

Online Resource D: Example two-dimensional simulation, and mean($StSE$) of two-dimensional ensemble

Belongs with: Model-based geostatistics from a Bayesian perspective: Investigating area-to-point kriging with small datasets
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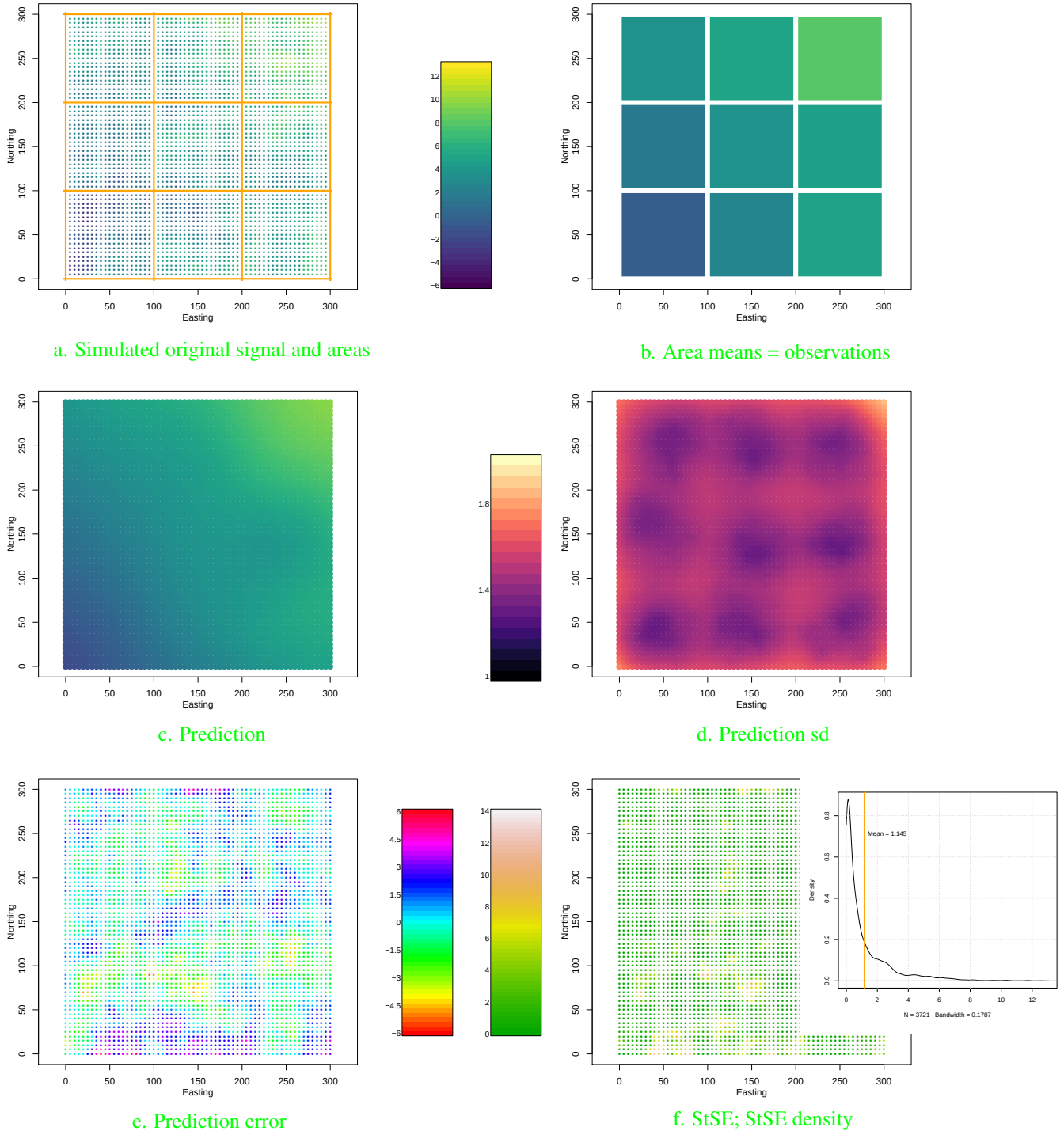


Figure D1: Example of two-dimensional simulation, with the same parameters as the 1-dimensional simulation in the paper. The signal is represented by a point realisation (a); as covariate, Easting+Northing is applied. Using the resulting area means (b), and the proposed Bayesian area-to-point kriging with an Uniform prior on ϕ , a prediction (c) and prediction uncertainty (d) are calculated. The plain prediction error (original signal minus prediction) is presented in (e). To compare the actual prediction error with the prediction uncertainty, the standardised squared error ($StSE$) is calculated and shown in f, as map and – inset – as density. For this simulation, $mean(StSE) = 1.145$ suggests that the approach is slightly optimistic.

Table D1: Mean standardised squared error ($mean(StSE)$) as resulting from the two-dimensional simulation ensemble, comparing restricted maximum likelihood (REML) with maximum marginal likelihood (MML) and three full Bayesian approaches with different priors for ϕ . Values $mean(StSE)$ larger (smaller) than one, indicate a underestimated (overestimated) prediction uncertainty. All simulations concern square areas in a 3×3 , 5×5 or 7×7 grid; the number of observed areas is indicated by m . The size of the total area remains 300×300 abstract units. The standard settings equal those of the one-dimensional simulations, with the covariate defined as Easting+Northing. Each presented value is the average over 128 runs, where the small font numbers indicate the corresponding standard deviation. Colours are used to highlight larger and smaller values for the readers' convenience. For further explanation, see Table 3 in the mean paper and its accompanying text.

Session	m	REML	MML	InvGam	Uniform	Reference
1: Standard settings	9	1.85 _{2.17}	1.32 _{1.55}	1.04 _{0.76}	1.27 _{1.04}	0.42 _{0.47}
	25	1.20 _{0.72}	1.09 _{0.66}	1.05 _{0.45}	1.19 _{0.62}	0.79 _{0.57}
	49	1.04 _{0.33}	0.99 _{0.31}	1.00 _{0.24}	1.08 _{0.31}	0.92 _{0.31}
2: Small upper limit: $\phi_u = 100$	9	1.52 _{1.64}	1.09 _{1.17}		0.72 _{0.54}	
	25	1.15 _{0.67}	1.05 _{0.61}		0.94 _{0.47}	
	49	1.02 _{0.31}	0.98 _{0.30}		0.96 _{0.26}	
3: Large upper limit: $\phi_u = 2,000$	9	1.93 _{2.36}	1.38 _{1.68}		1.96 _{1.59}	
	25	1.20 _{0.73}	1.10 _{0.66}		1.41 _{0.69}	
	49	1.04 _{0.33}	0.99 _{0.32}		1.18 _{0.33}	
4: Trend is Gaussian Random field	9	1.76 _{1.92}	1.25 _{1.38}	1.02 _{0.78}	1.22 _{0.95}	0.45 _{0.42}
	25	1.17 _{0.65}	1.07 _{0.60}	1.03 _{0.44}	1.15 _{0.57}	0.81 _{0.52}
	49	1.04 _{0.32}	1.00 _{0.31}	1.00 _{0.25}	1.07 _{0.31}	0.94 _{0.31}
5: No trend	9	1.95 _{2.12}	1.46 _{1.59}	1.04 _{0.79}	1.32 _{1.12}	0.47 _{0.55}
	25	1.21 _{0.69}	1.11 _{0.64}	1.06 _{0.46}	1.20 _{0.62}	0.86 _{0.57}
	49	1.04 _{0.32}	1.00 _{0.30}	1.00 _{0.24}	1.08 _{0.31}	0.94 _{0.30}
6: Matérn simulation, smooth; $v_{sim} = 2$	9	0.82 _{1.04}	0.59 _{0.75}	0.46 _{0.37}	0.57 _{0.51}	0.20 _{0.22}
	25	0.40 _{0.17}	0.37 _{0.15}	0.32 _{0.12}	0.38 _{0.15}	0.28 _{0.14}
	49	0.27 _{0.09}	0.26 _{0.08}	0.22 _{0.07}	0.26 _{0.08}	0.24 _{0.08}
7: Matérn simulation, unsmooth; $v_{sim} = .25$	9	3.20 _{3.56}	2.29 _{2.54}	1.80 _{1.29}	2.18 _{1.70}	0.72 _{0.73}
	25	2.25 _{1.46}	2.05 _{1.33}	2.06 _{0.89}	2.27 _{1.24}	1.45 _{1.12}
	49	2.01 _{0.71}	1.93 _{0.68}	2.07 _{0.48}	2.12 _{0.67}	1.77 _{0.68}
8: Added nugget: $\tau^2 = 1$	9	2.65 _{3.11}	1.89 _{2.22}	1.48 _{1.10}	1.80 _{1.50}	0.60 _{0.65}
	25	1.97 _{1.24}	1.80 _{1.13}	1.73 _{0.76}	1.95 _{1.05}	1.30 _{0.97}
	49	1.95 _{0.61}	1.87 _{0.58}	1.88 _{0.43}	2.03 _{0.57}	1.73 _{0.57}
9: Extreme short range in simulation: $\phi_{sim} = 5$	9	49.22 _{122.06}	35.20 _{87.22}	38.44 _{39.84}	41.95 _{56.09}	12.83 _{18.79}
	25	6.47 _{6.00}	5.91 _{5.45}	17.20 _{7.30}	10.01 _{6.69}	2.75 _{2.62}
	49	3.78 _{1.58}	3.61 _{1.50}	9.65 _{2.95}	4.87 _{1.82}	1.41 _{1.35}
10: Extreme long range in simulation: $\phi_{sim} = 600$	9	1.15 _{1.03}	0.82 _{0.74}	0.49 _{0.39}	0.67 _{0.52}	0.26 _{0.24}
	25	1.08 _{0.49}	0.99 _{0.44}	0.73 _{0.32}	0.95 _{0.42}	0.82 _{0.41}
	49	1.01 _{0.25}	0.96 _{0.24}	0.82 _{0.19}	0.96 _{0.23}	0.91 _{0.24}