

Online Resource B: Reference prior

Belongs with: Model-based geostatistics from a Bayesian perspective: Investigating area-to-point kriging with small datasets
Mathematical Geosciences Luc Steinbuch, Tom Orton, Dick Brus ✉ luc.steinbuch@wur.nl, Wageningen UR

Berger et al. [2001] proposed as independent non-informative reference priors for σ^2 and ϕ (their equations 2 and 6, in our notation) respectively:

$$f_0(\sigma^2) = \frac{1}{\sigma^2} \quad (\text{B1})$$

and

$$f_0(\phi) \propto \left\{ \text{tr}[\mathbf{W}(\phi)^2] - \frac{1}{m-k} (\text{tr}[\mathbf{W}(\phi)])^2 \right\}^{1/2}, \quad (\text{B2})$$

the latter with:

$$\mathbf{W}(\phi) = \left(\frac{\delta \mathbf{C}}{\delta \phi} \right) \mathbf{C}^{-1} (I - \mathbf{X}(\mathbf{X}^T \mathbf{C}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{C}^{-1}) \quad (\text{B3})$$

where $\left(\frac{\delta \mathbf{C}}{\delta \phi} \right)$ indicates the partial derivative of $\mathbf{C}$ with respect to ϕ .

The probability distribution of the reference prior $f_0(\phi)$ for two settings of our 1D simulation, with smaller between-area distances compared with the larger between-area distances (respectively 30 areas and 10 areas, both evenly spaced over the same total length of 300 au) is shown in Figure B1. Larger between-area distances will change the covariance values in the scaled covariance matrix $\mathbf{C}$ such that the prior probability density puts more weight on larger values for ϕ .

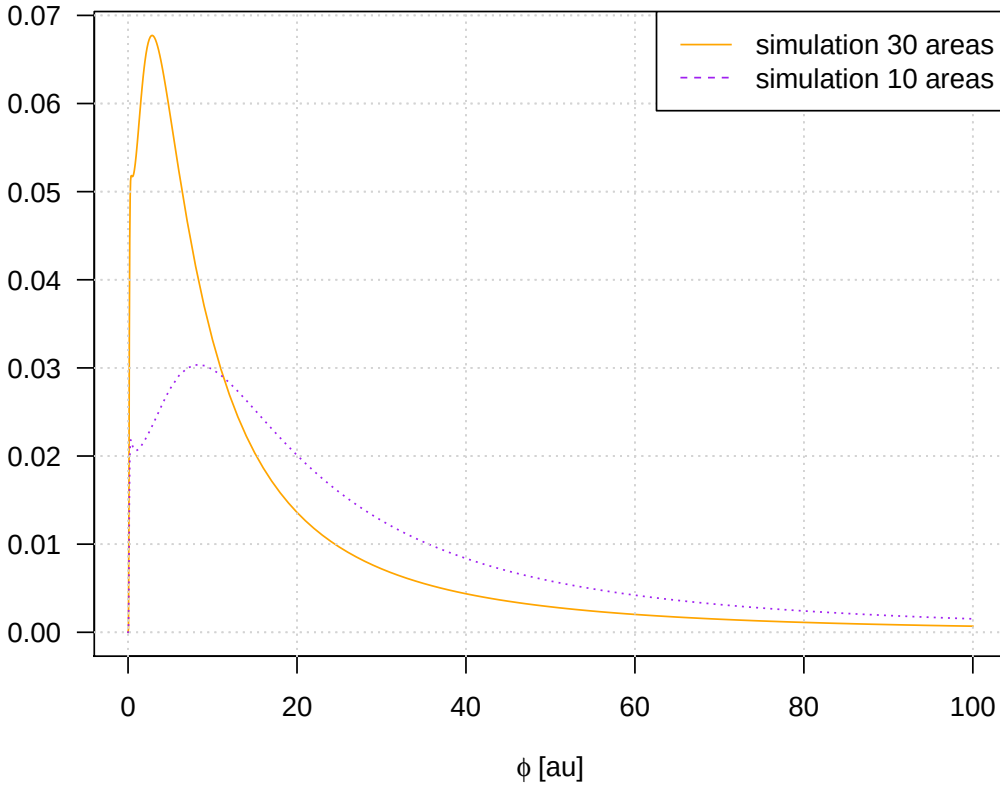


Figure B1: Reference prior for the simulated data. Distance is measured in abstract units (au).

Figure B2 shows the reference priors $f_0(\phi)$ for our crop yield case, comparing sorghum (4 covariates) and millet (no covariates), using identical spatial areas. It seems that – at least in this case – the difference caused by the different regression models, which enter the equation via the design matrix $\mathbf{X}$, is minimal.

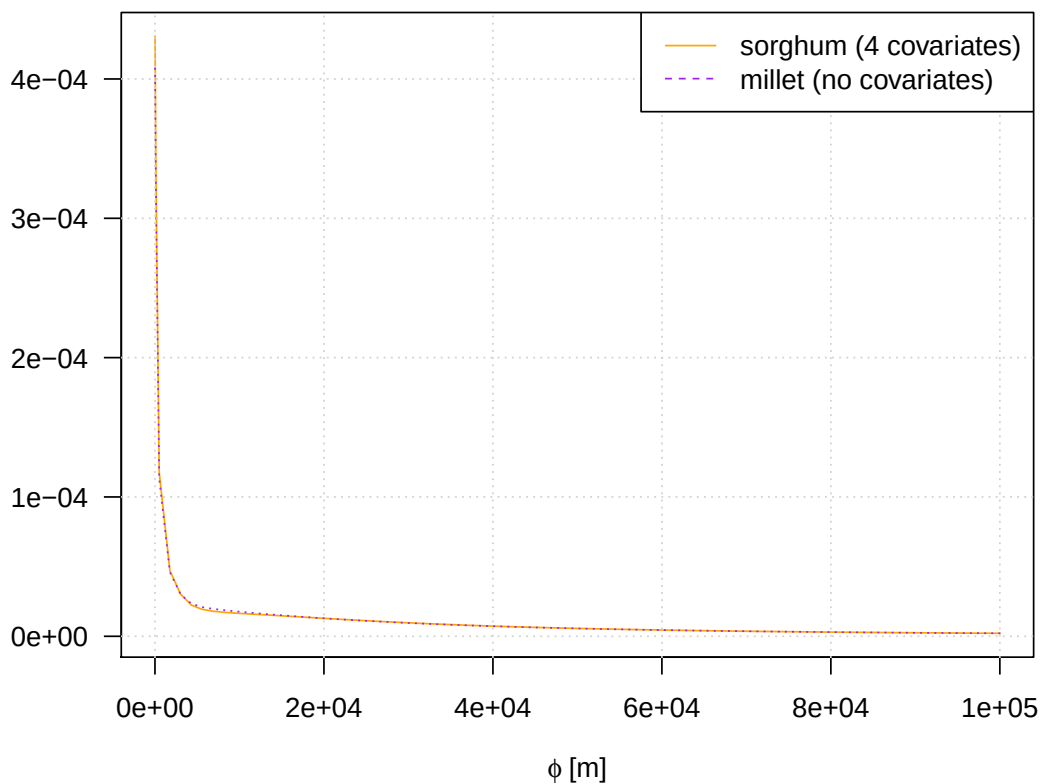


Figure B2: Reference priors for the case data.

References

James O. Berger, Victor de Oliveira, and Bruno Sansó. Objective bayesian analysis of spatially correlated data. *Journal of the American Statistical Association*, 96(456):1361–1374, 2001. ISSN 01621459. URL <http://www.jstor.org/stable/3085905>.