Supplement to "Partitioned Hybrid Learning of Bayesian Network Structures"

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This document contains supplementary information for the paper "Partitioned Hybrid Learning of Bayesian Network Structures," which we will refer to in this document as the main text.

Contents

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S1 Gaussian Results

In this section, we report and briefly discuss some preliminary results on the application of HGI to ARGES.

Adaptively restricted greedy equivalence search (ARGES) is a hybrid adaptation of

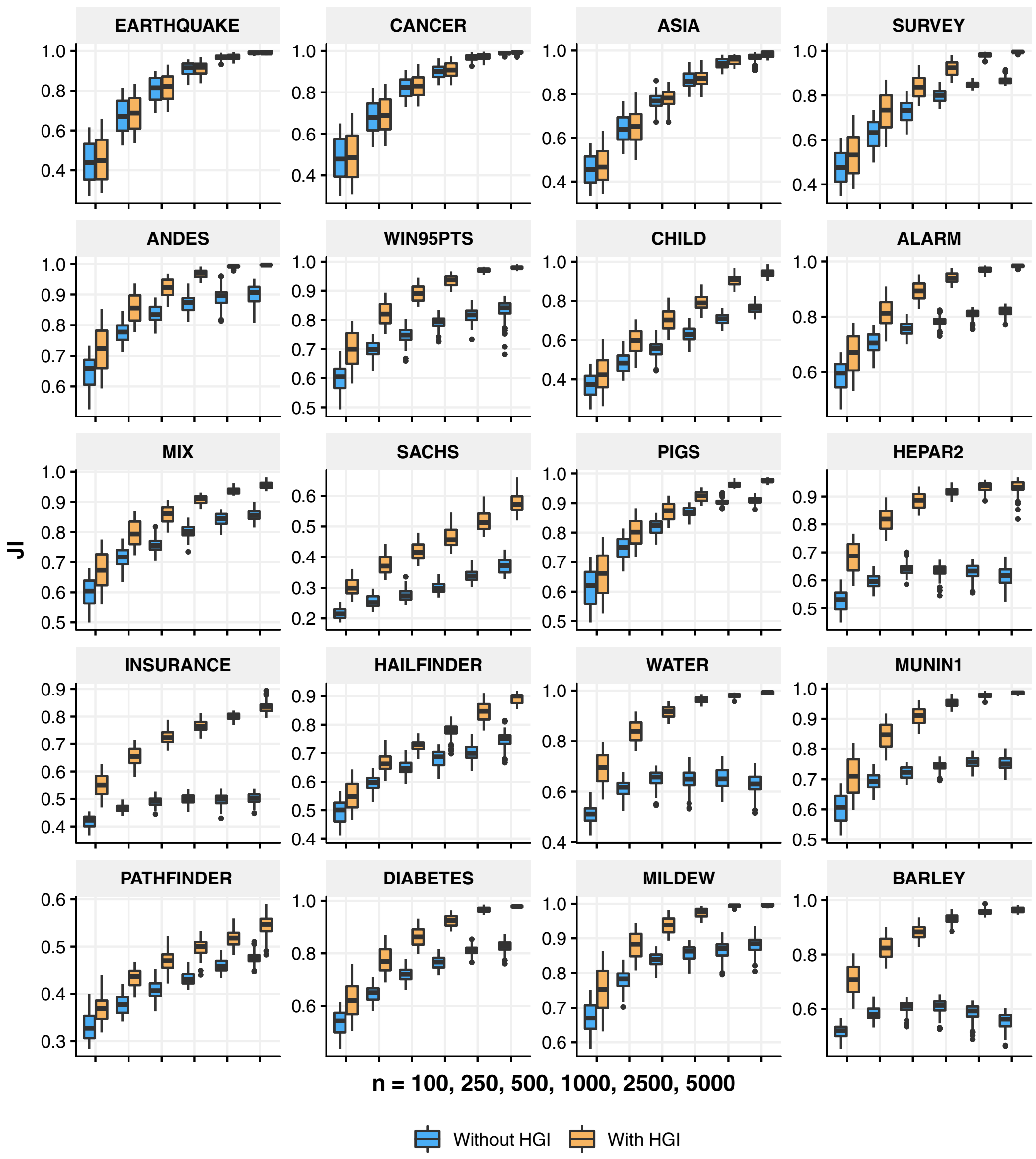

Fig. S1 Boxplots comparing the JI of ARGES without and with HGI for various networks and sample sizes, restricted to the skeleton or CIG of the underlying DAG greedy equivalence search (GES) that achieves asymptotic recovery of the CPDAG of the underlying DAG given either a consistent estimation algorithm for the skeleton or the conditional independence graph (CIG), the two variants respectively referred to as ARGES-skeleton and ARGES-CIG (?). The CIG of the joint distribution of variables $\mathbf{X} = \{X_1, \ldots, X_p\}$ represented by nodes $\mathbf{V} = \{1, \ldots, p\}$ is the undirected graph where i and j are adjacent if and only if $(X_i \not\perp\!\!\!\perp X_j \,|\, \mathbf{X} \setminus \{X_i, X_j\})_P$, and is a supergraph of the skeleton.

For our simulations, we again considered the network structures in Table 1 of the main text. For each DAG structure, the edge weights β_{ij} were sampled uniformly from $[-1, -0.1] \cup [0.1, 1]$ for $i \to j \in \mathbf{E}$, with $\beta_{ij} = 0$ otherwise. Similarly, the zero mean error variances σ_i^2, $i = 1, \ldots, p$ were drawn uniformly from $[1, 2]$. For each network configuration, we generated $N = 10$ datasets with sample sizes $n \in \{100, 250, 500, 1000, 2500, 5000\}$. For each dataset and algorithm, we applied the algorithm with ten score penalties $\lambda \in \mathcal{L} \coloneqq \{0.5, 0.7, \ldots, 2.3\}$ (BIC penalty being $\lambda = 0.5$). Note that due to the restriction to the true skeleton, λ functions largely to adjust the search traversal through the space of equivalence classes rather than control the sparsity. The detection of v-structures for HGI was accomplished with a single significance level threshold $\alpha = 0.01$.

The results are summarized in Fig. S1. For ARGES without HGI, we report the highest JI of ARGES-skeleton and ARGES-CIG. For ARGES with HGI, we report the JI obtained by executing ARGES-skeleton initialized with the CPDAG of the initial DAG obtained by HGI. Both ARGES and ARGES with HGI achieve greater structural accuracy as n increases, though ARGES with HGI is generally more accurate and approaches JI = 1 more quickly. The addition of HGI consistently results in typically 14-18% improvement in JI, typically slightly increasing with sample size. ARGES with HGI only obtained worse JI results than ARGES for 2.7% of the executions, with fewer than 0.4% executions more than 4% worse.

S2 Effect of pPC Clustering Quality

The pPC algorithm endeavors to discover some block dependence structure amongst variables by way of clustering in order to impose a partitioned ordering on the conditional independence investigation. As such, it is of interest to investigate the sensitivity of the pPC algorithm to the clustering quality in terms of both accuracy and efficiency. We conducted a simulation study, varying the partition by randomly permuting a proportion of

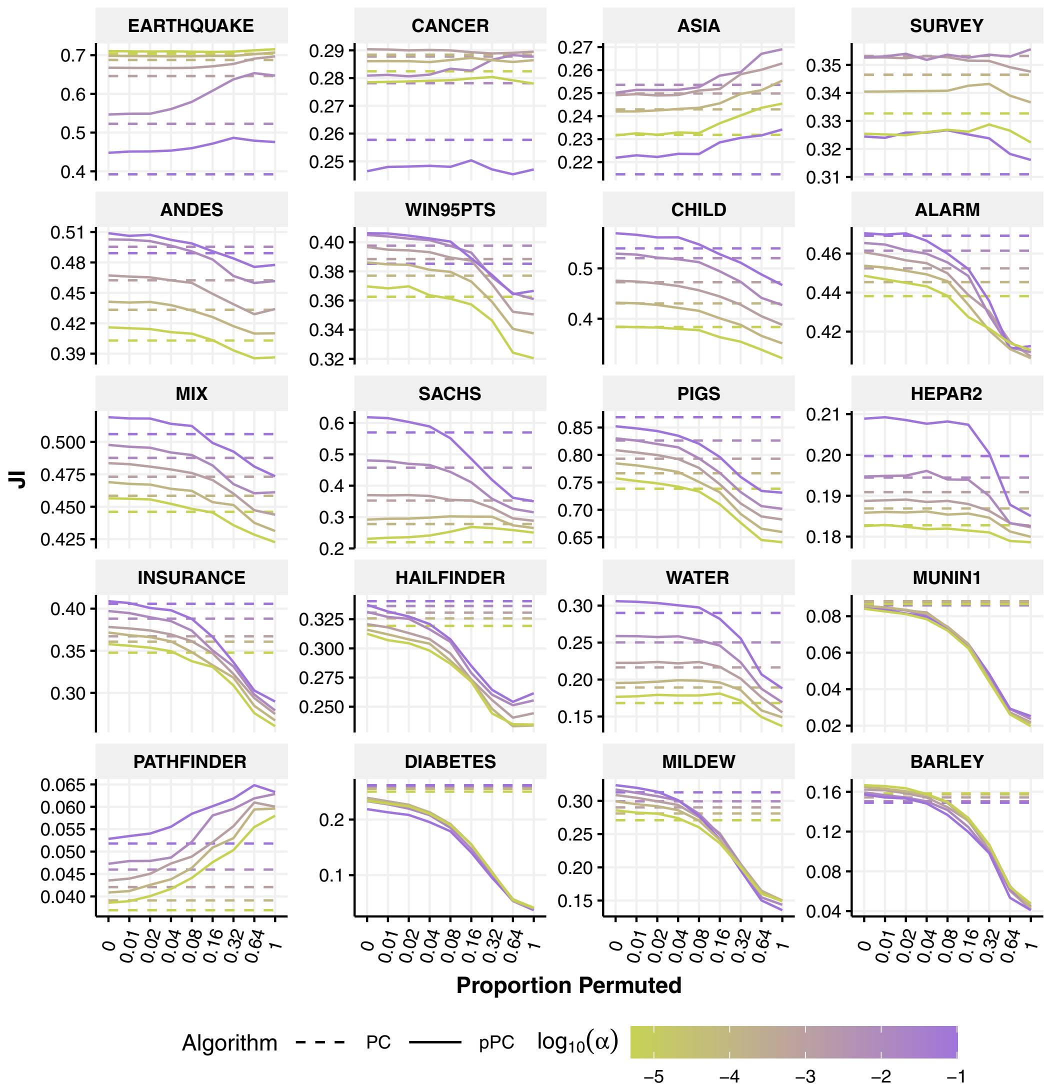

Fig. S2 Accuracy (JI) results for the pPC algorithm across a varying proportion of the partition randomly permuted. Five α were investigated, and for reference, the dashed lines represent the results for the PC algorithm

group labels obtained by the clustering algorithm. By such a design, the group sizes are still determined by the clustering algorithm, but some groups may be perturbed. A permuted proportion of 0 corresponds to the default pPC algorithm, and 1 corresponds to randomly rearranging all group labels in the partition. The simulation was executed with $\alpha \in \{0.1, 0.01, 0.001, 0.0001, 0.000005\}$ on a limited set of 20 datasets for each of the 20 networks. The accuracy results are visualized in Fig. S2.

For the majority of networks, the accuracy of estimated structures drops sharply following permuted proportions of greater than 0.08, unsurprisingly. Several exceptions exist, with CANCER and SURVEY appearing to be largely agnostic to the clustering quality, and EARTHQUAKE, ASIA, and PATHFINDER even preferring randomly rearranged par-

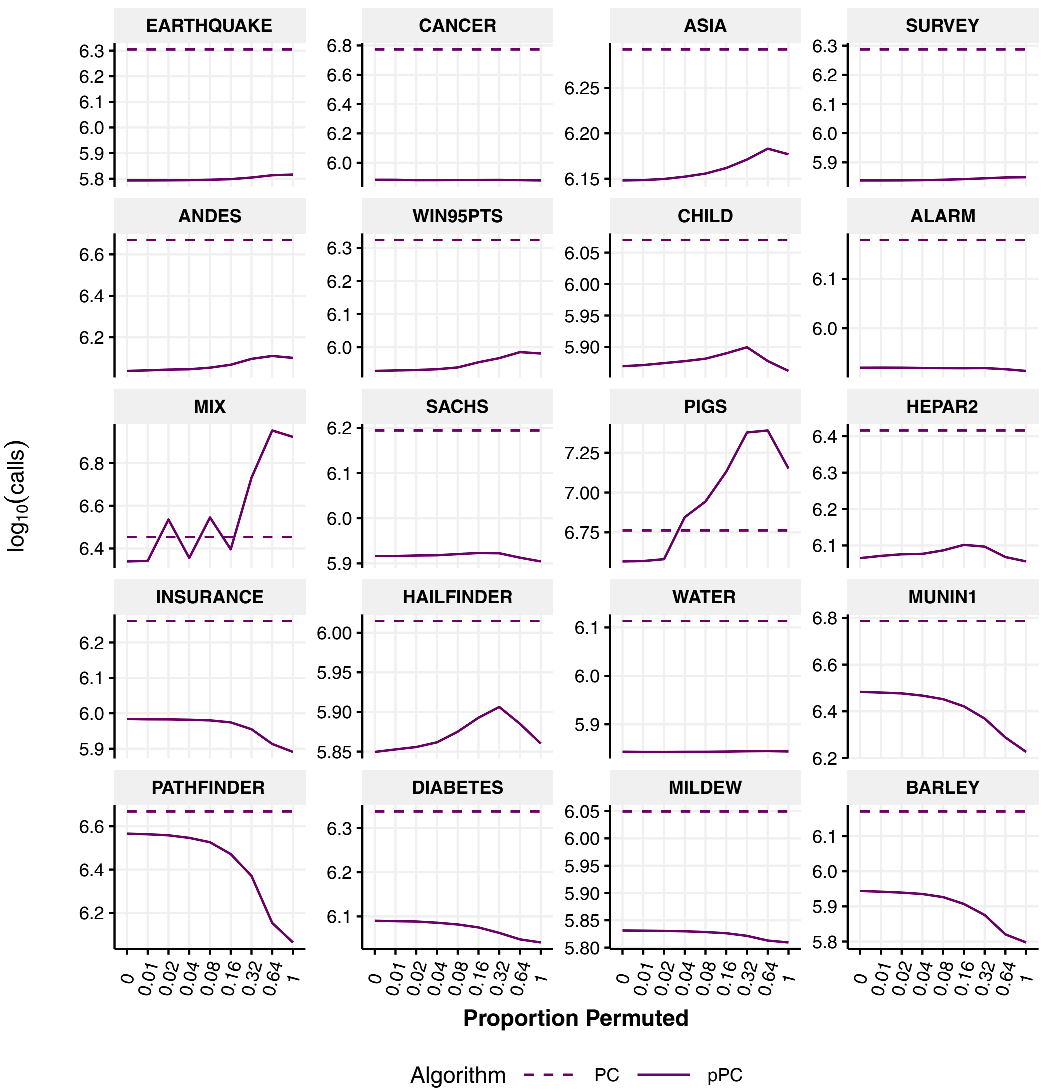

Fig. S3 Efficiency ($\log_{10}$(calls)) results for the pPC algorithm with $\alpha = 0.1$ across a varying proportion of the partition randomly permuted. For reference, the dashed lines represent the PC algorithm titions for some settings. Generally, the performance of pPC only drops below that of PC (represented by the dashed lines) when the permuted proportion is 0.16 or greater, if at all. This result shows that pPC is quite robust against varying clustering quality.

The corresponding computational efficiency results for $\alpha = 0.1$ are visualized in Fig. S3. Again, for the majority of networks, the clustering quality does not appear to significantly influence the computational expense of pPC relative to that of PC. Notable exceptions are the PIGS network which becomes significantly slower than PC and the MIX network which contains the PIGS network. Many high complexity networks even exhibit varying degrees of increased computational efficiency with higher permute proportions, likely due to increased false negatives. Regardless, the effect of clustering quality on the computational expense of pPC appears to vary according to the particular Bayesian network model.

Note that while there may be a *correct* partition according to the tiling of smaller Bayesian networks to create a large network (as discussed in Sect. 5.1 of the main text), that is not necessarily the optimal clustering, and multiple partitions that result in well-performance may exist.

S3 Extended Empirical Validation of PATH

In this section, we provide extended results supporting the performance of PATH compared to parameter tuning. In Sect. 5.2 of the main text, we demonstrated that for $n = 25000$ data samples, pPC and PC with PATH using parameters $\alpha = 0.1$, $\tau = 10$, and $\alpha^{(\tau)} = 10^{-5}$ are able to obtain estimates of competitive quality compared to optimal parameter tuning, in particular the best estimates from multiple executions with various α. Here, we discuss an extended simulation study that we conducted, wherein we generated 20 datasets at sample sizes

$$n \in \{1000, 2500, 5000, 10000, 25000, 50000\}$$

for 16 out of the 20 networks in Table 1 of the main text. The networks WIN95PTS, INSURANCE, PATHFINDER, and BARLEY were excluded due to minute marginal probabilities that made it difficult to generate datasets with at least two unique values for each variable at smaller sample sizes.

The results for pPC and PC are reported in figures Fig. S4 and Fig. S5, respectively. For parameter tuning, we executed algorithms pPC and PC on these datasets with the same ten α specified in Sect. 5.2 of the main text, in particular

$$\alpha \in \mathcal{A} := \{0.1, 0.05, 0.01, 0.005, 0.001, 0.0005, 0.0001, 0.00005, 0.00001, 0.000005\},$$

and we report the highest JI obtained for each dataset across the parameters as (p)PC* (JI). For PATH, we executed pPC and PC with $\tau = 10$ and $\alpha^{(\tau)} = 10^{-5}$, and we report the

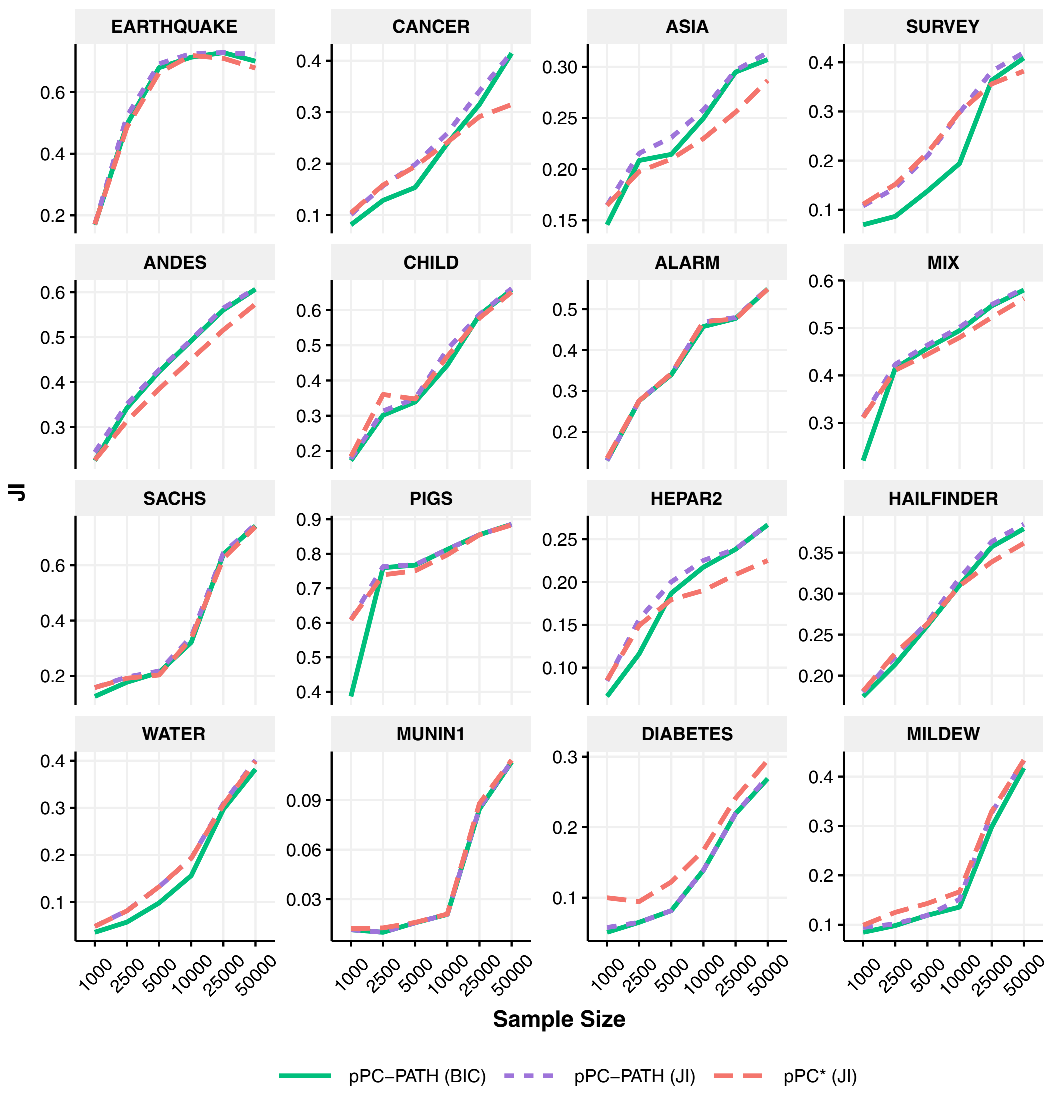

Fig. S4 Accuracy (JI) of PATH compared to parameter tuning applied to the pPC algorithm with varying sample sizes

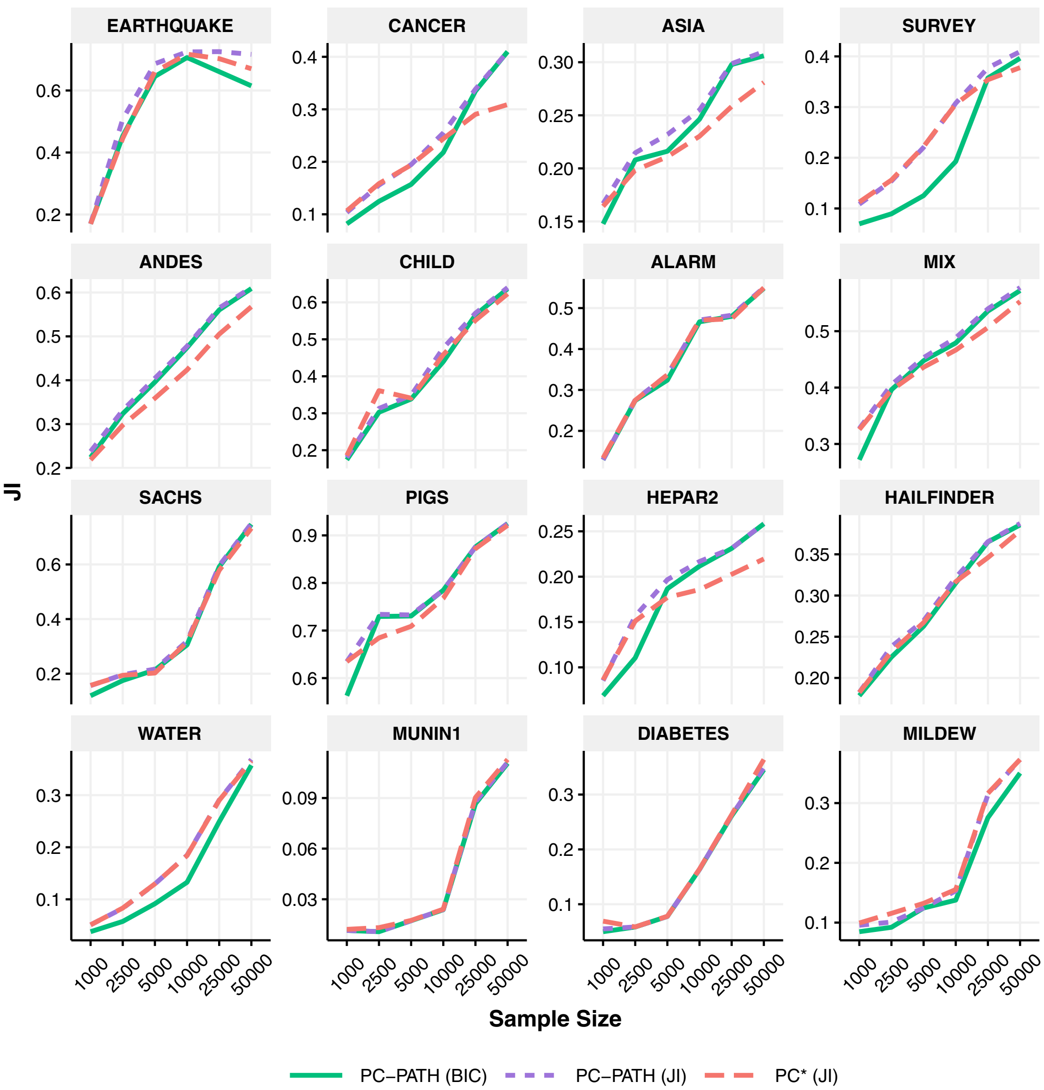

Fig. S5 Accuracy (JI) of PATH compared to parameter tuning applied to the PC algorithm with varying sample sizes

JI of the estimate selected by PATH as (p)PC-PATH (BIC) as well as the highest JI out of all estimates in the solution path generated by PATH as (p)PC-PATH (JI).

For almost every case, (p)PC-PATH (JI) performs at least as well as (p)PC* (JI), demonstrating that the solution path generated by PATH contains highly accurate structures. (p)PC-PATH (BIC) performs approximately equivalently but sometimes slightly worse than (p)PC* (JI), which is expected behavior due to the dependence on BIC selection. Note also that BIC selection in PATH selects from valid CPDAG estimates only if any are available, while (p)PC* (JI) chooses the highest JI regardless of whether the estimates are valid CPDAGs or not (see Remark 2 and Sect. 5.2 of the main text).

S4 Additional Results

In this section, we provide additional figures and tables further summarizing the results from the simulation study described in Sect. 5 of the main text. In particular, Fig. S6 visualizes the accuracy and efficiency of pPC compared against popular constraint-based algorithms, and Fig. S7 visualizes the accuracy of pHGS against HC and GSC algorithms MMHC and H2PC. The tables report extensive results containing numerous metrics comparing pPC with PATH against PC, MMPC, and HPC in Tables S1-S4, pHGS against HC, MMHC, and H2PC in Tables S5-S8, and pHGS against GES and FGES in Tables S9-S12.

We report the Jaccard index (JI), $\log_{10}$ number of statistical calls (Calls), number of predicted edges (P), true positives (TP), edges correctly present but with incorrect orientation (R), false positives (FP), and the structural Hamming distance (SHD), which may be interpreted as the number of edge additions, deletions, and reversals required to traverse from one graph to another. In Tables S1-S12, the means are provided alongside standard deviations (provided in parenthesis) across the $N = 100$ datasets for each of the 20 networks in Table 1 of the main text.

The results for pPC are that of pPC with PATH with parameters $\alpha = 0.1$, $\tau = 10$, nd $\alpha^{(\tau)} = 10^{-5}$, and the results for pHGS are with $\alpha = 0.01$, $\tau = 10$, and $\alpha^{(\tau)} = 10^{-5}$. The results for the estimates with the highest JI out of the ten executions with $\alpha \in \mathcal{A}$ for each dataset are reported for PC, MMPC, HPC, MMHC, and H2PC, alongside the $\log_{10}$ total number of statistical calls for all ten executions.

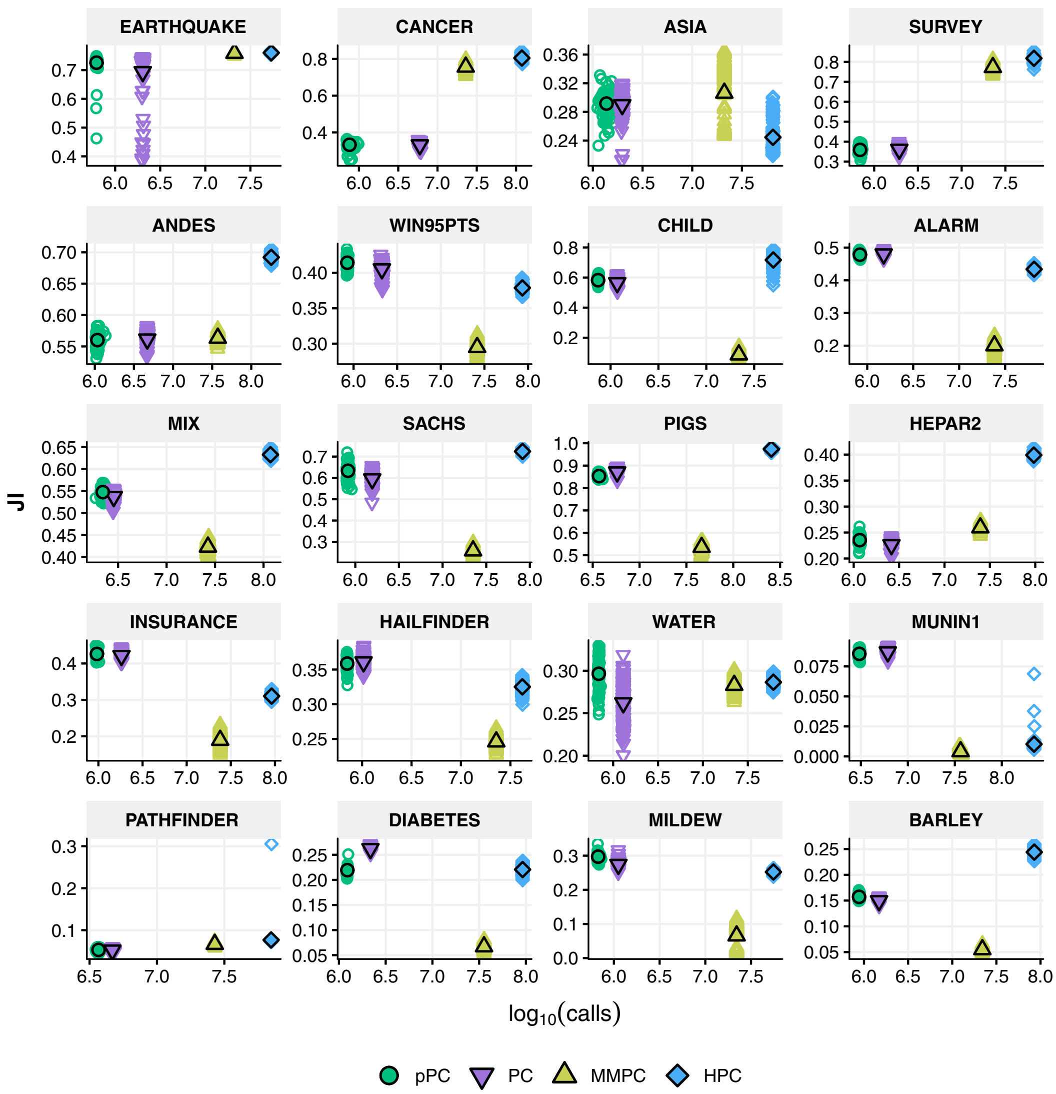

Fig. S6 Visual accuracy (JI) and efficiency ($\log_{10}$(calls)) comparisons amongst pPC, PC, MMPC, and HPC. Filled shapes with black outlines indicate averages, whereas colored outlines represent results for individual datasets

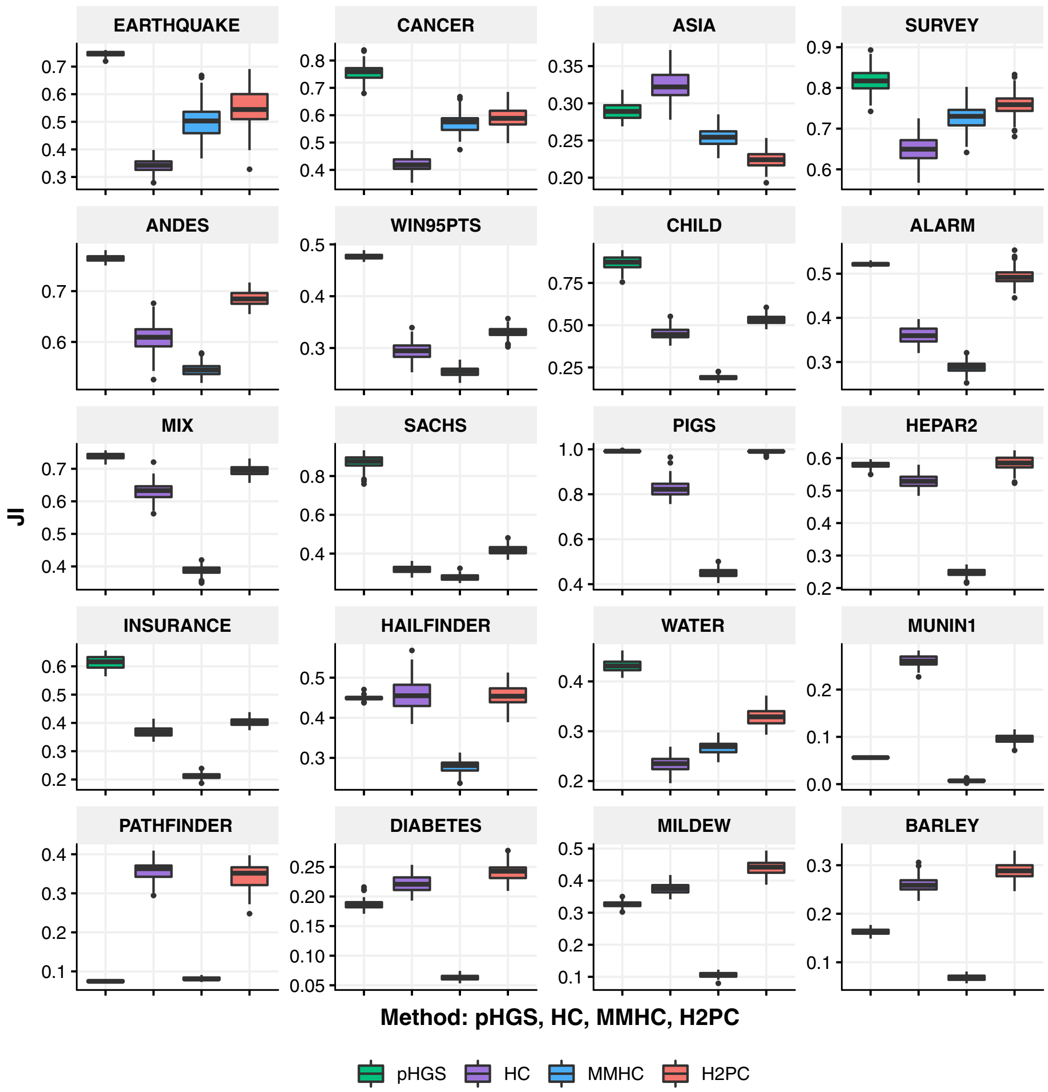

Fig. S7 Visual accuracy (JI) comparisons amongst pHGS, HC, MMHC, and H2PC

Table S1 Detailed results for networks 1-5 comparing pPC with PATH against PC, MMPC, and HPC

	pPC		PC		MMPC		HPC	
	EARTHQUAKE							
P	850.8	(23.8)	852.8	(2.4)	874.0	(9.0)	858.2	(4.7)
TP	820.3	(19.1)	809.8	(5.0)	853.1	(4.8)	847.0	(3.3)
R	27.6	(26.3)	43.0	(5.9)	16.6	(2.3)	8.8	(1.2)
FP	2.8	(16.0)	0.0	(0.2)	4.3	(3.4)	2.3	(2.2)
SHD	285.5	(34.6)	293.2	(5.0)	254.3	(3.7)	258.3	(3.2)
JI	0.725	(0.036)	0.707	(0.008)	0.759	(0.003)	0.760	(0.003)
Calls	5.793	(0.005)	6.849	(0.002)	7.323	(0.000)	7.731	(0.003)
	CANCER							
P	1064.4	(75.1)	1051.2	(9.5)	1073.0	(10.7)	1120.4	(13.1)
TP	546.1	(19.1)	488.5	(8.0)	945.8	(14.5)	1000.4	(11.9)
R	500.1	(18.5)	561.5	(6.0)	112.9	(13.2)	78.2	(10.7)
FP	18.2	(62.7)	1.3	(3.4)	14.3	(7.7)	41.8	(11.7)
SHD	595.1	(68.8)	635.8	(7.1)	191.5	(13.5)	164.4	(13.8)
JI	0.333	(0.019)	0.292	(0.004)	0.757	(0.018)	0.805	(0.016)
Calls	5.876	(0.026)	7.071	(0.004)	7.360	(0.001)	8.074	(0.004)
	ASIA							
P	805.6	(78.6)	774.6	(18.7)	906.8	(46.1)	712.9	(28.9)
TP	451.5	(19.3)	425.8	(15.2)	503.3	(62.6)	384.0	(31.1)
R	333.4	(49.7)	348.2	(17.7)	342.2	(53.9)	299.1	(25.2)
FP	20.7	(40.8)	0.6	(3.8)	61.2	(34.6)	29.8	(13.0)
SHD	812.2	(52.8)	817.8	(15.2)	800.8	(37.1)	888.7	(25.3)
JI	0.284	(0.023)	0.261	(0.009)	0.306	(0.042)	0.245	(0.022)
Calls	6.137	(0.040)	6.848	(0.002)	7.323	(0.000)	7.812	(0.003)
	SURVEY							
P	1290.6	(61.9)	1291.1	(13.9)	1307.2	(4.8)	1346.2	(20.5)
TP	700.2	(31.7)	688.5	(15.2)	1156.9	(14.3)	1211.8	(14.4)
R	578.2	(20.0)	602.3	(11.7)	146.9	(13.3)	117.7	(13.1)
FP	12.2	(34.2)	0.3	(1.2)	3.4	(2.0)	16.7	(18.3)
SHD	659.0	(44.5)	658.7	(14.9)	193.5	(14.5)	151.9	(16.4)
JI	0.362	(0.018)	0.360	(0.007)	0.773	(0.016)	0.818	(0.015)
Calls	5.837	(0.010)	6.851	(0.002)	7.357	(0.001)	7.827	(0.003)
	ANDES							
P	1790.5	(34.3)	1770.4	(38.6)	1557.5	(19.6)	1773.2	(22.6)
TP	1448.0	(19.0)	1401.9	(29.8)	1370.2	(12.8)	1643.3	(13.5)
R	312.9	(26.2)	359.5	(43.2)	149.5	(8.2)	118.1	(7.1)
FP	29.5	(5.3)	9.1	(3.6)	37.7	(11.4)	11.8	(11.3)
SHD	826.5	(18.8)	852.2	(30.7)	912.5	(12.0)	613.5	(11.0)
JI	0.560	(0.010)	0.505	(0.010)	0.563	(0.006)	0.692	(0.006)
Calls	6.028	(0.019)	7.133	(0.003)	7.572	(0.004)	8.255	(0.004)

Table S2 Detailed results for networks 6-10 comparing pPC with PATH against PC, MMPC, and HPC

	pPC		PC		MMPC		HPC	
	WIN95PTS							
P	1368.4	(42.7)	1297.6	(33.4)	1025.6	(17.3)	1174.5	(13.7)
TP	1035.0	(11.8)	992.5	(16.2)	731.0	(14.9)	922.6	(10.7)
R	269.3	(20.0)	277.1	(14.3)	273.7	(12.2)	236.1	(9.8)
FP	64.1	(22.9)	28.0	(11.9)	20.9	(7.3)	15.9	(5.7)
SHD	1213.1	(23.7)	1219.5	(12.7)	1474.0	(14.7)	1277.3	(11.1)
JI	0.411	(0.008)	0.398	(0.006)	0.295	(0.007)	0.379	(0.005)
Calls	5.927	(0.005)	6.964	(0.002)	7.417	(0.001)	7.935	(0.003)
	CHILD							
P	1299.4	(2.8)	1303.3	(3.8)	977.2	(17.5)	1310.9	(9.0)
TP	956.3	(20.9)	939.5	(22.3)	183.2	(33.5)	1092.4	(42.3)
R	342.1	(21.1)	361.9	(22.6)	777.1	(32.6)	207.5	(41.1)
FP	1.1	(1.8)	1.9	(2.9)	17.0	(10.5)	11.1	(10.0)
SHD	353.8	(20.9)	371.3	(22.8)	1142.8	(30.0)	227.6	(50.6)
JI	0.579	(0.020)	0.548	(0.019)	0.087	(0.017)	0.717	(0.049)
Calls	5.869	(0.004)	6.840	(0.001)	7.340	(0.001)	7.700	(0.001)
	ALARM							
P	1267.4	(22.3)	1287.4	(14.3)	1011.8	(21.0)	1403.3	(16.4)
TP	956.2	(12.8)	962.1	(11.1)	450.4	(30.7)	935.5	(12.0)
R	309.2	(11.3)	325.0	(10.9)	549.1	(26.0)	451.2	(15.6)
FP	2.0	(4.8)	0.2	(0.8)	12.3	(4.1)	16.7	(5.4)
SHD	734.8	(11.8)	727.1	(10.8)	1250.8	(30.3)	770.2	(14.4)
JI	0.478	(0.006)	0.473	(0.007)	0.200	(0.015)	0.434	(0.008)
Calls	5.921	(0.007)	6.914	(0.001)	7.383	(0.001)	7.815	(0.002)
	MIX							
P	1476.3	(25.2)	1483.1	(25.1)	1167.3	(14.5)	1567.7	(16.6)
TP	1184.8	(13.9)	1166.2	(16.9)	904.9	(16.5)	1334.8	(10.0)
R	285.5	(22.7)	313.5	(24.8)	244.8	(13.5)	223.7	(9.1)
FP	6.0	(3.6)	3.4	(2.8)	17.6	(6.2)	9.2	(6.6)
SHD	698.3	(14.0)	714.3	(16.5)	989.7	(18.0)	551.4	(10.5)
JI	0.546	(0.010)	0.506	(0.009)	0.423	(0.010)	0.633	(0.006)
Calls	6.342	(0.015)	7.249	(0.006)	7.427	(0.006)	8.072	(0.007)
	SACHS							
P	1791.2	(6.8)	1798.6	(3.4)	1372.0	(8.0)	1809.4	(2.6)
TP	1283.3	(38.0)	1241.7	(37.9)	656.0	(25.3)	1525.3	(9.9)
R	508.0	(39.1)	556.9	(38.6)	714.6	(25.3)	283.7	(10.0)
FP	0.0	(0.1)	0.0	(0.0)	1.3	(1.2)	0.4	(1.4)
SHD	538.8	(38.0)	580.3	(37.9)	1167.3	(25.4)	297.1	(10.0)
JI	0.551	(0.025)	0.508	(0.022)	0.259	(0.013)	0.724	(0.008)
Calls	5.916	(0.010)	6.912	(0.001)	7.349	(0.001)	7.916	(0.002)

Table S3 Detailed results for networks 11-15 comparing pPC with PATH against PC, MMPC, and HPC

	pPC		PC		MMPC		HPC	
	PIGS							
P	2171.2	(1.9)	2172.3	(1.2)	1533.1	(15.0)	2192.7	(10.6)
TP	2003.2	(9.5)	2027.1	(11.2)	1295.9	(26.2)	2158.1	(3.4)
R	167.9	(9.1)	145.2	(11.2)	213.6	(16.5)	14.6	(2.0)
FP	0.1	(0.3)	0.0	(0.0)	23.7	(7.0)	20.0	(10.3)
SHD	178.9	(9.5)	154.9	(11.2)	909.8	(28.0)	43.9	(10.4)
JI	0.852	(0.007)	0.867	(0.009)	0.536	(0.014)	0.974	(0.005)
Calls	6.567	(0.010)	7.594	(0.005)	7.663	(0.005)	8.407	(0.006)
	HEPAR2							
P	1286.6	(7.5)	1283.6	(8.0)	1015.3	(45.9)	1687.4	(54.7)
TP	636.3	(18.3)	598.9	(29.6)	636.9	(13.9)	1073.6	(20.6)
R	650.3	(21.0)	684.7	(34.7)	310.1	(10.8)	589.8	(15.7)
FP	0.0	(0.1)	0.0	(0.1)	68.3	(38.9)	24.0	(25.6)
SHD	1440.7	(18.2)	1478.2	(29.6)	1508.4	(32.8)	1027.3	(13.7)
JI	0.233	(0.008)	0.199	(0.005)	0.259	(0.006)	0.399	(0.005)
Calls	6.066	(0.006)	7.058	(0.002)	7.395	(0.001)	7.984	(0.003)
	INSURANCE							
P	1434.8	(10.5)	1438.5	(6.3)	991.2	(13.9)	1609.4	(39.5)
TP	1058.1	(19.3)	1050.7	(19.2)	495.0	(40.0)	883.2	(17.5)
R	375.6	(20.5)	387.8	(20.5)	473.4	(33.0)	696.0	(23.2)
FP	1.1	(0.5)	0.0	(0.0)	22.8	(4.6)	30.1	(16.1)
SHD	1063.0	(19.2)	1069.3	(19.2)	1647.8	(41.0)	1266.9	(17.7)
JI	0.424	(0.011)	0.404	(0.011)	0.189	(0.018)	0.310	(0.007)
Calls	5.983	(0.006)	6.978	(0.001)	7.379	(0.001)	7.962	(0.002)
	HAILFINDER							
P	927.1	(23.8)	911.1	(5.6)	830.5	(13.0)	1113.5	(19.1)
TP	653.7	(10.1)	643.1	(14.2)	468.1	(12.8)	651.1	(15.5)
R	218.8	(11.2)	257.1	(16.0)	316.2	(12.1)	461.9	(13.4)
FP	54.6	(17.9)	10.9	(3.0)	46.1	(11.5)	0.4	(1.1)
SHD	942.0	(20.8)	908.8	(14.6)	1119.0	(16.5)	890.4	(15.4)
JI	0.360	(0.009)	0.346	(0.008)	0.246	(0.008)	0.325	(0.008)
Calls	5.849	(0.003)	6.831	(0.001)	7.357	(0.001)	7.621	(0.001)
	WATER							
P	1338.6	(44.0)	1328.6	(62.8)	1313.6	(13.1)	1481.5	(46.6)
TP	836.5	(44.2)	776.7	(65.4)	797.7	(19.1)	842.8	(16.6)
R	498.4	(24.1)	551.0	(21.7)	513.4	(19.7)	616.3	(19.4)
FP	3.6	(4.2)	0.8	(1.7)	2.5	(1.7)	22.4	(24.1)
SHD	1470.0	(42.1)	1527.1	(64.6)	1507.8	(19.0)	1482.6	(20.6)
JI	0.298	(0.017)	0.291	(0.010)	0.283	(0.008)	0.287	(0.006)
Calls	5.840	(0.006)	6.849	(0.001)	7.349	(0.000)	7.785	(0.002)

Table S4 Detailed results for networks 16-20 comparing pPC with PATH against PC, MMPC, and HPC

	pPC		PC		MMPC		HPC	
	MUNIN1							
P	529.8	(6.8)	529.4	(13.0)	267.2	(10.0)	713.6	(14.5)
TP	182.2	(5.5)	191.0	(4.2)	8.3	(3.4)	25.1	(16.3)
R	307.4	(7.7)	309.7	(8.9)	216.0	(10.0)	636.2	(18.9)
FP	40.3	(2.6)	28.7	(5.1)	42.9	(6.1)	52.3	(6.4)
SHD	1634.1	(5.3)	1613.7	(5.2)	1810.6	(7.5)	1803.2	(17.1)
JI	0.086	(0.003)	0.090	(0.002)	0.004	(0.002)	0.010	(0.007)
Calls	6.484	(0.005)	7.609	(0.005)	7.562	(0.008)	8.343	(0.003)
	PATHFINDER							
P	303.5	(5.9)	295.4	(5.5)	330.6	(11.0)	1185.3	(28.6)
TP	114.4	(7.2)	112.4	(6.4)	143.5	(5.7)	223.7	(51.1)
R	182.6	(6.7)	178.5	(6.2)	170.3	(8.4)	916.9	(58.3)
FP	6.5	(3.1)	4.6	(2.5)	16.9	(4.2)	44.7	(7.5)
SHD	1860.1	(7.2)	1860.2	(6.4)	1841.4	(6.7)	1789.0	(51.9)
JI	0.053	(0.003)	0.053	(0.003)	0.067	(0.003)	0.077	(0.023)
Calls	6.567	(0.009)	7.560	(0.007)	7.429	(0.001)	7.847	(0.002)
	DIABETES							
P	1030.5	(9.2)	1057.7	(4.5)	700.4	(11.0)	1495.9	(15.2)
TP	551.4	(13.8)	644.4	(7.1)	171.9	(11.7)	638.0	(18.9)
R	458.9	(9.7)	404.7	(6.8)	465.0	(13.5)	673.6	(16.3)
FP	20.2	(2.9)	8.6	(0.6)	63.5	(7.0)	184.2	(7.1)
SHD	1503.8	(14.5)	1399.2	(7.2)	1926.6	(15.5)	1581.2	(20.7)
JI	0.219	(0.006)	0.263	(0.004)	0.067	(0.005)	0.221	(0.008)
Calls	6.090	(0.003)	7.154	(0.001)	7.552	(0.001)	7.964	(0.001)
	MILDEW							
P	1048.7	(18.7)	1032.2	(19.5)	783.3	(14.8)	1493.0	(9.1)
TP	678.7	(20.7)	636.3	(23.6)	163.7	(82.2)	686.0	(12.0)
R	363.5	(13.9)	395.9	(12.4)	613.5	(76.9)	806.0	(13.6)
FP	6.6	(1.9)	0.0	(0.0)	6.2	(2.4)	0.9	(1.3)
SHD	1240.9	(20.0)	1276.7	(23.6)	1755.6	(82.3)	1227.9	(12.0)
JI	0.297	(0.010)	0.313	(0.007)	0.066	(0.034)	0.252	(0.005)
Calls	5.828	(0.008)	6.839	(0.000)	7.343	(0.000)	7.745	(0.001)
	BARLEY							
P	733.9	(10.9)	738.8	(7.6)	585.2	(10.6)	1322.6	(10.8)
TP	385.4	(10.9)	390.9	(8.4)	140.0	(9.7)	671.6	(16.1)
R	328.6	(9.4)	328.9	(7.8)	421.7	(13.7)	580.4	(16.6)
FP	19.9	(1.7)	19.1	(1.2)	23.5	(5.1)	70.6	(5.4)
SHD	1735.6	(10.7)	1729.2	(8.0)	1984.6	(10.4)	1500.0	(17.1)
JI	0.157	(0.005)	0.160	(0.004)	0.055	(0.004)	0.244	(0.007)
Calls	5.944	(0.005)	6.930	(0.001)	7.341	(0.000)	7.930	(0.001)

Table S5 Detailed results for networks 1-5 comparing pHGS against HC, MMHC, and H2PC

	pHGS		HC		MMHC		H2PC	
	EARTHQUAKE							
P	911.3	(6.8)	1210.6	(9.6)	874.7	(26.7)	865.3	(21.3)
TP	860.7	(4.7)	588.1	(27.6)	657.2	(49.4)	693.8	(59.2)
R	21.9	(3.8)	313.4	(26.5)	208.2	(54.4)	161.1	(61.5)
FP	28.7	(5.9)	309.1	(10.7)	9.3	(15.6)	10.4	(14.4)
SHD	271.1	(7.7)	824.0	(33.0)	455.1	(56.0)	419.6	(63.2)
JI	0.746	(0.007)	0.341	(0.022)	0.500	(0.060)	0.548	(0.073)
Calls	5.724	(0.000)	6.378	(0.038)	6.321	(0.000)	6.732	(0.003)
	CANCER							
P	1124.1	(7.4)	1306.4	(10.0)	1107.0	(39.1)	1094.6	(16.4)
TP	947.3	(20.7)	716.7	(29.4)	808.9	(33.8)	822.2	(34.7)
R	126.5	(20.7)	364.3	(28.8)	255.9	(32.0)	252.3	(34.2)
FP	50.2	(7.2)	225.5	(10.4)	42.2	(33.0)	20.0	(12.2)
SHD	225.9	(23.4)	631.8	(36.9)	356.3	(39.3)	320.8	(34.6)
JI	0.729	(0.028)	0.419	(0.026)	0.570	(0.036)	0.590	(0.039)
Calls	5.739	(0.001)	6.444	(0.048)	6.326	(0.000)	7.074	(0.004)
	ASIA							
P	876.3	(6.5)	1432.0	(14.5)	852.3	(39.7)	702.0	(21.4)
TP	466.3	(14.0)	653.3	(25.3)	424.4	(20.0)	356.2	(15.6)
R	383.7	(13.7)	446.1	(24.6)	407.7	(21.2)	326.0	(15.0)
FP	26.3	(4.7)	332.5	(17.9)	20.2	(14.3)	19.8	(8.3)
SHD	803.1	(14.2)	922.2	(38.8)	838.9	(17.3)	906.6	(14.9)
JI	0.282	(0.011)	0.323	(0.018)	0.254	(0.012)	0.224	(0.011)
Calls	5.745	(0.002)	6.510	(0.048)	6.323	(0.000)	6.812	(0.003)
	SURVEY							
P	1329.2	(4.0)	1432.2	(7.1)	1327.6	(7.3)	1341.7	(5.6)
TP	1177.5	(24.7)	1093.4	(29.7)	1124.7	(27.8)	1160.7	(25.9)
R	145.2	(23.3)	241.2	(28.2)	190.1	(27.1)	171.6	(25.0)
FP	6.5	(2.3)	97.5	(8.4)	12.8	(5.1)	9.4	(4.0)
SHD	175.9	(24.9)	351.1	(36.1)	235.1	(28.0)	195.8	(25.6)
JI	0.786	(0.029)	0.649	(0.031)	0.726	(0.030)	0.760	(0.029)
Calls	5.742	(0.001)	6.506	(0.046)	6.327	(0.000)	6.823	(0.003)
	ANDES							
P	1853.6	(8.4)	2572.1	(35.8)	1549.8	(10.7)	1770.7	(16.9)
TP	1751.8	(10.5)	1823.7	(41.2)	1337.3	(21.4)	1633.3	(24.0)
R	57.8	(5.6)	270.2	(32.6)	185.2	(18.7)	130.5	(19.2)
FP	44.0	(5.7)	478.3	(39.7)	27.3	(5.3)	6.9	(5.1)
SHD	537.2	(12.4)	899.6	(77.8)	935.0	(21.2)	618.6	(23.3)
JI	0.746	(0.006)	0.610	(0.027)	0.544	(0.012)	0.686	(0.015)
Calls	5.924	(0.004)	6.776	(0.047)	6.442	(0.000)	7.201	(0.004)

Table S6 Detailed results for networks 6-10 comparing pHGS against HC, MMHC, and H2PC

	pHGS		HC		MMHC		H2PC	
	WIN95PTS							
P	1308.6	(7.5)	2334.8	(24.8)	1024.0	(13.9)	1167.4	(12.6)
TP	1111.9	(8.1)	1028.4	(36.9)	649.9	(19.3)	832.0	(21.0)
R	158.2	(5.8)	501.5	(24.7)	358.2	(18.4)	323.8	(18.8)
FP	38.5	(4.1)	804.8	(36.6)	15.9	(4.4)	11.6	(4.3)
SHD	1110.6	(9.5)	1960.4	(71.0)	1550.0	(19.2)	1363.6	(20.9)
JI	0.467	(0.004)	0.295	(0.015)	0.254	(0.009)	0.330	(0.010)
Calls	5.864	(0.002)	6.627	(0.033)	6.402	(0.001)	6.926	(0.003)
	CHILD							
P	1298.8	(1.1)	1438.7	(11.6)	969.6	(12.4)	1298.0	(2.4)
TP	1198.1	(35.1)	851.6	(39.4)	364.8	(21.8)	905.3	(27.7)
R	100.7	(34.9)	449.2	(38.9)	597.8	(17.3)	391.5	(27.5)
FP	0.1	(0.3)	137.9	(12.2)	7.0	(3.3)	1.1	(2.0)
SHD	111.0	(35.2)	595.4	(45.2)	951.2	(22.1)	404.8	(27.9)
JI	0.851	(0.046)	0.450	(0.032)	0.191	(0.013)	0.532	(0.025)
Calls	5.798	(0.001)	6.486	(0.046)	6.338	(0.001)	6.698	(0.001)
	ALARM							
P	1282.9	(4.8)	1828.8	(15.5)	1020.6	(16.4)	1399.9	(11.4)
TP	1010.1	(5.5)	932.5	(32.0)	606.5	(21.2)	1020.0	(28.5)
R	269.9	(3.9)	511.8	(28.3)	403.2	(17.8)	371.8	(24.3)
FP	3.0	(1.5)	384.6	(19.6)	10.9	(3.8)	8.1	(3.4)
SHD	681.9	(5.5)	1141.1	(46.1)	1093.5	(22.2)	677.1	(28.3)
JI	0.515	(0.003)	0.361	(0.018)	0.288	(0.012)	0.493	(0.019)
Calls	5.837	(0.002)	6.577	(0.047)	6.382	(0.001)	6.812	(0.002)
	MIX							
P	1547.8	(6.3)	1937.4	(22.7)	1171.8	(11.6)	1563.5	(10.8)
TP	1440.1	(11.5)	1473.2	(32.5)	851.8	(22.5)	1410.0	(19.0)
R	94.2	(9.0)	231.7	(28.0)	303.2	(19.2)	150.2	(19.0)
FP	13.5	(3.4)	232.5	(26.7)	16.8	(5.7)	3.3	(2.6)
SHD	450.4	(12.3)	636.3	(56.2)	1042.0	(23.8)	470.3	(19.0)
JI	0.726	(0.009)	0.630	(0.027)	0.388	(0.013)	0.695	(0.015)
Calls	6.223	(0.008)	6.631	(0.051)	6.397	(0.005)	6.950	(0.003)
	SACHS							
P	1797.5	(2.5)	1944.2	(10.7)	1373.0	(8.2)	1779.1	(6.5)
TP	1615.7	(40.7)	911.6	(34.9)	690.9	(28.4)	1061.1	(43.7)
R	181.9	(40.4)	855.0	(32.3)	680.8	(28.6)	718.0	(41.1)
FP	0.0	(0.0)	177.6	(12.6)	1.3	(1.2)	0.0	(0.2)
SHD	206.3	(40.7)	1088.0	(42.5)	1132.4	(28.5)	760.9	(43.7)
JI	0.807	(0.036)	0.320	(0.017)	0.276	(0.014)	0.418	(0.024)
Calls	5.847	(0.002)	6.577	(0.038)	6.346	(0.000)	6.905	(0.001)

Table S7 Detailed results for networks 11-15 comparing pHGS against HC, MMHC, and H2PC

	pHGS		HC		MMHC		H2PC	
	PIGS							
P	2170.6	(1.3)	2298.6	(27.5)	1533.8	(15.3)	2172.9	(2.1)
TP	2166.0	(1.7)	2026.6	(39.3)	1152.9	(35.4)	2166.2	(5.5)
R	4.6	(0.6)	152.5	(38.7)	357.2	(28.5)	6.7	(4.6)
FP	0.0	(0.0)	119.5	(28.0)	23.6	(7.0)	0.0	(0.0)
SHD	16.0	(1.7)	275.0	(66.1)	1052.7	(38.0)	15.8	(5.5)
JI	0.991	(0.001)	0.827	(0.039)	0.450	(0.019)	0.990	(0.004)
Calls	6.523	(0.009)	6.856	(0.052)	6.650	(0.005)	7.228	(0.003)
	HEPAR2							
P	1424.5	(7.7)	1781.5	(12.7)	988.3	(27.9)	1576.6	(15.9)
TP	1245.9	(15.6)	1335.3	(29.9)	606.1	(25.3)	1347.6	(32.6)
R	158.5	(14.2)	296.4	(26.4)	336.4	(23.9)	219.4	(30.5)
FP	20.1	(5.2)	149.8	(14.6)	45.7	(20.1)	9.6	(4.1)
SHD	851.2	(16.8)	891.6	(41.5)	1516.5	(28.0)	739.0	(32.1)
JI	0.552	(0.010)	0.529	(0.020)	0.247	(0.012)	0.585	(0.021)
Calls	5.978	(0.003)	6.517	(0.028)	6.377	(0.000)	6.912	(0.002)
	INSURANCE							
P	1512.0	(4.8)	2003.5	(9.9)	995.0	(13.2)	1536.9	(20.4)
TP	1375.2	(33.5)	1108.5	(35.6)	545.4	(22.8)	1051.4	(26.8)
R	133.8	(32.2)	582.6	(31.5)	426.8	(18.6)	469.7	(25.4)
FP	3.0	(1.3)	312.4	(15.4)	22.8	(4.6)	15.8	(6.3)
SHD	747.8	(33.6)	1323.9	(45.1)	1597.5	(24.4)	1084.4	(28.3)
JI	0.610	(0.023)	0.368	(0.016)	0.212	(0.010)	0.404	(0.014)
Calls	5.910	(0.002)	6.584	(0.046)	6.376	(0.001)	6.958	(0.002)
	HAILFINDER							
P	986.8	(8.9)	1574.8	(15.0)	838.9	(13.1)	1101.1	(18.2)
TP	783.3	(7.9)	974.6	(55.0)	518.7	(22.5)	825.3	(34.9)
R	117.1	(5.0)	328.6	(48.8)	273.1	(19.1)	275.5	(31.2)
FP	86.4	(6.7)	271.6	(21.0)	47.1	(11.5)	0.3	(0.6)
SHD	844.1	(9.7)	838.0	(71.8)	1069.4	(23.9)	715.9	(34.9)
JI	0.449	(0.006)	0.456	(0.040)	0.279	(0.014)	0.455	(0.026)
Calls	5.790	(0.004)	6.524	(0.050)	6.354	(0.001)	6.617	(0.001)
	WATER							
P	1321.1	(7.6)	1842.9	(15.6)	1312.7	(9.7)	1449.6	(9.9)
TP	1032.5	(25.9)	786.0	(40.0)	763.9	(29.8)	926.1	(36.4)
R	288.3	(24.0)	632.6	(30.7)	547.4	(28.4)	517.4	(33.0)
FP	0.3	(0.5)	424.3	(22.8)	1.4	(1.3)	6.1	(2.6)
SHD	1270.7	(26.0)	1941.4	(58.3)	1540.4	(29.9)	1383.0	(36.2)
JI	0.399	(0.014)	0.234	(0.015)	0.268	(0.013)	0.328	(0.017)
Calls	5.779	(0.002)	6.553	(0.036)	6.344	(0.000)	6.779	(0.001)

Table S8 Detailed results for networks 16-20 comparing pHGS against HC, MMHC, and H2PC

	pHGS		HC		MMHC		H2PC	
	MUNIN1							
P	518.2	(5.4)	1692.7	(10.3)	267.8	(10.0)	710.7	(14.1)
TP	120.8	(3.0)	717.4	(23.4)	14.3	(4.5)	217.8	(19.7)
R	364.2	(4.9)	318.1	(17.1)	210.8	(9.8)	444.4	(17.7)
FP	33.3	(2.2)	657.2	(17.9)	42.7	(6.3)	48.5	(5.7)
SHD	1688.5	(3.9)	1715.8	(37.8)	1804.4	(8.3)	1606.6	(20.5)
JI	0.056	(0.001)	0.261	(0.011)	0.007	(0.002)	0.096	(0.009)
Calls	6.437	(0.005)	6.449	(0.020)	6.562	(0.008)	7.343	(0.003)
	PATHFINDER							
P	290.6	(3.7)	1430.4	(7.6)	330.9	(7.4)	1107.7	(17.9)
TP	151.6	(5.1)	894.3	(41.1)	172.4	(7.7)	787.1	(55.0)
R	135.6	(4.5)	348.0	(41.8)	142.8	(6.7)	274.2	(54.9)
FP	3.4	(1.9)	188.2	(7.5)	15.7	(3.1)	46.4	(5.8)
SHD	1819.8	(5.5)	1261.8	(42.0)	1811.3	(8.5)	1227.2	(54.9)
JI	0.072	(0.003)	0.358	(0.022)	0.081	(0.004)	0.345	(0.032)
Calls	6.531	(0.009)	6.437	(0.029)	6.429	(0.001)	6.844	(0.002)
	DIABETES							
P	1053.3	(9.3)	2209.5	(14.2)	702.5	(12.0)	1422.5	(12.4)
TP	508.6	(14.3)	770.0	(42.0)	161.8	(11.2)	671.2	(33.4)
R	523.0	(8.7)	741.0	(31.1)	477.7	(14.7)	602.1	(28.5)
FP	21.7	(3.2)	698.5	(22.9)	63.1	(6.8)	149.1	(6.5)
SHD	1548.1	(14.1)	1963.5	(59.3)	1936.3	(13.6)	1512.8	(36.0)
JI	0.197	(0.006)	0.222	(0.015)	0.063	(0.005)	0.241	(0.015)
Calls	6.040	(0.002)	6.650	(0.029)	6.552	(0.001)	6.964	(0.001)
	MILDEW							
P	1094.8	(7.8)	1599.2	(11.1)	778.1	(11.7)	1314.7	(15.2)
TP	729.1	(15.7)	958.0	(31.6)	257.7	(17.6)	986.4	(37.8)
R	365.4	(11.6)	399.3	(21.9)	515.3	(15.8)	327.9	(26.2)
FP	0.2	(0.5)	241.9	(11.3)	5.1	(2.4)	0.4	(0.8)
SHD	1184.2	(15.7)	1196.9	(40.9)	1660.4	(18.1)	927.0	(37.9)
JI	0.320	(0.008)	0.375	(0.017)	0.106	(0.008)	0.440	(0.022)
Calls	5.772	(0.002)	6.529	(0.049)	6.343	(0.000)	6.744	(0.001)
	BARLEY							
P	758.6	(10.1)	1603.1	(7.5)	587.9	(10.9)	1170.5	(9.9)
TP	404.6	(12.6)	763.1	(35.1)	171.7	(13.8)	732.5	(35.9)
R	332.6	(8.0)	468.2	(23.5)	393.5	(13.8)	400.8	(30.1)
FP	21.3	(1.8)	371.9	(16.3)	22.7	(5.1)	37.2	(5.7)
SHD	1717.7	(12.0)	1709.8	(50.2)	1952.0	(14.3)	1405.7	(38.0)
JI	0.165	(0.005)	0.260	(0.015)	0.068	(0.006)	0.289	(0.018)
Calls	5.869	(0.002)	6.507	(0.050)	6.342	(0.000)	6.929	(0.001)

Table S9 Detailed results for networks 1-5 comparing pHGS against GES and FGES

	pHGS		GES		FGES	
	EARTHQUAKE					
P	911.3	(6.8)	1099.5	(7.9)	1082.0	(6.9)
TP	860.7	(4.7)	680.3	(18.8)	686.8	(19.0)
R	21.9	(3.8)	227.5	(17.5)	220.1	(17.6)
FP	28.7	(5.9)	191.7	(9.0)	175.1	(8.6)
SHD	271.1	(7.7)	614.4	(26.5)	591.2	(26.8)
JI	0.746	(0.007)	0.447	(0.020)	0.459	(0.020)
Calls	5.724	(0.000)	6.395	(0.046)	5.735	(0.002)
	CANCER					
P	1124.1	(7.4)	1203.3	(9.2)	1179.2	(8.1)
TP	947.3	(20.7)	949.5	(15.5)	947.0	(16.3)
R	126.5	(20.7)	135.0	(15.0)	131.3	(15.9)
FP	50.2	(7.2)	118.8	(9.4)	101.0	(8.2)
SHD	225.9	(23.4)	292.4	(21.7)	276.9	(21.5)
JI	0.729	(0.028)	0.690	(0.021)	0.699	(0.022)
Calls	5.739	(0.001)	6.326	(0.038)	5.724	(0.001)
	ASIA					
P	876.3	(6.5)	1199.7	(6.1)	1176.4	(5.8)
TP	466.3	(14.0)	947.9	(17.9)	941.1	(18.8)
R	383.7	(13.7)	177.4	(16.9)	175.6	(18.0)
FP	26.3	(4.7)	74.4	(7.8)	59.7	(6.8)
SHD	803.1	(14.2)	369.6	(22.2)	361.7	(22.5)
JI	0.282	(0.011)	0.634	(0.020)	0.637	(0.021)
Calls	5.745	(0.002)	6.374	(0.052)	5.751	(0.001)
	SURVEY					
P	1329.2	(4.0)	1431.0	(7.8)	1418.7	(7.9)
TP	1177.5	(24.7)	992.4	(23.5)	987.3	(24.0)
R	145.2	(23.3)	340.5	(22.2)	340.6	(22.9)
FP	6.5	(2.3)	98.2	(7.6)	90.7	(7.6)
SHD	175.9	(24.9)	452.8	(27.5)	450.4	(28.2)
JI	0.786	(0.029)	0.556	(0.021)	0.556	(0.022)
Calls	5.742	(0.001)	6.237	(0.013)	5.722	(0.000)
	ANDES					
P	1853.6	(8.4)	2398.4	(27.0)	2341.8	(27.4)
TP	1751.8	(10.5)	2009.2	(17.0)	1934.6	(16.1)
R	57.8	(5.6)	83.8	(11.1)	94.3	(10.7)
FP	44.0	(5.7)	305.4	(28.1)	312.9	(28.2)
SHD	537.2	(12.4)	541.1	(41.5)	623.4	(40.8)
JI	0.746	(0.006)	0.763	(0.016)	0.730	(0.015)
Calls	5.924	(0.004)	6.438	(0.010)	5.918	(0.002)

Table S10 Detailed results for networks 6-10 comparing pHGS against GES and FGES

	pHGS		GES		FGES	
	WIN95PTS					
P	1308.6	(7.5)	2023.0	(13.4)	1973.0	(12.1)
TP	1111.9	(8.1)	1403.3	(19.1)	1366.5	(21.0)
R	158.2	(5.8)	225.3	(10.7)	224.1	(10.9)
FP	38.5	(4.1)	394.4	(17.4)	382.4	(18.6)
SHD	1110.6	(9.5)	1175.1	(33.8)	1199.9	(37.5)
JI	0.467	(0.004)	0.501	(0.010)	0.490	(0.012)
Calls	5.864	(0.002)	6.327	(0.008)	5.893	(0.001)
	CHILD					
P	1298.8	(1.1)	1312.0	(1.8)	1304.6	(5.8)
TP	1198.1	(35.1)	1239.0	(11.5)	1200.5	(67.8)
R	100.7	(34.9)	66.7	(11.6)	99.4	(62.7)
FP	0.1	(0.3)	6.3	(1.7)	4.7	(1.3)
SHD	111.0	(35.2)	76.3	(12.0)	113.2	(67.5)
JI	0.851	(0.046)	0.897	(0.016)	0.853	(0.079)
Calls	5.798	(0.001)	6.277	(0.052)	5.785	(0.001)
	ALARM					
P	1282.9	(4.8)	1649.4	(9.9)	1631.2	(9.4)
TP	1010.1	(5.5)	1322.2	(18.4)	1313.2	(17.9)
R	269.9	(3.9)	203.8	(15.3)	203.5	(15.1)
FP	3.0	(1.5)	123.4	(9.5)	114.5	(8.7)
SHD	681.9	(5.5)	490.2	(25.5)	490.3	(24.0)
JI	0.515	(0.003)	0.656	(0.016)	0.654	(0.015)
Calls	5.837	(0.002)	6.306	(0.016)	5.862	(0.001)
	MIX					
P	1547.8	(6.3)	1850.7	(14.8)	1829.6	(15.0)
TP	1440.1	(11.5)	1592.2	(14.9)	1567.6	(13.4)
R	94.2	(9.0)	126.0	(10.6)	130.2	(9.8)
FP	13.5	(3.4)	132.5	(17.1)	131.8	(15.5)
SHD	450.4	(12.3)	417.4	(28.7)	441.3	(24.6)
JI	0.726	(0.009)	0.746	(0.014)	0.733	(0.012)
Calls	6.223	(0.008)	6.317	(0.012)	5.907	(0.002)
	SACHS					
P	1797.5	(2.5)	1864.4	(6.8)	1862.3	(7.0)
TP	1615.7	(40.7)	1544.3	(47.4)	1537.8	(47.1)
R	181.9	(40.4)	264.3	(46.6)	270.0	(46.5)
FP	0.0	(0.0)	55.8	(7.3)	54.4	(7.2)
SHD	206.3	(40.7)	333.6	(52.5)	338.6	(51.7)
JI	0.807	(0.036)	0.722	(0.040)	0.717	(0.039)
Calls	5.847	(0.002)	6.451	(0.028)	5.855	(0.005)

Table S11 Detailed results for networks 11-15 comparing pHGS against GES and FGES

	pHGS		GES		FGES	
	PIGS					
P	2170.6	(1.3)	2280.2	(15.7)	2280.6	(16.0)
TP	2166.0	(1.7)	2124.7	(7.5)	2122.2	(7.7)
R	4.6	(0.6)	49.7	(6.8)	51.2	(7.0)
FP	0.0	(0.0)	105.7	(15.2)	107.2	(15.5)
SHD	16.0	(1.7)	163.0	(21.8)	167.0	(22.3)
JI	0.991	(0.001)	0.909	(0.011)	0.907	(0.012)
Calls	6.523	(0.009)	6.441	(0.009)	6.139	(0.002)
	HEPAR2					
P	1424.5	(7.7)	1745.2	(12.5)	1685.5	(12.5)
TP	1245.9	(15.6)	1375.9	(19.2)	1325.1	(18.1)
R	158.5	(14.2)	239.2	(14.3)	243.1	(14.7)
FP	20.1	(5.2)	130.1	(13.3)	117.3	(12.0)
SHD	851.2	(16.8)	831.3	(28.9)	869.2	(26.8)
JI	0.552	(0.010)	0.563	(0.013)	0.544	(0.012)
Calls	5.978	(0.003)	6.317	(0.010)	5.822	(0.002)
	INSURANCE					
P	1512.0	(4.8)	1899.5	(7.8)	1848.4	(7.9)
TP	1375.2	(33.5)	1198.9	(15.9)	1178.4	(15.7)
R	133.8	(32.2)	479.0	(15.0)	479.5	(14.7)
FP	3.0	(1.3)	221.6	(8.0)	190.4	(7.1)
SHD	747.8	(33.6)	1142.8	(22.2)	1131.9	(20.9)
JI	0.610	(0.023)	0.425	(0.009)	0.422	(0.008)
Calls	5.910	(0.002)	6.368	(0.006)	5.868	(0.002)
	HAILFINDER					
P	986.8	(8.9)	1493.8	(5.0)	1480.8	(5.2)
TP	783.3	(7.9)	1191.0	(29.4)	1192.9	(28.5)
R	117.1	(5.0)	143.6	(24.1)	135.1	(23.0)
FP	86.4	(6.7)	159.3	(12.0)	152.8	(13.0)
SHD	844.1	(9.7)	509.3	(37.6)	500.9	(37.3)
JI	0.449	(0.006)	0.646	(0.026)	0.653	(0.026)
Calls	5.790	(0.004)	6.267	(0.013)	5.819	(0.001)
	WATER					
P	1321.1	(7.6)	1672.4	(10.7)	1642.3	(10.6)
TP	1032.5	(25.9)	1048.8	(13.6)	1033.8	(13.9)
R	288.3	(24.0)	427.3	(10.2)	427.5	(10.4)
FP	0.3	(0.5)	196.3	(12.4)	181.0	(12.6)
SHD	1270.7	(26.0)	1450.5	(23.5)	1450.2	(24.3)
JI	0.399	(0.014)	0.358	(0.007)	0.355	(0.007)
Calls	5.779	(0.002)	6.250	(0.006)	5.783	(0.001)

Table S12 Detailed results for networks 16-20 comparing pHGS against GES and FGES

	pHGS		GES		FGES	
	MUNIN1					
P	518.2	(5.4)	1629.6	(5.8)	1614.4	(5.8)
TP	120.8	(3.0)	890.5	(17.7)	891.3	(17.1)
R	364.2	(4.9)	215.7	(12.1)	216.0	(11.6)
FP	33.3	(2.2)	523.4	(13.3)	507.1	(12.5)
SHD	1688.5	(3.9)	1408.8	(29.0)	1391.8	(27.5)
JI	0.056	(0.001)	0.354	(0.010)	0.357	(0.009)
Calls	6.437	(0.005)	6.423	(0.004)	6.194	(0.003)
	PATHFINDER					
P	290.6	(3.7)	1458.5	(5.1)	1458.5	(5.3)
TP	151.6	(5.1)	1017.9	(36.8)	1006.9	(62.9)
R	135.6	(4.5)	262.0	(32.8)	274.5	(63.5)
FP	3.4	(1.9)	178.7	(10.4)	177.1	(9.9)
SHD	1819.8	(5.5)	1128.8	(43.6)	1138.2	(64.2)
JI	0.072	(0.003)	0.423	(0.022)	0.417	(0.034)
Calls	6.531	(0.009)	6.560	(0.061)	6.328	(0.049)
	DIABETES					
P	1053.3	(9.3)	1988.2	(8.7)	1972.2	(8.7)
TP	508.6	(14.3)	1303.6	(37.7)	1305.0	(41.4)
R	523.0	(8.7)	399.1	(33.8)	395.2	(37.4)
FP	21.7	(3.2)	285.5	(11.2)	272.0	(11.1)
SHD	1548.1	(14.1)	1016.8	(46.0)	1002.0	(49.4)
JI	0.197	(0.006)	0.480	(0.021)	0.483	(0.023)
Calls	6.040	(0.002)	6.496	(0.011)	6.043	(0.001)
	MILDEW					
P	1094.8	(7.8)	1526.1	(10.5)	1502.7	(10.6)
TP	729.1	(15.7)	1241.5	(14.2)	1232.8	(14.2)
R	365.4	(11.6)	178.2	(7.0)	178.0	(7.2)
FP	0.2	(0.5)	106.4	(8.0)	91.9	(7.0)
SHD	1184.2	(15.7)	777.9	(19.5)	772.1	(17.8)
JI	0.320	(0.008)	0.565	(0.009)	0.565	(0.009)
Calls	5.772	(0.002)	6.138	(0.016)	5.752	(0.000)
	BARLEY					
P	758.6	(10.1)	1481.1	(8.5)	1462.7	(8.2)
TP	404.6	(12.6)	1120.8	(24.4)	1117.6	(24.7)
R	332.6	(8.0)	212.9	(14.8)	213.6	(15.3)
FP	21.3	(1.8)	147.4	(10.1)	131.5	(8.2)
SHD	1717.7	(12.0)	1127.7	(33.3)	1114.9	(31.8)
JI	0.165	(0.005)	0.455	(0.014)	0.457	(0.014)
Calls	5.869	(0.002)	6.171	(0.016)	5.782	(0.001)