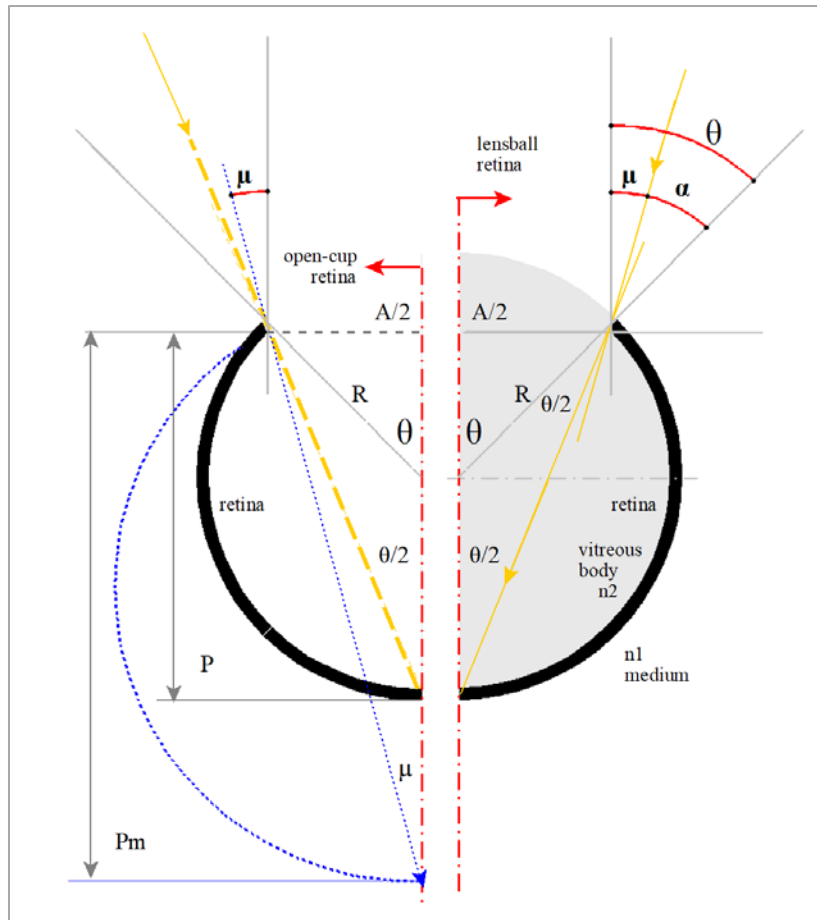


A possible origin of the inverted vertebrate retina revealed by physical modeling
Journal of Biological Physics, Jan M.M. Oomens, independent researcher
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Derivation of the modified cup depth to analyse the influence of a positive lens in the aperture of a retina cup on the maximum detectable spatial resolution.



The figure presents the cross-sections of two types of retina cup.

The left side is half the cross-section of an open-cup retina.

On the right is half of the cross-section of a lensball retina.

The radius (symbol: R) and the aperture diameter (symbol: A) is the same size for both retina cups.

The solid angle (symbol: $\theta/2$) is a measure of the amount of the field of view from the posterior pole of the open-cup retina.

The field of view from the posterior pole of the open-cup retina, with solid angle $\theta/2$, is bigger than the field of view from the posterior pole of the lensball retina cup, with solid angle μ , caused by refraction on the lensball ($n_2 > n_1$ then $\theta/2 > \mu$).

Symbol list

- α = The angle to the normal at which the photon must strike the incident point to hit the posterior pole
- θ = Polar angle of aperture rim
- μ = The angle between the optical axis and the trajectory of the incoming photon at angle α
- n_1 = Refractive index medium
- n_2 = Refractive index lensball
- P = Posterior nodal distance or cup depth
- P_m = Modified posterior nodal distance
- R = Radius of the retina cup
- $A/2$ = Aperture radius of the retina cup

The smaller field of view from the posterior pole of the lensball retina cup means a higher detectable spatial resolution.

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Derivation of the modified pit depth P_m to account for lens influence

Open pit retina cup: *pit depth* $P = R \cdot (1 + \cos \theta)$

Modified pit depth for the lensball retina cup:

$\theta = \mu + \alpha$; The relation between $\frac{\theta}{2}$ and α is given

by Snellius law: $\left(n_1 \cdot \sin \alpha = n_2 \cdot \sin \left(\frac{\theta}{2} \right) \right)$; $\sin \alpha = \frac{n_2}{n_1} \cdot \sin \left(\frac{\theta}{2} \right)$

$\alpha = \arcsin \left(\frac{n_2}{n_1} \cdot \sin \left(\frac{\theta}{2} \right) \right)$; $0 \leq \theta \leq \frac{\pi}{2}$

The distance from the aperture to the posterior nodal point in the modified open retina cup is the modified pit depth P_m .

$\tan \mu = \frac{A/2}{P_m} = \frac{R \cdot \sin \theta}{P_m}$; $P_m = R \cdot \frac{\sin \theta}{\tan \mu} = R \cdot \frac{\sin \theta}{\tan(\theta - \alpha)}$

$P_m = \frac{R \cdot \sin \theta}{\tan \left(\theta - \arcsin \left(\frac{n_2}{n_1} \cdot \sin \left(\frac{\theta}{2} \right) \right) \right)}$ equation 6

boundary condition: $\frac{n_2}{n_1} \cdot \sin \left(\frac{\theta}{2} \right) \in [-1, 1]$; θ in (rad)

The field of view from the posterior pole of the lensball retina cup, with solid angle μ , can be simulated in the open-cup retina by a modified cup size with a posterior nodal distance P_m ($n_2 > n_1$ then $P_m > P$).

Equation 6, modified aperture depth

The influence on the maximum detectable spatial frequency of a positive lens in the aperture in a cup shaped retina can be calculated by a parameter P_m representing a modified posterior nodal distance P .

Supersede parameter P in eqn 5 by P_m

$\nu_{\max} = 0.375 \cdot \left(\frac{P}{A} \right) \cdot \sqrt{\ln \left(0.746 \cdot A^2 \cdot \sqrt{I} \right)}$ equation 5

(Nilsson & Pelger 1994) to calculate the maximum detectable spatial frequency in a cup shaped retina with a positive lens in the aperture.

symbol list:

$\nu_{\max}$ = maximum detectable spatial frequency

A = diameter aperture

P = posterior nodal distance or cup depth

I = photons/nodal distance

The maximum spatial resolution of the lensball retina cup can be calculated by superseding P_m (equation 6) in equation 5 (Nilsson & Pelger 1994) while keeping the light entry opening constant.