

Income taxation and equity:
new dominance criteria with a microsimulation application:
APPENDIXES

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Appendix A

Proof of Claim 1

Let $w_i(v, \delta^{(\tau)}) = \int_0^1 v_i(p) \delta_i^{(\tau)}(p) dp$ and assume that $\delta_i^{(\tau)}(p) = x \geq 0$ for all $p \in [0, 1]$, then we can rewrite $w_i(v, \delta^{(\tau)}) = \int_0^1 v_i(p) x dp = x \int_0^1 v_i(p) dp$ since $\int_0^1 v_i(p) dp = 1$, it follow that $w_i(v, \delta^{(\tau)}) = x$ so that $w_i(v, \delta^{(\tau)})$ satisfies Standardization.

Let $w_i(v, \delta^{(\tau)}) = \int_0^1 v_i(p) \delta_i^{(\tau)}(p) dp$ and $w_i(v, \delta_i^{(\tau')}) = \int_0^1 v_i(p) \delta_i^{(\tau')}(p) dp$ such that $\delta_i^{(\tau)}(p) = \delta_i^{(\tau')}(p) + \alpha$ with $\alpha \geq 0$ and $\delta_i^{(\tau)}(q) = \delta_i^{(\tau')}(q)$ for all $p, q \in [0, 1]$ and $p \neq q$. Then we rewrite $w_i(v, \delta_i^{(\tau)}) = \int_0^1 v_i(p) \delta_i^{(\tau')}(p) dp - v_i(p) \delta_i^{(\tau')}(p) + v_i(p) (\delta_i^{(\tau')}(p) + \alpha)$. It follows that $w_i(v, \delta_i^{(\tau)}) = \int_0^1 v_i(p) \delta_i^{(\tau')}(p) dp = \int_0^1 v_i(p) \delta_i(p)^{(\tau')} dp + \alpha v_i(p)$, so that $w_i(v, \delta_i^{(\tau)}) = w_i(v, \delta_i^{(\tau')}) + \alpha v_i(p)$. Given that $\alpha, v_i(p) \in \mathbb{R}_+$, $w_i(v, \delta^{(\tau)})$ satisfies Monotonicity.

Let $w_i(v, \delta^{(\tau)}) = \int_0^1 v_i(p) \delta_i^{(\tau)}(p) dp$. $\frac{\partial w_i}{\partial \delta_i^{(\tau)}(p)} = v_i(p)$ and $\frac{\partial^2 w_i}{\partial \delta_i^{(\tau)}(p) \partial \delta_i^{(\tau)}(q)} = 0$. Therefore, $w_i(v, \delta^{(\tau)})$ satisfies independence.

Proof of Claim 2

Let $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau)})$ and assume that $w_i(v, \delta^{(\tau)}) = 0$ for all $i = 1, \dots, n$ than we can rewrite $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i 0 = 0$. It follows that $W(\delta^{(\tau)}, \Omega)$ satisfies Normalization.

Let $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau)})$ and $W(\delta^{(\tau')}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau')})$. Assume that $w_i(v, \delta^{(\tau)}) \geq w_i(v, \delta^{(\tau')})$ for all $i = 1, \dots, n$. Given that $q_i \geq 0$, $W(\delta^{(\tau)}, \Omega) \geq W(\delta^{(\tau')}, \Omega)$. Hence $W(\delta^{(\tau)}, \Omega)$ satisfies Pareto.

Let $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau)})$. $\frac{\partial W(\delta^{(\tau)}, \Omega)}{\partial w_i(v, \delta^{(\tau)})} = q_i$ and $\frac{\partial^2 W(\delta^{(\tau)}, \Omega)}{\partial w_i(v, \delta^{(\tau)}) \partial w_j(v, \delta^{(\tau)})} = 0$. So, $W(\delta^{(\tau)}, \Omega)$ satisfies Decomposability.

Let $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau)})$. $\frac{\partial W((v, \delta^{(\tau)}))}{\partial w_i(v, \delta^{(\tau)})} = q_i$ and $\frac{\partial^2 W(\delta^{(\tau)}, \Omega)}{\partial w_i(v, \delta^{(\tau)})} = 0$. Thus, $W(\delta^{(\tau)}, \Omega)$ satisfies Utilitarianism.

Let $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i w_i(v, \delta^{(\tau)})$ and $W(w^r(v, \delta^{(\tau)})) = \sum_{i=1}^n q_i w_i^r(v, \delta^{(\tau)})$. Since $q_i = \frac{n_i}{N}$, where n_i is the number of individuals in type i and N the total number of individuals in the population, rewrite $W(\delta^{(\tau)}, \Omega^r) = \sum_{i=1}^n r \frac{n_i}{N} w_i(v, \delta^{(\tau)}) = \sum_{i=1}^n \frac{n_i}{N} w_i(v, \delta^{(\tau)})$, which implies that $W(\delta^{(\tau)}, \Omega^r) = W(\delta^{(\tau)}, \Omega)$. Thus, $W(\delta^{(\tau)}, \Omega)$ satisfies Population.

Proof of Proposition 1

Before proving Proposition 1 we state Abel's Lemma.¹

Abel's Lemma. If $v_1 \geq \dots \geq v_i \geq \dots \geq v_n \geq 0$, a sufficient condition for $\sum_{i=1}^n v_i w_i \geq 0$ is $\sum_{i=1}^j w_i \geq 0 \forall j = 1, \dots, n$. If $v_1 \leq \dots \leq v_i \leq \dots \leq v_n \leq 0$, the same condition is sufficient for $\sum_{i=1}^n v_i w_i \leq 0$.

We can now proceed with the proof of Proposition 1.

We now want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^n \int_0^1 v_i(p) \left[q_i \delta_i^{(\tau_A)}(p) - q_i \delta_i^{(\tau_B)}(p) \right] dp \geq 0, \quad \forall W \in \mathbf{W}_3 \quad (1)$$

Sufficiency can be shown as follows. First, reverse the order of integration and summation, such that

$$\Delta W = \int_0^1 \sum_{i=1}^n v_i(p) \left[q_i \delta_i^{(\tau_A)}(p) - q_i \delta_i^{(\tau_B)}(p) \right] dp \geq 0 \quad (2)$$

Letting $S_i(p) = q_i \delta_i^{(\tau_A)}(p) - q_i \delta_i^{(\tau_B)}(p)$ and rewriting (12):

$$\Delta W = \int_0^1 \sum_{i=1}^n v_i(p) S_i(p) dp \geq 0 \quad (3)$$

Now, notice that by property 3 we obtain that $v_i(p) \geq v_{i+1}(p) \geq 0$ as follows. Rewrite $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i \int_0^1 v(p) \delta_i^{(\tau')}(p) dp - v_i(p) \delta_i^{(\tau')}(p) + v_i(p) (\delta_i^{(\tau')}(p) + \alpha) - v_{i+1}(p) \delta_{i+1}^{(\tau')}(p) + v_{i+1}(p) (\delta_{i+1}^{(\tau')}(p) - \alpha) = \sum_{i=1}^n q_i \int_0^1 v_i(p) \delta_i^{(\tau')}(p) dp + \alpha v_i(p) - \alpha v_{i+1}(p)$, then $W(\delta^{(\tau)}, \Omega) \geq W(\delta^{(\tau)}, \Omega)$ if and only if $v_i(p) \geq v_{i+1}(p) \geq 0, \forall i = 1, \dots, n-1$ and $\forall p \in [0, 1]$. Hence, we can apply the Abel's Lemma and obtain that $\sum_{i=1}^n v_i(p) S_i(p) \geq 0$ if $\sum_{i=1}^k S_i(p) \geq 0, \forall k = 1, \dots, n$ and $\forall p \in [0, 1]$. It follows that $\sum_{i=1}^n v_i(p) S_i(p) \geq 0, \forall p \in [0, 1]$, implies that, integrating with respect to p , $\int_0^1 \sum_{i=1}^n v_i(p) S_i(p) dp \geq 0$.

For the necessity, suppose for a contradiction that $\Delta W \geq 0, \forall W \in \mathbf{W}_3$, but there is a type $h \in \{1, \dots, n\}$ and an interval $I \equiv [a, b] \subseteq [0, 1]$ such that $\sum_{i=1}^h S_i(p) < 0, \forall p \in I$. Now, by Abel's

¹See Jenkins and Lambert (1993) for a formal proof.

Lemma, there exists a set of functions $\{v_i(p) \geq 0\}$, such that $\sum_{i=1}^n v_i(p) S_i(p) < 0, \forall p \in I$. Writing $\sum_{i=1}^n v_i(p) S_i(p) = T(p)$, ΔW reduces to $\int_0^1 T(p) dp$, where $T(p) < 0, \forall p \in I$. Selecting a set of function $T(p)$, such that $T(p) \rightarrow 0, \forall p \in [0, 1] \setminus I$, ΔW would reduce to $\int_a^b T(p) dp < 0$, a contradiction. **QED**

Proof of Proposition 2

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^n \left(q_i \int_0^1 v_i(p_t) \delta_i^{(\tau_A)}(p_t) dp - q_i \int_0^1 v_i(p_t) \delta_i^{(\tau_B)}(p_t) dp \right) \geq 0, \forall W \in \mathbf{W}_{1,3} \quad (4)$$

For the sufficiency, by property 1 we obtain that $v_i(p) = \beta_i \geq 0$ for all $i = 1, \dots, n$ as follows: rewrite $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i \int_0^1 v_i(p) \delta_i^{(\tau')}(p) - v_i(p) \delta_i^{(\tau')}(p) + v_i(p) (\delta_i^{(\tau')}(p) + \alpha) - v_i(p + \pi) \delta_i^{(\tau')}(p + \pi) + v_i(p + \pi) (\delta_i^{(\tau')}(p + \pi) - \alpha)$, which reduces to $W(\delta^{(\tau)}, \Omega) = \sum_{i=1}^n q_i \int_0^1 v_i(p) \delta_i^{(\tau')}(p) + \alpha v_i(p) - \alpha v_i(p + \pi)$. Hence $W(\delta^{(\tau)}, \Omega) = W(\delta^{(\tau')}, \Omega)$ if and only if $\alpha v_i(p) = \alpha v_i(p + \pi)$, simplifying by α and for π very small, we get that $v_i'(p) = 0$, thus we can write $v_i(p) = \beta_i$ for all $i = 1, \dots, n$ and for all $p \in [0, 1]$. Therefore we can write eq. (16) as follows:

$$\begin{aligned} \Delta W &= \sum_{i=1}^n \beta_i \left[q_i \int_0^1 \delta_i^{(\tau_A)}(p_t) dp - q_i \int_0^1 \delta_i^{(\tau_B)}(p_t) dp \right] = \\ &= \sum_{i=1}^n \beta_i \left[q_i \mu_i^{(\tau_A)} - q_i \mu_i^{(\tau_B)} \right] \geq 0. \end{aligned} \quad (5)$$

Given that $v_i(p_t) = \beta_i \geq 0 \forall p_t \in [0, 1]$ and $\forall i = 1, 2, \dots, n$, and given that by property 3 $\beta_i \geq \beta_{i+1}$, by Abel's Lemma we obtain that $\Delta W \geq 0$ if $\sum_{i=1}^j (q_i \mu_i^{(\tau_A)} - q_i \mu_i^{(\tau_B)}) \geq 0, \forall j = 1, \dots, n$.

For the necessity, suppose that

$$\Delta W = \sum_{i=1}^n \beta_i \left[q_i \mu_i^{(\tau_A)} - q_i \mu_i^{(\tau_B)} \right] \geq 0$$

but $\exists k = 1, \dots, n$ such that $\sum_{j=1}^k (q_j \mu_j^{(\tau_A)} - q_j \mu_j^{(\tau_B)}) < 0$. We can choose a set of numbers $\{\beta_i\}_{i=1, \dots, n}$

such that $\beta_i \searrow 0, \forall i > k$. ΔW would reduce to $\sum_{j=1}^k \beta_k (q_k \mu_k^{(\tau_A)} - q_k \mu_k^{(\tau_B)}) < 0$, a contradiction.

QED

Proof of Proposition 3

We want to find a necessary and sufficient condition for

$$\Delta W = \sum_{i=1}^n \int_0^1 v_i(p) \left[q_i \delta_i^{(\tau_A)}(p) - q_i \delta_i^{(\tau_B)}(p) \right] dp \geq 0, \quad \forall W \in \mathbf{W}_{2,3,4} \quad (6)$$

For sufficiency, by property 2 and 4, consider two identical progressive transfers. One transfer of an amount $\rho > 0$ from individuals ranked at the $100(p + \pi)^{th}$ percentile of the distribution of income change of type i to individuals belonging to the same type but ranked at the $100p^{th}$ percentile. The other transfers of an amount $\rho > 0$ from individuals ranked at the $100(p + \pi)^{th}$ percentile of the distribution of income change of type $i + 1$ to individuals belonging to the same type but ranked at the $100p^{th}$ percentile. The impact of these transfers on social welfare change applied is expressed as follows:

$$\Delta_i W(\delta^{(\tau)}, \Omega) \geq \Delta_{i+1} W(\delta^{(\tau)}, \Omega) \quad \text{for any } p, \pi, \rho, i,$$

which corresponds to

$$-\rho v_i(p + \pi) + \rho v_i(p) \geq -\rho v_{i+1}(p + \pi) + \rho v_{i+1}(p) \quad \text{for any } p, \pi, \rho, i.$$

Simplifying for ρ and for small π , we get

$$v'_i(p) \leq v'_{i+1}(p) \leq 0 \quad \text{for any } p, i.$$

Sufficiency can now be shown as follows. First, letting $S_i(p) = q_1 \delta_i^{(1)}(p) - q_i \delta_i^{(2)}(p)$ and integrating

by parts eq. (18)

$$\Delta W = \sum_{i=1}^n \left[v_i(1) \int_0^1 S_i(p) dp \right] - \sum_{i=1}^n \int_0^1 v'_i(p) \int_0^p S_i(q) dq dp \quad (7)$$

reversing the order of integration and summation in the second part of eq. (19):

$$\Delta W = \sum_{i=1}^n \left[v_i(1) \int_0^1 S_i(p) dp \right] - \int_0^1 \sum_{i=1}^n v'_i(p) \int_0^p S_i(q) dq dp \quad (8)$$

By property 3 and 4 and by Abel's Lemma we get that $\sum_{i=1}^j \int_0^p S_i(q) dq \geq 0, \forall j = 1, \dots, n$ and $\forall p \in [0, 1]$ implies $-\int_0^1 \sum_{i=1}^n v'_i(p) \int_0^p S_i(q) dq dp \geq 0$, but given property 3, this also implies that $\sum_{i=1}^n \left[v_i(1) \int_0^1 S_i(p) dp \right] \geq 0$, hence it is sufficient for $\Delta W \geq 0$.

For the necessity, by application of Abel's decomposition, write eq. (18) as follows:

$$\Delta W = \sum_{i=1}^n \left[v_i(1) \int_0^1 S_i(p) dp \right] + \int_0^1 \varepsilon_n(p) \sum_{i=1}^n \int_0^p S_i(q) dq dp + \int_0^1 \sum_{i=1}^{n-1} \omega_i(p) \sum_{j=1}^i \int_0^p S_j(q) dq dp$$

with $\varepsilon_n(p) = -v'_n(p) \forall p$ and $\omega_i(p) = -(v'_i(p) - v'_{i+1}(p)) \forall i$ and $\forall p$. Now, suppose for a contradiction that $\Delta W \geq 0$, but $\exists h$ and interval $I \equiv [a, b] \subseteq [0, 1]$ such that $\sum_{j=1}^h \int_0^p S_j(q) dq < 0, \forall p \in I$. Applying Lemma 1 in Chambaz and Maurin (1998)², that there exists a set of non-negative functions $\{\phi_i(p)\}_{i \in \{1, \dots, n-1\}}$ such that $\sum_{i=1}^{n-1} \phi_i(p) \left(\sum_{j=1}^i \int_0^p S_j(q) dq \right) < 0 \forall p \in I$. By Lemma 2 in Chambaz and Maurin (1998) there exists a set of non-negative functions $\tau(p)$ such that $\int_0^1 \tau(p) \sum_{j=1}^n \phi_j(p) \sum_{i=1}^j \int_0^p S_i(q) dq dp < 0$. Now, write $\tau(p) \phi_i(p) = \omega_i(p)$, given that $\tau(p) \phi_i(p) \geq 0$, $\omega_i(p)$ satisfies eq. (6) and (7). Hence, $\int_0^1 \sum_{i=1}^{n-1} \omega_i(p) \left(\sum_{j=1}^i \int_0^p S_j(q) dq \right) \leq 0$. Applying again Lemma 2 and following a similar argument as above, we will have that $\int_0^1 \varepsilon_n(p) \sum_{j=1}^n \int_0^p S_j(q) dp \leq 0$. Thus, it is possible to choose a combination of $v_i(1)$ and $\int_0^1 S_j(p) dp$ to contradict eq.(18) **QED**.

²Chambaz, C., Maurin, E. (1998). Atkinson and Bourguignon's dominance criteria: Extended and Applied to the Measurement of Poverty in France, *Review of Income and Wealth*, 44(4), 497-513.

1 Appendix B

Table 1: Bootstrapped standard errors for proposition 1 with absolute income change

	quartile	Reform-2012	0.95 c.i.	0.95 c.i.
type 1	1	-0.01	-0.01	-0.01
	2	0.18	0.18	0.18
	3	0.10	0.10	0.10
	4	0.04	0.04	0.04
type 2	1	-3.93	-5.61	-2.57
	2	0.74	0.65	0.83
	3	2.05	1.97	2.14
	4	3.66	3.43	3.95
type 3	1	-6.47	-8.50	-4.81
	2	1.14	1.03	1.26
	3	3.05	2.96	3.15
	4	5.50	5.24	5.78
type 4	1	-14.20	-17.15	-11.81
	2	3.69	3.53	3.85
	3	8.21	8.06	8.33
	4	13.93	13.61	14.28
type 5	1	-25.49	-30.04	-21.99
	2	6.24	6.04	6.44
	3	13.45	13.29	13.62
	4	22.96	22.55	23.42
type 6	1	-29.56	-34.31	-25.80
	2	6.24	6.04	6.44
	3	13.98	13.81	14.17
	4	24.55	24.12	25.01
type 7	1	-29.58	-34.34	-25.82
	2	6.24	6.04	6.44
	3	14.00	13.82	14.18
	4	24.58	24.14	25.04
type 8	1	-31.68	-36.42	-27.78
	2	6.59	6.37	6.80
	3	14.83	14.66	15.02
	4	25.87	25.44	26.35
type 9	1	-32.62	-37.41	-28.70
	2	6.85	6.61	7.08
	3	15.41	15.23	15.60
	4	26.77	26.33	27.26
type 10	1	-39.75	-44.85	-35.10
	2	7.12	6.85	7.37
	3	16.80	16.60	17.02
	4	29.16	28.67	29.67
type 11	1	-43.49	-48.89	-38.93
	1	5.89	5.47	6.34
	2	17.03	16.82	17.26
	3	29.82	29.34	30.32
type 12	1	-57.44	-63.22	-51.98
	2	4.46	3.87	5.09
	3	18.01	17.77	18.25
	4	32.22	31.71	32.78

Table 2: Bootstrapped standard errors for proposition 1 with relative income change: difference in equivalent income from baseline to proposed reform

	<i>quartile</i>	<i>Reform-2012</i>	0.95 c.i.	0.95 c.i.
type 1	1	0.00	0.00	0.00
	2	0.00	0.00	0.00
	3	0.00	0.00	0.00
	4	0.00	0.00	0.00
type 2	1	-0.00	-0.00	-0.00
	2	0.00	0.00	0.00
	3	0.00	0.00	0.00
	4	0.00	0.00	0.00
type 3	1	-0.00	-0.00	-0.00
	2	0.00	0.00	0.00
	3	0.00	0.00	0.00
	4	0.00	0.00	0.00
type 4	1	-0.01	-0.01	-0.00
	2	0.01	0.01	0.01
	3	0.01	0.01	0.01
	4	0.01	0.01	0.01
type 5	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.01	0.01	0.02
	4	0.02	0.02	0.02
type 6	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 7	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 8	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 9	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 10	1	-0.01	-0.01	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 11	1	-0.01	-0.02	-0.01
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02
type 12	1	-0.02	-0.02	-0.02
	2	0.01	0.01	0.01
	3	0.02	0.02	0.02
	4	0.02	0.02	0.02

Table 3: Bootstrapped standard errors for proposition 2, absolute income change

<i>type</i>	<i>Reform-2012</i>	0.95 c.i.	0.95 c.i.
1	0.0009	-0.0000	0.0016
2	0.0601	0.0245	0.0928
3	0.0798	0.0405	0.1157
4	0.6118	0.4752	0.7395
5	1.0911	0.8527	1.2987
6	1.0793	0.8400	1.2857
7	1.0793	0.8400	1.2857
8	1.0781	0.8370	1.2870
9	1.0822	0.8409	1.2899
10	1.0232	0.7732	1.2373
11	0.9979	0.7495	1.2130
12	0.7775	0.5205	1.0028

Table 4: Bootstrapped standard errors for proposition 2, relative income change

<i>type</i>	<i>Reform-2012</i>	0.95 c.i.	0.95 c.i.
1	0.000002	0.000001	0.000002
2	0.000130	0.000114	0.000147
3	0.000171	0.000153	0.000189
4	0.001000	0.000921	0.001071
5	0.001908	0.001805	0.002007
6	0.001924	0.001819	0.002022
7	0.001924	0.001819	0.002022
8	0.001935	0.001830	0.002034
9	0.001939	0.001835	0.002039
10	0.001948	0.001845	0.002048
11	0.001942	0.001840	0.002042
12	0.001913	0.001809	0.002014

Table 5: Bootstrapped standard errors for proposition 3, absolute income change

	<i>quartile</i>	<i>Reform-2012</i>	0.95 c.i.	0.95 c.i.
type 1	1	-0.0088	-0.0087	-0.0087
	2	0.1681	0.1683	0.1683
	3	0.2687	0.2689	0.2689
	4	0.3131	0.3131	0.3131
type 2	1	-3.6172	-5.2998	-2.2599
	2	-2.8789	-4.5721	-1.5050
	3	-0.8313	-2.5549	0.5608
	4	2.8315	1.0539	4.2423
type 3	1	-3.6431	-7.2562	-0.8676
	2	-2.5007	-6.0278	0.2609
	3	0.5540	-3.0187	3.3287
	4	6.0520	2.3919	8.8899
type 4	1	-8.1517	-14.1249	-3.6771
	2	-4.4585	-10.3935	0.0298
	3	3.7476	-2.2436	8.2584
	4	17.6804	11.6324	22.2341
type 5	1	-7.8128	-16.9805	-0.7101
	2	-1.5711	-10.7225	5.5808
	3	11.8829	2.6335	18.9969
	4	34.8454	25.4995	41.9752
type 6	1	5.2883	-8.2299	15.5970
	2	11.5300	-1.9736	21.7831
	3	25.5113	11.9063	35.8058
	4	50.0657	36.6219	60.3783
type 7	1	20.4890	2.5848	34.4817
	2	26.7331	8.8548	40.6459
	3	40.7284	22.7248	54.7051
	4	65.3094	47.4113	79.2540
type 8	1	33.6253	11.5224	51.2655
	2	40.2150	18.1273	57.8608
	3	55.0463	32.8530	72.7351
	4	80.9128	58.9543	98.6384
type 9	1	48.2904	22.0546	69.6664
	2	55.1372	28.9400	76.4689
	3	70.5506	44.2744	91.7646
	4	97.3181	71.1979	118.4907
type 10	1	57.5679	26.8341	83.5475
	2	64.6845	33.9379	90.5895
	3	81.4863	50.6235	107.3038
	4	110.6472	79.5839	136.3734
type 11	1	67.1599	30.8630	97.4495
	2	73.0531	36.7151	103.2175
	3	90.0849	53.7811	120.0970
	4	119.9013	83.3289	149.9001
type 12	1	62.4576	21.1756	97.1253
	2	66.9217	25.5490	101.6260
	3	84.9269	43.6518	119.5319
	4	117.1482	75.8654	151.9744

Table 6: Bootstrapped standard errors for proposition 3, relative income change

	<i>quartile</i>	<i>Reform-2012</i>	0.95 c.i.	0.95 c.i.
type 1	1	0.0000	0.0000	0.0000
	2	0.0002	0.0002	0.0002
	3	0.0005	0.0005	0.0005
	4	0.0006	0.0006	0.0006
type 2	1	-0.0007	-0.0011	-0.0003
	2	0.0009	0.0004	0.0013
	3	0.0034	0.0029	0.0039
	4	0.0058	0.0053	0.0064
type 3	1	0.0036	0.0026	0.0044
	2	0.0057	0.0047	0.0067
	3	0.0096	0.0085	0.0107
	4	0.0134	0.0123	0.0146
type 4	1	0.0083	0.0064	0.0099
	2	0.0148	0.0127	0.0166
	3	0.0242	0.0221	0.0261
	4	0.0338	0.0317	0.0359
type 5	1	0.0250	0.0220	0.0278
	2	0.0356	0.0324	0.0385
	3	0.0504	0.0472	0.0536
	4	0.0663	0.0629	0.0695
type 6	1	0.0561	0.0518	0.0601
	2	0.0667	0.0623	0.0709
	3	0.0823	0.0778	0.0867
	4	0.0994	0.0948	0.1039
type 7	1	0.0892	0.0833	0.0947
	2	0.0998	0.0939	0.1054
	3	0.1154	0.1093	0.1211
	4	0.1325	0.1264	0.1383
type 8	1	0.1216	0.1142	0.1284
	2	0.1327	0.1250	0.1398
	3	0.1489	0.1413	0.1561
	4	0.1669	0.1592	0.1741
type 9	1	0.1556	0.1465	0.1639
	2	0.1670	0.1577	0.1755
	3	0.1836	0.1744	0.1922
	4	0.2021	0.1928	0.2107
type 10	1	0.1889	0.1782	0.1987
	2	0.2005	0.1898	0.2105
	3	0.2182	0.2073	0.2285
	4	0.2380	0.2270	0.2484
type 11	1	0.2238	0.2116	0.2351
	2	0.2349	0.2227	0.2466
	3	0.2528	0.2403	0.2647
	4	0.2730	0.2604	0.2849
type 12	1	0.2556	0.2418	0.2685
	2	0.2662	0.2523	0.2793
	3	0.2850	0.2709	0.2982
	4	0.3066	0.2923	0.3198