Trade (Dis)integration: The Sudden Death of NAFTA

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Online Appendix

Appendix I: Estimation Strategy

As mentioned in the empirical model section, our PVAR model follows the structural factor approach developed by Canova and Ciccarelli (2006, 2013). The Bayesian Panel VAR model is estimated using the BEAR toolbox for MATLAB developed by Dieppe *et al*. (2016).

The advantage of this structural factor modelling approach is that the parameter space of the Panel VAR model which is estimated is significantly reduced. Specifically, from a total number of $Nn(Nnp + m) = 156$ coefficients, with N countries, n endogenous variables, p lags and m exogenous variables, the parameters are dropped to $(1 + N + n + (p - 1) + m) = 9$ with the factor approach. The idea is to express the parameters that need estimation as a function of a few structural factors:

$$\beta = \Gamma_1\varphi_1 + \Gamma_2\varphi_2 + \Gamma_3\varphi_3 + \Gamma_4\varphi_4 + \Gamma_5\varphi_5, \tag{A1}$$

where β is the vector of the parameters, φ are the vectors containing the structural factors, with φ_1 is a common factor, φ_2 is the unit-specific factor that captures the components which are specific to each country, φ_3 is the variable-specific factor that captures the components of each endogenous variable, φ_4 is the lag-specific component, and φ_5 is the factor that captures the exogenous variable. Finally, Γ are the selection matrices whose entries are 0 or 1. We can rewrite Eq. (A1) in a stacked form as:

$$\beta = \Gamma\varphi. \tag{A2}$$

The PVAR model of Eq. (1), in the main paper, can be written as $y_t = \tilde{X}_t * \beta + \varepsilon_t$, with $\tilde{X}_t$ contains both the vector of lagged endogenous variables ($y_{i,t-1}$) and the exogenous variable (x_t). In this way, using Eq. (A2) we can rewrite the PVAR model as:

$$\begin{aligned} y_t &= \tilde{X}_t * \Gamma\varphi + \varepsilon_t \\ &= Z\varphi + \varepsilon_t, \end{aligned} \tag{A3}$$

where $Z = \tilde{X}_t * \Gamma$ and $\varepsilon_t \sim \mathrm{N}(0, \Sigma)$, with $\Sigma = \sigma\tilde{\Sigma} = \begin{pmatrix} \sigma\tilde{\Sigma}_{11} & \cdots & \sigma\tilde{\Sigma}_{1N} \\ \vdots & \ddots & \vdots \\ \sigma\tilde{\Sigma}_{N1} & \cdots & \sigma\tilde{\Sigma}_{NN} \end{pmatrix} = \begin{pmatrix} \Sigma_{11} & \cdots & \Sigma_{1N} \\ \vdots & \ddots & \vdots \\ \Sigma_{N1} & \cdots & \Sigma_{NN} \end{pmatrix}$ and σ is a scaling random variable which follows an inverse Gamma distribution; $\sigma \sim IG(\frac{\alpha}{2}, \frac{\delta}{2})$.

The benefits of this procedure become evident when we consider the dimensions of each factor. More precisely, the common factor φ_1 is a scalar, and therefore, its dimension is $d_1 = 1$. φ_2 is the country specific factor and therefore $d_2 = N = 3$. φ_3 is the factor that is specific to each variable and thus $d_3 = n = 4$. φ_4 is the lag-specific factor and $d_4 = p - 1 = 0$. Finally, φ_5 is the factor related to the exogenous variable and, therefore, $d_5 = m = 1$.

In order to understand the benefits of this transformation, it is useful to analyse the first element of the y_t vector; that is the variable $y_{11,t}$, which is now written as:

$$
\begin{aligned}
y_{11,t} = (&y_{11,t-1} + y_{12,t-1} + y_{13,t-1} + y_{14,t-1} + y_{21,t-1} + y_{22,t-1} + y_{23,t-1} + y_{24,t-1} \\
&+y_{31,t-1} + y_{32,t-1} + y_{33,t-1} + y_{34,t-1} + y_{41,t-1} + y_{42,t-1} + y_{43,t-1} + y_{44,t-1}) * \varphi_{11} \\
&+(y_{11,t-1} + y_{12,t-1} + y_{13,t-1} + y_{14,t-1}) * \varphi_{21} \\
&+(y_{11,t-1} + y_{21,t-1} + y_{31,t-1}) * \varphi_{31} \\
&+(x_{1,t}) * \varphi_{51} + \varepsilon_{11,t}.
\end{aligned} \tag{A4}
$$

So,

$$
y_{11,t} = \Psi_{11,t}\varphi_{11} + \Psi_{21,t}\varphi_{21} + \Psi_{31,t}\varphi_{31} + x_{1,t} * \varphi_{51} + \varepsilon_{11,t}. \tag{A5}
$$

Doing the same for all equations, the system can be rewritten as:

$$
\begin{pmatrix} y11,t \\ y12,t \\ y13,t \\ y14,t \\ y15,t \\ y16,t \\ y17,t \\ y18,t \\ y19,t \\ y110,t \\ y111,t \\ y112,t \end{pmatrix} = \begin{pmatrix} \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \\ \Psi11,t \end{pmatrix} (\varphi 11) + \begin{pmatrix} \Psi_{21,t} & 0 & 0 \\ \Psi_{21,t} & 0 & 0 \\ \Psi_{21,t} & 0 & 0 \\ \Psi_{21,t} & 0 & 0 \\ 0 & \Psi_{22,t} & 0 \\ 0 & \Psi_{22,t} & 0 \\ 0 & \Psi_{22,t} & 0 \\ 0 & \Psi_{22,t} & 0 \\ 0 & 0 & \Psi_{23,t} \\ 0 & 0 & \Psi_{23,t} \\ 0 & 0 & \Psi_{23,t} \\ 0 & 0 & \Psi_{23,t} \end{pmatrix} \begin{pmatrix} \varphi 21 \\ \varphi 22 \\ \varphi 23 \end{pmatrix}
$$

$$
+\begin{pmatrix} \Psi_{31,t} & 0 & 0 & 0 \\ 0 & \Psi_{32,t} & 0 & 0 \\ 0 & 0 & \Psi_{33,t} & 0 \\ 0 & 0 & 0 & \Psi_{34,t} \\ \Psi_{31,t} & 0 & 0 & 0 \\ 0 & \Psi_{32,t} & 0 & 0 \\ 0 & 0 & \Psi_{33,t} & 0 \\ 0 & 0 & 0 & \Psi_{34,t} \\ \Psi_{31,t} & 0 & 0 & 0 \\ 0 & \Psi_{32,t} & 0 & 0 \\ 0 & 0 & \Psi_{33,t} & 0 \\ 0 & 0 & 0 & \Psi_{34,t} \end{pmatrix} \begin{pmatrix} \varphi 31 \\ \varphi 32 \\ \varphi 33 \\ \varphi 34 \end{pmatrix} + \begin{pmatrix} x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \\ x_{1,t} \end{pmatrix} (\varphi 51) + \begin{pmatrix} \varepsilon 11, t \\ \varepsilon 12, t \\ \varepsilon 13, t \\ \varepsilon 14, t \\ \varepsilon 15, t \\ \varepsilon 16, t \\ \varepsilon 17, t \\ \varepsilon 18, t \\ \varepsilon 19, t \\ \varepsilon 110, t \\ \varepsilon 111, t \\ \varepsilon 112, t \end{pmatrix}. \qquad \text{(A6)}
$$

In a typical fashion, the key objective is to find the posterior distributions for φ, $\tilde{\Sigma}$ and σ. To compute the posterior distribution, an expression for the Bayes rule is:

$$
\pi(\varphi, \tilde{\Sigma}, \sigma | y) \propto f(y | \varphi, \tilde{\Sigma}, \sigma) \pi(\varphi) \pi(\tilde{\Sigma}) \pi(\sigma). \qquad \text{(A7)}
$$

The likelihood function is given as:

$$
f(y | \varphi, \tilde{\Sigma}, \sigma) \propto (\sigma)^{-TNn/2} \left| \tilde{\Sigma} \right|^{-T/2} \prod_{t=1}^{T} \left\{ \exp\left(-\frac{1}{2} \sigma^{-1} (y_t - \tilde{X}_t \varphi)' \tilde{\Sigma}^{-1} (y_t - \tilde{X}_t \varphi) \right) \right\}, \qquad \text{(A8)}
$$

while the prior for φ is given as:

$$
\pi(\varphi | \varphi_0, \varphi_0) \propto \exp\left(-\frac{1}{2} (\varphi - \varphi_0)' {\Phi_0}^{-1} (\varphi - \varphi_0) \right). \qquad \text{(A9)}
$$

For $\tilde{\Sigma}$ we use an uninformative prior as follows:

$$
\pi(\tilde{\Sigma}) \propto \left| \tilde{\Sigma} \right|^{-(Nn+1)/2}. \qquad \text{(A10)}
$$

Finally, σ follows an inverse Gamma distribution, that is:

$$\pi(\sigma) \propto \sigma^{-(\frac{a_0}{2})-1} \exp\left(\frac{-\delta_0}{2\sigma}\right). \tag{A11}$$

Using the Bayes rule from Eq. (A7), and combining the likelihood, in Eq. (A8), with the priors, Eq. (A9), (A10) and (A11), the joint posterior is given as:

$$f(\varphi, \tilde{\Sigma}, \sigma | y) \propto \prod_{t=1}^{T} \left\{ \exp\left(-\frac{1}{2}\sigma^{-1}(y_t - \tilde{X}_t\varphi)' \tilde{\Sigma}^{-1}(y_t - \tilde{X}_t\varphi)\right)\right\} \times \exp\left(\frac{-\delta_0}{2\sigma}\right)$$
$$\times (\sigma)^{-(NnT+a_0)/2-1} \times |\tilde{\Sigma}|^{-(T+Nn+1)/2} \times \exp\left(-\frac{1}{2}(\varphi - \varphi_0)' \Phi {\Theta_0}^{-1}(\varphi - \varphi_0)\right). \tag{A12}$$

References

Canova, F. and Ciccarelli, M. (2006). *Estimating Multi-Country VAR Models*, Working Paper Series 603, European Central Bank.

Canova, F. and Ciccarelli, M. (2013). Panel Vector Autoregressive Models: A Survey, in: *VAR Models in Macroeconomics-New Developments and Applications: Essays in Honor of Christopher A. Sims*, Emerald, 205-246.

Dieppe, A., van Roye, B. and Legrand, R. (2016). *The BEAR Toolbox*. Working Paper Series 1934: European Central Bank.

Appendix II: Additional figures for the two sub-samples (pre-NAFTA vs. post-NAFTA period) using the export share measure

Figure A1: Impulse responses of the Canadian consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between Canada-US and Canada-Mexico for the pre-NAFTA period.

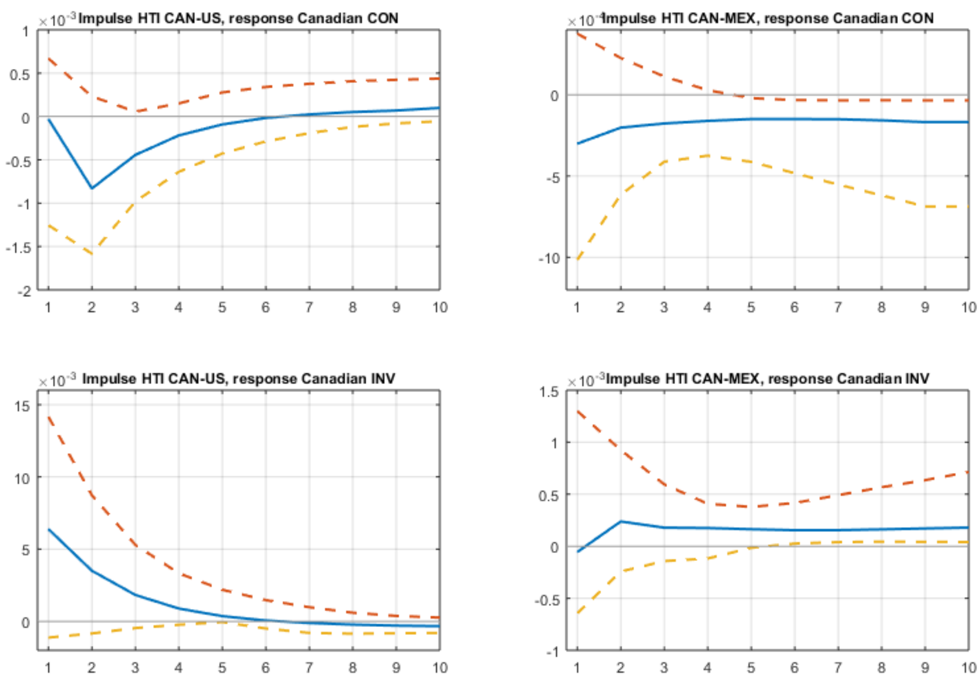

Figure A2: Impulse responses of the Canadian consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between Canada-US and Canada-Mexico for the post-NAFTA period.

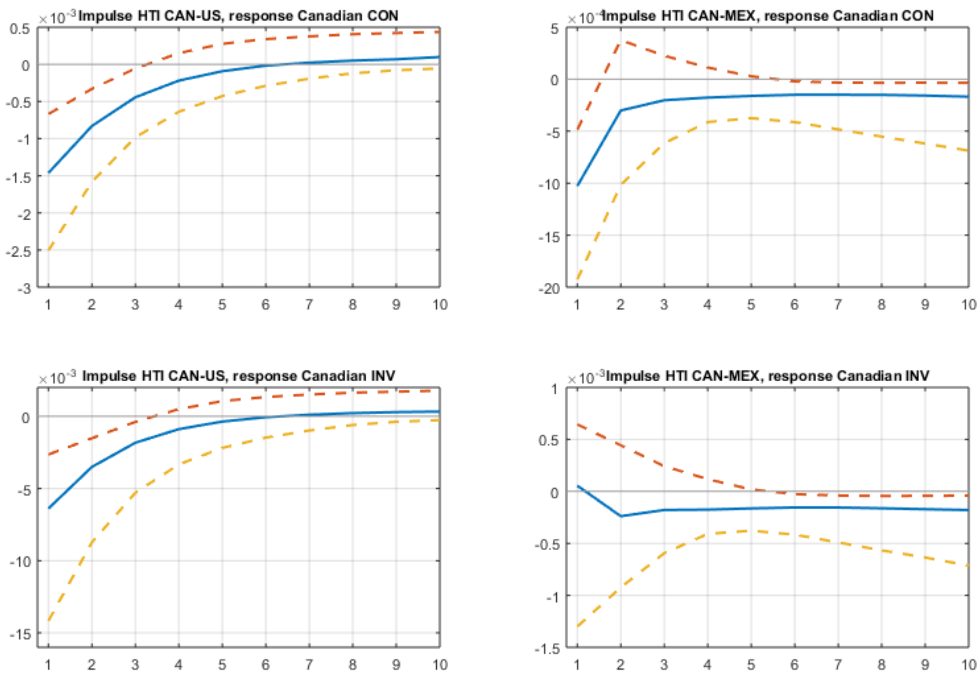

Figure A3: Impulse responses of the Mexican consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between Mexico-Canada and Mexico-US for the pre-NAFTA period.

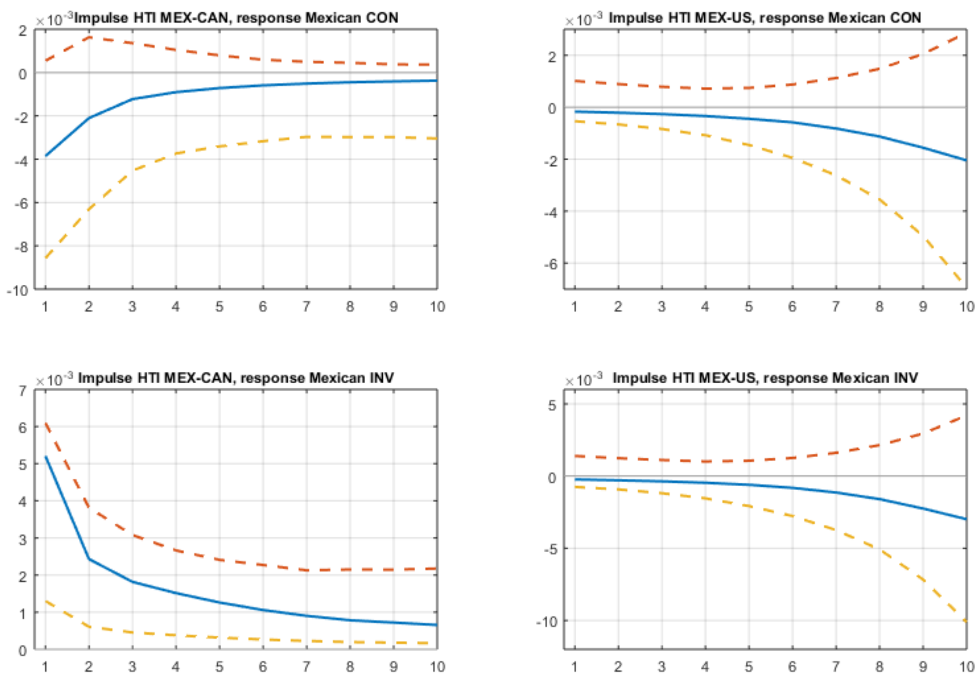

Figure A4: Impulse responses of the Mexican consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between Mexico-Canada and Mexico-US for the post-NAFTA period.

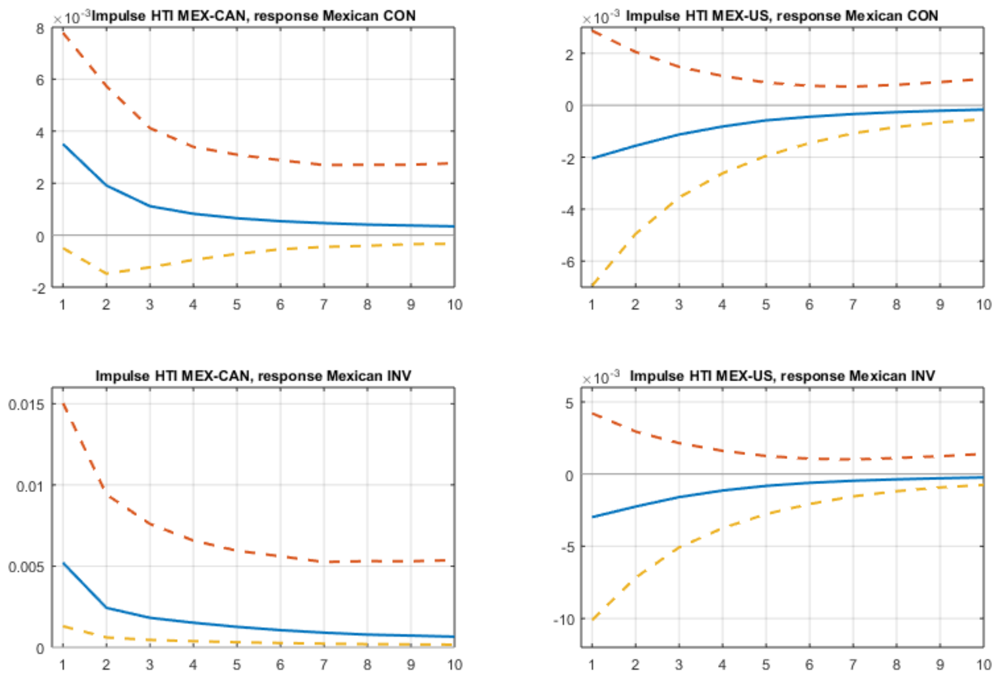

Figure A5: Impulse responses of the US consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between US-Canada and US-Mexico for the pre-NAFTA period.

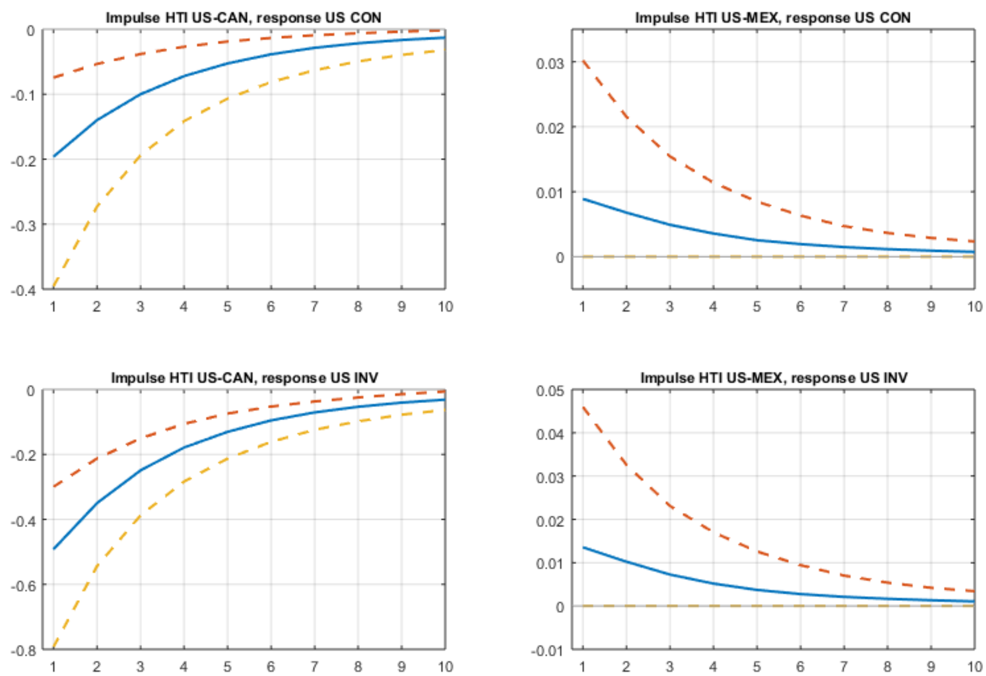

Figure A6: Impulse responses of the US consumption (CON) and investment (INV) to a negative shock to the export share measure (XS) between US-Canada and US-Mexico for the post-NAFTA period.