Derivative-free optimization: A review of algorithms and comparison of software implementations

Material for On-line Supplement

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1 Fraction of problems solved as a function of allowable number of function evaluations

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.018	0.020	0.020	0.022	0.024	0.024
BOBYQA	0.112	0.203	0.249	0.283	0.297	0.317
CMA-ES	0.042	0.145	0.217	0.239	0.269	0.297
DAKOTA/DIRECT	0.129	0.143	0.147	0.147	0.147	0.147
DAKOTA/EA	0.026	0.032	0.036	0.038	0.040	0.040
DAKOTA/PATTERN	0.042	0.074	0.074	0.078	0.082	0.082
DAKOTA/SOLIS-WETS	0.042	0.082	0.090	0.092	0.092	0.094
DFO	0.058	0.072	0.125	0.145	0.159	0.181
FMINSEARCH	0.084	0.197	0.225	0.235	0.235	0.237
GLOBAL	0.082	0.125	0.129	0.131	0.131	0.131
HOPSPACK	0.118	0.171	0.187	0.191	0.197	0.201
IMFIL	0.072	0.082	0.086	0.086	0.086	0.086
MCS	0.281	0.404	0.442	0.482	0.508	0.514
NEWUOA	0.076	0.159	0.207	0.237	0.261	0.269
NOMAD	0.062	0.165	0.203	0.217	0.229	0.231
PSWARM	0.032	0.112	0.173	0.199	0.211	0.225
SID-PSM	0.139	0.303	0.337	0.367	0.371	0.378
SNOBFIT	0.137	0.219	0.283	0.315	0.335	0.343
TOMLAB/GLCCLUSTER	0.257	0.406	0.504	0.556	0.592	0.622
TOMLAB/LGO	0.131	0.337	0.400	0.430	0.456	0.478
TOMLAB/MULTIMIN	0.145	0.392	0.512	0.578	0.610	0.637
TOMLAB/OQNLP	0.137	0.303	0.392	0.440	0.470	0.490

Table 1 Fraction of all problems solved as a function of allowable number of function evaluation

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.026	0.115	0.218	0.321	0.385	0.474
CMA-ES	0.000	0.000	0.051	0.077	0.090	0.128
DAKOTA/DIRECT	0.000	0.000	0.013	0.013	0.013	0.013
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.013	0.038
FMINSEARCH	0.000	0.064	0.077	0.077	0.077	0.077
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.026	0.064	0.077	0.103	0.115
IMFIL	0.000	0.000	0.000	0.000	0.000	0.000
MCS	0.077	0.321	0.462	0.628	0.756	0.756
NEWUOA	0.026	0.090	0.192	0.244	0.333	0.359
NOMAD	0.000	0.000	0.026	0.051	0.064	0.064
PSWARM	0.000	0.000	0.000	0.013	0.026	0.026
SID-PSM	0.064	0.256	0.372	0.436	0.436	0.449
SNOBFIT	0.128	0.321	0.513	0.577	0.577	0.577
TOMLAB/GLCCLUSTER	0.077	0.385	0.590	0.756	0.782	0.795
TOMLAB/LGO	0.064	0.192	0.269	0.333	0.397	0.462
TOMLAB/MULTIMIN	0.000	0.231	0.372	0.500	0.564	0.628
TOMLAB/OQNLP	0.064	0.231	0.385	0.462	0.564	0.654

Table 2 Fraction of convex smooth problems solved as a function of allowable number of function evaluations

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.019	0.062	0.062	0.075	0.075	0.075
CMA-ES	0.000	0.050	0.087	0.087	0.130	0.149
DAKOTA/DIRECT	0.012	0.012	0.012	0.012	0.012	0.012
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.006	0.006	0.012	0.012	0.012
DAKOTA/SOLIS-WETS	0.000	0.019	0.019	0.025	0.025	0.025
DFO	0.006	0.019	0.037	0.037	0.037	0.043
FMINSEARCH	0.037	0.075	0.081	0.087	0.087	0.087
GLOBAL	0.006	0.025	0.025	0.025	0.025	0.025
HOPSPACK	0.037	0.037	0.037	0.037	0.043	0.043
IMFIL	0.062	0.062	0.062	0.062	0.062	0.062
MCS	0.075	0.087	0.093	0.112	0.118	0.124
NEWUOA	0.012	0.050	0.062	0.075	0.075	0.081
NOMAD	0.006	0.056	0.081	0.087	0.099	0.099
PSWARM	0.000	0.006	0.025	0.037	0.043	0.050
SID-PSM	0.000	0.106	0.106	0.130	0.137	0.137
SNOBFIT	0.000	0.025	0.050	0.068	0.081	0.087
TOMLAB/GLCCLUSTER	0.037	0.168	0.298	0.342	0.404	0.429
TOMLAB/LGO	0.019	0.106	0.161	0.174	0.199	0.217
TOMLAB/MULTIMIN	0.019	0.174	0.292	0.366	0.416	0.441
TOMLAB/OQNLP	0.000	0.062	0.118	0.149	0.180	0.199

Table 3 Fraction of convex nonsmooth problems solved as a function of allowable number of function evaluations

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.037	0.041	0.041	0.045	0.049	0.049
BOBYQA	0.208	0.339	0.396	0.424	0.433	0.445
CMA-ES	0.086	0.257	0.355	0.392	0.416	0.449
DAKOTA/DIRECT	0.245	0.273	0.278	0.278	0.278	0.278
DAKOTA/EA	0.053	0.065	0.073	0.078	0.082	0.082
DAKOTA/PATTERN	0.086	0.147	0.147	0.151	0.159	0.159
DAKOTA/SOLIS-WETS	0.086	0.155	0.171	0.171	0.171	0.176
DFO	0.114	0.135	0.229	0.265	0.290	0.322
FMINSEARCH	0.143	0.318	0.367	0.384	0.384	0.388
GLOBAL	0.163	0.241	0.249	0.253	0.253	0.253
HOPSPACK	0.216	0.314	0.335	0.339	0.339	0.343
IMFIL	0.102	0.122	0.131	0.131	0.131	0.131
MCS	0.490	0.657	0.682	0.694	0.702	0.710
NEWUOA	0.139	0.261	0.318	0.355	0.376	0.380
NOMAD	0.122	0.294	0.343	0.359	0.371	0.376
PSWARM	0.065	0.220	0.327	0.359	0.371	0.396
SID-PSM	0.265	0.453	0.486	0.506	0.510	0.522
SNOBFIT	0.241	0.327	0.380	0.412	0.441	0.453
TOMLAB/GLCCLUSTER	0.469	0.592	0.637	0.657	0.682	0.722
TOMLAB/LGO	0.229	0.543	0.600	0.633	0.649	0.661
TOMLAB/MULTIMIN	0.278	0.592	0.714	0.755	0.767	0.784
TOMLAB/OQNLP	0.257	0.498	0.584	0.637	0.645	0.645

Table 4 Fraction of nonconvex smooth problems solved as a function of allowable number of function evaluations

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.000	0.056	0.056	0.056	0.056
CMA-ES	0.000	0.111	0.222	0.222	0.278	0.278
DAKOTA/DIRECT	0.167	0.167	0.167	0.167	0.167	0.167
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.056	0.111	0.111	0.111
FMINSEARCH	0.056	0.222	0.222	0.222	0.222	0.222
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.056	0.056	0.056	0.056	0.056
IMFIL	0.056	0.056	0.056	0.056	0.056	0.056
MCS	0.167	0.167	0.222	0.278	0.278	0.278
NEWUOA	0.000	0.056	0.056	0.056	0.056	0.056
NOMAD	0.000	0.111	0.167	0.167	0.167	0.167
PSWARM	0.000	0.056	0.167	0.278	0.333	0.333
SID-PSM	0.000	0.222	0.222	0.278	0.278	0.278
SNOBFIT	0.000	0.056	0.056	0.056	0.111	0.111
TOMLAB/GLCCLUSTER	0.111	0.111	0.167	0.222	0.222	0.222
TOMLAB/LGO	0.111	0.222	0.389	0.389	0.389	0.389
TOMLAB/MULTIMIN	0.111	0.333	0.333	0.389	0.389	0.444
TOMLAB/OQNLP	0.056	0.111	0.278	0.278	0.278	0.278

Table 5 Fraction of nonconvex nonsmooth problems solved as a function of allowable number of function evaluations

2 Fraction of problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.008	0.010	0.012	0.014	0.016	0.018
BOBYQA	0.118	0.191	0.237	0.265	0.279	0.299
CMA-ES	0.034	0.131	0.205	0.239	0.265	0.289
DAKOTA/DIRECT	0.151	0.135	0.137	0.137	0.137	0.137
DAKOTA/EA	0.016	0.020	0.028	0.030	0.030	0.030
DAKOTA/PATTERN	0.042	0.058	0.062	0.062	0.062	0.064
DAKOTA/SOLIS-WETS	0.034	0.070	0.072	0.074	0.074	0.074
DFO	0.056	0.068	0.118	0.137	0.151	0.173
FMINSEARCH	0.078	0.193	0.221	0.233	0.231	0.231
GLOBAL	0.076	0.100	0.106	0.108	0.108	0.108
HOPSPACK	0.112	0.159	0.171	0.173	0.177	0.185
IMFIL	0.064	0.070	0.072	0.072	0.072	0.072
MCS	0.392	0.422	0.436	0.472	0.492	0.498
NEWUOA	0.070	0.149	0.193	0.221	0.243	0.253
NOMAD	0.060	0.157	0.185	0.201	0.207	0.213
PSWARM	0.024	0.104	0.163	0.195	0.203	0.221
SID-PSM	0.135	0.301	0.333	0.351	0.361	0.367
SNOBFIT	0.131	0.213	0.279	0.307	0.319	0.323
TOMLAB/GLCCLUSTER	0.412	0.496	0.538	0.578	0.618	0.635
TOMLAB/LGO	0.139	0.339	0.390	0.422	0.448	0.464
TOMLAB/MULTIMIN	0.454	0.673	0.715	0.739	0.741	0.751
TOMLAB/OQNLP	0.141	0.303	0.382	0.430	0.458	0.476

Table 6 Fraction of problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.026	0.115	0.218	0.321	0.385	0.474
CMA-ES	0.000	0.000	0.051	0.077	0.090	0.128
DAKOTA/DIRECT	0.013	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.013	0.038
FMINSEARCH	0.000	0.064	0.077	0.077	0.077	0.077
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.026	0.064	0.077	0.103	0.115
IMFIL	0.000	0.000	0.000	0.000	0.000	0.000
MCS	0.205	0.346	0.474	0.628	0.756	0.756
NEWUOA	0.026	0.090	0.179	0.244	0.321	0.359
NOMAD	0.000	0.000	0.026	0.051	0.064	0.064
PSWARM	0.000	0.000	0.000	0.013	0.026	0.026
SID-PSM	0.064	0.256	0.372	0.423	0.436	0.449
SNOBFIT	0.128	0.321	0.513	0.577	0.577	0.577
TOMLAB/GLCCLUSTER	0.269	0.615	0.667	0.795	0.808	0.808
TOMLAB/LGO	0.064	0.192	0.269	0.333	0.397	0.462
TOMLAB/MULTIMIN	0.538	0.526	0.603	0.679	0.731	0.769
TOMLAB/OQNLP	0.064	0.231	0.385	0.462	0.564	0.654

Table 7 Fraction of convex smooth problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.019	0.031	0.031	0.031	0.031	0.031
CMA-ES	0.000	0.050	0.062	0.087	0.118	0.143
DAKOTA/DIRECT	0.019	0.012	0.012	0.012	0.012	0.012
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.006	0.006	0.006	0.006	0.006
DAKOTA/SOLIS-WETS	0.000	0.019	0.019	0.019	0.019	0.019
DFO	0.006	0.019	0.025	0.025	0.025	0.031
FMINSEARCH	0.025	0.068	0.075	0.087	0.081	0.081
GLOBAL	0.000	0.025	0.019	0.019	0.019	0.019
HOPSPACK	0.012	0.019	0.019	0.019	0.019	0.025
IMFIL	0.062	0.062	0.062	0.062	0.062	0.062
MCS	0.118	0.081	0.062	0.068	0.075	0.075
NEWUOA	0.012	0.043	0.056	0.056	0.056	0.056
NOMAD	0.000	0.043	0.050	0.050	0.056	0.062
PSWARM	0.000	0.000	0.019	0.037	0.043	0.056
SID-PSM	0.000	0.075	0.093	0.099	0.118	0.112
SNOBFIT	0.000	0.019	0.043	0.043	0.050	0.050
TOMLAB/GLCCLUSTER	0.248	0.267	0.360	0.404	0.472	0.491
TOMLAB/LGO	0.019	0.093	0.118	0.143	0.155	0.174
TOMLAB/MULTIMIN	0.565	0.658	0.640	0.652	0.634	0.634
TOMLAB/OQNLP	0.000	0.068	0.099	0.137	0.149	0.168

Table 8 Fraction of convex nonsmooth problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.016	0.020	0.024	0.029	0.033	0.037
BOBYQA	0.220	0.335	0.396	0.420	0.429	0.441
CMA-ES	0.069	0.233	0.347	0.392	0.416	0.433
DAKOTA/DIRECT	0.286	0.261	0.265	0.265	0.265	0.265
DAKOTA/EA	0.033	0.041	0.057	0.061	0.061	0.061
DAKOTA/PATTERN	0.086	0.114	0.122	0.122	0.122	0.127
DAKOTA/SOLIS-WETS	0.069	0.131	0.135	0.139	0.139	0.139
DFO	0.110	0.127	0.220	0.261	0.286	0.318
FMINSEARCH	0.139	0.314	0.363	0.380	0.380	0.380
GLOBAL	0.155	0.188	0.204	0.208	0.208	0.208
HOPSPACK	0.216	0.306	0.318	0.318	0.318	0.327
IMFIL	0.086	0.098	0.102	0.102	0.102	0.102
MCS	0.637	0.682	0.682	0.702	0.702	0.714
NEWUOA	0.127	0.249	0.302	0.339	0.359	0.367
NOMAD	0.122	0.286	0.335	0.359	0.363	0.371
PSWARM	0.049	0.204	0.314	0.355	0.359	0.388
SID-PSM	0.253	0.465	0.486	0.498	0.502	0.514
SNOBFIT	0.229	0.318	0.376	0.412	0.433	0.437
TOMLAB/GLCCLUSTER	0.584	0.633	0.645	0.649	0.682	0.706
TOMLAB/LGO	0.249	0.555	0.604	0.629	0.653	0.657
TOMLAB/MULTIMIN	0.351	0.727	0.800	0.824	0.824	0.837
TOMLAB/OQNLP	0.265	0.494	0.584	0.629	0.645	0.641

Table 9 Fraction of nonconvex smooth problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.000	0.000	0.000	0.000	0.000
CMA-ES	0.000	0.056	0.222	0.222	0.278	0.333
DAKOTA/DIRECT	0.111	0.111	0.111	0.111	0.111	0.111
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.056	0.056	0.056	0.056
FMINSEARCH	0.056	0.222	0.222	0.222	0.222	0.222
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.056	0.000	0.000	0.000	0.000	0.000
IMFIL	0.056	0.056	0.056	0.056	0.056	0.056
MCS	0.333	0.278	0.278	0.278	0.222	0.222
NEWUOA	0.000	0.000	0.000	0.000	0.000	0.000
NOMAD	0.000	0.111	0.056	0.056	0.056	0.056
PSWARM	0.000	0.111	0.111	0.222	0.278	0.278
SID-PSM	0.056	0.278	0.222	0.278	0.278	0.278
SNOBFIT	0.000	0.056	0.056	0.056	0.056	0.111
TOMLAB/GLCCLUSTER	0.167	0.167	0.111	0.222	0.222	0.222
TOMLAB/LGO	0.056	0.222	0.444	0.500	0.500	0.444
TOMLAB/MULTIMIN	0.500	0.722	0.722	0.611	0.611	0.556
TOMLAB/OQNLP	0.056	0.111	0.167	0.222	0.222	0.222

Table 10 Fraction of nonconvex nonsmooth problems, as a function of allowable number of function evaluations, for which a solver found the best solution among all solvers

3 Fraction of problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

	τ				
Solver	1E-1	1E-2	1E-3	1E-6	0E+0
ASA	0.488	0.424	0.384	0.327	0.124
BOBYQA	0.831	0.807	0.789	0.631	0.375
CMA-ES	0.867	0.757	0.637	0.542	0.325
DAKOTA/DIRECT	0.865	0.823	0.817	0.743	0.283
DAKOTA/EA	0.540	0.472	0.428	0.349	0.155
DAKOTA/PATTERN	0.725	0.562	0.512	0.432	0.201
DAKOTA/SOLIS-WETS	0.847	0.711	0.594	0.458	0.201
DFO	0.578	0.528	0.486	0.422	0.227
FMINSEARCH	0.614	0.512	0.480	0.446	0.265
GLOBAL	0.813	0.649	0.600	0.528	0.239
HOPSPACK	0.797	0.731	0.620	0.490	0.247
IMFIL	0.839	0.769	0.659	0.410	0.211
MCS	0.869	0.849	0.839	0.819	0.514
NEWUOA	0.815	0.733	0.709	0.558	0.333
NOMAD	0.647	0.568	0.532	0.474	0.279
PSWARM	0.835	0.669	0.582	0.490	0.295
SID-PSM	0.761	0.679	0.651	0.606	0.406
SNOBFIT	0.765	0.701	0.681	0.622	0.412
TOMLAB/GLCCLUSTER	0.992	0.984	0.978	0.859	0.492
TOMLAB/LGO	0.849	0.787	0.747	0.643	0.452
TOMLAB/MULTIMIN	0.994	0.984	0.982	0.888	0.512
TOMLAB/OQNLP	0.930	0.851	0.809	0.735	0.448

Table 11 Fraction of problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

	τ				
Solver	1E-1	1E-2	1E-3	1E-6	0E+0
ASA	0.103	0.000	0.000	0.000	0.000
BOBYQA	0.923	0.923	0.923	0.769	0.397
CMA-ES	0.846	0.756	0.436	0.256	0.103
DAKOTA/DIRECT	1.000	1.000	1.000	1.000	0.026
DAKOTA/EA	0.205	0.038	0.000	0.000	0.000
DAKOTA/PATTERN	0.705	0.321	0.179	0.077	0.026
DAKOTA/SOLIS-WETS	0.897	0.718	0.397	0.154	0.013
DFO	0.321	0.192	0.141	0.038	0.038
FMINSEARCH	0.436	0.179	0.090	0.077	0.077
GLOBAL	0.936	0.526	0.423	0.359	0.013
HOPSPACK	0.846	0.782	0.474	0.244	0.064
IMFIL	0.846	0.718	0.526	0.141	0.000
MCS	1.000	1.000	1.000	1.000	0.654
NEWUOA	0.859	0.795	0.782	0.590	0.359
NOMAD	0.423	0.244	0.179	0.115	0.064
PSWARM	0.872	0.538	0.321	0.128	0.013
SID-PSM	0.705	0.577	0.513	0.462	0.449
SNOBFIT	0.744	0.603	0.603	0.590	0.577
TOMLAB/GLCCLUSTER	1.000	1.000	1.000	1.000	0.603
TOMLAB/LGO	0.923	0.833	0.731	0.551	0.462
TOMLAB/MULTIMIN	1.000	1.000	1.000	1.000	0.513
TOMLAB/OQNLP	1.000	0.949	0.872	0.769	0.397

Table 12 Fraction of convex smooth problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

	τ				
Solver	1E-1	1E-2	1E-3	1E-6	0E+0
ASA	0.161	0.118	0.075	0.037	0.012
BOBYQA	0.571	0.528	0.509	0.205	0.019
CMA-ES	0.708	0.522	0.354	0.205	0.037
DAKOTA/DIRECT	0.665	0.565	0.559	0.534	0.043
DAKOTA/EA	0.236	0.143	0.106	0.043	0.025
DAKOTA/PATTERN	0.491	0.236	0.186	0.137	0.025
DAKOTA/SOLIS-WETS	0.696	0.472	0.286	0.137	0.019
DFO	0.267	0.230	0.174	0.112	0.025
FMINSEARCH	0.323	0.186	0.174	0.149	0.031
GLOBAL	0.615	0.342	0.267	0.205	0.037
HOPSPACK	0.534	0.447	0.329	0.143	0.031
IMFIL	0.652	0.534	0.391	0.124	0.087
MCS	0.634	0.584	0.571	0.571	0.081
NEWUOA	0.621	0.472	0.447	0.199	0.019
NOMAD	0.391	0.255	0.199	0.137	0.025
PSWARM	0.609	0.373	0.248	0.149	0.031
SID-PSM	0.534	0.410	0.385	0.323	0.043
SNOBFIT	0.509	0.460	0.435	0.342	0.031
TOMLAB/GLCCLUSTER	1.000	0.994	0.994	0.714	0.056
TOMLAB/LGO	0.615	0.509	0.491	0.342	0.050
TOMLAB/MULTIMIN	1.000	0.994	0.994	0.739	0.056
TOMLAB/OQNLP	0.820	0.627	0.547	0.497	0.062

Table 13 Fraction of convex nonsmooth problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

	τ				
Solver	1E-1	1E-2	1E-3	1E-6	0E+0
ASA	0.812	0.759	0.722	0.637	0.245
BOBYQA	0.967	0.951	0.931	0.886	0.629
CMA-ES	0.971	0.902	0.873	0.853	0.596
DAKOTA/DIRECT	0.951	0.939	0.931	0.816	0.531
DAKOTA/EA	0.829	0.820	0.776	0.678	0.298
DAKOTA/PATTERN	0.873	0.833	0.812	0.747	0.388
DAKOTA/SOLIS-WETS	0.927	0.857	0.845	0.776	0.392
DFO	0.849	0.816	0.800	0.759	0.437
FMINSEARCH	0.841	0.820	0.800	0.771	0.494
GLOBAL	0.902	0.890	0.873	0.816	0.457
HOPSPACK	0.951	0.894	0.849	0.800	0.461
IMFIL	0.955	0.935	0.886	0.690	0.371
MCS	0.980	0.976	0.963	0.943	0.780
NEWUOA	0.922	0.878	0.853	0.796	0.555
NOMAD	0.869	0.853	0.845	0.812	0.531
PSWARM	0.963	0.894	0.865	0.824	0.576
SID-PSM	0.914	0.878	0.861	0.841	0.653
SNOBFIT	0.927	0.886	0.861	0.837	0.637
TOMLAB/GLCCLUSTER	0.992	0.984	0.971	0.931	0.763
TOMLAB/LGO	0.971	0.947	0.918	0.873	0.731
TOMLAB/MULTIMIN	0.992	0.984	0.980	0.963	0.837
TOMLAB/OQNLP	0.980	0.967	0.959	0.894	0.739

Table 14 Fraction of nonconvex smooth problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

	τ				
Solver	1E-1	1E-2	1E-3	1E-6	0E+0
ASA	0.667	0.444	0.222	0.111	0.000
BOBYQA	0.889	0.833	0.778	0.389	0.000
CMA-ES	0.944	0.889	0.833	0.556	0.167
DAKOTA/DIRECT	0.889	0.778	0.778	0.500	0.167
DAKOTA/EA	0.778	0.556	0.444	0.111	0.056
DAKOTA/PATTERN	0.889	0.833	0.778	0.333	0.000
DAKOTA/SOLIS-WETS	0.889	0.833	0.778	0.333	0.056
DFO	0.778	0.722	0.500	0.278	0.000
FMINSEARCH	0.889	0.667	0.556	0.278	0.056
GLOBAL	0.833	0.667	0.611	0.222	0.056
HOPSPACK	0.833	0.833	0.722	0.444	0.056
IMFIL	0.889	0.833	0.556	0.333	0.056
MCS	0.889	0.833	0.833	0.556	0.167
NEWUOA	0.889	0.833	0.778	0.389	0.000
NOMAD	0.889	0.889	0.778	0.444	0.056
PSWARM	0.944	0.833	0.833	0.556	0.056
SID-PSM	0.944	0.833	0.778	0.556	0.111
SNOBFIT	0.944	0.778	0.778	0.333	0.056
TOMLAB/GLCCLUSTER	0.889	0.833	0.833	0.556	0.222
TOMLAB/LGO	0.944	0.889	0.778	0.611	0.222
TOMLAB/MULTIMIN	0.944	0.833	0.833	0.722	0.167
TOMLAB/OQNLP	0.944	0.833	0.833	0.556	0.167

Table 15 Fraction of nonconvex nonsmooth problems, as a function of τ values, for which starting points were improved within 2500 function evaluations

4 Minimum number of solvers required to solve test problems for various limits of function evaluations (best solver performance)

In the following tables, a zero value indicates that the corresponding solver is not part of the minimum number of solvers. A nonzero value indicates the percentage of problems solved by the solver. The sum of nonzero entries across a column provides the percentage of problems that can be solved by the minimal set of solvers for the corresponding number of function evaluations.

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.002	0.004	0.000	0.000	0.000
CMA-ES	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/DIRECT	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.000	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.000	0.000	0.000	0.000
IMFIL	0.022	0.022	0.018	0.012	0.012	0.010
MCS	0.281	0.018	0.002	0.002	0.002	0.000
NEWUOA	0.014	0.000	0.000	0.000	0.000	0.000
NOMAD	0.000	0.000	0.000	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.042	0.060	0.008	0.026	0.008	0.006
SNOBFIT	0.002	0.004	0.068	0.002	0.002	0.002
TOMLAB/GLCCLUSTER	0.026	0.424	0.014	0.078	0.080	0.072
TOMLAB/LGO	0.004	0.008	0.006	0.004	0.002	0.002
TOMLAB/MULTIMIN	0.000	0.034	0.534	0.598	0.625	0.655
TOMLAB/OQNLP	0.000	0.002	0.028	0.012	0.024	0.020

Table 16 Minimum number of solvers required to solve test problems for various limits of function evaluations (best solver performance)

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.000	0.000	0.000	0.000	0.000
CMA-ES	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/DIRECT	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.000	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.000	0.000	0.000	0.000
IMFIL	0.000	0.000	0.000	0.000	0.000	0.000
MCS	0.000	0.000	0.000	0.000	0.000	0.000
NEWUOA	0.000	0.000	0.000	0.000	0.000	0.000
NOMAD	0.000	0.000	0.000	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.000	0.000	0.000	0.000	0.000	0.000
SNOBFIT	0.128	0.026	0.051	0.000	0.000	0.000
TOMLAB/GLCCLUSTER	0.013	0.385	0.590	0.756	0.782	0.795
TOMLAB/LGO	0.000	0.000	0.000	0.000	0.000	0.000
TOMLAB/MULTIMIN	0.000	0.000	0.000	0.000	0.000	0.000
TOMLAB/OQNLP	0.000	0.000	0.000	0.000	0.000	0.000

Table 17 Minimum number of solvers required to solve convex smooth test problems for various limits of function evaluations (best solver performance)

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.000	0.000	0.000	0.000	0.000
CMA-ES	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/DIRECT	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.019	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.000	0.000	0.000	0.000
IMFIL	0.062	0.068	0.056	0.037	0.037	0.031
MCS	0.075	0.006	0.000	0.000	0.000	0.000
NEWUOA	0.025	0.000	0.000	0.000	0.006	0.006
NOMAD	0.000	0.000	0.000	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.000	0.000	0.000	0.000	0.000	0.000
SNOBFIT	0.000	0.000	0.000	0.000	0.000	0.000
TOMLAB/GLCCLUSTER	0.012	0.199	0.311	0.062	0.062	0.056
TOMLAB/LGO	0.000	0.000	0.019	0.019	0.000	0.000
TOMLAB/MULTIMIN	0.000	0.037	0.050	0.366	0.416	0.447
TOMLAB/OQNLP	0.000	0.000	0.000	0.006	0.006	0.012

Table 18 Minimum number of solvers required to solve convex nonsmooth test problems for various limits of function evaluations (best solver performance)

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.008	0.008	0.012	0.000	0.000	0.000
CMA-ES	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/DIRECT	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.000	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.000	0.000	0.000	0.000
IMFIL	0.000	0.000	0.000	0.000	0.000	0.000
MCS	0.490	0.657	0.004	0.004	0.004	0.000
NEWUOA	0.000	0.000	0.000	0.000	0.000	0.000
NOMAD	0.000	0.000	0.000	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.041	0.037	0.069	0.073	0.073	0.065
SNOBFIT	0.000	0.000	0.000	0.008	0.004	0.004
TOMLAB/GLCCLUSTER	0.065	0.000	0.004	0.004	0.004	0.004
TOMLAB/LGO	0.012	0.020	0.000	0.000	0.000	0.000
TOMLAB/MULTIMIN	0.000	0.073	0.751	0.796	0.800	0.812
TOMLAB/OQNLP	0.000	0.000	0.029	0.012	0.024	0.024

Table 19 Minimum number of solvers required to solve nonconvex smooth test problems for various limits of function evaluations (best solver performance)

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.000	0.000	0.000	0.000	0.000
CMA-ES	0.000	0.000	0.000	0.444	0.444	0.500
DAKOTA/DIRECT	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.000	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.000	0.000	0.000	0.000
IMFIL	0.056	0.056	0.056	0.056	0.056	0.056
MCS	0.000	0.000	0.000	0.000	0.000	0.000
NEWUOA	0.222	0.000	0.000	0.000	0.056	0.000
NOMAD	0.000	0.056	0.056	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.000	0.000	0.000	0.000	0.000	0.000
SNOBFIT	0.000	0.000	0.000	0.000	0.000	0.000
TOMLAB/GLCCLUSTER	0.000	0.000	0.000	0.000	0.000	0.000
TOMLAB/LGO	0.000	0.000	0.389	0.000	0.000	0.000
TOMLAB/MULTIMIN	0.056	0.333	0.000	0.056	0.000	0.056
TOMLAB/OQNLP	0.000	0.000	0.000	0.000	0.000	0.000

Table 20 Minimum number of solvers required to solve nonconvex nonsmooth test problems for various limits of function evaluations (best solver performance)

5 Fraction of problems solved as a function of problem size

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.071	0.081	0.081	0.091	0.101	0.101
BOBYQA	0.343	0.515	0.566	0.586	0.586	0.586
CMA-ES	0.162	0.566	0.717	0.747	0.747	0.747
DAKOTA/DIRECT	0.384	0.404	0.404	0.404	0.404	0.404
DAKOTA/EA	0.101	0.131	0.141	0.152	0.162	0.162
DAKOTA/PATTERN	0.162	0.242	0.242	0.242	0.253	0.253
DAKOTA/SOLIS-WETS	0.172	0.303	0.323	0.323	0.323	0.333
DFO	0.202	0.222	0.465	0.545	0.576	0.586
FMINSEARCH	0.384	0.707	0.727	0.727	0.727	0.727
GLOBAL	0.354	0.465	0.465	0.475	0.475	0.475
HOPSPACK	0.475	0.556	0.556	0.556	0.556	0.556
IMFIL	0.172	0.172	0.172	0.172	0.172	0.172
MCS	0.808	0.889	0.899	0.899	0.899	0.909
NEWUOA	0.273	0.475	0.525	0.535	0.535	0.535
NOMAD	0.242	0.626	0.646	0.657	0.667	0.677
PSWARM	0.121	0.404	0.636	0.697	0.707	0.717
SID-PSM	0.394	0.727	0.737	0.747	0.747	0.758
SNOBFIT	0.303	0.485	0.606	0.657	0.707	0.727
TOMLAB/GLCCLUSTER	0.717	0.798	0.879	0.899	0.899	0.949
TOMLAB/LGO	0.444	0.879	0.879	0.899	0.899	0.899
TOMLAB/MULTIMIN	0.616	0.879	0.939	0.939	0.939	0.939
TOMLAB/OQNLP	0.434	0.788	0.869	0.879	0.889	0.899

Table 21 Fraction of problems with one to two variables that were solved

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.015	0.015	0.015	0.015	0.015	0.015
BOBYQA	0.143	0.263	0.331	0.346	0.361	0.368
CMA-ES	0.038	0.120	0.263	0.323	0.406	0.474
DAKOTA/DIRECT	0.105	0.135	0.143	0.143	0.143	0.143
DAKOTA/EA	0.023	0.023	0.030	0.030	0.030	0.030
DAKOTA/PATTERN	0.038	0.083	0.083	0.090	0.098	0.098
DAKOTA/SOLIS-WETS	0.030	0.075	0.090	0.090	0.090	0.090
DFO	0.060	0.083	0.105	0.120	0.150	0.226
FMINSEARCH	0.030	0.211	0.301	0.338	0.338	0.338
GLOBAL	0.045	0.113	0.120	0.120	0.120	0.120
HOPSPACK	0.083	0.211	0.233	0.233	0.241	0.241
IMFIL	0.060	0.090	0.090	0.090	0.090	0.090
MCS	0.338	0.549	0.579	0.624	0.624	0.624
NEWUOA	0.083	0.195	0.256	0.301	0.323	0.323
NOMAD	0.053	0.150	0.263	0.301	0.331	0.331
PSWARM	0.030	0.113	0.165	0.211	0.241	0.278
SID-PSM	0.218	0.429	0.459	0.466	0.466	0.481
SNOBFIT	0.218	0.271	0.293	0.338	0.376	0.391
TOMLAB/GLCCLUSTER	0.383	0.549	0.602	0.617	0.662	0.692
TOMLAB/LGO	0.158	0.466	0.602	0.654	0.669	0.692
TOMLAB/MULTIMIN	0.083	0.586	0.684	0.744	0.752	0.782
TOMLAB/OQNLP	0.173	0.368	0.481	0.541	0.541	0.541

Table 22 Fraction of problems with three to nine variables that were solved

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.025	0.092	0.168	0.252	0.294	0.328
CMA-ES	0.000	0.008	0.025	0.025	0.050	0.084
DAKOTA/DIRECT	0.059	0.067	0.076	0.076	0.076	0.076
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.017	0.017	0.025	0.025	0.025
DAKOTA/SOLIS-WETS	0.000	0.008	0.008	0.017	0.017	0.017
DFO	0.008	0.025	0.025	0.025	0.025	0.025
FMINSEARCH	0.000	0.008	0.008	0.008	0.008	0.017
GLOBAL	0.000	0.017	0.025	0.025	0.025	0.025
HOPSPACK	0.008	0.025	0.050	0.059	0.076	0.084
IMFIL	0.092	0.101	0.101	0.101	0.101	0.101
MCS	0.084	0.277	0.378	0.395	0.420	0.437
NEWUOA	0.000	0.059	0.143	0.193	0.252	0.286
NOMAD	0.000	0.008	0.025	0.034	0.042	0.042
PSWARM	0.000	0.008	0.017	0.025	0.034	0.042
SID-PSM	0.017	0.176	0.277	0.378	0.395	0.403
SNOBFIT	0.084	0.218	0.277	0.277	0.277	0.277
TOMLAB/GLCCLUSTER	0.059	0.412	0.605	0.630	0.647	0.647
TOMLAB/LGO	0.008	0.143	0.261	0.311	0.395	0.412
TOMLAB/MULTIMIN	0.008	0.218	0.462	0.622	0.664	0.681
TOMLAB/OQNLP	0.025	0.176	0.336	0.403	0.462	0.479

Table 23 Fraction of problems with 10 to 30 variables that were solved

	Iterations					
Solver	100	200	500	1000	2000	2500
ASA	0.000	0.000	0.000	0.000	0.000	0.000
BOBYQA	0.000	0.033	0.033	0.053	0.053	0.086
CMA-ES	0.000	0.000	0.000	0.000	0.007	0.013
DAKOTA/DIRECT	0.040	0.040	0.040	0.040	0.040	0.040
DAKOTA/EA	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/PATTERN	0.000	0.000	0.000	0.000	0.000	0.000
DAKOTA/SOLIS-WETS	0.000	0.000	0.000	0.000	0.000	0.000
DFO	0.000	0.000	0.000	0.000	0.000	0.000
FMINSEARCH	0.000	0.000	0.000	0.000	0.000	0.000
GLOBAL	0.000	0.000	0.000	0.000	0.000	0.000
HOPSPACK	0.000	0.000	0.013	0.020	0.020	0.026
IMFIL	0.000	0.000	0.013	0.013	0.013	0.013
MCS	0.040	0.060	0.073	0.152	0.219	0.219
NEWUOA	0.000	0.000	0.007	0.020	0.033	0.033
NOMAD	0.000	0.000	0.000	0.000	0.000	0.000
PSWARM	0.000	0.000	0.000	0.000	0.000	0.000
SID-PSM	0.000	0.013	0.013	0.020	0.020	0.020
SNOBFIT	0.000	0.000	0.066	0.099	0.099	0.099
TOMLAB/GLCCLUSTER	0.000	0.020	0.093	0.219	0.285	0.325
TOMLAB/LGO	0.000	0.020	0.020	0.020	0.026	0.066
TOMLAB/MULTIMIN	0.000	0.040	0.119	0.159	0.225	0.278
TOMLAB/OQNLP	0.000	0.026	0.046	0.093	0.139	0.185

Table 24 Fraction of problems with 31 to 300 variables that were solved

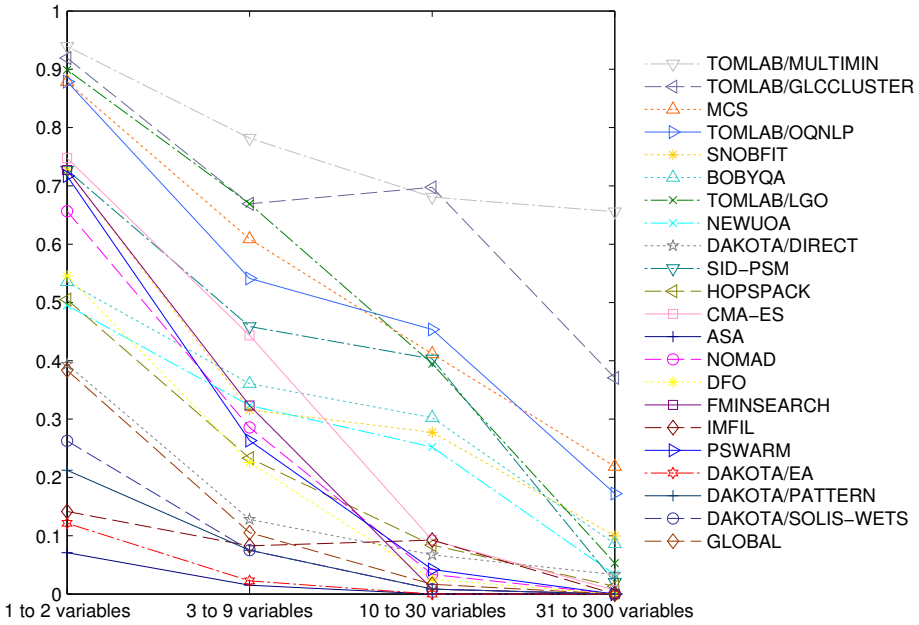

Fig. 1 Fraction of all problems, as a function of problem size, for which a solver found the best solution among all solvers

6 Best, mean, median and worst results over 10 runs and after 2500 function evaluations

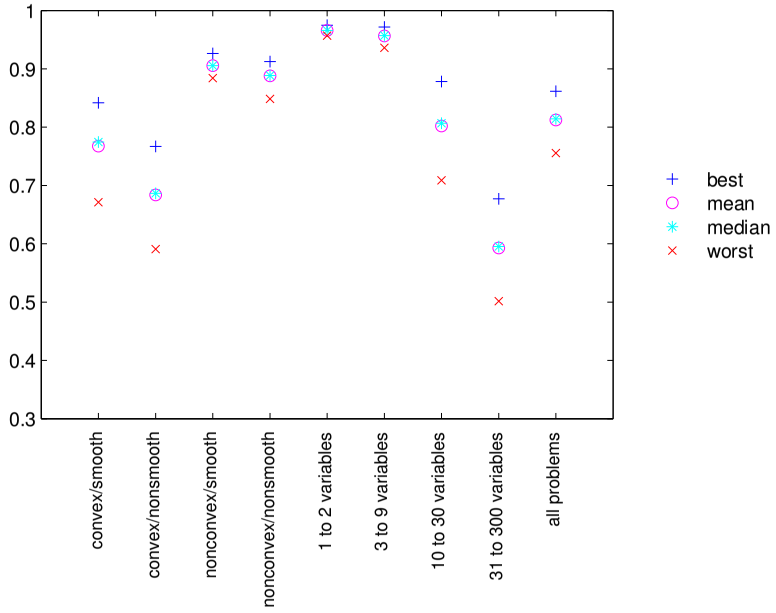

Fig. 2 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver ASA

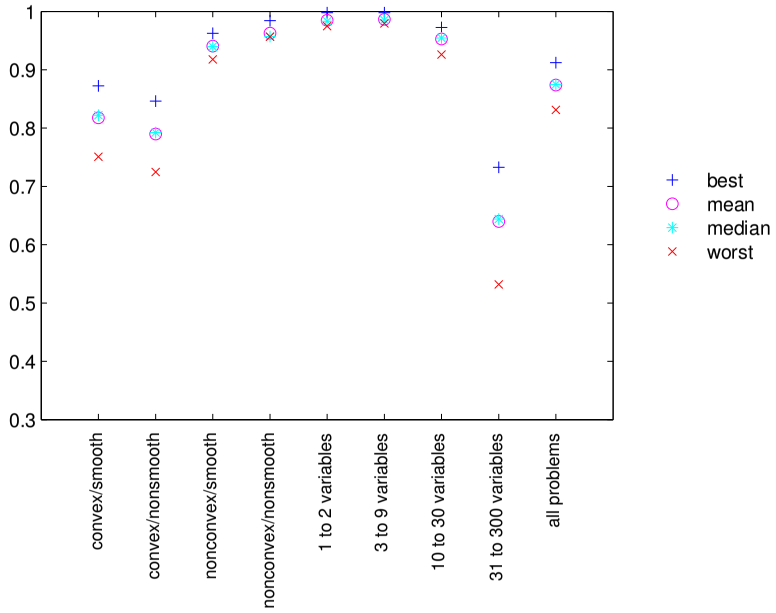

Fig. 3 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver NOMAD

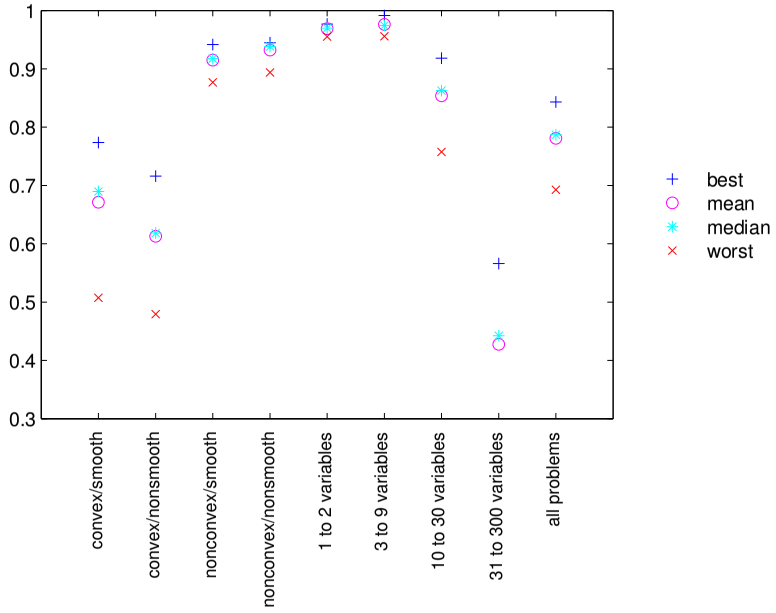

Fig. 4 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver DFO

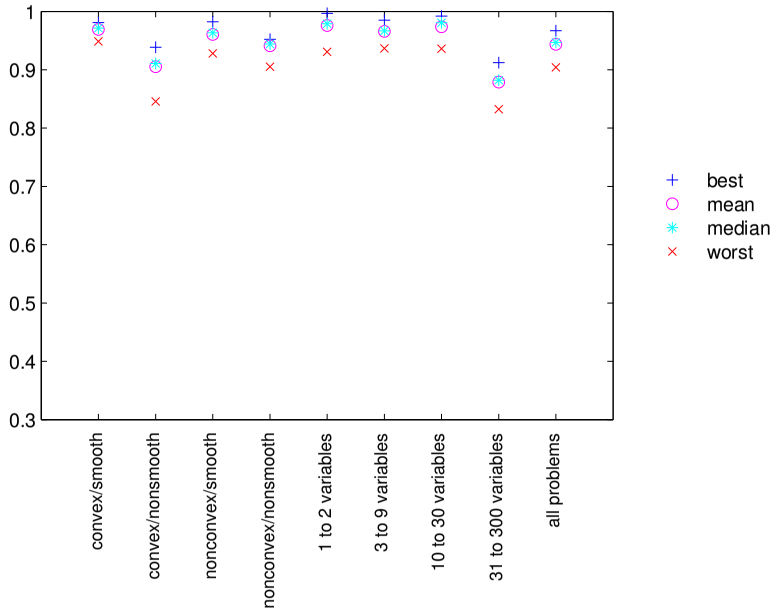

Fig. 5 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver NEWUOA

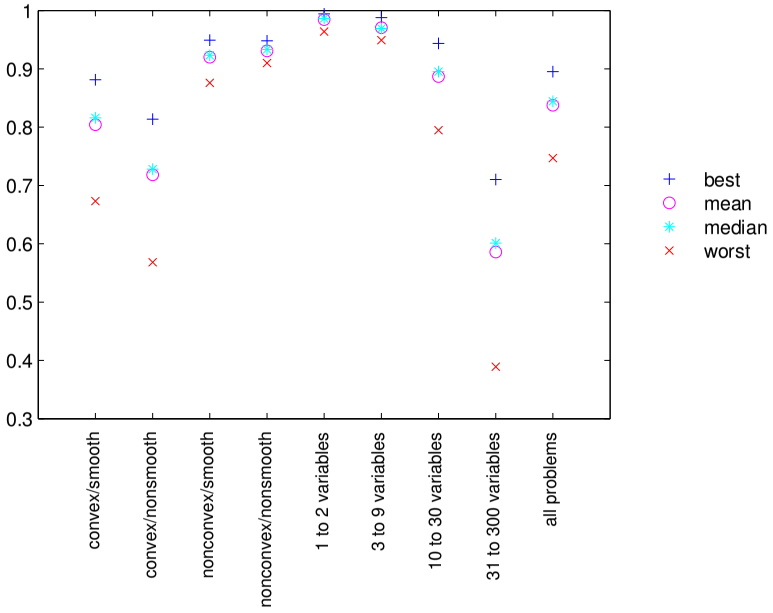

Fig. 6 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver FMINSEARCH

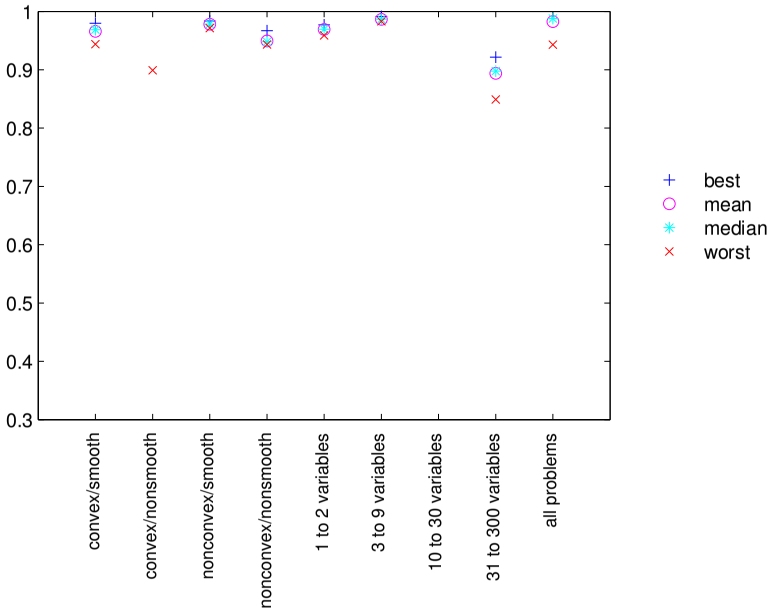

Fig. 7 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver IMFIL

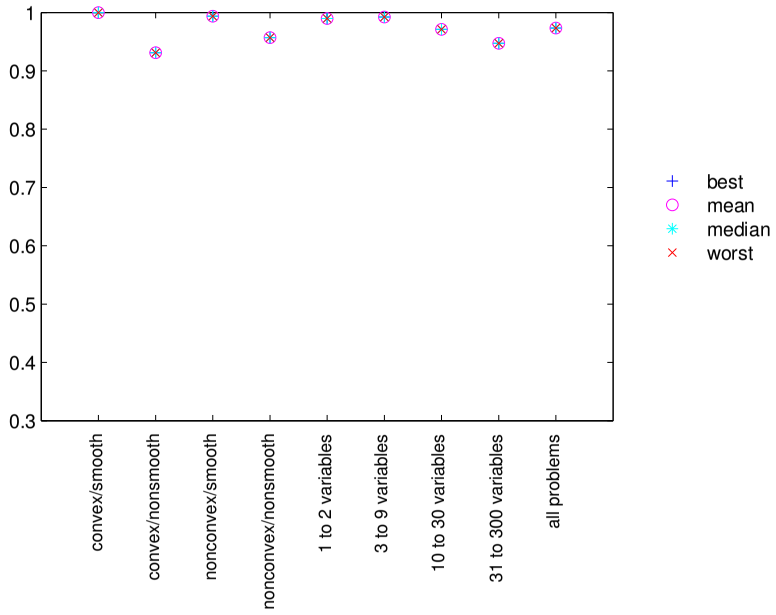

Fig. 8 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver MCS

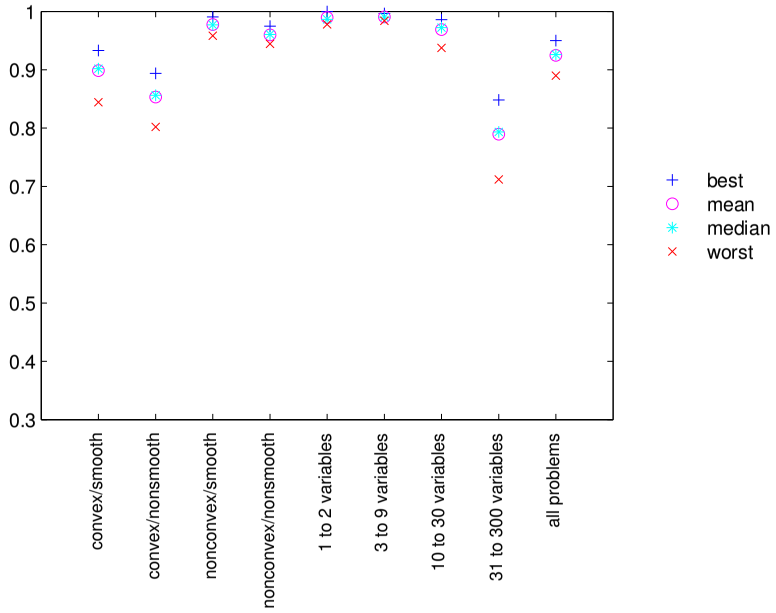

Fig. 9 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver SID-PSM

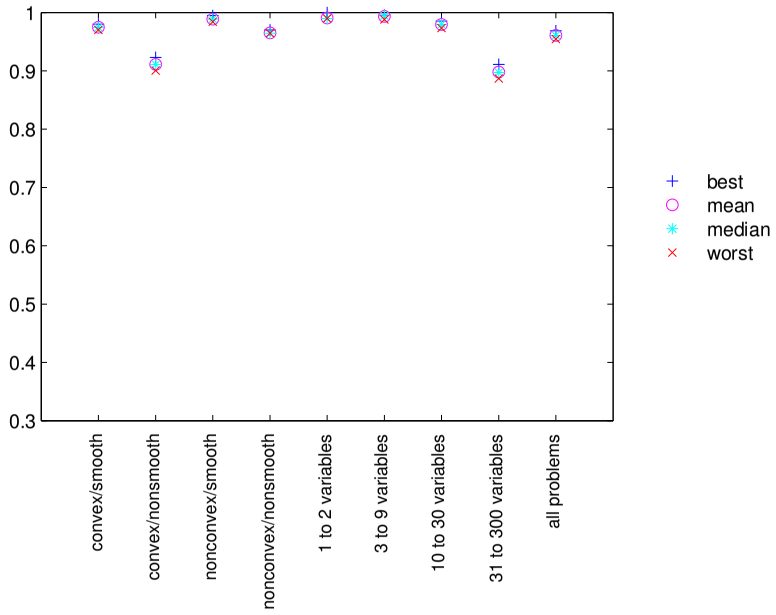

Fig. 10 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver PSWARM

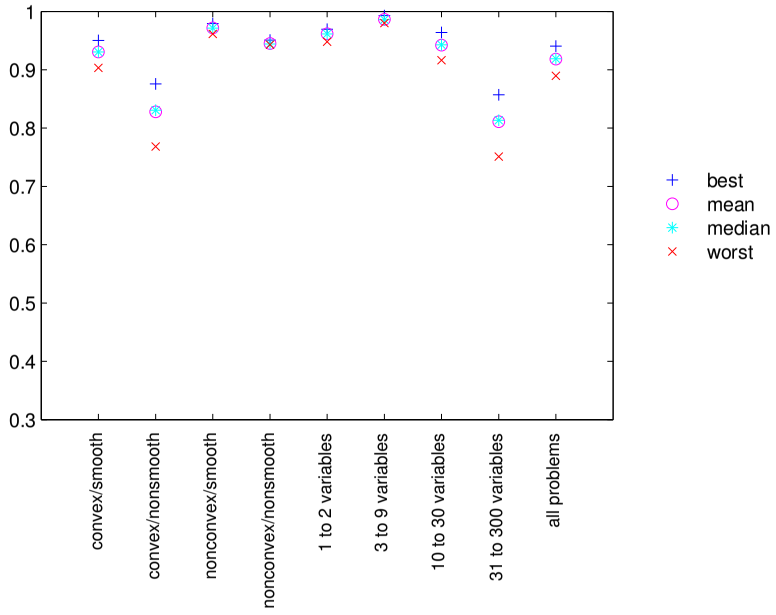

Fig. 11 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver HOPSPACK

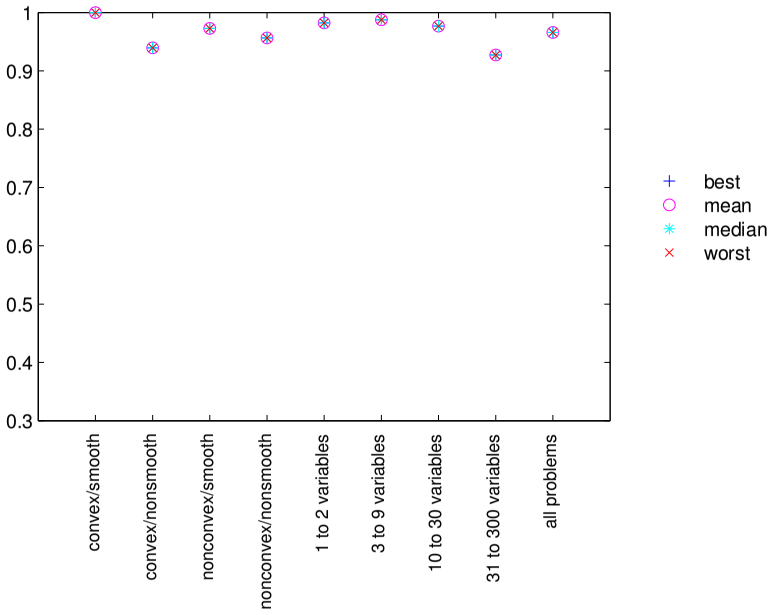

Fig. 12 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver DAKOTA/DIRECT

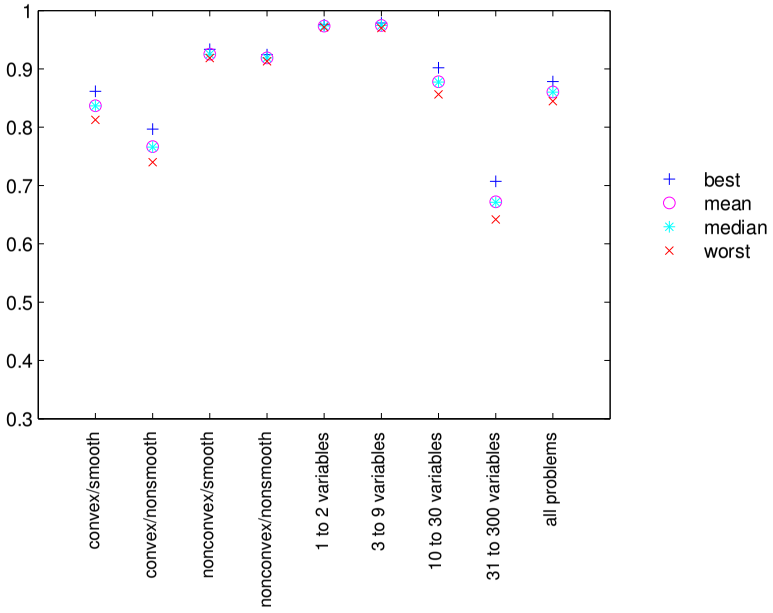

Fig. 13 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver DAKOTA/EA

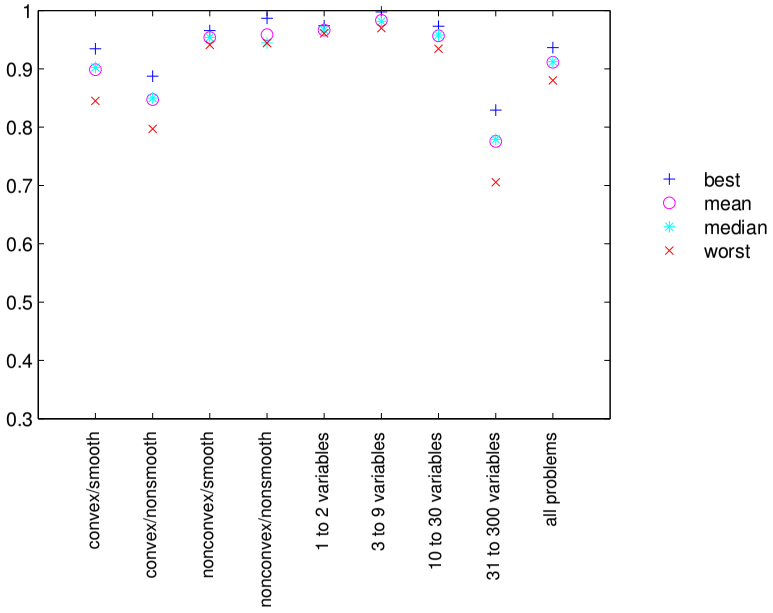

Fig. 14 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver DAKOTA/PATTERN

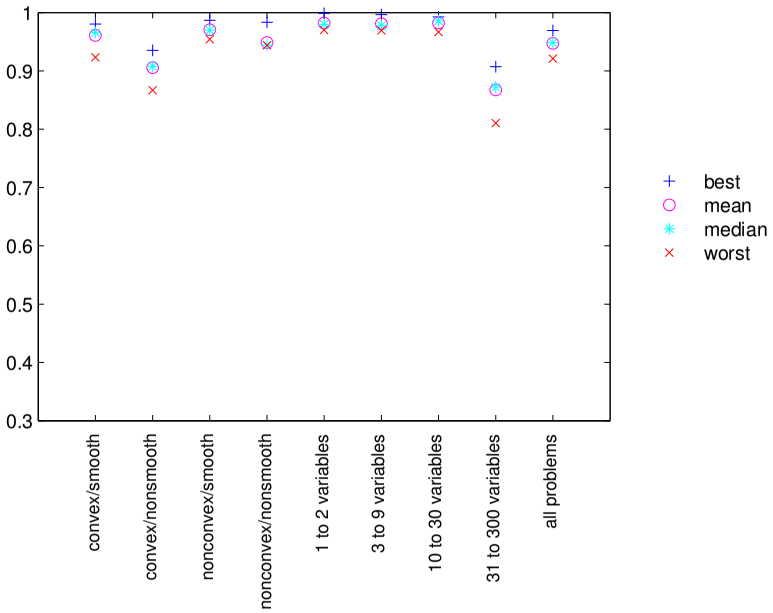

Fig. 15 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver DAKOTA/SOLIS-WETS

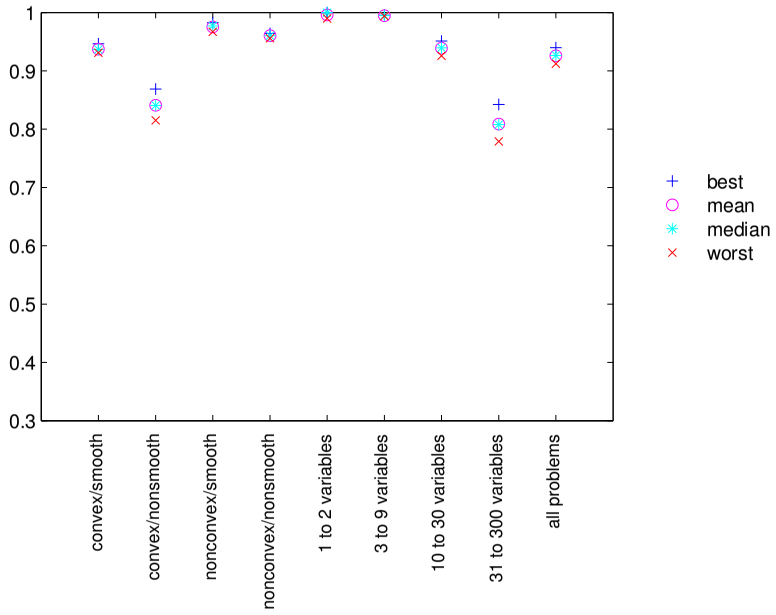

Fig. 16 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver SNOBFIT

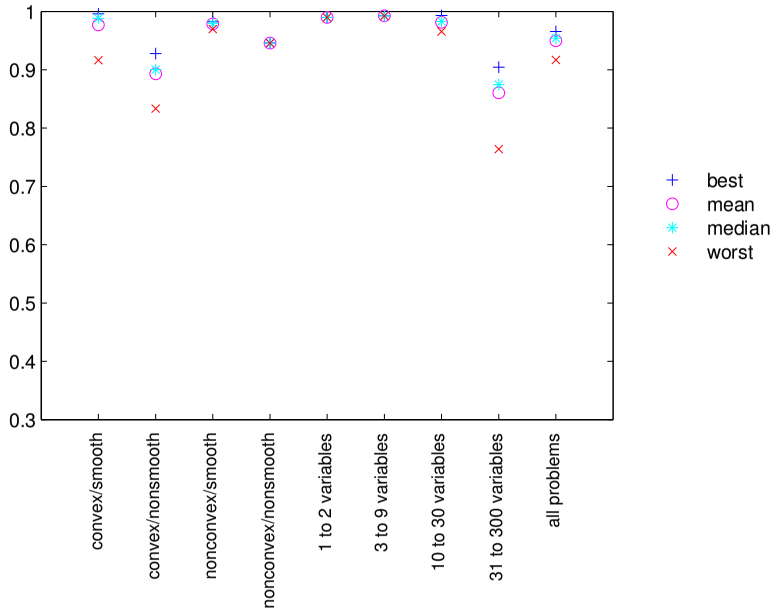

Fig. 17 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver TOMLAB/LGO

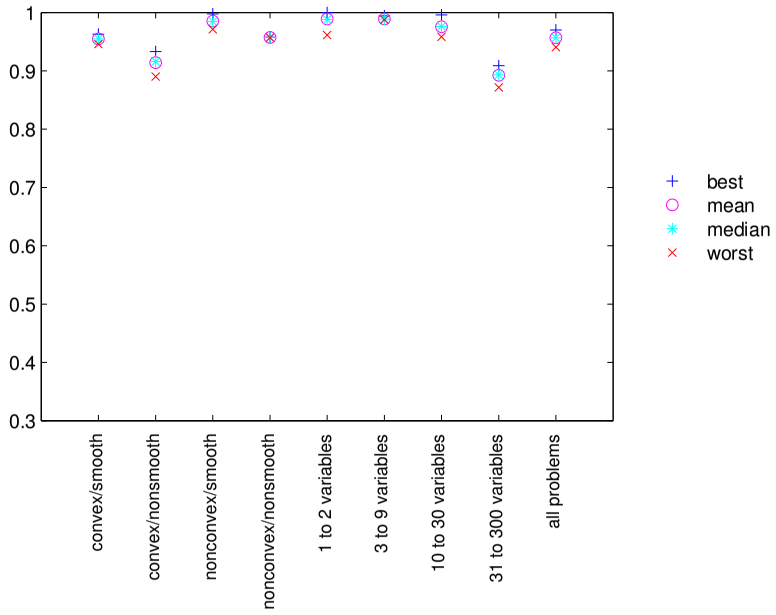

Fig. 18 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver CMA-ES

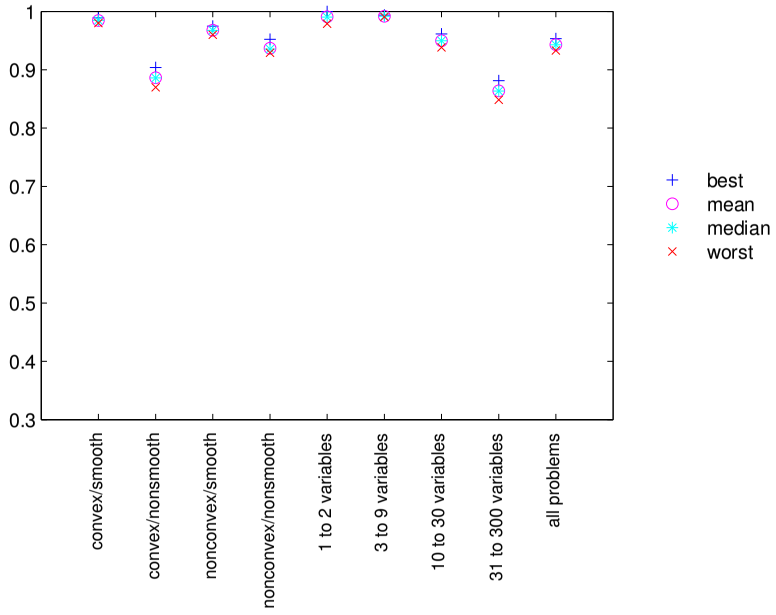

Fig. 19 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver GLOBAL

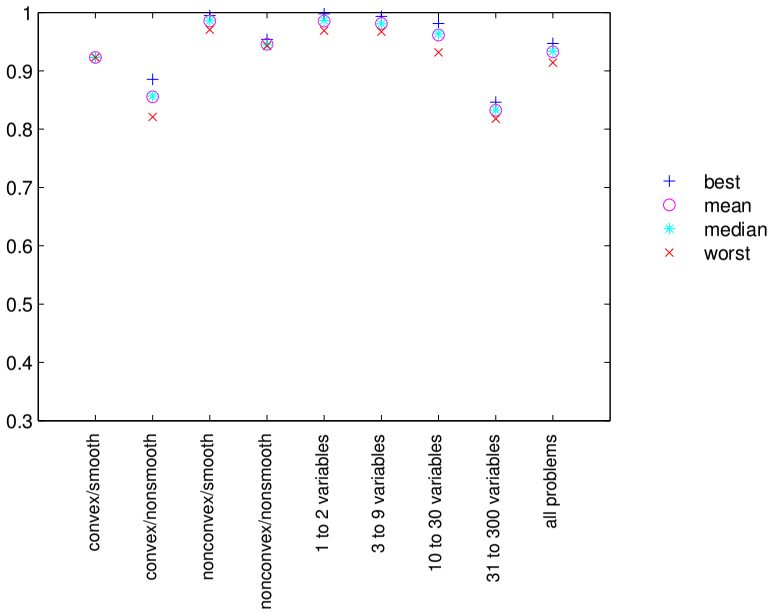

Fig. 20 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver BOBYQA

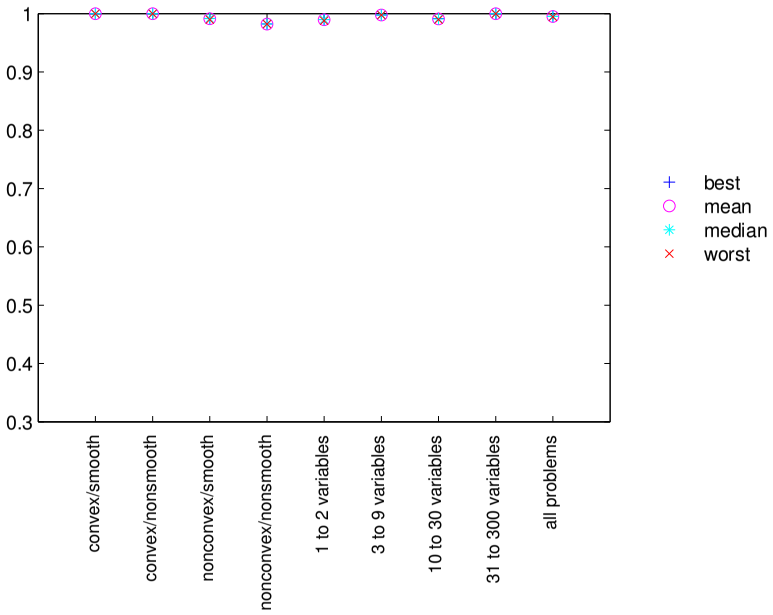

Fig. 21 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver TOMLAB/MULTIMIN

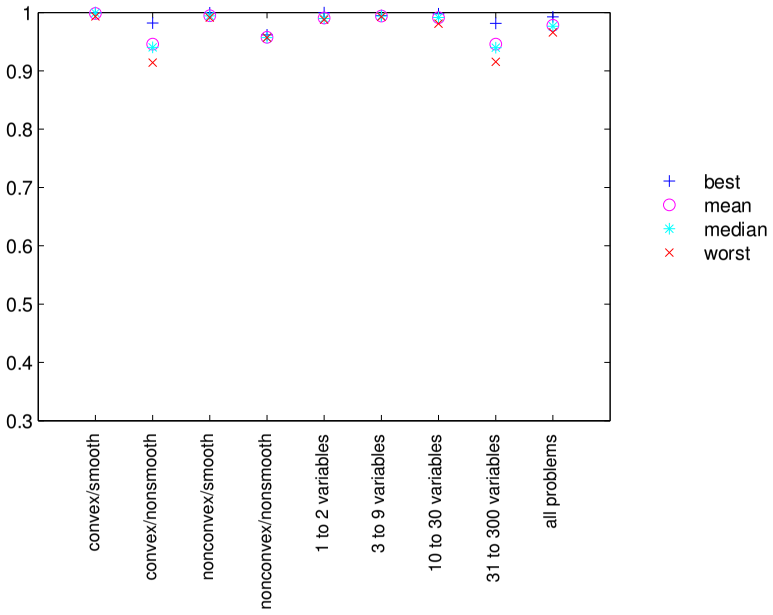

Fig. 22 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver TOMLAB/OQNLP

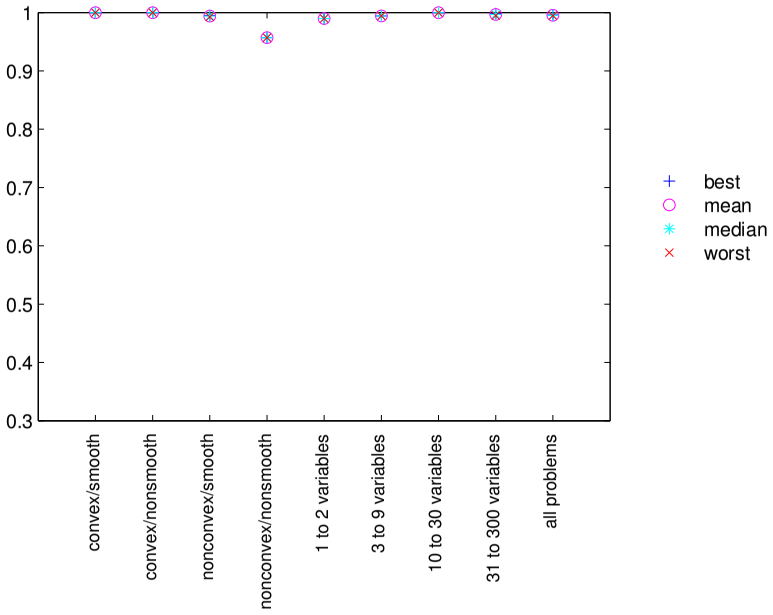

Fig. 23 Best, mean, median and worst results over 10 runs and after 2500 function evaluations for solver TOMLAB/GLCCLUSTER

7 Refinement ability.

Solver	Number of variables			
	1 to 2 variables	3 to 9 variables	10 to 30 variables	31 to 300 variables
ASA	0.838	0.474	0.067	0.066
BOBYQA	0.960	0.880	0.655	0.510
CMA-ES	0.980	0.940	0.672	0.318
DAKOTA/DIRECT	0.939	0.684	0.471	0.132
DAKOTA/EA	0.939	0.594	0.118	0.073
DAKOTA/PATTERN	0.939	0.767	0.555	0.106
DAKOTA/SOLIS-WETS	0.960	0.850	0.639	0.258
DFO	0.960	0.835	0.597	0.086
FMINSEARCH	0.960	0.692	0.437	0.146
GLOBAL	0.970	0.835	0.336	0.073
HOPSPACK	0.899	0.662	0.605	0.331
IMFIL	0.919	0.759	0.647	0.417
MCS	0.980	0.789	0.613	0.351
NEWUOA	0.970	0.895	0.672	0.490
NOMAD	0.960	0.835	0.479	0.086
PSWARM	0.970	0.835	0.622	0.179
SID-PSM	0.980	0.940	0.655	0.232
SNOBFIT	0.980	0.917	0.588	0.166
TOMLAB/GLCCLUSTER	0.970	0.955	0.697	0.391
TOMLAB/LGO	0.980	0.962	0.689	0.464
TOMLAB/MULTIMIN	0.960	0.917	0.689	0.391
TOMLAB/OQNLP	0.970	0.925	0.689	0.530

Table 25 Fraction of all problems solved from a near-optimal solution

Solver	Number of variables			
	1 to 2 variables	3 to 9 variables	10 to 30 variables	31 to 300 variables
ASA	0.000	1.000	0.000	0.000
BOBYQA	0.000	1.000	0.833	0.524
CMA-ES	0.000	1.000	0.833	0.190
DAKOTA/DIRECT	0.000	1.000	0.500	0.095
DAKOTA/EA	0.000	1.000	0.033	0.000
DAKOTA/PATTERN	0.000	1.000	0.600	0.000
DAKOTA/SOLIS-WETS	0.000	1.000	0.833	0.095
DFO	0.000	1.000	0.833	0.000
FMINSEARCH	0.000	1.000	0.767	0.238
GLOBAL	0.000	1.000	0.333	0.000
HOPSPACK	0.000	1.000	0.833	0.286
IMFIL	0.000	1.000	0.833	0.405
MCS	0.000	1.000	0.833	0.452
NEWUOA	0.000	1.000	0.833	0.548
NOMAD	0.000	1.000	0.533	0.000
PSWARM	0.000	1.000	0.767	0.071
SID-PSM	0.000	1.000	0.767	0.143
SNOBFIT	0.000	1.000	0.833	0.286
TOMLAB/GLCCLUSTER	0.000	1.000	0.833	0.548
TOMLAB/LGO	0.000	1.000	0.833	0.500
TOMLAB/MULTIMIN	0.000	1.000	0.833	0.548
TOMLAB/OQNLP	0.000	1.000	0.833	0.524

Table 26 Fraction of convex smooth problems solved from a near-optimal solution

Solver	Number of variables			
	1 to 2 variables	3 to 9 variables	10 to 30 variables	31 to 300 variables
ASA	0.556	0.368	0.034	0.014
BOBYQA	0.667	0.684	0.424	0.324
CMA-ES	0.778	1.000	0.492	0.297
DAKOTA/DIRECT	0.778	0.632	0.407	0.068
DAKOTA/EA	0.778	0.579	0.034	0.027
DAKOTA/PATTERN	0.778	0.737	0.458	0.081
DAKOTA/SOLIS-WETS	0.778	0.842	0.458	0.270
DFO	0.778	0.579	0.356	0.027
FMINSEARCH	0.778	0.632	0.085	0.027
GLOBAL	0.778	0.684	0.220	0.027
HOPSPACK	0.778	0.632	0.407	0.270
IMFIL	0.778	0.737	0.441	0.324
MCS	0.778	0.579	0.390	0.203
NEWUOA	0.778	0.737	0.458	0.338
NOMAD	0.778	0.737	0.339	0.041
PSWARM	0.778	0.737	0.458	0.176
SID-PSM	0.778	0.842	0.475	0.108
SNOBFIT	0.778	0.895	0.390	0.041
TOMLAB/GLCCLUSTER	0.778	1.000	0.508	0.311
TOMLAB/LGO	0.778	0.947	0.492	0.324
TOMLAB/MULTIMIN	0.778	1.000	0.542	0.216
TOMLAB/OQNLP	0.778	1.000	0.492	0.338

Table 27 Fraction of convex nonsmooth problems solved from a near-optimal solution

Solver	Number of variables			
	1 to 2 variables	3 to 9 variables	10 to 30 variables	31 to 300 variables
ASA	0.872	0.454	0.222	0.257
BOBYQA	0.988	0.938	1.000	0.886
CMA-ES	1.000	0.918	0.889	0.514
DAKOTA/DIRECT	0.953	0.691	0.630	0.314
DAKOTA/EA	0.953	0.567	0.407	0.257
DAKOTA/PATTERN	0.953	0.763	0.778	0.286
DAKOTA/SOLIS-WETS	0.977	0.825	0.852	0.429
DFO	0.988	0.897	0.889	0.314
FMINSEARCH	0.977	0.680	0.852	0.286
GLOBAL	0.988	0.845	0.593	0.257
HOPSPACK	0.919	0.680	0.852	0.514
IMFIL	0.942	0.753	0.926	0.629
MCS	1.000	0.835	0.926	0.543
NEWUOA	0.988	0.918	1.000	0.743
NOMAD	0.977	0.825	0.778	0.286
PSWARM	0.988	0.825	0.852	0.314
SID-PSM	1.000	0.959	0.963	0.600
SNOBFIT	1.000	0.918	0.778	0.286
TOMLAB/GLCCLUSTER	0.988	0.969	1.000	0.371
TOMLAB/LGO	1.000	0.959	1.000	0.714
TOMLAB/MULTIMIN	0.977	0.897	0.926	0.571
TOMLAB/OQNLP	0.988	0.907	1.000	0.943

Table 28 Fraction of nonconvex smooth problems solved from a near-optimal solution

Solver	Number of variables			
	1 to 2 variables	3 to 9 variables	10 to 30 variables	31 to 300 variables
ASA	0.750	0.545	0.000	0.000
BOBYQA	1.000	0.636	0.333	0.000
CMA-ES	1.000	1.000	0.667	0.000
DAKOTA/DIRECT	1.000	0.545	0.000	0.000
DAKOTA/EA	1.000	0.636	0.000	0.000
DAKOTA/PATTERN	1.000	0.727	0.000	0.000
DAKOTA/SOLIS-WETS	1.000	1.000	0.333	0.000
DFO	0.750	0.636	0.333	0.000
FMINSEARCH	1.000	0.727	0.333	0.000
GLOBAL	1.000	0.909	0.333	0.000
HOPSPACK	0.750	0.364	0.000	0.000
IMFIL	0.750	0.727	0.333	0.000
MCS	1.000	0.636	0.000	0.000
NEWUOA	1.000	0.909	0.333	0.000
NOMAD	1.000	1.000	0.000	0.000
PSWARM	1.000	1.000	0.333	0.000
SID-PSM	1.000	0.909	0.333	0.000
SNOBFIT	1.000	0.909	0.333	0.000
TOMLAB/GLCCLUSTER	1.000	0.727	0.333	0.000
TOMLAB/LGO	1.000	1.000	0.333	0.000
TOMLAB/MULTIMIN	1.000	0.909	0.000	0.000
TOMLAB/OQNLP	1.000	0.909	0.333	0.000

Table 29 Fraction of nonconvex nonsmooth problems solved from a near-optimal solution