

Online Resource E: results two-dimensional simulation ensemble

Belongs with: Model-based geostatistics from a Bayesian perspective: Investigating area-to-point kriging with small datasets
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Table. Results two-dimensional simulation ensemble with validating on original signal. Shown results are the average over, or the fraction of, 128 simulations. Smallfont numbers show the corresponding standard deviation.
m: number of area means, c.q. number of observations; *Algorithm*: explained in the text; *ME*: mean error; *RMSE*: root mean squared error; *FIR(1) or FIR(2)*: fraction of original signal inside region of +/- 1 or +/- 2 prediction sd; *StSE*: standardised squared error; *max(MPP)*: the maximal found difference with respect to the mass preserving property; $\hat{\sigma}^2$: estimated variance according to REML; *mode(σ^2)* mode of the variance according to the Bayesian approaches $\hat{\phi}$: estimated range parameter according to REML and MML; *mode(ϕ)* mode of the range parameter according to the Bayesian approaches; ϕ_{sim} in C.I.: ϕ_{sim} inside 90% credibility interval of posterior ϕ ; $\hat{\phi}$, *mode(ϕ) approx min, max* : $\hat{\phi}$ (REML, MML) or mode ϕ (Bayesian) very close to respectively minimal or maximal possible value.
Note that $\hat{\phi}$ of MML equals *mode(ϕ)* of Bayesian uniform by definition.

Session	<i>m</i>	Algorithm	Average over simulations									Fraction of simulations		
			ME	RMSE	FIR(1)	FIR(2)	mean(StSE)	med(StSE)	max(MPP)	$\hat{\sigma}^2$, mode(σ^2)	$\hat{\phi}$, mode(ϕ)	ϕ_{sim} in C.I.	$\hat{\phi}$, mode(ϕ) approx min	$\hat{\phi}$, mode(ϕ) approx max
1: Standard settings	9	REML	1.09e-03 _{2.95e-02}	1.58 _{0.14}	0.66 _{0.22}	0.88 _{0.14}	1.85 _{2.17}	0.85 _{0.99}	0.24 _{0.16}	8.16	109.59 _{105.7}	-	0.188	0.148
		MML	9.05e-04 _{2.93e-02}	1.58 _{0.14}	0.72 _{0.21}	0.92 _{0.11}	1.32 _{1.55}	0.61 _{0.71}	0.25 _{0.16}	-	109.43 _{105.9}	-	0.195	0.164
		InvGam	1.29e-03 _{2.9e-02}	1.56 _{0.13}	0.72 _{0.14}	0.94 _{0.07}	1.04 _{0.76}	0.48 _{0.35}	0.18 _{0.09}	3.63	50.47 _{2.6}	1.000	0.000	0.000
		Uniform	1.25e-03 _{2.93e-02}	1.57 _{0.13}	0.68 _{0.15}	0.93 _{0.09}	1.27 _{1.04}	0.58 _{0.47}	0.18 _{0.08}	4.71	109.43 _{105.9}	0.977	0.195	0.164
		Reference	9.45e-04 _{2.76e-02}	1.57 _{0.13}	0.90 _{0.12}	0.99 _{0.04}	0.42 _{0.47}	0.19 _{0.21}	0.26 _{0.13}	4.20	14.55 _{21.2}	0.992	0.000	0.000
		Baseline	-9.31e-18 _{1.07e-16}	1.81 _{0.15}	-	-	-	-	0.00 _{0.00}	-	-	-	-	-
	25	REML	-9e-05 _{1.94e-02}	1.27 _{0.07}	0.67 _{0.12}	0.93 _{0.07}	1.20 _{0.72}	0.55 _{0.33}	0.25 _{0.09}	5.83	81.22 _{74.4}	-	0.016	0.070
		MML	-9.53e-05 _{1.94e-02}	1.27 _{0.07}	0.69 _{0.12}	0.94 _{0.06}	1.09 _{0.66}	0.50 _{0.30}	0.25 _{0.09}	-	81.25 _{74.4}	-	0.016	0.070
		InvGam	-2.59e-04 _{1.87e-02}	1.27 _{0.07}	0.69 _{0.08}	0.95 _{0.05}	1.05 _{0.45}	0.48 _{0.21}	0.25 _{0.08}	4.27	51.52 _{5.8}	1.000	0.000	0.000
		Uniform	-1.53e-04 _{1.94e-02}	1.27 _{0.07}	0.67 _{0.10}	0.93 _{0.06}	1.19 _{0.62}	0.54 _{0.28}	0.25 _{0.08}	5.34	81.25 _{74.4}	0.859	0.016	0.070
		Reference	-8.73e-05 _{1.88e-02}	1.27 _{0.07}	0.77 _{0.14}	0.97 _{0.05}	0.79 _{0.57}	0.36 _{0.26}	0.27 _{0.09}	4.52	39.49 _{20.3}	0.938	0.000	0.000
		Baseline	-3.41e-18 _{6.71e-17}	1.44 _{0.09}	-	-	-	-	0.00 _{0.00}	-	-	-	-	-
	49	REML	-1.23e-04 _{1.73e-02}	1.08 _{0.04}	0.68 _{0.07}	0.95 _{0.03}	1.04 _{0.33}	0.47 _{0.15}	0.27 _{0.07}	5.61	71.61 _{58.2}	-	0.008	0.016
		MML	-1.25e-04 _{1.73e-02}	1.08 _{0.04}	0.69 _{0.07}	0.95 _{0.03}	0.99 _{0.31}	0.45 _{0.14}	0.27 _{0.07}	-	71.67 _{58.2}	-	0.008	0.016
		InvGam	-2.79e-04 _{1.7e-02}	1.08 _{0.04}	0.69 _{0.05}	0.95 _{0.03}	1.00 _{0.24}	0.46 _{0.11}	0.27 _{0.07}	4.44	51.71 _{7.4}	1.000	0.000	0.000
		Uniform	-2.58e-05 _{1.69e-02}	1.08 _{0.04}	0.67 _{0.06}	0.94 _{0.03}	1.08 _{0.31}	0.49 _{0.14}	0.27 _{0.07}	5.34	71.67 _{58.2}	0.820	0.008	0.016
		Reference	-1.77e-04 _{1.72e-02}	1.08 _{0.04}	0.71 _{0.08}	0.96 _{0.03}	0.92 _{0.31}	0.42 _{0.14}	0.28 _{0.09}	4.37	43.73 _{18.0}	0.891	0.008	0.000
		Baseline	-1.3e-18 _{4.58e-17}	1.25 _{0.06}	-	-	-	-	0.00 _{0.00}	-	-	-	-	-
2: Small upper limit: $\phi_u = 100$	9	REML	8.75e-04 _{2.92e-02}	1.58 _{0.14}	0.68 _{0.21}	0.90 _{0.12}	1.52 _{1.64}	0.70 _{0.75}	0.25 _{0.16}	7.38	60.78 _{36.9}	-	0.195	0.352
		MML	8.69e-04 _{2.92e-02}	1.58 _{0.14}	0.74 _{0.19}	0.94 _{0.09}	1.09 _{1.17}	0.50 _{0.53}	0.25 _{0.16}	-	60.79 _{36.9}	-	0.195	0.352
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	1.11e-03 _{2.85e-02}	1.57 _{0.13}	0.79 _{0.13}	0.97 _{0.05}	0.72 _{0.54}	0.33 _{0.25}	0.19 _{0.09}	3.77	60.79 _{36.9}	1.000	0.195	0.352
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-
	25	REML	-2.29e-04 _{1.91e-02}	1.27 _{0.07}	0.68 _{0.12}	0.94 _{0.06}	1.15 _{0.67}	0.52 _{0.31}	0.25 _{0.09}	4.95	60.03 _{27.8}	-	0.016	0.219
		MML	-2.27e-04 _{1.91e-02}	1.27 _{0.07}	0.70 _{0.11}	0.95 _{0.06}	1.05 _{0.61}	0.48 _{0.28}	0.25 _{0.09}	-	60.09 _{27.8}	-	0.016	0.219
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	-2.16e-04 _{1.87e-02}	1.27 _{0.07}	0.72 _{0.09}	0.96 _{0.05}	0.94 _{0.47}	0.43 _{0.22}	0.25 _{0.08}	4.46	60.09 _{27.8}	1.000	0.016	0.219
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-
	49	REML	-1.74e-04 _{1.72e-02}	1.08 _{0.04}	0.69 _{0.07}	0.95 _{0.03}	1.02 _{0.31}	0.47 _{0.14}	0.27 _{0.07}	4.90	58.80 _{25.2}	-	0.008	0.156
		MML	-1.76e-04 _{1.72e-02}	1.08 _{0.04}	0.70 _{0.07}	0.95 _{0.03}	0.98 _{0.30}	0.45 _{0.13}	0.27 _{0.07}	-	58.79 _{25.2}	-	0.008	0.156
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	-2.14e-04 _{1.71e-02}	1.08 _{0.04}	0.70 _{0.06}	0.96 _{0.03}	0.96 _{0.26}	0.44 _{0.12}	0.27 _{0.07}	4.58	58.79 _{25.2}	0.977	0.008	0.156
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-
3: Large upper limit: $\phi_u = 2,000$	9	REML	8.75e-04 _{2.94e-02}	1.58 _{0.14}	0.66 _{0.23}	0.88 _{0.15}	1.93 _{2.36}	0.88 _{1.07}	0.25 _{0.16}	11.71	266.59 _{537.5}	-	0.211	0.070
		MML	8.77e-04 _{2.94e-02}	1.58 _{0.14}	0.72 _{0.21}	0.91 _{0.12}	1.38 _{1.68}	0.63 _{0.76}	0.25 _{0.16}	-	266.75 _{537.6}	-	0.195	0.070
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	1.31e-03 _{3.01e-02}	1.57 _{0.13}	0.59 _{0.15}	0.87 _{0.12}	1.96 _{1.59}	0.90 _{0.73}	0.18 _{0.08}	6.30	266.75 _{537.6}	0.289	0.195	0.070
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-
	25	REML	-6.52e-05 _{1.94e-02}	1.27 _{0.07}	0.67 _{0.12}	0.93 _{0.07}	1.20 _{0.73}	0.55 _{0.33}	0.25 _{0.09}	7.12	118.83 _{264.1}	-	0.031	0.016
		MML	-6e-05 _{1.94e-02}	1.27 _{0.07}	0.69 _{0.12}	0.94 _{0.06}	1.10 _{0.66}	0.50 _{0.30}	0.25 _{0.09}	-	119.23 _{264.5}	-	0.016	0.016
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	-1.23e-04 _{2e-02}	1.27 _{0.06}	0.62 _{0.09}	0.91 _{0.07}	1.41 _{0.69}	0.64 _{0.32}	0.26 _{0.08}	6.34	119.23 _{264.5}	0.414	0.016	0.016
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-
	49	REML	-1.16e-04 _{1.73e-02}	1.08 _{0.04}	0.68 _{0.07}	0.95 _{0.03}	1.04 _{0.33}	0.47 _{0.15}	0.27 _{0.07}	6.75	98.17 _{245.8}	-	0.008	0.016
		MML	-1.05e-04 _{1.73e-02}	1.08 _{0.04}	0.69 _{0.07}	0.95 _{0.03}	0.99 _{0.32}	0.45 _{0.14}	0.27 _{0.07}	-	98.21 _{245.9}	-	0.008	0.016
		InvGam	-	-	-	-	-	-	-	-	-	-	-	-
		Uniform	8.08e-05 _{1.68e-02}	1.08 _{0.04}	0.65 _{0.06}	0.93 _{0.04}	1.18 _{0.33}	0.54 _{0.15}	0.28 _{0.07}	6.05	98.21 _{245.9}	0.516	0.008	0.016
		Reference	-	-	-	-	-	-	-	-	-	-	-	-
		Baseline	-	-	-	-	-	-	-	-	-	-	-	-

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Session	m	Algorithm	ME	RMSE	FIR(1)	FIR(2)	mean(StSE)	med(StSE)	max(MPP)	$\hat{\sigma}^2, mode(\sigma^2)$	$\hat{\phi}, mode(\phi)$	ϕ_{sim} in C.I.	$\hat{\phi}, mode(\phi) \approx min$	$\hat{\phi}, mode(\phi) \approx max$
4: Trend is Gaussian Random field	9	REML	2.21e-03 ^{3.17e-02}	1.80 ^{0.44}	0.66 ^{0.22}	0.88 ^{0.14}	1.76 ^{1.92}	0.85 ^{0.97}	0.25 ^{0.16}	8.28	108.06 ^{104.7}	-	0.172	0.172
		MML	2.11e-03 ^{3.17e-02}	1.81 ^{0.44}	0.72 ^{0.21}	0.92 ^{0.11}	1.25 ^{1.38}	0.61 ^{0.69}	0.25 ^{0.16}	-	107.86 ^{104.8}	-	0.180	0.172
		InvGam	2.34e-03 ^{3e-02}	1.80 ^{0.46}	0.72 ^{0.15}	0.95 ^{0.08}	1.02 ^{0.78}	0.50 ^{0.39}	0.19 ^{0.09}	3.75	50.51 ^{2.6}	1.000	0.000	0.000
		Uniform	2.66e-03 ^{3.11e-02}	1.80 ^{0.47}	0.68 ^{0.16}	0.93 ^{0.09}	1.22 ^{0.95}	0.60 ^{0.48}	0.19 ^{0.08}	4.60	107.86 ^{104.8}	0.961	0.180	0.172
		Reference	2.25e-03 ^{2.97e-02}	1.80 ^{0.45}	0.88 ^{0.13}	0.99 ^{0.03}	0.45 ^{0.42}	0.22 ^{0.21}	0.26 ^{0.11}	4.28	15.36 ^{22.4}	1.000	0.000	0.000
		Baseline	1.52e-18 ^{3.45e-17}	2.55 ^{0.19}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	4.48e-04 ^{1.9e-02}	1.30 ^{0.08}	0.67 ^{0.12}	0.93 ^{0.06}	1.17 ^{0.65}	0.54 ^{0.30}	0.25 ^{0.08}	5.53	75.96 ^{61.8}	-	0.016	0.031
		MML	4.48e-04 ^{1.89e-02}	1.30 ^{0.08}	0.70 ^{0.11}	0.94 ^{0.06}	1.07 ^{0.60}	0.49 ^{0.27}	0.25 ^{0.08}	-	75.99 ^{61.7}	-	0.008	0.031
		InvGam	2e-04 ^{1.83e-02}	1.30 ^{0.09}	0.69 ^{0.09}	0.95 ^{0.05}	1.03 ^{0.44}	0.47 ^{0.20}	0.24 ^{0.08}	4.36	51.95 ^{5.9}	1.000	0.000	0.000
		Uniform	3.09e-04 ^{1.91e-02}	1.30 ^{0.08}	0.67 ^{0.10}	0.94 ^{0.06}	1.15 ^{0.57}	0.53 ^{0.26}	0.25 ^{0.08}	5.32	75.99 ^{61.7}	0.852	0.008	0.031
		Reference	4.59e-04 ^{1.85e-02}	1.30 ^{0.09}	0.76 ^{0.13}	0.96 ^{0.05}	0.81 ^{0.52}	0.37 ^{0.24}	0.26 ^{0.08}	4.64	42.60 ^{19.9}	0.969	0.000	0.000
		Baseline	-1.47e-18 ^{2.21e-17}	2.19 ^{0.12}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	49	REML	1.75e-05 ^{1.75e-02}	1.09 ^{0.05}	0.68 ^{0.07}	0.95 ^{0.03}	1.04 ^{0.32}	0.47 ^{0.15}	0.27 ^{0.07}	5.35	67.27 ^{45.2}	-	0.008	0.008
		MML	1.42e-05 ^{1.75e-02}	1.09 ^{0.05}	0.69 ^{0.07}	0.95 ^{0.03}	1.00 ^{0.31}	0.45 ^{0.14}	0.27 ^{0.07}	-	67.12 ^{45.0}	-	0.008	0.008
		InvGam	-8.79e-05 ^{1.73e-02}	1.09 ^{0.05}	0.69 ^{0.06}	0.95 ^{0.03}	1.00 ^{0.25}	0.46 ^{0.11}	0.27 ^{0.06}	4.51	52.55 ^{7.8}	0.992	0.000	0.000
		Uniform	1.07e-04 ^{1.73e-02}	1.09 ^{0.05}	0.67 ^{0.07}	0.95 ^{0.03}	1.07 ^{0.31}	0.49 ^{0.14}	0.27 ^{0.07}	5.28	67.12 ^{45.0}	0.852	0.008	0.008
		Reference	-1.15e-05 ^{1.74e-02}	1.09 ^{0.05}	0.71 ^{0.08}	0.96 ^{0.03}	0.94 ^{0.31}	0.43 ^{0.14}	0.27 ^{0.09}	4.51	46.53 ^{18.7}	0.867	0.008	0.000
		Baseline	9.28e-19 ^{1.85e-17}	1.97 ^{0.08}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
5: No trend	9	REML	2.7e-03 ^{3.09e-02}	1.57 ^{0.14}	0.63 ^{0.20}	0.87 ^{0.13}	1.95 ^{2.12}	0.89 ^{0.96}	0.24 ^{0.15}	7.13	107.63 ^{95.5}	-	0.086	0.148
		MML	2.69e-03 ^{3.09e-02}	1.57 ^{0.14}	0.68 ^{0.19}	0.91 ^{0.11}	1.46 ^{1.59}	0.67 ^{0.72}	0.24 ^{0.15}	-	107.59 ^{95.5}	-	0.086	0.148
		InvGam	3.07e-03 ^{2.97e-02}	1.55 ^{0.13}	0.72 ^{0.14}	0.95 ^{0.07}	1.04 ^{0.79}	0.48 ^{0.36}	0.20 ^{0.09}	3.88	50.95 ^{3.1}	1.000	0.000	0.000
		Uniform	3.37e-03 ^{3.07e-02}	1.56 ^{0.13}	0.67 ^{0.15}	0.92 ^{0.09}	1.32 ^{1.12}	0.60 ^{0.51}	0.20 ^{0.09}	4.73	107.59 ^{95.5}	0.938	0.086	0.148
		Reference	2.99e-03 ^{2.9e-02}	1.56 ^{0.13}	0.88 ^{0.13}	0.99 ^{0.04}	0.47 ^{0.55}	0.21 ^{0.24}	0.27 ^{0.13}	4.36	21.64 ^{26.8}	0.984	0.000	0.000
		Baseline	-1.46e-18 ^{2.43e-17}	1.63 ^{0.14}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	3.36e-04 ^{1.94e-02}	1.27 ^{0.06}	0.67 ^{0.11}	0.93 ^{0.07}	1.21 ^{0.69}	0.55 ^{0.32}	0.25 ^{0.08}	5.54	77.40 ^{62.3}	-	0.016	0.031
		MML	3.36e-04 ^{1.95e-02}	1.27 ^{0.06}	0.69 ^{0.11}	0.94 ^{0.06}	1.11 ^{0.64}	0.51 ^{0.29}	0.25 ^{0.08}	-	77.45 ^{62.4}	-	0.008	0.031
		InvGam	2.58e-05 ^{1.87e-02}	1.26 ^{0.06}	0.69 ^{0.08}	0.95 ^{0.05}	1.06 ^{0.46}	0.48 ^{0.21}	0.25 ^{0.08}	4.35	52.32 ^{6.3}	1.000	0.000	0.000
		Uniform	1.77e-04 ^{1.95e-02}	1.26 ^{0.06}	0.66 ^{0.10}	0.93 ^{0.06}	1.20 ^{0.62}	0.55 ^{0.28}	0.25 ^{0.08}	5.34	77.45 ^{62.4}	0.852	0.008	0.031
		Reference	3.28e-04 ^{1.9e-02}	1.27 ^{0.06}	0.75 ^{0.13}	0.96 ^{0.05}	0.86 ^{0.57}	0.39 ^{0.26}	0.26 ^{0.09}	4.65	44.29 ^{20.4}	0.945	0.000	0.000
		Baseline	2.92e-18 ^{1.74e-17}	1.36 ^{0.08}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	49	REML	3.23e-05 ^{1.73e-02}	1.08 ^{0.04}	0.68 ^{0.07}	0.95 ^{0.03}	1.04 ^{0.32}	0.47 ^{0.14}	0.27 ^{0.07}	5.35	67.23 ^{43.4}	-	0.008	0.008
		MML	3.31e-05 ^{1.73e-02}	1.08 ^{0.04}	0.69 ^{0.07}	0.95 ^{0.03}	1.00 ^{0.30}	0.45 ^{0.14}	0.27 ^{0.07}	-	67.22 ^{43.3}	-	0.008	0.008
		InvGam	-1.2e-04 ^{1.7e-02}	1.07 ^{0.04}	0.69 ^{0.05}	0.95 ^{0.03}	1.00 ^{0.24}	0.45 ^{0.11}	0.27 ^{0.07}	4.51	52.38 ^{7.7}	1.000	0.000	0.000
		Uniform	1.03e-04 ^{1.7e-02}	1.08 ^{0.04}	0.67 ^{0.06}	0.94 ^{0.03}	1.08 ^{0.31}	0.49 ^{0.14}	0.27 ^{0.07}	5.29	67.22 ^{43.3}	0.859	0.008	0.008
		Reference	-5.56e-08 ^{1.72e-02}	1.08 ^{0.04}	0.71 ^{0.07}	0.96 ^{0.03}	0.94 ^{0.30}	0.43 ^{0.14}	0.28 ^{0.09}	4.53	46.60 ^{18.1}	0.875	0.008	0.000
		Baseline	1.14e-18 ^{1.27e-17}	1.21 ^{0.06}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
6: Matérn simulation, smooth; $\nu_{sim} = 2$	9	REML	4.13e-03 ^{3.39e-02}	1.28 ^{0.21}	0.81 ^{0.19}	0.96 ^{0.09}	0.82 ^{1.04}	0.39 ^{0.52}	0.29 ^{0.19}	11.62	106.15 ^{102.8}	-	0.188	0.133
		MML	4.13e-03 ^{3.39e-02}	1.28 ^{0.21}	0.86 ^{0.17}	0.97 ^{0.07}	0.59 ^{0.75}	0.28 ^{0.37}	0.29 ^{0.19}	-	106.23 ^{102.8}	-	0.188	0.133
		InvGam	3.95e-03 ^{3.38e-02}	1.25 ^{0.19}	0.88 ^{0.12}	0.99 ^{0.03}	0.46 ^{0.37}	0.22 ^{0.18}	0.21 ^{0.10}	5.15	50.48 ^{2.7}	1.000	0.000	0.000
		Uniform	4.16e-03 ^{3.42e-02}	1.25 ^{0.20}	0.85 ^{0.14}	0.98 ^{0.05}	0.57 ^{0.51}	0.27 ^{0.25}	0.21 ^{0.10}	6.66	106.23 ^{102.8}	0.977	0.188	0.133
		Reference	3.77e-03 ^{3.21e-02}	1.26 ^{0.20}	0.96 ^{0.08}	1.00 ^{0.01}	0.20 ^{0.22}	0.09 ^{0.10}	0.31 ^{0.14}	5.95	14.30 ^{20.6}	1.000	0.000	0.000
		Baseline	1.07e-17 ^{9.92e-17}	1.59 ^{0.22}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	7.33e-04 ^{2.22e-02}	0.75 ^{0.08}	0.89 ^{0.07}	1.00 ^{0.01}	0.40 ^{0.17}	0.18 ^{0.08}	0.28 ^{0.11}	7.73	114.17 ^{84.8}	-	0.000	0.109
		MML	7.34e-04 ^{2.22e-02}	0.75 ^{0.08}	0.90 ⁰									

Session	m	Algorithm	ME	RMSE	FIR(1)	FIR(2)	mean(StSE)	med(StSE)	max(MPP)	$\hat{\sigma}^2, mode(\sigma^2)$	$\hat{\phi}, mode(\phi)$	ϕ_{sim} in C.I.	$\hat{\phi}, mode(\phi) \approx min$	$\hat{\phi}, mode(\phi) \approx max$
8: Added nugget: $\tau^2 = 1$	9	REML	7.66e-04 ^{2.97e-02}	1.87 ^{0.12}	0.59 ^{0.23}	0.83 ^{0.16}	2.65 ^{3.11}	1.20 ^{1.39}	0.24 ^{0.16}	8.08	110.13 ^{105.6}	-	0.156	0.156
		MML	7.69e-04 ^{2.97e-02}	1.87 ^{0.12}	0.65 ^{0.22}	0.88 ^{0.14}	1.89 ^{2.22}	0.86 ^{0.99}	0.24 ^{0.16}	-	110.19 ^{105.6}	-	0.156	0.156
		InvGam	9.68e-04 ^{2.93e-02}	1.86 ^{0.11}	0.64 ^{0.14}	0.91 ^{0.09}	1.48 ^{1.10}	0.68 ^{0.50}	0.18 ^{0.09}	3.65	50.49 ^{2.6}	1.000	0.000	0.000
		Uniform	8.9e-04 ^{2.95e-02}	1.86 ^{0.11}	0.60 ^{0.15}	0.88 ^{0.11}	1.80 ^{1.50}	0.82 ^{0.67}	0.18 ^{0.08}	4.71	110.19 ^{105.6}	0.977	0.156	0.156
		Reference	6.07e-04 ^{2.78e-02}	1.86 ^{0.11}	0.85 ^{0.13}	0.98 ^{0.05}	0.60 ^{0.65}	0.27 ^{0.29}	0.26 ^{0.13}	4.22	14.28 ^{20.9}	0.992	0.000	0.000
		Baseline	7.51e-18 ^{1.06e-16}	2.06 ^{0.13}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	-2.6e-04 ^{1.94e-02}	1.62 ^{0.05}	0.56 ^{0.12}	0.86 ^{0.10}	1.97 ^{1.24}	0.89 ^{0.56}	0.25 ^{0.09}	5.82	80.64 ^{72.8}	-	0.016	0.070
		MML	-2.61e-04 ^{1.95e-02}	1.62 ^{0.05}	0.58 ^{0.12}	0.87 ^{0.09}	1.80 ^{1.13}	0.81 ^{0.51}	0.25 ^{0.09}	-	80.55 ^{72.7}	-	0.016	0.070
		InvGam	-4.87e-04 ^{1.88e-02}	1.61 ^{0.05}	0.57 ^{0.08}	0.88 ^{0.07}	1.73 ^{0.76}	0.78 ^{0.34}	0.24 ^{0.08}	4.30	51.60 ^{5.8}	1.000	0.000	0.000
		Uniform	-3.4e-04 ^{1.95e-02}	1.61 ^{0.05}	0.55 ^{0.09}	0.86 ^{0.08}	1.95 ^{1.05}	0.88 ^{0.47}	0.25 ^{0.08}	5.36	80.55 ^{72.7}	0.867	0.016	0.070
		Reference	-2.85e-04 ^{1.89e-02}	1.62 ^{0.05}	0.67 ^{0.15}	0.92 ^{0.08}	1.30 ^{0.97}	0.59 ^{0.44}	0.27 ^{0.09}	4.55	39.36 ^{19.9}	0.938	0.000	0.000
		Baseline	4.63e-19 ^{5.88e-17}	1.75 ^{0.07}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	49	REML	-2.2e-04 ^{1.74e-02}	1.47 ^{0.03}	0.54 ^{0.07}	0.85 ^{0.06}	1.95 ^{0.61}	0.88 ^{0.28}	0.27 ^{0.07}	5.63	71.57 ^{58.7}	-	0.008	0.016
		MML	-2.23e-04 ^{1.74e-02}	1.47 ^{0.03}	0.55 ^{0.07}	0.86 ^{0.06}	1.87 ^{0.58}	0.84 ^{0.26}	0.27 ^{0.07}	-	71.54 ^{58.7}	-	0.008	0.016
		InvGam	-3.81e-04 ^{1.72e-02}	1.47 ^{0.03}	0.54 ^{0.05}	0.86 ^{0.04}	1.88 ^{0.43}	0.85 ^{0.20}	0.27 ^{0.07}	4.47	51.67 ^{7.4}	1.000	0.000	0.000
		Uniform	-1.26e-04 ^{1.71e-02}	1.47 ^{0.03}	0.53 ^{0.06}	0.84 ^{0.05}	2.03 ^{0.57}	0.91 ^{0.26}	0.27 ^{0.07}	5.36	71.54 ^{58.7}	0.852	0.008	0.016
		Reference	-2.71e-04 ^{1.74e-02}	1.47 ^{0.03}	0.57 ^{0.08}	0.87 ^{0.06}	1.73 ^{0.57}	0.78 ^{0.26}	0.28 ^{0.10}	4.39	43.72 ^{17.9}	0.891	0.008	0.000
		Baseline	-2.18e-18 ^{4.42e-17}	1.60 ^{0.05}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
9: Extreme short range in simulation: $\phi_{sim} = 5$	9	REML	-1.74e-03 ^{8.9e-03}	2.22 ^{0.03}	0.24 ^{0.13}	0.44 ^{0.22}	49.22 ^{122.06}	21.69 ^{53.94}	0.09 ^{0.06}	0.72	52.34 ^{80.4}	-	0.445	0.070
		MML	-1.75e-03 ^{8.9e-03}	2.22 ^{0.03}	0.28 ^{0.16}	0.50 ^{0.24}	35.20 ^{87.22}	15.51 ^{38.54}	0.09 ^{0.06}	-	52.39 ^{80.4}	-	0.438	0.070
		InvGam	-1.54e-03 ^{9.39e-03}	2.22 ^{0.03}	0.16 ^{0.05}	0.30 ^{0.09}	38.44 ^{39.84}	17.14 ^{17.89}	0.04 ^{0.02}	0.22	48.71 ^{2.5}	0.000	0.000	0.000
		Uniform	-1.6e-03 ^{9.5e-03}	2.22 ^{0.03}	0.16 ^{0.06}	0.31 ^{0.11}	41.95 ^{56.09}	18.69 ^{25.08}	0.05 ^{0.02}	0.28	52.39 ^{80.4}	0.000	0.438	0.070
		Reference	-1.63e-03 ^{8.91e-03}	2.22 ^{0.03}	0.29 ^{0.11}	0.53 ^{0.17}	12.83 ^{18.79}	5.81 ^{8.58}	0.08 ^{0.04}	0.26	6.74 ^{14.6}	0.727	0.000	0.000
		Baseline	1.25e-17 ^{1.03e-16}	2.36 ^{0.04}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	-1.13e-03 ^{1.29e-02}	2.20 ^{0.03}	0.35 ^{0.08}	0.62 ^{0.12}	6.47 ^{6.00}	2.90 ^{2.57}	0.14 ^{0.05}	1.23	12.98 ^{6.4}	-	0.703	0.000
		MML	-1.13e-03 ^{1.29e-02}	2.20 ^{0.03}	0.36 ^{0.09}	0.64 ^{0.13}	5.91 ^{5.45}	2.65 ^{2.34}	0.14 ^{0.05}	-	12.97 ^{6.3}	-	0.672	0.000
		InvGam	-1.1e-03 ^{1.28e-02}	2.20 ^{0.03}	0.20 ^{0.04}	0.39 ^{0.06}	17.20 ^{7.30}	7.64 ^{3.18}	0.09 ^{0.03}	0.68	40.55 ^{2.9}	0.000	0.000	0.000
		Uniform	-1.05e-03 ^{1.28e-02}	2.20 ^{0.03}	0.27 ^{0.06}	0.51 ^{0.10}	10.01 ^{6.69}	4.48 ^{2.87}	0.10 ^{0.03}	0.74	12.97 ^{6.3}	0.000	0.672	0.000
		Reference	-9.66e-04 ^{1.28e-02}	2.20 ^{0.03}	0.51 ^{0.12}	0.81 ^{0.13}	2.75 ^{2.62}	1.24 ^{1.15}	0.25 ^{0.11}	0.78	3.19 ^{6.9}	0.930	0.000	0.000
		Baseline	-1.47e-18 ^{5.96e-17}	2.24 ^{0.03}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	49	REML	-1.1e-03 ^{1.1e-02}	2.16 ^{0.03}	0.41 ^{0.05}	0.71 ^{0.07}	3.78 ^{1.58}	1.71 ^{0.70}	0.17 ^{0.05}	1.73	10.70 ^{2.2}	-	0.844	0.000
		MML	-1.1e-03 ^{1.1e-02}	2.16 ^{0.03}	0.42 ^{0.06}	0.72 ^{0.07}	3.61 ^{1.50}	1.63 ^{0.67}	0.17 ^{0.05}	-	10.69 ^{2.1}	-	0.844	0.000
		InvGam	-1.09e-03 ^{1.24e-02}	2.17 ^{0.03}	0.26 ^{0.04}	0.50 ^{0.06}	9.65 ^{2.95}	4.31 ^{1.30}	0.15 ^{0.04}	1.33	32.68 ^{2.8}	0.000	0.000	0.000
		Uniform	-1.1e-03 ^{1.14e-02}	2.16 ^{0.03}	0.36 ^{0.05}	0.65 ^{0.07}	4.87 ^{1.82}	2.20 ^{0.81}	0.16 ^{0.05}	1.37	10.69 ^{2.1}	0.000	0.844	0.000
		Reference	-1.17e-03 ^{1.05e-02}	2.17 ^{0.03}	0.66 ^{0.14}	0.92 ^{0.10}	1.41 ^{1.35}	0.64 ^{0.61}	0.43 ^{0.19}	1.91	4.60 ^{5.3}	0.891	0.516	0.000
		Baseline	3.4e-18 ^{4.4e-17}	2.19 ^{0.04}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
10: Extreme long range in simulation: $\phi_{sim} = 600$	9	REML	1.04e-03 ^{1.41e-02}	0.55 ^{0.06}	0.71 ^{0.17}	0.93 ^{0.09}	1.15 ^{1.03}	0.53 ^{0.48}	0.10 ^{0.05}	2.10	193.36 ^{114.1}	-	0.078	0.430
		MML	1e-03 ^{1.41e-02}	0.55 ^{0.06}	0.78 ^{0.15}	0.96 ^{0.07}	0.82 ^{0.74}	0.38 ^{0.34}	0.10 ^{0.05}	-	193.13 ^{114.3}	-	0.078	0.438
		InvGam	7.97e-04 ^{1.35e-02}	0.56 ^{0.06}	0.87 ^{0.11}	0.99 ^{0.03}	0.49 ^{0.39}	0.22 ^{0.18}	0.09 ^{0.04}	1.03	52.62 ^{3.6}	0.000	0.000	0.000
		Uniform	1.06e-03 ^{1.39e-02}	0.55 ^{0.06}	0.81 ^{0.13}	0.98 ^{0.05}	0.67 ^{0.52}	0.31 ^{0.24}	0.09 ^{0.04}	1.26	193.13 ^{114.3}	0.000	0.078	0.438
		Reference	9.71e-04 ^{1.35e-02}	0.55 ^{0.06}	0.95 ^{0.08}	1.00 ^{0.02}	0.26 ^{0.24}	0.12 ^{0.11}	0.11 ^{0.05}	1.18	33.18 ^{36.1}	0.727	0.000	0.000
		Baseline	-6.56e-18 ^{9.6e-17}	0.99 ^{0.13}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	25	REML	4.42e-04 ^{8.28e-03}	0.42 ^{0.02}	0.69 ^{0.10}	0.94 ^{0.05}	1.08 ^{0.49}	0.49 ^{0.22}	0.11 ^{0.04}	1.77	204.91 ^{104.8}	-	0.000	0.492
		MML	4.43e-04 ^{8.28e-03}	0.42 ^{0.02}	0.71 ^{0.10}	0.95 ^{0.04}	0.99 ^{0.44}	0.45 ^{0.20}	0.11 ^{0.04}	-	204.79 ^{104.9}	-	0.000	0.492
		InvGam	2.64e-04 ^{7.62e-03}	0.42 ^{0.02}	0.77 ^{0.08}	0.98 ^{0.03}	0.73 ^{0.32}	0.33 ^{0.14}	0.11 ^{0.04}	0.83	62.95 ^{11.3}	0.000	0.000	0.000
		Uniform	3.96e-04 ^{8.1e-03}	0.42 ^{0.02}	0.71 ^{0.09}	0.96 ^{0.04}	0.95 ^{0.42}	0.43 ^{0.19}	0.11 ^{0.04}	1.28	204.79 ^{104.9}	0.000	0.000	0.492
		Reference	3.76e-04 ^{8.02e-03}	0.42 ^{0.02}	0.75 ^{0.11}	0.97 ^{0.04}	0.82 ^{0.41}	0.37 ^{0.19}	0.11 ^{0.04}	0.99	84.32 ^{50.2}	0.727	0.000	0.000
		Baseline	-6.09e-19 ^{5.72e-17}	0.66 ^{0.07}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-
	49	REML	-2.95e-04 ^{6.58e-03}	0.35 ^{0.01}	0.69 ^{0.06}	0.95 ^{0.03}	1.01 ^{0.25}	0.46 ^{0.11}	0.11 ^{0.03}	1.76	203.46 ^{98.8}	-	0.000	0.430
		MML	-2.96e-04 ^{6.58e-03}	0.35 ^{0.01}	0.70 ^{0.06}	0.96 ^{0.03}	0.96 ^{0.24}	0.44 ^{0.11}	0.11 ^{0.03}	-	203.48 ^{98.9}	-	0.000	0.438
		InvGam	-4.23e-04 ^{6.88e-03}	0.35 ^{0.01}	0.74 ^{0.05}	0.97 ^{0.02}	0.82 ^{0.19}	0.37 ^{0.08}	0.11 ^{0.03}	0.76	69.81 ^{16.8}	0.000	0.000	0.000
		Uniform	-3.17e-04 ^{6.56e-03}	0.35 ^{0.01}	0.70 ^{0.05}	0.96 ^{0.02}	0.96 ^{0.23}	0.43 ^{0.10}	0.11 ^{0.03}	1.37	203.48 ^{98.9}	0.000	0.000	0.438
		Reference	-3.29e-04 ^{6.66e-03}	0.35 ^{0.01}	0.71 ^{0.06}	0.96 ^{0.02}	0.91 ^{0.24}	0.41 ^{0.11}	0.11 ^{0.03}	0.97	98.01 ^{58.5}	0.727	0.000	0.000
		Baseline	1.1e-18 ^{4.23e-17}	0.53 ^{0.05}	-	-	-	-	0.00 ^{0.00}	-	-	-	-	-