

In this file we consider the vector field (P1,P2,P3) with given 6 idempotents and we will compute the 7th idempotent.

>

> restart;

> P1 := x1^2+(2*x1*x2*(-gamma[11]^2*gamma[21]*gamma[23]*gamma[32]*
gamma[33]+gamma[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma[33]+
gamma[11]*gamma[13]*gamma[21]^2*gamma[32]*gamma[33]-gamma[11]*
gamma[13]*gamma[22]*gamma[23]*gamma[31]^2-gamma[12]*gamma[13]*
gamma[21]^2*gamma[31]*gamma[33]+gamma[12]*gamma[13]*gamma[21]*
gamma[23]*gamma[31]^2-gamma[11]*gamma[13]*gamma[21]*gamma[32]*
gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]+gamma
[11]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[22]*
gamma[23]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma
[23]*gamma[31]+gamma[12]*gamma[13]*gamma[21]*gamma[31]*gamma[33])
-2*x2*x3*gamma[11]*gamma[21]*gamma[31]*(gamma[11]*gamma[22]*gamma
[33]-gamma[11]*gamma[23]*gamma[32]-gamma[12]*gamma[21]*gamma[33]+
gamma[12]*gamma[23]*gamma[31]+gamma[13]*gamma[21]*gamma[32]-gamma
[13]*gamma[22]*gamma[31]-gamma[12]*gamma[23]+gamma[12]*gamma[33]+
gamma[13]*gamma[22]-gamma[13]*gamma[32]-gamma[22]*gamma[33]+gamma
[23]*gamma[32])+2*x1*x3*(gamma[11]^2*gamma[21]*gamma[22]*gamma
[32]*gamma[33]-gamma[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma
[32]-gamma[11]*gamma[12]*gamma[21]^2*gamma[32]*gamma[33]+gamma
[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]^2+gamma[12]*gamma
[13]*gamma[21]^2*gamma[31]*gamma[32]-gamma[12]*gamma[13]*gamma
[21]*gamma[22]*gamma[31]^2+gamma[11]*gamma[12]*gamma[21]*gamma
[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]-
gamma[11]*gamma[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma
[22]*gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*
gamma[22]*gamma[31]-gamma[12]*gamma[13]*gamma[21]*gamma[31]*gamma
[32]))/(2*gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma
[33]-2*gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma
[33]-2*gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma
[33]+2*gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma
[32]+2*gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma
[33]-2*gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma
[32]):

>

> P2 := x2^2+(2*x1*x2*(gamma[11]*gamma[13]*gamma[22]^2*gamma[32]*
gamma[33]-gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[32]^2-
gamma[12]^2*gamma[21]*gamma[23]*gamma[32]*gamma[33]+gamma[12]^2*
gamma[22]*gamma[23]*gamma[31]*gamma[33]+gamma[12]*gamma[13]*gamma
[21]*gamma[23]*gamma[32]^2-gamma[12]*gamma[13]*gamma[22]^2*gamma
[31]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[32]-
gamma[11]*gamma[13]*gamma[22]*gamma[32]*gamma[33]-gamma[12]*gamma
[13]*gamma[21]*gamma[23]*gamma[32]+gamma[12]*gamma[13]*gamma[22]*
gamma[31]*gamma[33]+gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma
[33]-gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33])+2*x2*x3*
(-gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]^2+gamma[11]*
gamma[12]*gamma[22]^2*gamma[31]*gamma[33]+gamma[11]*gamma[13]*
gamma[21]*gamma[22]*gamma[32]^2-gamma[11]*gamma[13]*gamma[22]^2*
gamma[31]*gamma[32]-gamma[12]^2*gamma[21]*gamma[22]*gamma[31]*
gamma[33]+gamma[12]^2*gamma[21]*gamma[23]*gamma[31]*gamma[32]+

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gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]-gamma[11]*gamma
[12]*gamma[22]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*
gamma[22]*gamma[32]+gamma[11]*gamma[13]*gamma[22]*gamma[31]*gamma
[32]+gamma[12]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-gamma[12]*
gamma[21]*gamma[23]*gamma[31]*gamma[32])-2*x1*x3*gamma[12]*gamma
[22]*gamma[32]*(gamma[11]*gamma[22]*gamma[33]-gamma[11]*gamma[23]
*gamma[32]-gamma[12]*gamma[21]*gamma[33]+gamma[12]*gamma[23]*
gamma[31]+gamma[13]*gamma[21]*gamma[32]-gamma[13]*gamma[22]*gamma
[31]+gamma[11]*gamma[23]-gamma[11]*gamma[33]-gamma[13]*gamma[21]+
gamma[13]*gamma[31]+gamma[21]*gamma[33]-gamma[23]*gamma[31]))/(2*
gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-2*
gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33]-2*
gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]+2*
gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]+2*
gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-2*
gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32]):

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> P3 := collect((((x1*x2*(-gamma[21]*gamma[32]+gamma[22]*gamma[31])
*gamma[33]-gamma[31]*gamma[32]*x3*(-gamma[21]*x2+gamma[22]*x1))*
gamma[23]+gamma[33]*x3*gamma[21]*gamma[22]*(-gamma[31]*x2+gamma
[32]*x1))*gamma[13]^2+((-x1*x2*(-gamma[11]*gamma[32]+gamma[12]*
gamma[31])*gamma[33]+gamma[31]*gamma[32]*x3*(-gamma[11]*x2+gamma
[12]*x1))*gamma[23]^2+(-x1*x2*(gamma[11]*gamma[22]-gamma[12]*
gamma[21])*gamma[33]^2+x1*x2*(gamma[22]-gamma[32])*gamma[11]+
(gamma[31]-gamma[21])*gamma[12]-gamma[22]*gamma[31]+gamma[21]*
gamma[32])*gamma[33]-gamma[31]*gamma[32]*x3*(-gamma[11]*gamma
[22]+gamma[12]*gamma[21])*x3+gamma[12]*x1-gamma[22]*x1-gamma[11]*
x2+gamma[21]*x2))*gamma[23]-gamma[33]*x3*gamma[22]*((-gamma[11]*
x2+gamma[12]*x1)*gamma[33]+(gamma[11]*gamma[32]-gamma[12]*gamma
[31])*x3+gamma[32]*x1-gamma[12]*x1-gamma[31]*x2+gamma[11]*x2)*
gamma[21])*gamma[13]-gamma[33]*x3*(-gamma[31]*x2+gamma[32]*x1)*
gamma[23]+(gamma[21]*x2-gamma[22]*x1)*gamma[33]+(-gamma[21]*gamma
[32]+gamma[22]*gamma[31])*x3-gamma[32]*x1+gamma[22]*x1+gamma[31]*
x2-gamma[21]*x2)*gamma[23]*gamma[11]*gamma[12])/((gamma[31]*gamma
[32]*(gamma[11]*gamma[22]-gamma[12]*gamma[21])*gamma[23]+gamma
[33]*gamma[21]*gamma[22]*(-gamma[11]*gamma[32]+gamma[12]*gamma
[31]))*gamma[13]-gamma[33]*gamma[11]*gamma[12]*gamma[23]*(-gamma
[21]*gamma[32]+gamma[22]*gamma[31])) , x3, factor):

```

We are going to rewrite our system into the form

$$P1-x1=x1^2+A1(x2,x3)x1+A2(x2,x3)$$

$$P2-x2=B1(x2,x3)x1+B2(x2,x3)$$

$$P3-x3=C1(x2,x3)x1+C2(x2,x3)$$

with

```

> A1:=a11*x2+a12*x3-1:
  A2:=a13*x2*x3:
  B1:=b11*x2+b12*x3:
  B2:=x2^2-x2+b13*x2*x3:
  C1:=c11*x2+c12*x3:

```

```
C2:=x3^2-x3+c13*x2*x3:
```

We are looking the seventh idempotent, so we are looking for $p(x)-x=0$, with $x=(x_1, x_2, x_3)$. So we have the following equations

$$x_1^2 + A_1 x_1 + A_2 = 0$$

$B_1 x_1 + B_2 = 0 \rightarrow$ From this equation we have that $x_1 = -B_2/B_1$. This is the first component of the idempotent that we are looking for

$C_1 x_1 + C_2 = 0 \rightarrow$ We could also use this equation to obtain $x_1 = -C_2/C_1$.

Substituting $x_1 = -B_2/B_1$ to the other two equations we have

```
> eq1:=B1*C2-C1*B2:simplify(%):
eq2:=B2^2-A1*B1*B2+A2*B1^2:simplify(%):
```

Now in order to solve the system of the two equations $\{eq1, eq2\}$ we use the resultant.

```
>
> with(Groebner):
>
```

The following resultant is a polynomial of degree 12 in the variable x_3 . We divide the resultant $resx_2$ by some factors. the remaining polynomial r_2 is of degree 4 in the variable x_3 and we divide by the leading coefficient in order to obtain a monic polynomial.

```
> resx2:=factor(resultant(eq1,eq2,x2)):
r2:=resx2/(x3^5*(x3-1)*b12^2*(b11*b13*x3-b12*x3-b11)^2):r2coef:=
coeff(r2,x3,4):
rr2:=r2/r2coef:
coeff(rr2,x3,4):
x33:=coeff(rr2,x3,3):
```

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(1)

Note that for this monic polynomial of degree 4 the sum of the four roots is the negative trace of the polynomial. But the negative trace is the coefficient of x^3 . Additionally we know the three roots and we just need the fourth one, let's say s_3 . So we have a relation of the form $\gamma_{13} + \gamma_{23} + \gamma_{33} + s_3 = -\text{trace}$.

Now we repeat the same steps but for the resultant with respect the variable x_2

```
> res3x3:=factor(resultant(eq1,eq2,x3))/(x2^5*(x2-1)*b12*(b11*b13*
x2-b12*x2+b12)^2):
r3coef:=coeff(res3x3,x2,4):
rr3:=res3x3/r3coef:
coeff(rr3,x2,4):
x22:=coeff(rr3,x2,3):
```

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(2)

Similarly we calculate the fourth root, let's say s_2 from an equation of the form $\gamma_{12} + \gamma_{22} + \gamma_{32} + s_2 = -\text{trace}$.



Before we carry out these calculations we express the coefficients of A1,A2,B1,B2,C1,C2 in terms of the gamma[ij] by comparing terms of P1,P2,P3 with the ones using A1,A2,B1,B2,C1,C2.

First consider the equation of the first component of the vector field P1 without the term of x^2

```
> eqP1 := (gamma[11]^2*gamma[21]*gamma[22]*gamma[32]*gamma[33]*x3-
gamma[11]^2*gamma[21]*gamma[23]*gamma[32]*gamma[33]*x2-gamma[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma[32]*x3+gamma[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma[33]*x2-gamma[11]*gamma[12]*gamma[21]^2*gamma[32]*gamma[33]*x3+gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]^2*x3+gamma[11]*gamma[13]*gamma[21]^2*gamma[32]*gamma[33]*x2-gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]^2*x2+gamma[12]*gamma[13]*gamma[21]^2*gamma[31]*gamma[32]*x3-gamma[12]*gamma[13]*gamma[21]^2*gamma[31]*gamma[33]*x2-gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]^2*x3+gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]^2*x2+gamma[11]*gamma[12]*gamma[21]*gamma[32]*gamma[33]*x3-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*x3-gamma[11]*gamma[13]*gamma[21]*gamma[32]*gamma[33]*x2+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*x2-gamma[11]*gamma[21]*gamma[22]*gamma[32]*gamma[33]*x3+gamma[11]*gamma[21]*gamma[23]*gamma[32]*gamma[33]*x2+gamma[11]*gamma[22]*gamma[23]*gamma[31]*gamma[32]*x3-gamma[11]*gamma[22]*gamma[23]*gamma[31]*gamma[33]*x2+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*x3-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*x2-gamma[12]*gamma[13]*gamma[21]*gamma[31]*gamma[32]*x3+gamma[12]*gamma[13]*gamma[21]*gamma[31]*gamma[33]*x2)*x1/(gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32]) -x2*x3*gamma[11]*gamma[21]*gamma[31]*(gamma[11]*gamma[22]*gamma[33]-gamma[11]*gamma[23]*gamma[32]-gamma[12]*gamma[21]*gamma[33]+gamma[12]*gamma[23]*gamma[31]+gamma[13]*gamma[21]*gamma[32]-gamma[13]*gamma[22]*gamma[31]-gamma[12]*gamma[23]+gamma[12]*gamma[33]+gamma[13]*gamma[22]-gamma[13]*gamma[32]-gamma[22]*gamma[33]+gamma[23]*gamma[32])/ (gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32]) )
```

$$eqP1 := \left(\left(-x^2 \gamma_{11}^2 \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} + x^2 \gamma_{11}^2 \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} + x^2 \gamma_{11} \gamma_{13} \gamma_{21}^2 \gamma_{32} \gamma_{33} - x^2 \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31}^2 - x^2 \gamma_{12} \gamma_{13} \gamma_{21}^2 \gamma_{31} \gamma_{33} + x^2 \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31}^2 + x^3 \gamma_{11}^2 \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} - x^3 \gamma_{11}^2 \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} - x^3 \gamma_{11} \gamma_{12} \gamma_{21}^2 \gamma_{32} \gamma_{33} + x^3 \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31}^2 + x^3 \gamma_{12} \gamma_{13} \gamma_{21}^2 \gamma_{31} \gamma_{32} - x^3 \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31}^2 - x^2 \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{32} \gamma_{33} + x^2 \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} + x^2 \gamma_{11} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} - x^2 \gamma_{11} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} \right) \right)$$

$$eqPlx10 := - \left(\gamma_{11} \gamma_{21} \gamma_{31} \left(\gamma_{11} \gamma_{22} \gamma_{33} - \gamma_{11} \gamma_{23} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{33} + \gamma_{12} \gamma_{23} \gamma_{31} + \gamma_{13} \gamma_{21} \gamma_{32} \right. \right. \\ \left. \left. - \gamma_{13} \gamma_{22} \gamma_{31} - \gamma_{12} \gamma_{23} + \gamma_{12} \gamma_{33} + \gamma_{13} \gamma_{22} - \gamma_{13} \gamma_{32} - \gamma_{22} \gamma_{33} + \gamma_{23} \gamma_{32} \right) \right) / \\ \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} \right. \\ \left. + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right)$$

a13 x2 x3

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```
> a13=factor(-gamma[11]*gamma[21]*gamma[31]*(gamma[11]*gamma[22]*
gamma[33]-gamma[11]*gamma[23]*gamma[32]-gamma[12]*gamma[21]*gamma
[33]+gamma[12]*gamma[23]*gamma[31]+gamma[13]*gamma[21]*gamma[32]-
gamma[13]*gamma[22]*gamma[31]-gamma[12]*gamma[23]+gamma[12]*gamma
[33]+gamma[13]*gamma[22]-gamma[13]*gamma[32]-gamma[22]*gamma[33]+
gamma[23]*gamma[32])/(gamma[11]*gamma[12]*gamma[21]*gamma[23]*
gamma[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma
[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*
gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma
[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-
gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32])) ;
```

$$a13 = - \left(\gamma_{11} \gamma_{21} \gamma_{31} \left(\gamma_{11} \gamma_{22} \gamma_{33} - \gamma_{11} \gamma_{23} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{33} + \gamma_{12} \gamma_{23} \gamma_{31} + \gamma_{13} \gamma_{21} \gamma_{32} - \gamma_{13} \gamma_{22} \gamma_{31} \right. \right. \\ \left. \left. - \gamma_{12} \gamma_{23} + \gamma_{12} \gamma_{33} + \gamma_{13} \gamma_{22} - \gamma_{13} \gamma_{32} - \gamma_{22} \gamma_{33} + \gamma_{23} \gamma_{32} \right) \right) / \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} \right. \\ \left. - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \right. \\ \left. - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right)$$

(7)

```
> eqP2:=(2*x1*x2*(gamma[11]*gamma[13]*gamma[22]^2*gamma[32]*gamma
[33]-gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[32]^2-gamma
[12]^2*gamma[21]*gamma[23]*gamma[32]*gamma[33]+gamma[12]^2*gamma
[22]*gamma[23]*gamma[31]*gamma[33]+gamma[12]*gamma[13]*gamma[21]*
gamma[23]*gamma[32]^2-gamma[12]*gamma[13]*gamma[22]^2*gamma[31]*
gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[32]-gamma
[11]*gamma[13]*gamma[22]*gamma[32]*gamma[33]-gamma[12]*gamma[13]*
gamma[21]*gamma[23]*gamma[32]+gamma[12]*gamma[13]*gamma[22]*gamma
[31]*gamma[33]+gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-
gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33])+2*x2*x3*(-
gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]^2+gamma[11]*
gamma[12]*gamma[22]^2*gamma[31]*gamma[33]+gamma[11]*gamma[13]*
gamma[21]*gamma[22]*gamma[32]^2-gamma[11]*gamma[13]*gamma[22]^2*
gamma[31]*gamma[32]-gamma[12]^2*gamma[21]*gamma[22]*gamma[31]*
gamma[33]+gamma[12]^2*gamma[21]*gamma[23]*gamma[31]*gamma[32]+
gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]-gamma[11]*gamma
[12]*gamma[22]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*
gamma[22]*gamma[32]+gamma[11]*gamma[13]*gamma[22]*gamma[31]*gamma
[32]+gamma[12]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-gamma[12]*
gamma[21]*gamma[23]*gamma[31]*gamma[32])-2*x1*x3*gamma[12]*gamma
[22]*gamma[32]*(gamma[11]*gamma[22]*gamma[33]-gamma[11]*gamma[23]*
gamma[32]-gamma[12]*gamma[21]*gamma[33]+gamma[12]*gamma[23]*
gamma[31]+gamma[13]*gamma[21]*gamma[32]-gamma[13]*gamma[22]*gamma
[31]+gamma[11]*gamma[23]-gamma[11]*gamma[33]-gamma[13]*gamma[21]+
gamma[13]*gamma[31]+gamma[21]*gamma[33]-gamma[23]*gamma[31]))/(2*
gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-2*
gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33]-2*
gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]+2*
gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]+2*
```


$$\begin{aligned}
& -\gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \\
& - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32})
\end{aligned}$$

> b13:=factor(coeff(eqP2,x1,0)/(x2*x3));B2;

$$\begin{aligned}
b13 := & - \left(\gamma_{11}^2 \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32}^2 - \gamma_{11}^2 \gamma_{12} \gamma_{22}^2 \gamma_{31} \gamma_{33} - \gamma_{11}^2 \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32}^2 + \gamma_{11}^2 \gamma_{13} \gamma_{22}^2 \gamma_{31} \gamma_{32} + \right. \\
& \gamma_{12}^2 \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12}^2 \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} - \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} + \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \\
& \left. - \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{31} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) / \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} \right. \\
& - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \\
& \left. - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) \\
& - \left(\left(\gamma_{11}^2 \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32}^2 - \gamma_{11}^2 \gamma_{12} \gamma_{22}^2 \gamma_{31} \gamma_{33} - \gamma_{11}^2 \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32}^2 + \gamma_{11}^2 \gamma_{13} \gamma_{22}^2 \gamma_{31} \gamma_{32} + \right. \right. \\
& \gamma_{12}^2 \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12}^2 \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} - \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} + \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \\
& \left. - \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{31} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) x^2 x^3 \Big/ \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} \right. \\
& - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \\
& \left. - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) + x^2 - x^2
\end{aligned}$$

(11)

> P3;

$$\begin{aligned}
& x^3 + \left(\left(xI \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{33}^2 - xI \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{32} \gamma_{33} - xI \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33}^2 + xI \gamma_{12} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} \right. \right. \\
& + xI \gamma_{13}^2 \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} - xI \gamma_{13}^2 \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} - x^2 \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{33}^2 + x^2 \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{31} \gamma_{33} \\
& + x^2 \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33}^2 - x^2 \gamma_{11} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} - x^2 \gamma_{13}^2 \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} + x^2 \gamma_{13}^2 \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \\
& - xI \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{33} + xI \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{32} \gamma_{33} + xI \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33} - xI \gamma_{12} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} \\
& - xI \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + xI \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + x^2 \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{33} - x^2 \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{31} \gamma_{33} \\
& - x^2 \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33} + x^2 \gamma_{11} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} + x^2 \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - x^2 \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \Big) \\
& x^3 \Big/ \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} \right. \\
& + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \Big) \\
& - \left(\gamma_{13} \gamma_{23} xI x^2 \gamma_{33} \left(\gamma_{11} \gamma_{22} \gamma_{33} - \gamma_{11} \gamma_{23} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{33} + \gamma_{12} \gamma_{23} \gamma_{31} + \gamma_{13} \gamma_{21} \gamma_{32} - \gamma_{13} \gamma_{22} \gamma_{31} - \gamma_{11} \gamma_{22} + \gamma_{11} \gamma_{23} \right. \right. \\
& \left. \left. + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) \right)
\end{aligned}$$

> eqP3:=- (gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[33]^2*x2-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[33]^2*x1-gamma[11]*gamma[12]*gamma[23]^2*gamma[31]*gamma[33]*x2+gamma[11]*gamma[12]*gamma[23]^2*gamma[32]*gamma[33]*x1-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[33]^2*x2+gamma[11]*gamma[13]*gamma[23]^2*gamma[31]*gamma[32]*x2+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[33]^2*x1-gamma[12]*gamma[13]*gamma[23]^2*gamma[31]*gamma[32]*x1+gamma[13]^2*gamma[21]*gamma[22]*gamma[31]*gamma[33]*x2-gamma[13]^2*gamma[21]*gamma[22]*gamma[32]*gamma[33]*x1-gamma[13]^2*gamma

```

[21]*gamma[23]*gamma[31]*gamma[32]*x2+gamma[13]^2*gamma[22]*gamma
[23]*gamma[31]*gamma[32]*x1-gamma[11]*gamma[12]*gamma[21]*gamma
[23]*gamma[33]*x2+gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma
[33]*x1+gamma[11]*gamma[12]*gamma[23]*gamma[31]*gamma[33]*x2-
gamma[11]*gamma[12]*gamma[23]*gamma[32]*gamma[33]*x1+gamma[11]*
gamma[13]*gamma[21]*gamma[22]*gamma[33]*x2-gamma[11]*gamma[13]*
gamma[23]*gamma[31]*gamma[32]*x2-gamma[12]*gamma[13]*gamma[21]*
gamma[22]*gamma[33]*x1+gamma[12]*gamma[13]*gamma[23]*gamma[31]*
gamma[32]*x1-gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]*
x2+gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]*x1+gamma[13]
*gamma[21]*gamma[23]*gamma[31]*gamma[32]*x2-gamma[13]*gamma[22]*
gamma[23]*gamma[31]*gamma[32]*x1)*x3/(gamma[11]*gamma[12]*gamma
[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*
gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma
[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*
gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma
[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*
gamma[32])-gamma[13]*gamma[23]*x1*x2*gamma[33]*(gamma[11]*gamma
[22]*gamma[33]-gamma[11]*gamma[23]*gamma[32]-gamma[12]*gamma[21]*
gamma[33]+gamma[12]*gamma[23]*gamma[31]+gamma[13]*gamma[21]*gamma
[32]-gamma[13]*gamma[22]*gamma[31]-gamma[11]*gamma[22]+gamma[11]*
gamma[32]+gamma[12]*gamma[21]-gamma[12]*gamma[31]-gamma[21]*gamma
[32]+gamma[22]*gamma[31]))/(gamma[11]*gamma[12]*gamma[21]*gamma
[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*
gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma
[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*
gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma
[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32])
;

```

$$\begin{aligned}
 eqP3 := & - \left(\left(-xI \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{33}^2 + xI \gamma_{11} \gamma_{12} \gamma_{23}^2 \gamma_{32} \gamma_{33} + xI \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33}^2 - xI \gamma_{12} \gamma_{13} \right. \right. \\
 & \gamma_{23}^2 \gamma_{31} \gamma_{32} - xI \gamma_{13}^2 \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + xI \gamma_{13}^2 \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + x^2 \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{33}^2 - x^2 \gamma_{11} \gamma_{12} \\
 & \gamma_{23}^2 \gamma_{31} \gamma_{33} - x^2 \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33}^2 + x^2 \gamma_{11} \gamma_{13} \gamma_{23}^2 \gamma_{31} \gamma_{32} + x^2 \gamma_{13}^2 \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - x^2 \\
 & \gamma_{13}^2 \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} + xI \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{33} - xI \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{32} \gamma_{33} - xI \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33} \\
 & + xI \gamma_{12} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} + xI \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} - xI \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} - x^2 \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{33} \\
 & + x^2 \gamma_{11} \gamma_{12} \gamma_{23} \gamma_{31} \gamma_{33} + x^2 \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{33} - x^2 \gamma_{11} \gamma_{13} \gamma_{23} \gamma_{31} \gamma_{32} - x^2 \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \\
 & \left. \left. + x^2 \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) x^3 \right) / \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} \right. \\
 & - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \\
 & - \left(\gamma_{13} \gamma_{23} xI x^2 \gamma_{33} \left(\gamma_{11} \gamma_{22} \gamma_{33} - \gamma_{11} \gamma_{23} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{33} + \gamma_{12} \gamma_{23} \gamma_{31} + \gamma_{13} \gamma_{21} \gamma_{32} \right. \right. \\
 & \left. \left. - \gamma_{13} \gamma_{22} \gamma_{31} - \gamma_{11} \gamma_{22} + \gamma_{11} \gamma_{32} + \gamma_{12} \gamma_{21} - \gamma_{12} \gamma_{31} - \gamma_{21} \gamma_{32} + \gamma_{22} \gamma_{31} \right) \right) / \\
 & \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} \right. \\
 & \left. + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right)
 \end{aligned} \tag{13}$$

```

> eqP3x1:=coeff(eqP3,x1,1);C1;

```


$$\begin{aligned}
& - \left(\left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32}^2 - \gamma_{11} \gamma_{12} \gamma_{22}^2 \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32}^2 + \gamma_{11} \gamma_{13} \gamma_{22}^2 \gamma_{31} \gamma_{32} + \right. \right. \\
& \quad \gamma_{12}^2 \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} - \gamma_{12}^2 \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} - \gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} + \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \\
& \quad \left. - \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{31} \gamma_{32} - \gamma_{12} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} + \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) x^2 x^3 \Big/ \left(\gamma_{11} \gamma_{12} \gamma_{21} \gamma_{23} \gamma_{32} \gamma_{33} \right. \\
& \quad \left. - \gamma_{11} \gamma_{12} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{33} - \gamma_{11} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{32} \gamma_{33} + \gamma_{11} \gamma_{13} \gamma_{22} \gamma_{23} \gamma_{31} \gamma_{32} + \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{22} \gamma_{31} \gamma_{33} \right. \\
& \quad \left. - \gamma_{12} \gamma_{13} \gamma_{21} \gamma_{23} \gamma_{31} \gamma_{32} \right) + x^2^2 - x^2
\end{aligned} \tag{18}$$

> C1;C2;

$$\begin{aligned}
& c11 x^2 + c12 x^3 \\
& c13 x^2 x^3 + x^3^2 - x^3
\end{aligned}$$

(19)

So we have calculated all the coefficients of A1,A2,B1,B2,C1,C2:

```

> sol:={
a11 = (-gamma[11]^2*gamma[21]*gamma[23]*gamma[32]*gamma[33]+gamma
[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma[33]+gamma[11]*gamma
[13]*gamma[21]^2*gamma[32]*gamma[33]-gamma[11]*gamma[13]*gamma
[22]*gamma[23]*gamma[31]^2-gamma[12]*gamma[13]*gamma[21]^2*gamma
[31]*gamma[33]+gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]
^2-gamma[11]*gamma[13]*gamma[21]*gamma[32]*gamma[33]+gamma[11]*
gamma[13]*gamma[22]*gamma[23]*gamma[31]+gamma[11]*gamma[21]*gamma
[23]*gamma[32]*gamma[33]-gamma[11]*gamma[22]*gamma[23]*gamma[31]*
gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]+gamma
[12]*gamma[13]*gamma[21]*gamma[31]*gamma[33])/(gamma[11]*gamma
[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*
gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma
[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*
gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma
[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*
gamma[31]*gamma[32]),

a12 = (gamma[11]^2*gamma[21]*gamma[22]*gamma[32]*gamma[33]-gamma
[11]^2*gamma[22]*gamma[23]*gamma[31]*gamma[32]-gamma[11]*gamma
[12]*gamma[21]^2*gamma[32]*gamma[33]+gamma[11]*gamma[12]*gamma
[22]*gamma[23]*gamma[31]^2+gamma[12]*gamma[13]*gamma[21]^2*gamma
[31]*gamma[32]-gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]
^2+gamma[11]*gamma[12]*gamma[21]*gamma[32]*gamma[33]-gamma[11]*
gamma[12]*gamma[22]*gamma[23]*gamma[31]-gamma[11]*gamma[21]*gamma
[22]*gamma[32]*gamma[33]+gamma[11]*gamma[22]*gamma[23]*gamma[31]*
gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]-gamma
[12]*gamma[13]*gamma[21]*gamma[31]*gamma[32])/(gamma[11]*gamma
[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*
gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma
[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*
gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma
[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*
gamma[31]*gamma[32]),

a13 = -gamma[11]*gamma[21]*gamma[31]*(gamma[11]*gamma[22]*gamma
[33]-gamma[11]*gamma[23]*gamma[32]-gamma[12]*gamma[21]*gamma[33]+
gamma[12]*gamma[23]*gamma[31]+gamma[13]*gamma[21]*gamma[32]-gamma
[13]*gamma[22]*gamma[31]-gamma[12]*gamma[23]+gamma[12]*gamma[33]+
gamma[13]*gamma[22]-gamma[13]*gamma[32]-gamma[22]*gamma[33]+gamma
[23]*gamma[32])/(gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma

```



```
[22]*gamma[31]) / (gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma
[32]*gamma[33]-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[31]*
gamma[33]-gamma[11]*gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma
[33]+gamma[11]*gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]+
gamma[12]*gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]-gamma
[12]*gamma[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32]) ,
```

```
c12 = -(-gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[33]^2+
gamma[11]*gamma[12]*gamma[23]^2*gamma[32]*gamma[33]+gamma[12]*
gamma[13]*gamma[21]*gamma[22]*gamma[33]^2-gamma[12]*gamma[13]*
gamma[23]^2*gamma[31]*gamma[32]-gamma[13]^2*gamma[21]*gamma[22]*
gamma[32]*gamma[33]+gamma[13]^2*gamma[22]*gamma[23]*gamma[31]*
gamma[32]+gamma[11]*gamma[12]*gamma[22]*gamma[23]*gamma[33]-gamma
[11]*gamma[12]*gamma[23]*gamma[32]*gamma[33]-gamma[12]*gamma[13]*
gamma[21]*gamma[22]*gamma[33]+gamma[12]*gamma[13]*gamma[23]*gamma
[31]*gamma[32]+gamma[13]*gamma[21]*gamma[22]*gamma[32]*gamma[33]-
gamma[13]*gamma[22]*gamma[23]*gamma[31]*gamma[32]) / (gamma[11]*
gamma[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma
[12]*gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*
gamma[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma
[22]*gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*
gamma[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma
[23]*gamma[31]*gamma[32]) ,
```

```
c13 = -(gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[33]^2-gamma
[11]*gamma[12]*gamma[23]^2*gamma[31]*gamma[33]-gamma[11]*gamma
[13]*gamma[21]*gamma[22]*gamma[33]^2+gamma[11]*gamma[13]*gamma
[23]^2*gamma[31]*gamma[32]+gamma[13]^2*gamma[21]*gamma[22]*gamma
[31]*gamma[33]-gamma[13]^2*gamma[21]*gamma[23]*gamma[31]*gamma
[32]-gamma[11]*gamma[12]*gamma[21]*gamma[23]*gamma[33]+gamma[11]*
gamma[12]*gamma[23]*gamma[31]*gamma[33]+gamma[11]*gamma[13]*gamma
[21]*gamma[22]*gamma[33]-gamma[11]*gamma[13]*gamma[23]*gamma[31]*
gamma[32]-gamma[13]*gamma[21]*gamma[22]*gamma[31]*gamma[33]+gamma
[13]*gamma[21]*gamma[23]*gamma[31]*gamma[32]) / (gamma[11]*gamma
[12]*gamma[21]*gamma[23]*gamma[32]*gamma[33]-gamma[11]*gamma[12]*
gamma[22]*gamma[23]*gamma[31]*gamma[33]-gamma[11]*gamma[13]*gamma
[21]*gamma[22]*gamma[32]*gamma[33]+gamma[11]*gamma[13]*gamma[22]*
gamma[23]*gamma[31]*gamma[32]+gamma[12]*gamma[13]*gamma[21]*gamma
[22]*gamma[31]*gamma[33]-gamma[12]*gamma[13]*gamma[21]*gamma[23]*
gamma[31]*gamma[32])
```

```
} :
```

```
%%%%%%%%%
%%%%%%%%%
%%%%%%%%%
%%%%%%%%%
%%%%%%%%%
%%%%%%%%%
```

```
>
```

Now we are ready to compute $s_2 = x_{22} - \text{gamma}[12] - \text{gamma}[22] - \text{gamma}[32]$ and $s_3 = x_{33} - \text{gamma}[13] - \text{gamma}[23] - \text{gamma}[33]$.

In order to facilitate the calculations and keep the output manageable we fix some parameters for which we obtain the following vector field. There is no basic problem with choosing different parameters or keeping the parameters arbitrary, except for running time and output size.

$$\begin{aligned} &> \text{parameters} := \{\text{gamma}[11] = \text{gamma}[11], \text{gamma}[12] = 1, \text{gamma}[13] = 2, \text{gamma}[21] = 1, \text{gamma}[22] = 1, \text{gamma}[23] = 1, \text{gamma}[31] = 0, \text{gamma}[32] = 3, \text{gamma}[33] = 2\}; \\ &\text{parameters} := \{\gamma_{11} = \gamma_{11}, \gamma_{12} = 1, \gamma_{13} = 2, \gamma_{21} = 1, \gamma_{22} = 1, \gamma_{23} = 1, \gamma_{31} = 0, \gamma_{32} = 3, \gamma_{33} = 2\} \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{simplify}(\text{subs}(\text{parameters}, [\text{P1}, \text{P2}, \text{P3}])) ; \\ &\left[x1 \left((x2 - x3) \gamma_{11} + x1 - x2 + x3 \right), \frac{((2x2 - x3)x1 + x2(x2 - x3)) \gamma_{11} - 2x1(x2 - x3)}{\gamma_{11}}, \right. \\ &\quad \left. \frac{1}{3} \frac{((2x2 - x3)x1 - x3(x2 - 3x3)) \gamma_{11} + 4x1(x2 - x3)}{\gamma_{11}} \right] \end{aligned} \quad (21)$$

For these parameters the coefficients of A1,A2,B1,B2,C1,C2 become

$$\begin{aligned} &> \text{soltest} := \text{simplify}(\text{subs}(\text{parameters}, \text{sol})) ; \\ &\text{soltest} := \left\{ -1 = -1, a11 = \gamma_{11} - 1, a12 = -\gamma_{11} + 1, a13 = 0, b11 = \frac{2\gamma_{11} - 2}{\gamma_{11}}, b12 \right. \\ &\quad \left. = \frac{-\gamma_{11} + 2}{\gamma_{11}}, c11 = \frac{1}{3} \frac{2\gamma_{11} + 4}{\gamma_{11}}, c12 = \frac{1}{3} \frac{-\gamma_{11} - 4}{\gamma_{11}}, c13 = -\frac{1}{3} \right\} \end{aligned} \quad (22)$$

The trace of the resultant for these parameters that we fixed becomes

$$> \text{x222test} := \text{simplify}(\text{subs}(\text{soltest}, \text{x22})) :$$

and so we calculate the second component s22 of the idempotent.

$$\begin{aligned} &> \text{solve}(\text{gamma}[12] + \text{gamma}[22] + \text{gamma}[32] + \text{s2} = -\text{x222test}, \text{s2}) : \\ &\text{s22} := \text{subs}(\text{parameters}, \%); \\ &s22 := -\frac{1}{12} \frac{-2592\gamma_{11}^5 + 5184\gamma_{11}^4}{\gamma_{11}(360\gamma_{11}^5 - 1008\gamma_{11}^4 + 720\gamma_{11}^3 - 288\gamma_{11}^2)} \end{aligned} \quad (23)$$

Using the other resultant we calculate the third component s33 of the idempotent

$$\begin{aligned} &> \text{x333test} := \text{subs}(\text{parameters}, \text{simplify}(\text{subs}(\text{soltest}, \text{x33}))) ; \\ &x333test := \frac{1}{24} \frac{-21600\gamma_{11}^7 + 102816\gamma_{11}^6 - 162432\gamma_{11}^5 + 107136\gamma_{11}^4 - 41472\gamma_{11}^3}{\gamma_{11}(\gamma_{11} - 2)(180\gamma_{11}^5 - 504\gamma_{11}^4 + 360\gamma_{11}^3 - 144\gamma_{11}^2)} \\ &> \text{solve}(\text{gamma}[13] + \text{gamma}[23] + \text{gamma}[33] + \text{s3} = -\text{x333test}, \text{s3}) : \\ &\text{s33} := \text{subs}(\text{parameters}, \%); \end{aligned} \quad (24)$$

$$s33 := -\frac{-\gamma_{11} - 2}{5\gamma_{11}^2 - 4\gamma_{11} + 2} \quad (25)$$

```
> s11:=simplify(subs(soltest,simplify(subs({x2=s22,x3=s33},subs
(parameters,-C2/C1)))));
```

$$s11 := \frac{3\gamma_{11}^2}{5\gamma_{11}^2 - 4\gamma_{11} + 2} \quad (26)$$

Now we can calculate the first component s11 of the idempotent

```
> t:=subs(parameters, simplify(subs({x1=s11,x2=s22,x3=s33},[P1,P2,
P3]))):
```

Finally, we do a control: we check $p(v)-v=0$, with $v=(s11,s22,s33)$.

```
> factor(simplify(t[1]-s11));
```

$$0 \quad (27)$$

```
> simplify(t[2]-s22);
```

$$0 \quad (28)$$

```
> simplify(t[3]-s33);
```

$$0 \quad (29)$$