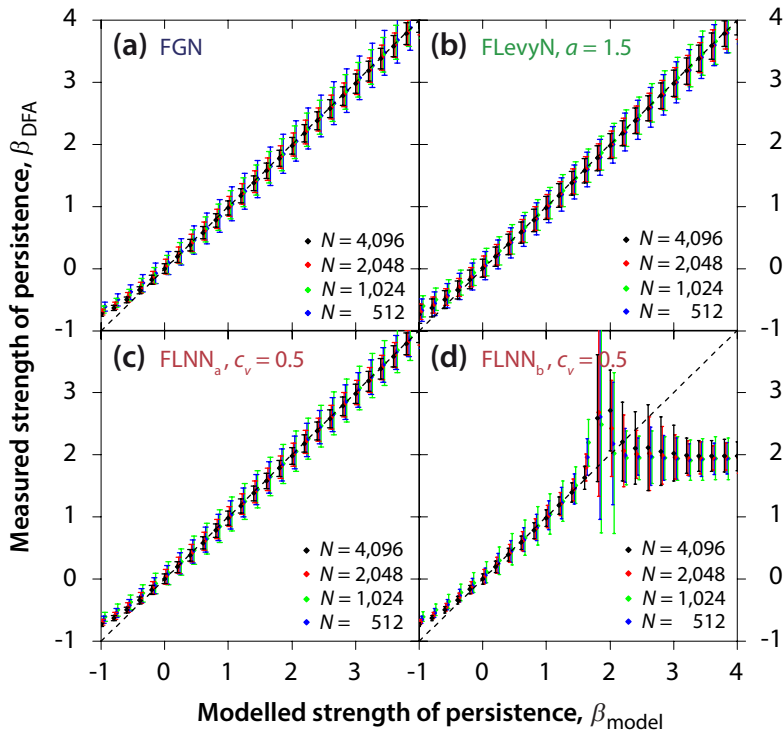


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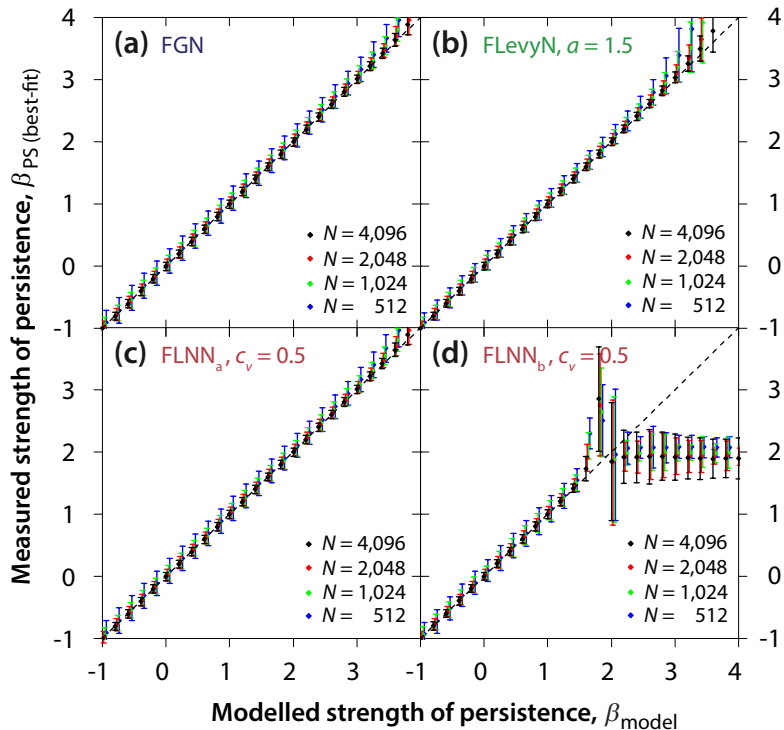


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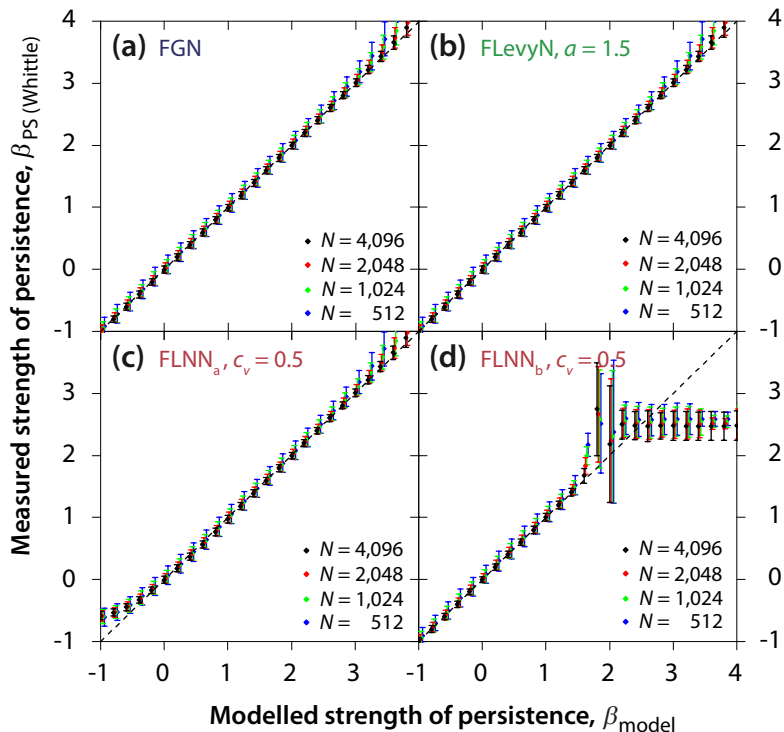


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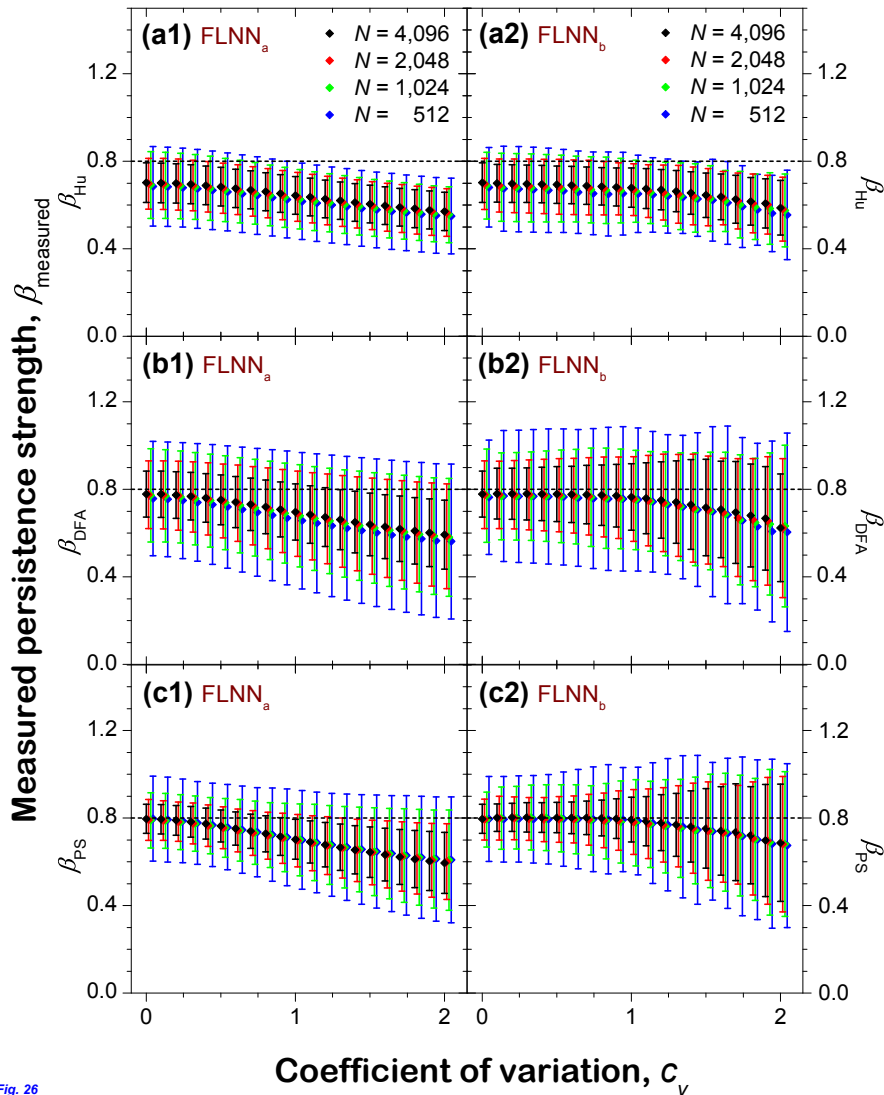


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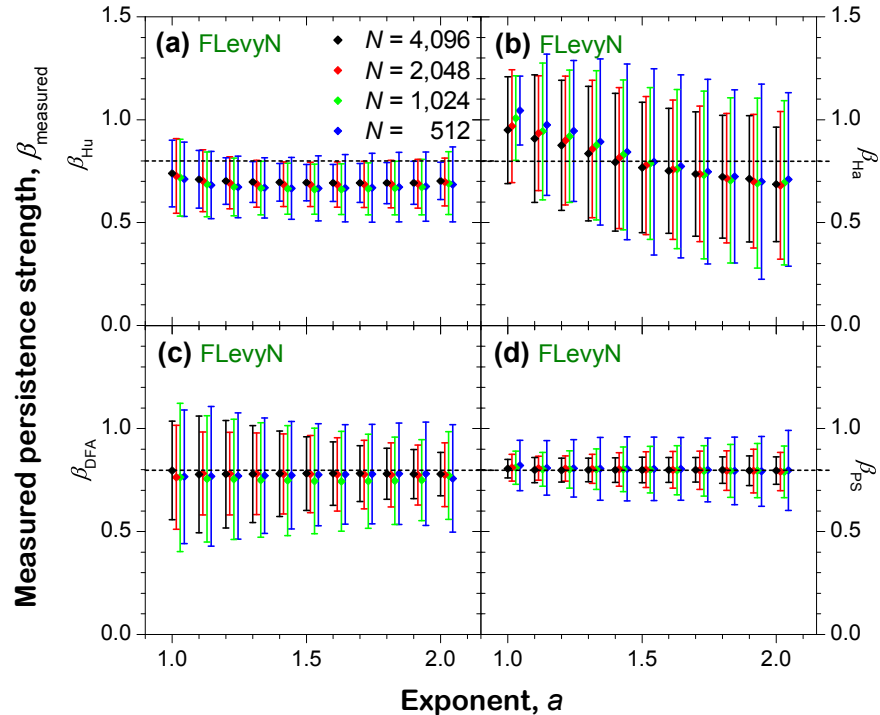


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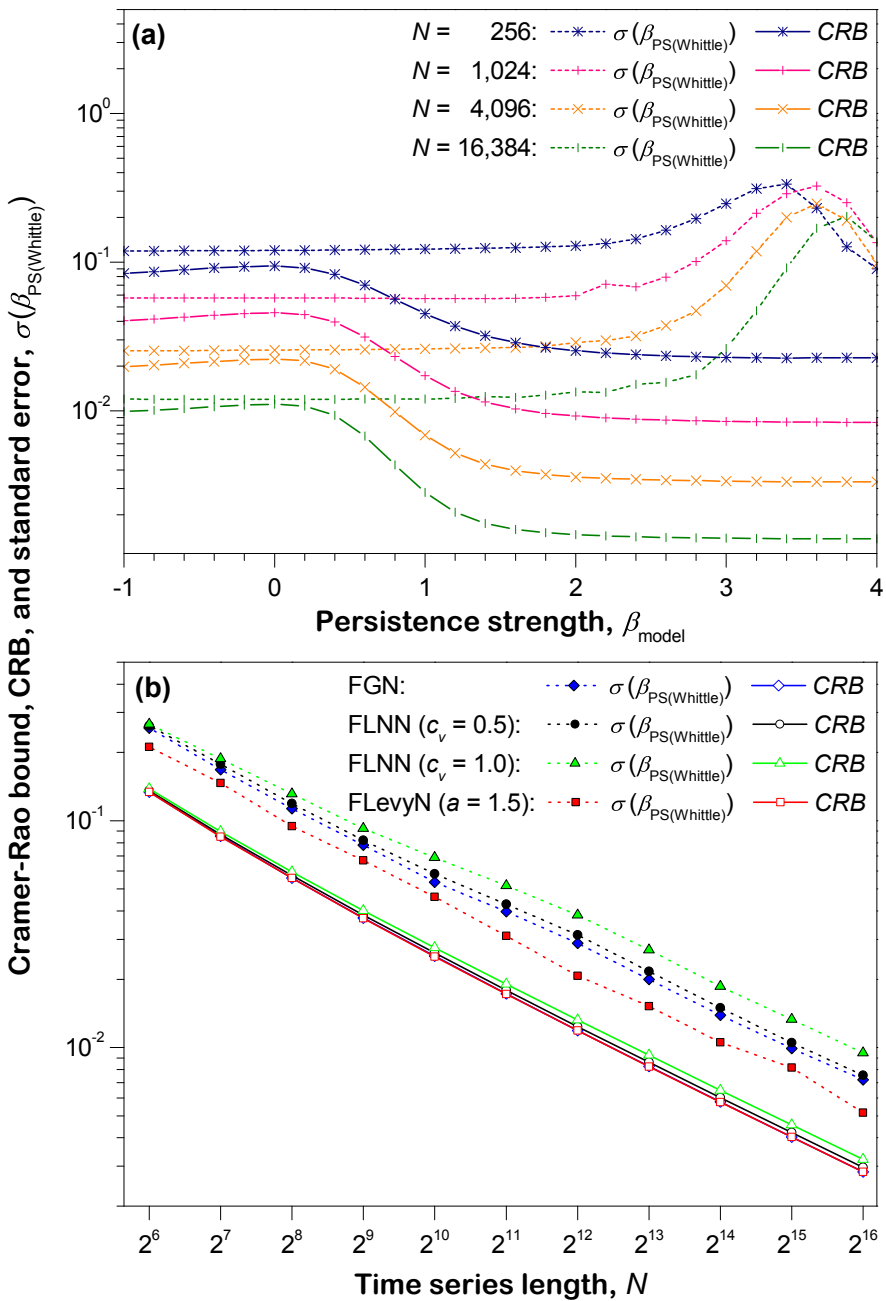


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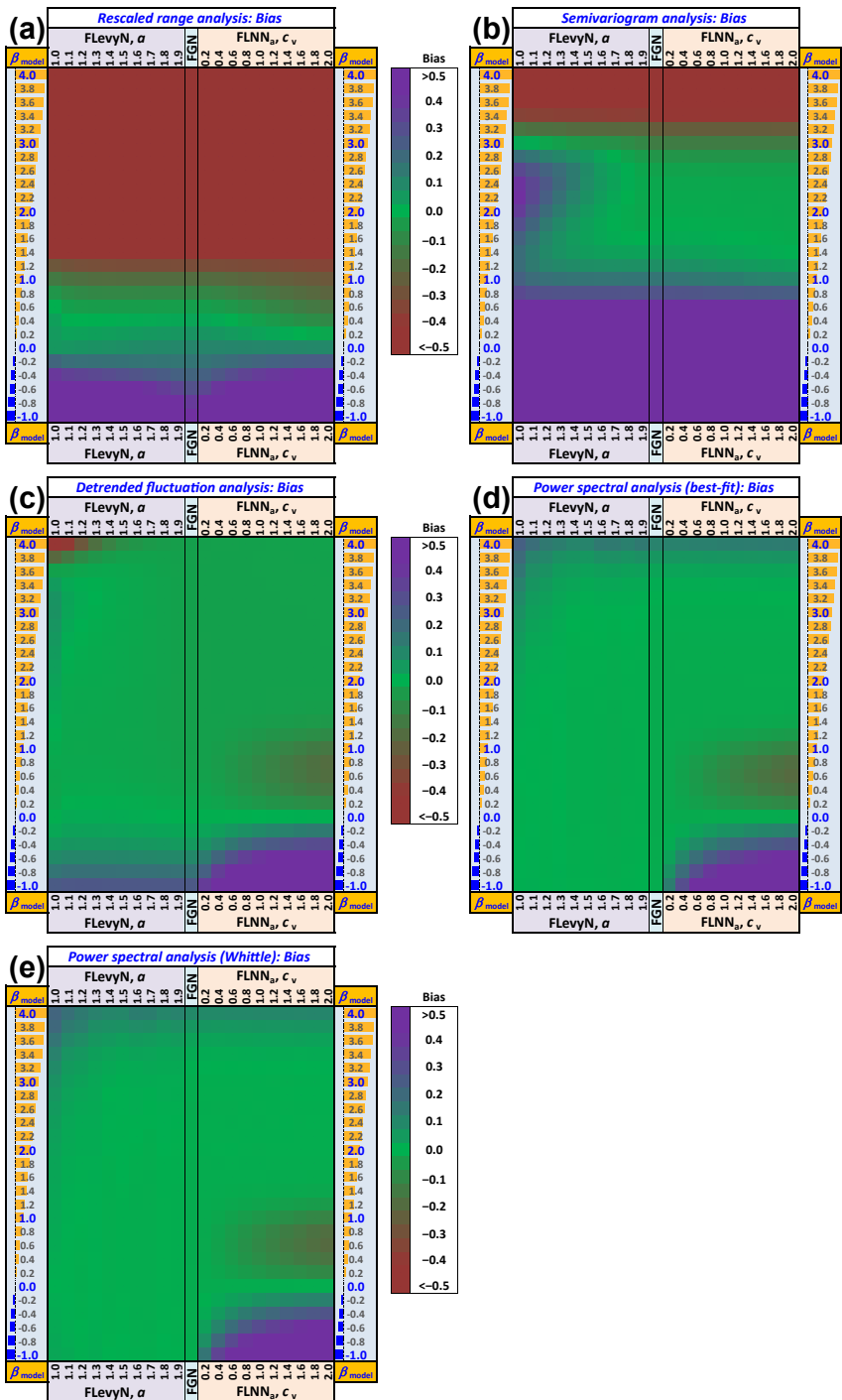


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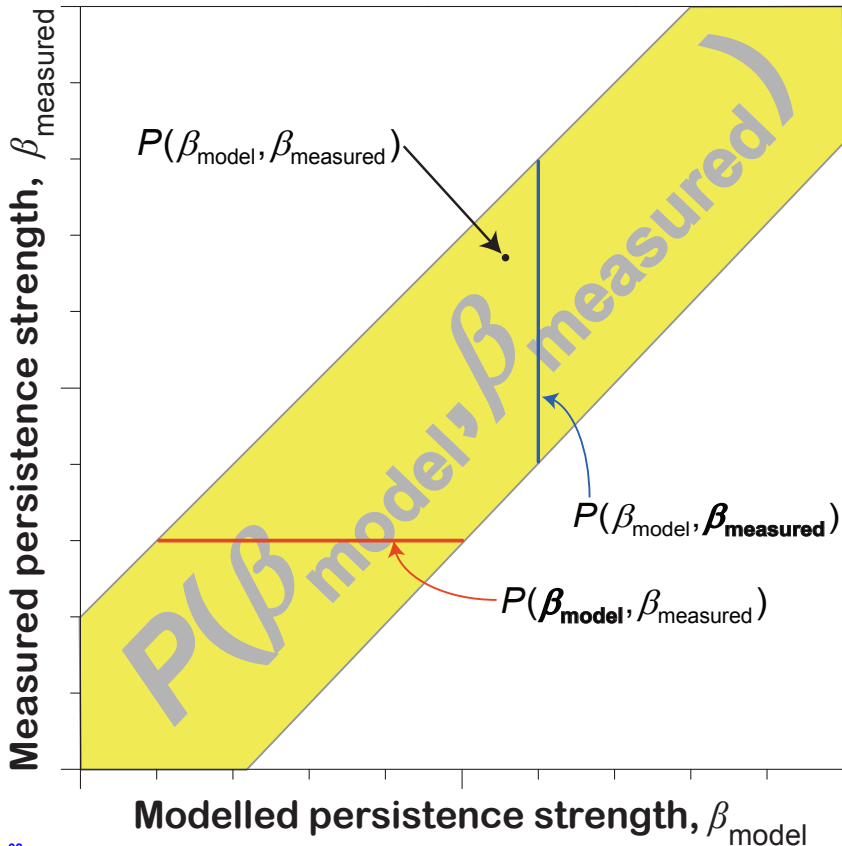
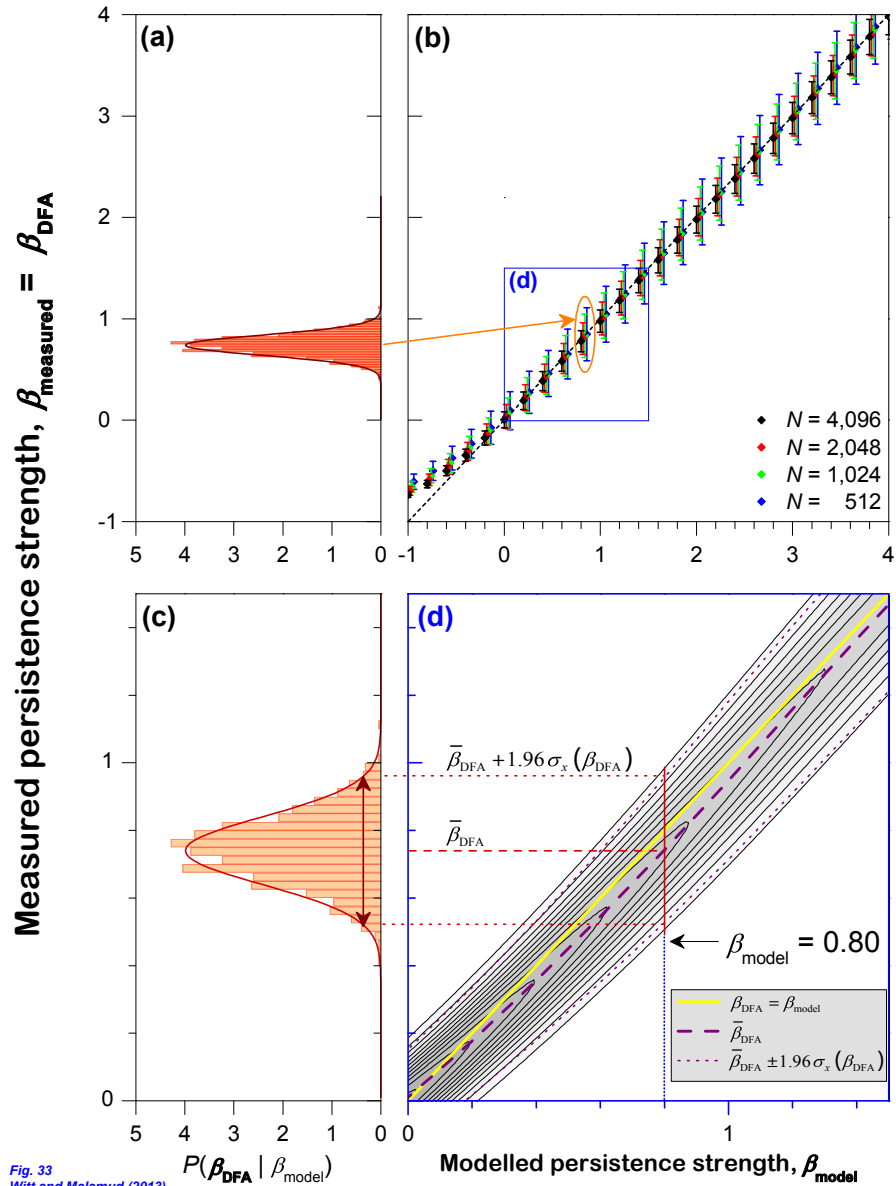


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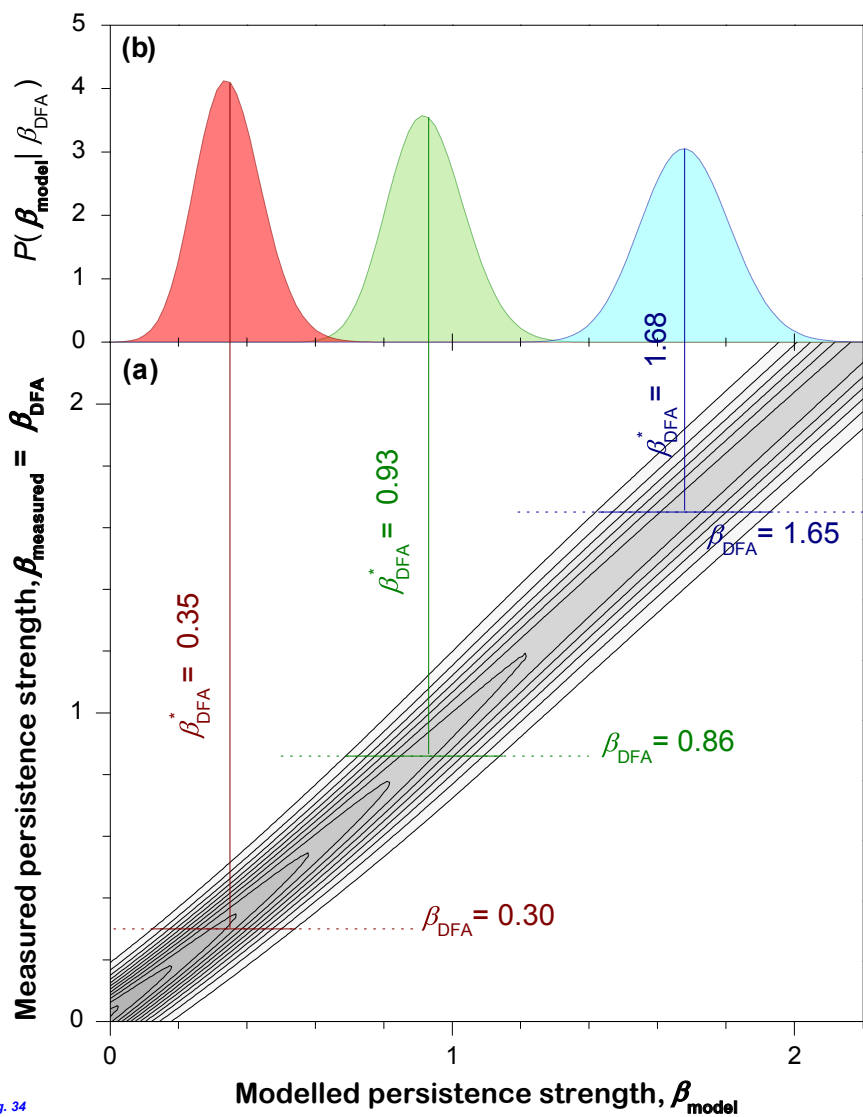


Fig. 34
Witt and Malamud (2013)

(a)

A_PerfTestResults Worksheet: Results of performance tests using benchmark synthetic time series with modeled strengths of long-range persistence and given one-point probability distributions, for time series lengths $N = 64, 128, 256, \dots, 131,072$.

Instructions: Using the 'Filter' arrow in the header row (row 6), select the one-point probability distribution, time series length and technique applied, closest to your situation, to display the summary statistics (for each β_{model}) of 100 realisations of the process.

Citation: Witt, AW and Malamud, BD (2013) Quantification of Long-Range Persistence in Geophysical Time Series: Conventional and Benchmark-Based Improvement Techniques. *Surveys in Geophysics*.

Benchmark Time Series & Technique Applied							Performance Test Results								
	One-Point Probability Distribution Type	Value of α or c_v	Time series length, N	# Realisations	Technique applied	β_{model}	mean β_{measured}	standard deviation β_{measured}	Bias = $\text{mean}(\beta_{\text{measured}}) - \beta_{\text{model}}$	RMSE = Root Mean Squared Error	min β_{measured}	1st quantile β_{measured}	median β_{measured}	3rd quantile β_{measured}	max β_{measured}
27671	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	-1.000	-0.482	0.098	0.518	0.528	-0.698	-0.548	-0.486	-0.429	-0.205
27672	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	-0.800	-0.436	0.096	0.364	0.376	-0.654	-0.507	-0.438	-0.375	-0.178
27673	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	-0.600	-0.373	0.096	0.227	0.247	-0.606	-0.437	-0.372	-0.305	-0.142
27674	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	-0.400	-0.278	0.098	0.122	0.157	-0.494	-0.337	-0.268	-0.207	-0.038
27675	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	-0.200	-0.154	0.102	0.046	0.112	-0.517	-0.215	-0.146	-0.086	0.089
27676	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	0.000	-0.006	0.102	-0.006	0.102	-0.340	-0.064	0.007	0.056	0.224
27677	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	0.200	0.156	0.102	-0.044	0.111	-0.211	0.088	0.170	0.224	0.380
27678	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	0.400	0.335	0.102	-0.065	0.121	-0.027	0.273	0.342	0.403	0.550
27679	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	0.600	0.526	0.104	-0.074	0.128	0.196	0.466	0.528	0.592	0.760
27680	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	0.800	0.735	0.107	-0.065	0.125	0.418	0.667	0.730	0.799	1.002
27681	FLNN _a	$c_v = 0.8$	512	100	PS_BestFit	1.000	0.964	0.107	-0.036	0.113	0.599	0.910	0.959	1.034	1.230

	A	B	C	D	E	F	G	H	I	J	K	L	M
(b)	B_InterpolSheet Worksheet: This worksheet creates $P(\beta_{\text{model}} \beta_{\text{measured}})$ distribution based on user measured beta for a time series, giving mean P, and 2.5/97.5 Percentiles for that P.												
1	Instructions: Input below (in yellow boxes) the mean and standard deviation from the A_PerfTestResults Worksheet for a process that most closely match the parameters of your time series (for the one-point probability distribution type, number of values N , and technique used), along with the β_{measured} for your specific time series.												
2	Citation: Witt, AW and Malamud, BD (2013) Quantification of Long-Range Persistence in Geophysical Time Series: Conventional and Benchmark-Based Improvement Techniques. <i>Surveys in Geophysics</i>												
3													
5		β_{measured}	0.75	Do not change any of they below, they will adjust themselves and will be plotted on C_CalibratedProbChart.									
6	Interpolated, step size $\Delta = 0.01$												
7	Spreadsheet input							Linear Interpolation	P without c_2	Non-Cumulative P	Cumulative P		
8	β_{model}	mean β_{measured}	standard deviation β_{measured}	β_{model}	Array location	mean β_{measured}	standard deviation β_{measured}	P/c_2	$P(\beta_{\text{model}} \beta_{\text{measured}})$	Normalized Running Sum $P * \Delta$			
9	-1.0	-0.4815	0.0976	-1.00	1	-4.82E-01	9.76E-02	2.87E-35	1.19E-34	1.19E-36			
10	-0.8	-0.4364	0.0957	-0.99	1	-4.79E-01	9.75E-02	3.28E-35	1.35E-34	2.54E-36			
11	-0.6	-0.3726	0.0965	-0.98	1	-4.77E-01	9.75E-02	3.74E-35	1.54E-34	4.08E-36			
509				4.00	26	4.15E+00	1.51E-01	7.80E-111	3.22E-110	1.00E+00			
510						$c_2 = \Delta * \text{sum}(P/c_2)$		4.12					
511						Check integral $P = 1.00$		1.000					
512						Mean $P = \Delta * \text{sum}(\beta_{\text{model}} * P)$		0.808					
513						2.5 Percentile of P distribution				0.610			
514						97.5 Percentile of P distribution				0.990			

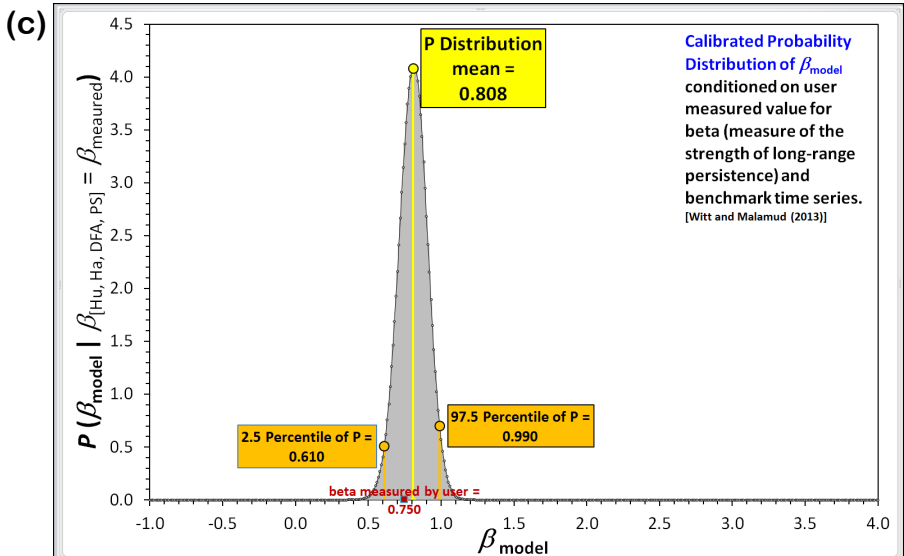


Fig. 35 A_PerfTestResults B_InterpolSheet C_CalibratedProbChart
Witt and Malamud (2013)

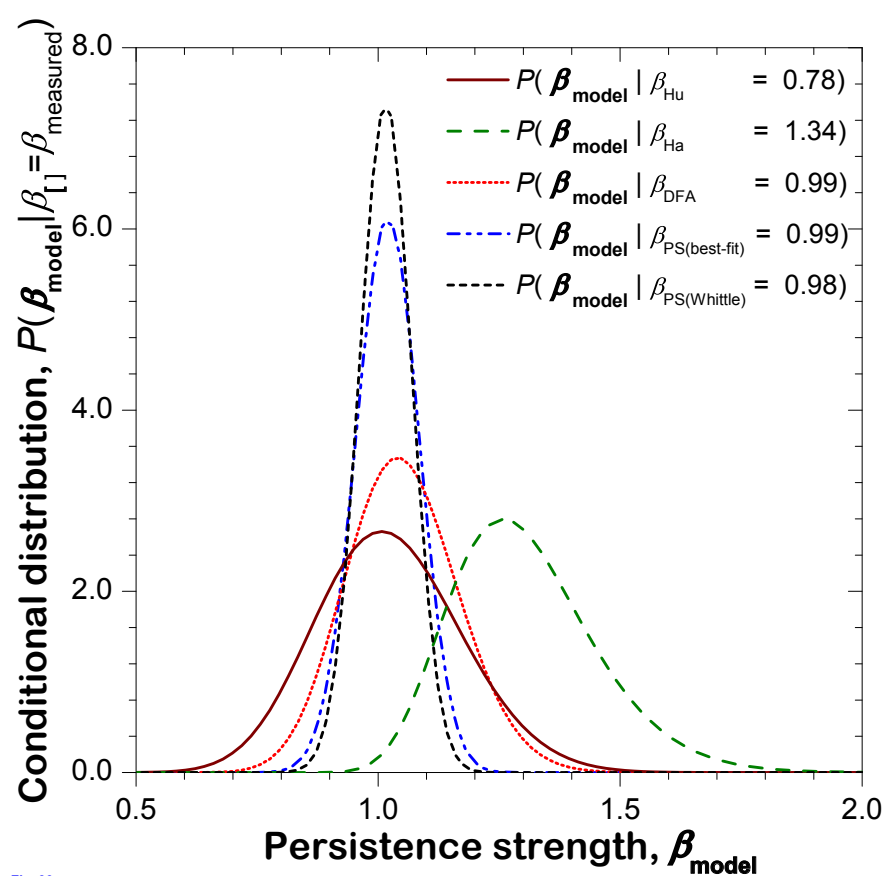


Fig. 36
Witt and Malamud (2013)

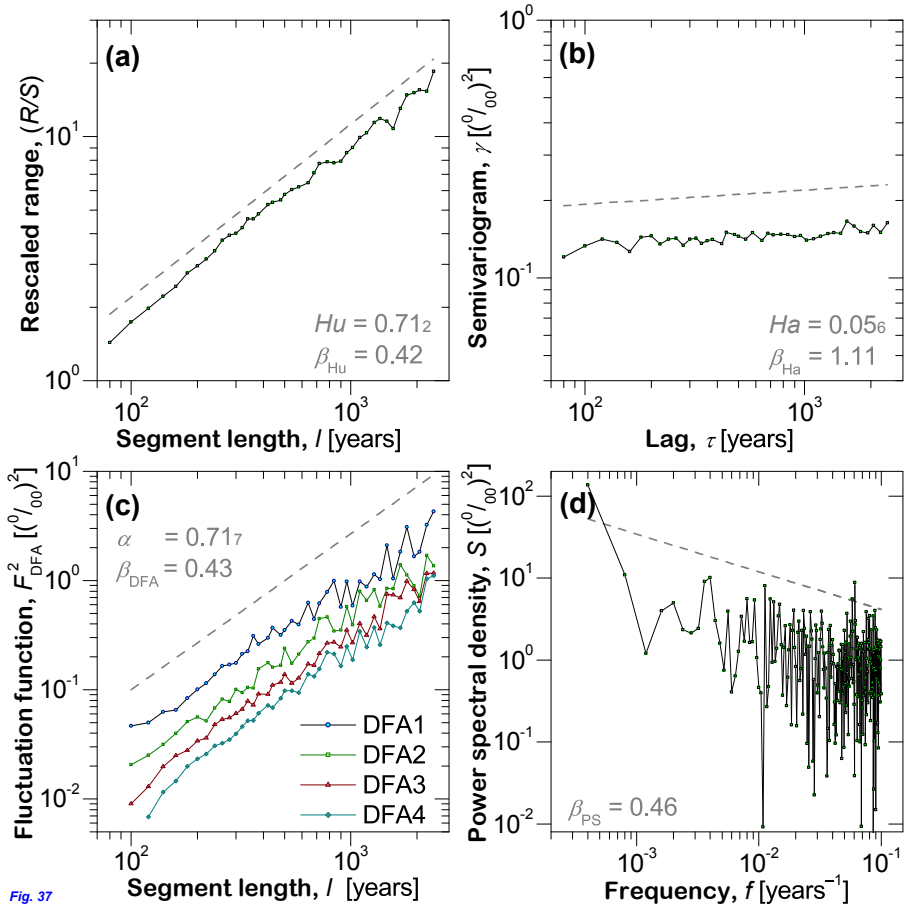


Fig. 37
Witt and Malamud (2013)

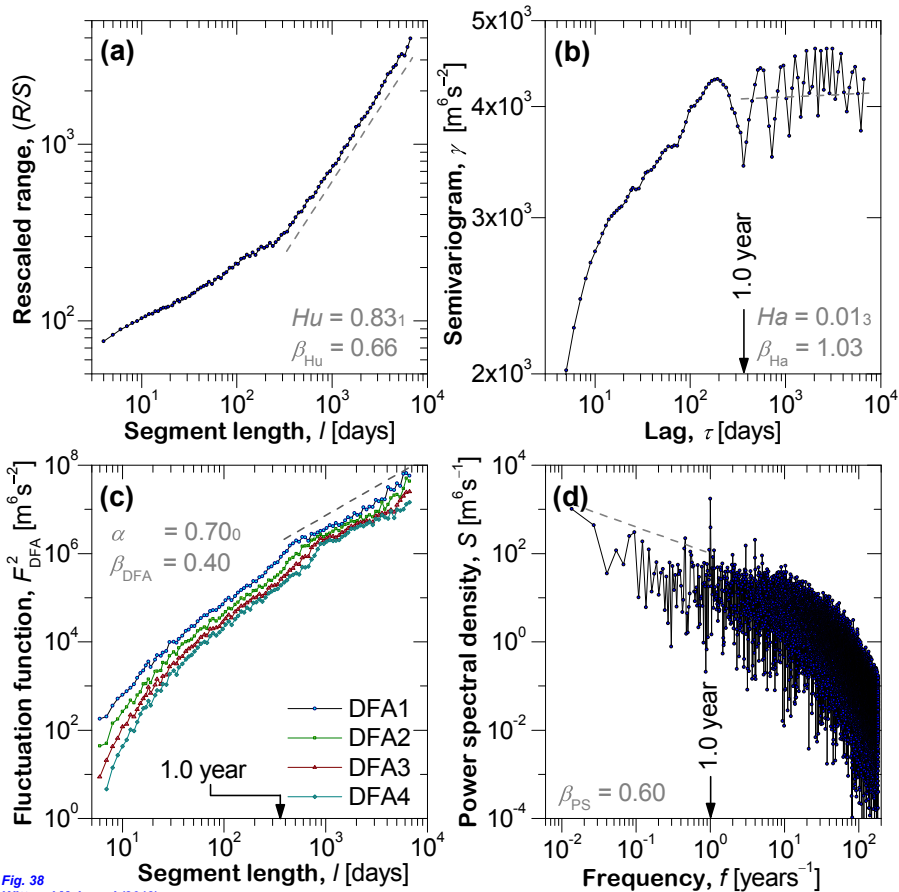


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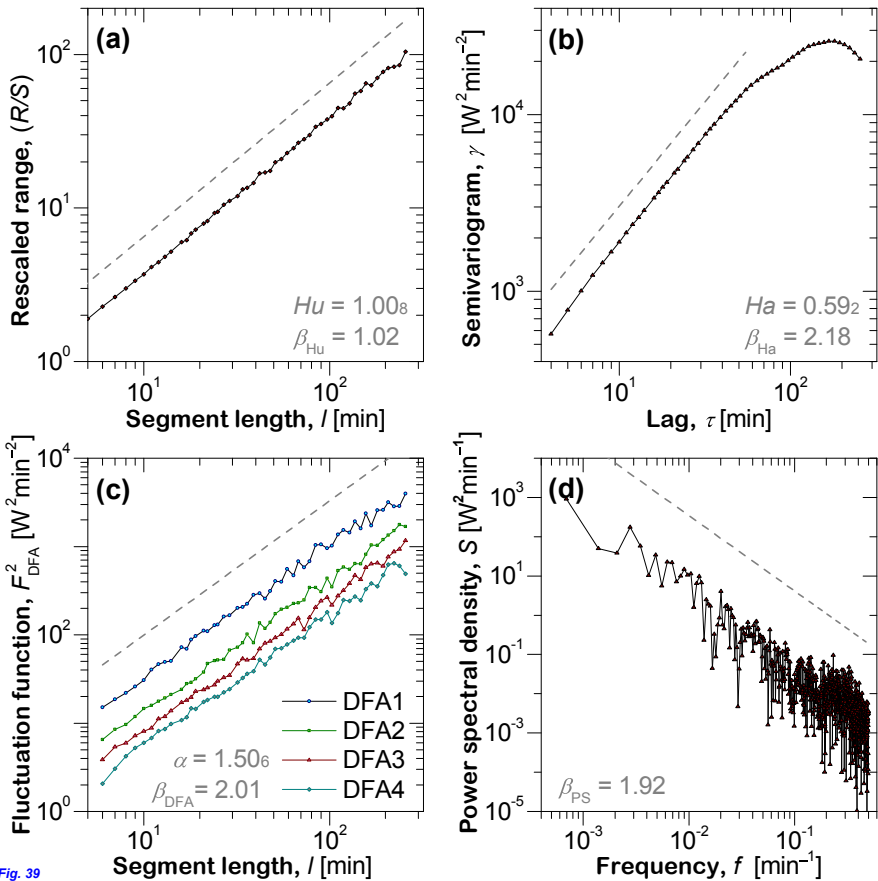


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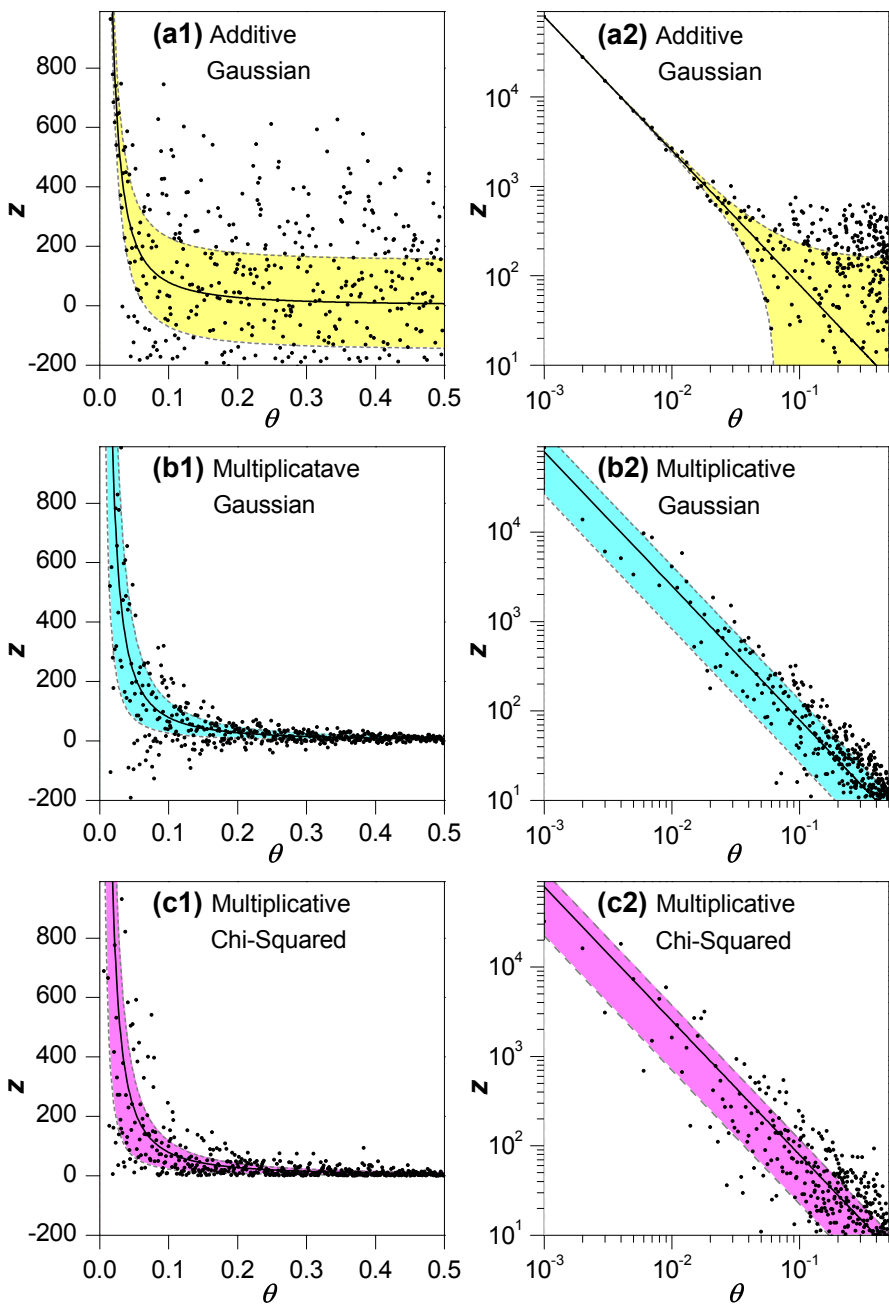


Fig. 40
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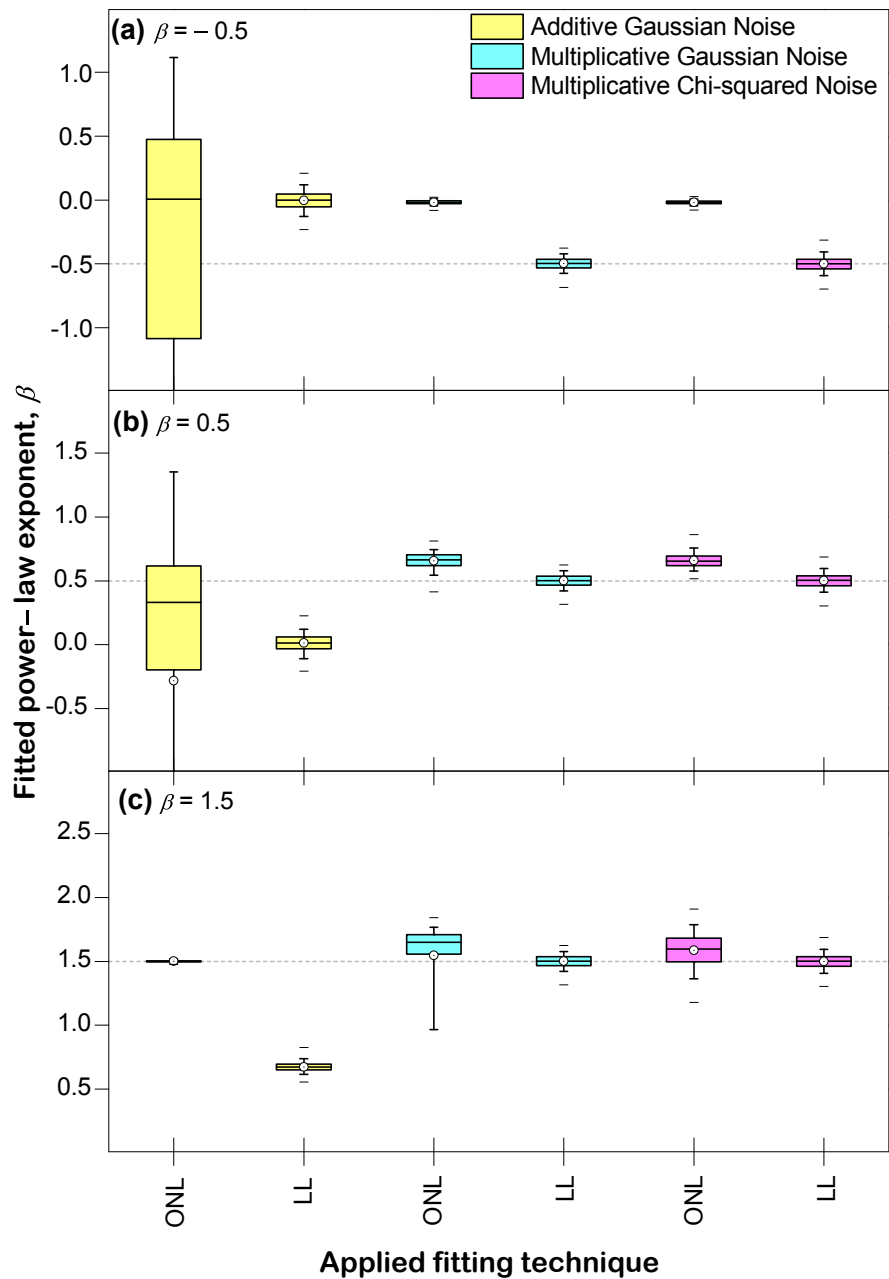


Fig. 41
Witt and Malamud (2013)