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Supplementary material - Approach of generating candidate initial reference points and boxplots of all the results of the four reference point based interactive methods

Lu Chen · Kaisa Miettinen · Bin Xin · Vesa Ojalehto

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Abstract In this material, the approach of generating candidate initial reference points is introduced first. Then, the boxplots of the results obtained by the four reference point based methods (i.e., the reference point method, R-NSGA-II, g-NSGA-II, r-NSGA-II) when utilizing ADM1 or ADM2 on each test problem are presented for an intuitive comparison of the four methods.

1 Generating candidate initial reference points

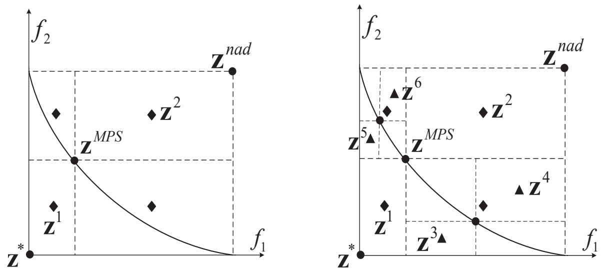
To see the influence of different initial reference points on an ADM, one can adopt initial reference points with different features, e.g., reference points whose objective values can or cannot be improved simultaneously, reference points close to or far from the MPS, and reference points close to or far from the Pareto front. We propose a systematic approach to generate multiple candidate initial reference points in the hypercube constituted by the ideal and nadir objective vectors. The objective values of a half of these reference points can be improved at the same time. For the other candidate initial reference points, achieving the aspiration level of one objective will lead to the sacrifice of some other objectives.

The systematic approach of generating candidate initial reference points is as follows:

1. First, calculate the MPS of the ADM according to the utility function. Obtain a representative set of Pareto optimal solutions by using some multiobjective optimization methods. Define the hypercube constituted by the ideal and nadir

Lu Chen · Bin Xin
School of Automation, Beijing Institute of Technology, 100081, Beijing, China; State Key Laboratory of Intelligent Control and Decision of Complex Systems, 100081, Beijing, China; Beijing Advanced Innovation Center for Intelligent Robots and Systems, 100081, Beijing, China
E-mail: brucebin@bit.edu.cn

Kaisa Miettinen · Vesa Ojalehto
University of Jyvaskyla, Faculty of Information Technology, FI-40014 University of Jyvaskyla, Finland



(a) (b)

Fig. 1 The generation of candidate initial reference points: (a) shows the center points (marked by diamonds) of the second-level rectangles which are determined at the first step. $\mathbf{z}^1$ and $\mathbf{z}^2$ are two candidate initial reference points. (b) exhibits four more candidate initial reference points $\mathbf{z}^3 \sim \mathbf{z}^6$ which are obtained at the second step.

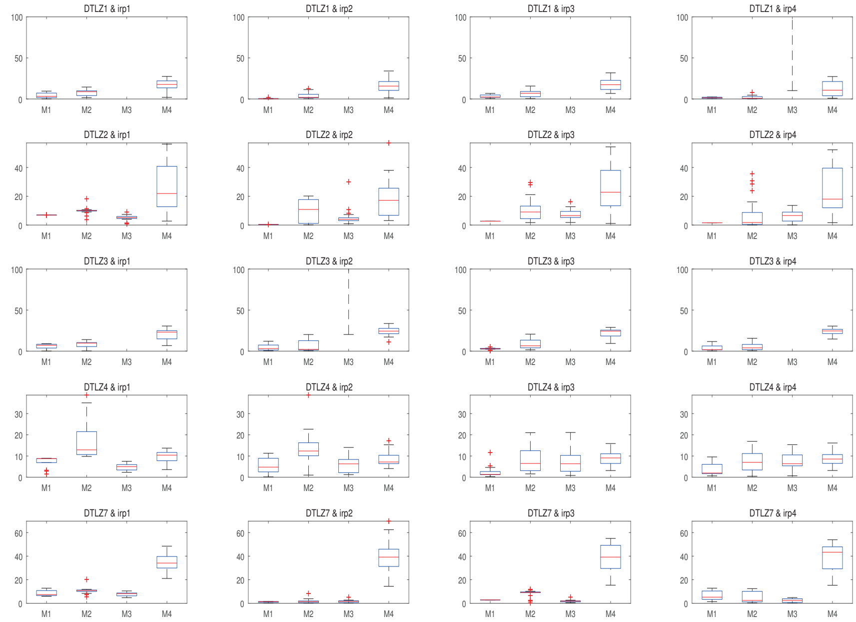
objective vectors as the first-level hypercube. Use the MPS to divide this hypercube into 2^k second-level hypercubes. Find the centers of all second-level hypercubes. The center point of the ideal point and the MPS is a candidate initial reference point. Obviously, it dominates the MPS and cannot be achieved. The center point of the MPS and the nadir objective vector is also a candidate initial reference point. It is dominated by the MPS, which means that all its components can be improved at the same time.

2. Next, for each of the remaining $2^k - 2$ center points, find the solution closest to it from the set of Pareto optimal solutions according to the normalized Euclidean distance. Up to $2^k - 2$ Pareto optimal solutions can be identified and for each of them, the middle point between it and the bottom left vertex of the second-level hypercube where it is located is a candidate initial reference point which dominates it. The middle point between it and the upper right vertex of the corresponding second-level hypercube is also a candidate initial reference point which is dominated by it.

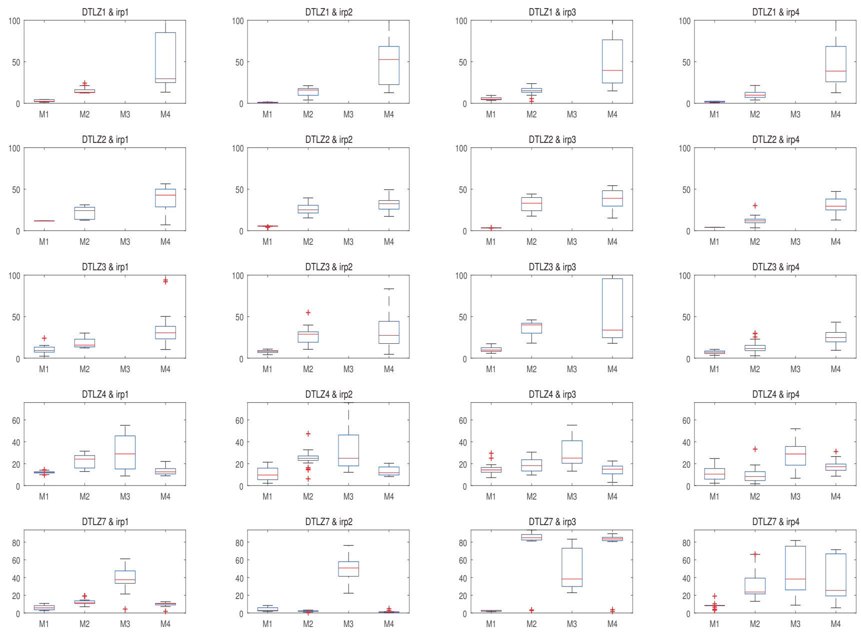
With this approach, we can obtain up to $2(2^k - 2) + 2$ candidate initial reference points. Each of a half of them dominates at least one Pareto optimal solution and cannot be achieved. Each of the other half is dominated by at least one Pareto optimal solution and therefore, all its components can be improved. Fig. 1 illustrates how to generate candidate initial reference points in a bi-objective case. Six candidates are generated. We can test with all of them or a part of them according to our needs.

2 Boxplots of the experimental results

The boxplots of the *difference/distance* values over 21 independent runs of each method are presented in Figs. 2-5. M1, M2, M3, and M4 represent the reference point method, R-NSGA-II, g-NSGA-II, and r-NSGA-II, respectively. The maximum *difference* value and *distance* value of all boxes are set to be no more than 100% and 2, respectively. Values exceeding these two numbers are not shown. From Figs. 2-5, we can obtain similar observations to what we have derived in the main body of the paper.

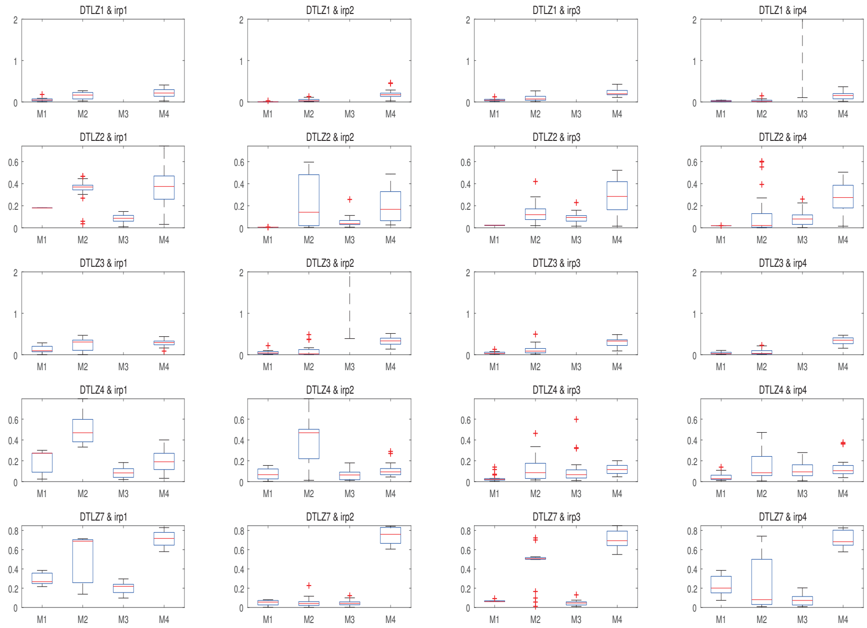


(a)

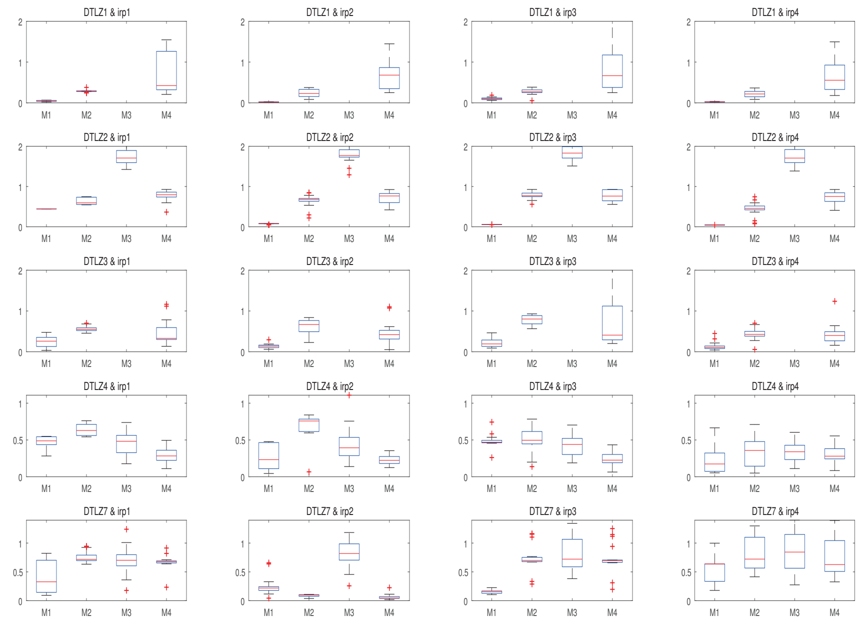


(b)

Fig. 2 Boxplots of *difference* values (%) over 21 independent runs for each method when using ADM1 to compare methods: (a) 3-objective test problems. (b) 5-objective test problems.

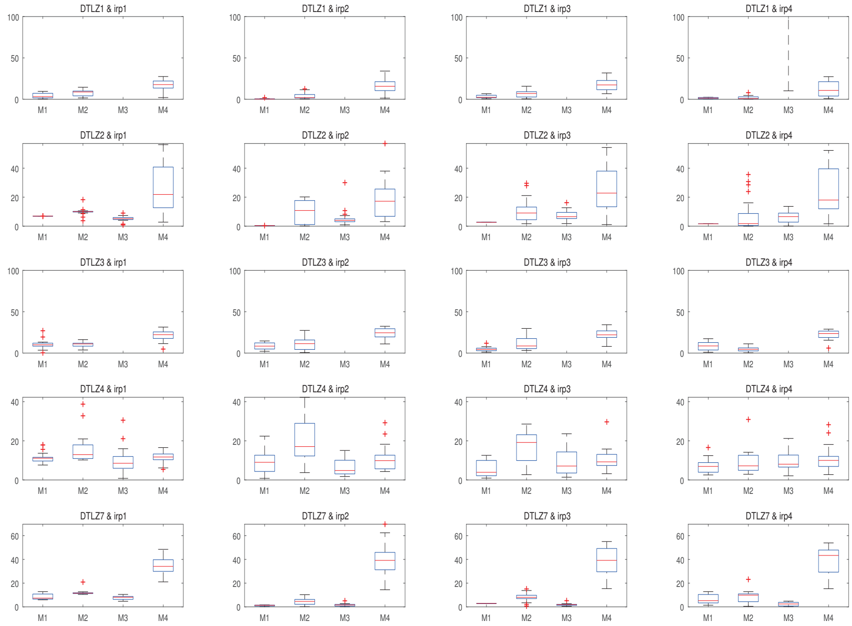


(a)

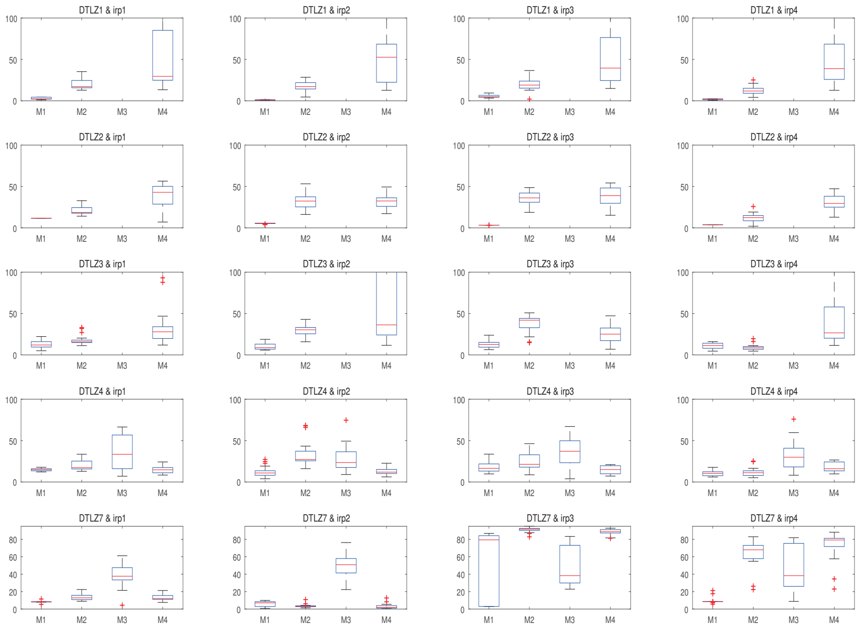


(b)

Fig. 3 Boxplots of *distance* values (%) over 21 independent runs for each method when using ADM1 to compare methods: (a) 3-objective test problems. (b) 5-objective test problems.

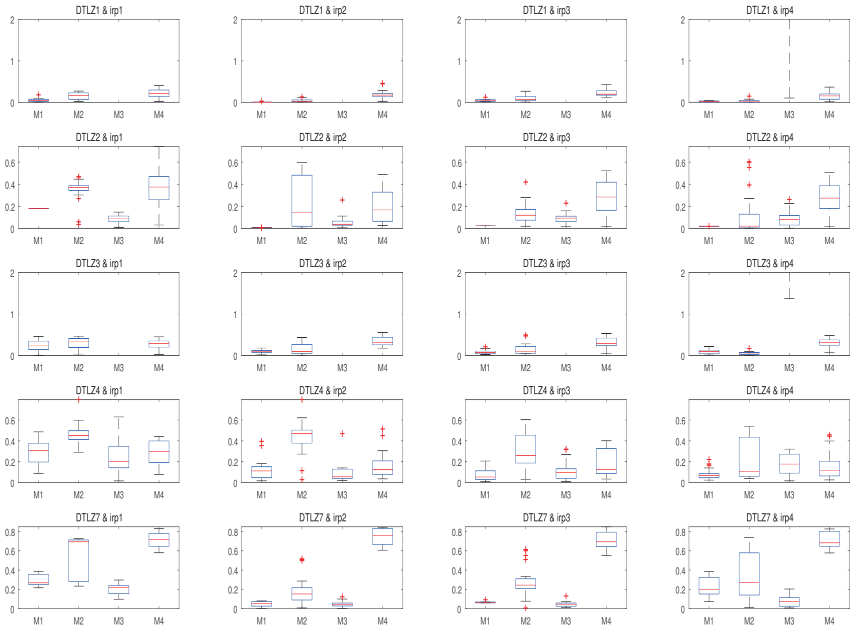


(a)



(b)

Fig. 4 Boxplots of *difference* values (%) over 21 independent runs for each method when using ADM2 to compare methods: (a) 3-objective test problems. (b) 5-objective test problems.



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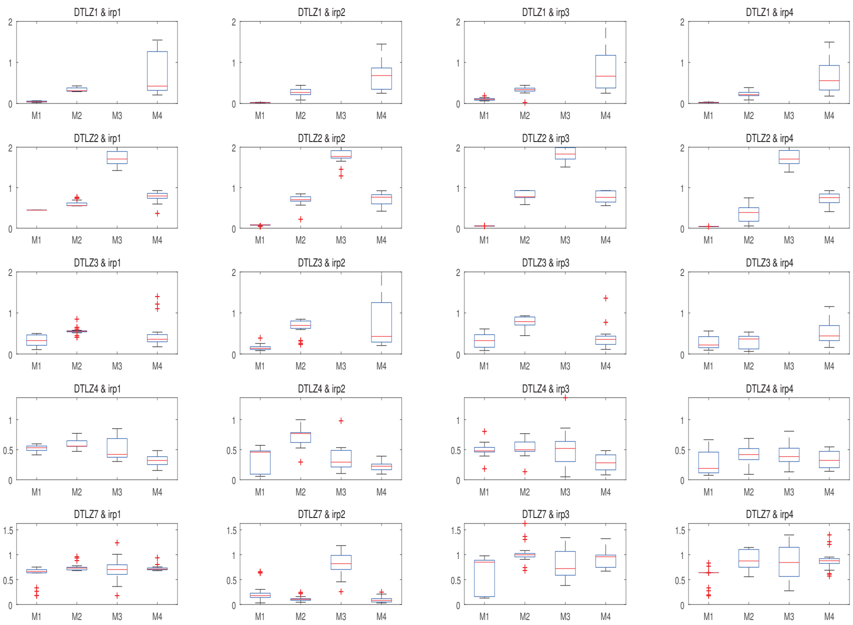

(b)

Fig. 5 Boxplots of *distance* values (%) over 21 independent runs for each method when using ADM2 to compare methods: (a) 3-objective test problems. (b) 5-objective test problems.