

Internet Appendix

“A First Look at the Historical Performance of the New NAV REITs”

This Internet Appendix contains tables referred to (but not included) in the main text. It also provides a reply to McCann and Qin ([2024](#))

A Replying to McCann and Qin (2024)

This section provides a reply to McCann and Qin (2024) (henceforth M&Q), which criticizes certain aspects of the original version of this paper (dated January 10, 2024).¹ We focus on the three primary limitations they raise in their section 3 (titled “Summary of Comments”). The other points they discuss are more tangential.²

Before addressing each of these limitations directly, it is important to put them into context. First, one of the limitations they discuss is a minor issue that can be easily addressed (and which we do address in the main text of this paper). Second, we properly highlight the other two limitations and properly discuss them in detail in the original paper they critiqued (which we posted publicly in January of 2024). These later limitations are simply an inherent consequence of analyzing a new asset class that is subject to lagged pricing updates. Any paper evaluating the early historical performance of a new asset class with these characteristics would also be subject to the same limitations. This is why we put so much attention and focus on highlighting them in our original paper.

Unfortunately, M&Q presented these limitations in such a way as to make it appear as though raising these limitations was a novel finding on their part, giving the reader the false impression that we did not highlight and recognize them in our own research. Again, these limitations exist in analyzing any new asset class with lagged pricing updates, making it naturally important to discuss them.

As an example, the conclusion from M&Q starts with, “We identify several limitations in Coutts and Goncalves (2024)’s analysis of NAV REIT index returns and discuss their implications for C&G 2024’s main result...” whereas, in reality, the main limitations were

¹It is also important to point out that the title of M&Q implicitly suggests we argue NAV REITs belong to investors’ portfolios. This is not the case. As our title (and writing more broadly) suggests, our paper is providing a historical analysis of the performance of NAV REITs. Whether an asset class belongs to investors’ portfolios depends on a forward looking analysis of the asset class and market conditions, which is something we do not tackle in this paper (as discussed in Section 5.6, which was also present in our original draft).

²While our reply to M&Q focuses on their three primary criticisms to avoid distracting the reader with the less relevant points brought up by M&Q, this fact should not be misconstrued as indicating that we endorse other claims in their paper.

already identified and discussed in our own original work. Our replies to their specific points clarify this aspect.³ Our replies also highlight a few issues with how M&Q attempted to address some of these limitations.

A.1 Sample Period

The first major argument in M&Q is that our sample period “is far too short.” Our original sample covered from January of 2016 to September 2023, which reflects the full history of data available to us at the time we started this project. M&Q added 6 months of data (to cover until March 2024) and show that this has a non-trivial effect on alphas because NAV REITs performed poorly from October 2023 to March 2024. For example, in our original analysis, the alpha of the value-weighted index of NAV REITs (VW) based on unsmoothed returns was 5.3% whereas the analogous alpha of the VW index in their analysis with the added six months is 4.0%. So, there is a non-trivial decline in alpha (of 1.3 percentage points), albeit the VW index alpha remains large in their analysis (at 4.0%).⁴

M&Q conspicuously failed to recognize that the short sample nature of our analysis was fully discussed in our original draft (thereby positioning their argument as if it was original to their work). In particular, page 3 in the introduction of our original draft had the following paragraph (there is a similar paragraph in the current main draft):

“The main implication of our results is that the early performance of NAV REITs has

³The first paragraph in M&Q contains the sentence, “While the authors identify important limitations in their empirical work, the paper’s conclusions are being used without qualification to influence public policy, warranting a second look at its empirical support.” This statement clearly does not inform the reader that the most important limitations they claim to have identified had already been discussed in our own paper.

⁴We focus our discussion on the economic significance of alphas instead of statistical significance because (i) alphas are often statistically insignificant in our main text so that t-stats are less relevant than the alphas themselves and (ii) the reported t-statistics are not comparable across our paper and theirs. As is standard in the asset pricing literature, our original paper reported t-statistics that are robust to residual heteroskedasticity and autocorrelation (our current paper reports the same t-statistics as well as t-statistics that are robust to residual heteroskedasticity but not autocorrelation). In contrast, their analysis relies on t-statistics that are not robust to residual heteroskedasticity or autocorrelation (it is not entirely clear from their writing, but that seems to be the case). In fact, they claimed their analysis suggested we had errors in our t-statistics, but the reason is that they failed to recognize that we used t-statistics that are robust to residual heteroskedasticity and autocorrelation. If they had contacted us for clarification (as is customary among researchers in finance and real estate), we would have pointed out their confusion.

been positive even after adjusting for risk. However, further analysis is needed to provide a final answer to the important question of whether NAV REITs will consistently generate positive risk-adjusted returns to their investors. In particular, our evidence on the return autocorrelations of NAV REITs suggests that a conclusive evaluation of the risks and returns of NAV REITs would reasonably require a longer sample than what is currently available for these new funds. For instance, to the extent any recent negative shocks to CRE (such as higher interest rates or slower than expected rent growth) have not been incorporated into private real estate valuations, it would bias risk-adjusted returns upward and negatively impact future performance relative to publicly traded REITs. More research is needed to further analyze how these funds incorporate market pricing information into their NAVs.”

Moreover, the original draft had a section on the limitations of our analysis, with one subsection (titled “Limited Times Series & No Extended Downturns in the Sample”) detailing the limitations associated with the short sample nature of our analysis and another subsection (titled “Historical Analysis”) explaining that our work should be understood as a historical analysis and not as a forward looking analysis, and highlighting that “the limited time-series of the return data and the smoothness of returns in NAV REITs pose a challenge even for the evaluation of the historical performance of NAV REITs.”

Importantly, Subsections 5.5 and 5.6 in the current main text are almost identical to these two subsections we had in the original draft. Even though we have updated our sample to December 2024 during the editorial process (with alphas declining and becoming statistically insignificant due to the poor performance of NAV REITs in 2024), it remains the case that our analysis is based on a short sample and should be interpreted accordingly. This is an inherent limitation of analyzing the historical performance of any emerging asset class, such as NAV REITs.

A.2 Return Unsmoothing

The second major critique in M&Q relates to return unsmoothing and can be split into two points. The first point is about autocorrelation in the unsmoothed returns and was already

discussed in our original draft (something that is, again, not recognized in M&Q). We further reinforce these points in Subsection A.2.1 below. The second point is about selecting the optimal number of moving average (MA) lags empirically. As we show in Subsection A.2.2 below, alphas are (if anything) higher when standard statistical methods are used (instead of the ad-hoc algorithm suggested by M&Q) to select the optimal number of MA lags.

A.2.1 Autocorrelation in Unsmoothed Returns

First, M&Q argue that, “C&G 2024’s unsmoothing method reduces autocorrelation in observed returns but its unsmoothed returns remain smooth,” highlighting the small but persistent autocorrelations that remains in unsmoothed returns. Again, this is not a novel point. Our original draft highlighted this issue. For instance, we had the following paragraph in the section that discusses autocorrelations:

“In summary, the results from Table 2 indicate that while reported returns for NAV REITs have substantial autocorrelation, which likely reflects lagged price reactions, the unsmoothed returns of NAV REITs do not. That said, it is important to understand that even the unsmoothed returns of NAV REITs display some anomalous autocorrelation patterns. For instance, most autocorrelations of NAV REIT indices are (small but) positive, which deviates from the typical autocorrelation patterns observed in public indices. As such, the proper inference is that return unsmoothing goes a long way in alleviating the potential underestimation of NAV REIT risk exposures, but it likely does not fully mitigate the issue. For practical purposes, however, the magnitudes of the autocorrelations are small enough to limit the magnitude of potential risk exposure underestimation, which implies our alpha analyses in the next sections are informative about the true historical alpha of NAV REITs over the time period analyzed.”

Similarly, right before moving to the alpha analysis, the original draft contained the following:

“...it is important to highlight a limitation of return unsmoothing methods. While return unsmoothing has a large effect of risk measurement, roughly doubling measures of risk such

as volatility and exposure to public REITs, it is unlikely to provide a full solution to the issue of smoothed returns. For instance, we still observe small but consistently positive autocorrelations for NAV REITs, a pattern that is unlikely to arise from typical sampling variation, and thus indicates our unsmoothing method does not fully unsmooth the returns of these funds.”

Furthermore, right after providing the alphas from the traditional alpha analysis, the original draft contained the following:

“...Still, it is important to keep in mind that some spurious autocorrelation might persist even with our unsmoothing analysis, and that it might have some effect on our performance analysis. Our return unsmoothing method addresses the issue, but it is likely not a full solution.”

So, in a nutshell, the limitation raised by M&Q was already discussed in detail in our original draft (with M&Q failing to recognize this). Of course, return unsmoothing (as any method) is an imperfect solution for the lagged pricing updates present in illiquid assets. However, it is an important step in performance analysis. Without it, estimated alphas would be much higher, but incorrectly so.

A.2.2 Choice of the Number of MA Lags during Return Unsmoothing

Second, M&Q argue that using a larger number of MA lags (instead of the 12 MA lags we use in our original draft) reduces alphas. For instance, for the VW index, the alpha declines from 5.3% in our original analysis (which uses a MA(12)) to 3.8% in their analysis (which uses a MA(17)). The problem with this argument is that estimating 18 parameters (mean return plus 17 MA lags) from a sample of 99 months (as they have) or even 108 months (as in our main text) is statistically unsound.

Interestingly, they propose an ad-hoc algorithm to justify the high number of MA lags even though standard methods to select the number of MA lags exist in the time series literature. The most common methods in the time series literature are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). These methods are designed to trade-

off model fit (through the log-likelihood function) and statistical power (through the number of parameters and sample size) to guard against overfitting. Applying these methods to the VW index in our original sample period leads to 5 MA lags under the AIC and 2 MA lags under the BIC, as can be seen from Table IA.6 (with VW alphas that are higher than the ones we reported in the original draft). The AIC and BIC recommend the same number of lags in our updated sample, as can be seen in Table 7. Clearly, even our original choice of 12 MA lags required many more parameters than suggested by the AIC and BIC.⁵ So, if anything, selecting the optimal number of lags using standard statistical methods leads to alphas that are higher (not lower) than the ones we reported in our original draft.

Our interpretation of the AIC and BIC results is not that we should use a MA(5) or MA(2). Rather, we interpret these findings as evidence that there is not enough statistical power in our short sample to estimate a MA model with a large number of lags. To avoid this problem, we follow Coutts, S Gonçalves, and Rossi (2024) in focusing on an autoregressive (AR) unsmoothing process. We use an AR(2), which is recommended by the AIC (with BIC recommending an AR(1)) and also consistent with the choice in Coutts, S Gonçalves, and Rossi (2024).⁶ The key advantage of an AR process (relative to an MA process) is that it captures long-lived autocorrelations regardless of the number of lags included. For instance, an AR(1) is equivalent to a MA(∞) with exponentially decaying MA lags. Using an AR(2), as we do, allows us to still consider a MA(∞) while deviating from the baseline exponentially decaying parametrization. In contrast, a MA(L) process only accounts for autocorrelations up to lag L, but is fully flexible on the format of the autocorrelation function. Unfortunately, as the AIC and BIC indicate, flexibly estimating a MA(L) with a large L is suboptimal from a statistical perspective as it would lead to low statistical power in our short sample.

Beyond reporting results for an AR(2), we also report results using many alternative MA and AR specifications (see Table IA.6 for results associated with our original sample and

⁵Originally, we explored moving average (MA) processes with 1 to 12 lags as well as autoregressive (AR) processes with 1 to 2 lags. We reported results based on a MA(12) because the alphas tended to be the smallest among the specifications we explored (so, we were reporting conservative results).

⁶Early work on real estate investments focused on an autoregressive process of order one (see Geltner (1991, 1993)). So, our AR(2) specification nests the specification in Geltner (1991, 1993) as a special case.

Table 7 for results associated with our updated sample). This fact should not be misconstrued as an indication that all unsmoothing models are equally reasonable. An AR(2) seems to be the best model in that it considers long-lived autocorrelations (as it is a MA(∞)) while keeping the number of parameters in line with prior literature (Couts, S Gonçalves, and Rossi (2024)) as well as with what is suggested by the AIC (and only slightly higher than what is suggested by the BIC).

A.3 Alternative Alpha Analysis

The third major critique in M&Q is that “the positive ‘alternative’ alpha estimated by C&G 2024 is meaningless.” This argument is based on two points, both of which are addressed below (and neither of which justifies the claim that alternative alphas are meaningless). However, the reader should note that our alternative alpha analysis leads to lower alphas than the traditional alpha analysis commonly applied in the literature (which we also apply). So, a reader who entirely discards our alternative alpha analysis is left with alpha estimates that are higher (not lower) than the ones obtained from our alternative alpha analysis. Consequently, the third major argument in M&Q goes against their own overall point (which we understand to be that the alphas of NAV REITs are lower than the alphas we report).

A.3.1 Relation Between Alphas and Idiosyncratic Volatility

Our alternative alpha is given by Equation 14, reproduced below for convenience:

$$\alpha_j = \text{sign}(\Delta) \cdot \sigma_\epsilon \cdot \sqrt{|\text{SR}[r_{p,j}]^2 - \text{SR}[r_p]^2|} \quad (\text{IA.6})$$

where $\Delta = \text{SR}[r_{p,j}] - \text{SR}[r_p]$.

M&Q highlights that the alternative alpha (α_j) is a function of idiosyncratic volatility (σ_ϵ) directly and indirectly through $\text{SR}[r_{p,j}]$. Moreover, they point out that α_j can increase with σ_ϵ in some regions of the parameter space (in particular, when σ_ϵ and the allocation to NAV REITs are small). So, α_j has a property that is economically counterintuitive in some regions of the parameter space.

The reason is as follows. In the traditional alpha analysis, the allocation to NAV REITs is ex-post optimal given the σ_ϵ value (and other parameters). Therefore, an increase in σ_ϵ leads to a lower NAV REIT allocation so that α_j is independent of σ_ϵ . In contrast, in the alternative alpha analysis the allocation to NAV REITs is held fixed, which makes α_j a function of σ_ϵ .

While this property represents a limitation of the alternative alpha analysis, it is easy to address. Specifically, we have that the Information Ratio (IR) is given by

$$IR_j = \frac{\alpha_j}{\sigma_\epsilon} = \text{sign}(\Delta) \cdot \sqrt{|\text{SR}[r_{p,j}]^2 - \text{SR}[r_p]^2|} \quad (\text{IA.6})$$

which represents alpha per unit of idiosyncratic volatility. In contrast to α_j , IR_j is decreasing in σ_ϵ .

Note that alphas and information ratios have the same sign. They only differ in units. Moreover, information ratios are commonly used by practitioners to assess the value added from trading strategies. So, this issue highlighted by M&Q does not justify their argument that alternative alphas are meaningless. We continue to report alternative alphas in the main text (since they are easy to compare with traditional alphas) while also reporting information ratios in the Internet Appendix for interested readers (see Table [IA.4](#)).

A.3.2 Statistical Significance for Alphas

M&Q argue that we do not report statistical significance for the alternative alphas of NAV REIT indices. Clearly, this does not imply alternative alphas are meaningless as suggested by M&Q. This criticism is also easy to address. Table [IA.7](#) provides alternative alphas over our original sample period (January 2016 to September 2023) with bootstrap t-statistics for the VW index. Alternative alphas were generally statistically significant over our original sample period. As we highlight in Section [A.1](#), NAV REITs performed poorly in 2024, so alternative alphas (just like traditional alphas) are not statistically significant over our extended sample that goes from January 2016 to December 2024 (see Tables 8 and 10 in the main text).

References for Internet Appendix

- Andrews, D. W. K. and J. C. Monahan (1992). “An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator”. In: *Econometrica* 60.4, pp. 953–966.
- Couts, S. J., A. S. Gonçalves, and A. Rossi (2024). “Unsmoothing returns of illiquid funds”. In: *The Review of Financial Studies* 37.7, pp. 2110–2155.
- Geltner, D. (1991). “Smoothing in Appraisal-Based Returns”. In: *Journal of Real Estate Finance and Economics* 4, pp. 327–345.
- Geltner, D. (1993). “Estimating Market Values from Appraised Values without Assuming an Efficient Market”. In: *Journal of Real Estate Research* 8.3, pp. 325–345.
- McCann, C. and C. Qin (2024). “No, NAV REITs Do Not Belong in Investors’ Portfolios: Comments on Coutts and Goncalves (2024)”. Working Paper.
- Newey, W. K. and K. D. West (1987). “A Simple, Positive-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix”. In: *Econometrica* 55.3, pp. 703–708.
- Newey, W. K. and K. D. West (1994). “Automatic Lag Selection in Covariance Matrix Estimation”. In: *Review of Economic Studies* 61.4, pp. 631–653.
- White, H. (1980). “A Heteroskedastic Consistent Covariance Matrix and a Direct Test for Heteroskedasticity”. In: *Econometrica* 48.4, pp. 817–838.

Table IA.1
Initial Month for each Share Class of each NAV REIT

This table provides the date of the first month we have returns available for each share class of each NAV REIT. We provide columns for each of the common share classes (D, S, T, I) as well as generic columns that contain information on other share classes (which are organized in alphabetic order within each NAV REIT). See Table 1 for a detailed description of the NAV REITs in this table and Table 2 for basic information on each NAV REIT.

NAV REIT	Retail Share Classes (Min Inv < \$1M)							Institutional Share Classes (Min Inv ≥ \$1M)				
	D	S	T	Other1	Other2	Other3	Other4	I	Other2	Other2	Other3	Other4
AREIT	1/2016	09/2017	1/2016	1/2016				1/2016				
JLLIPT				1/2016	1/2016				1/2016	1/2016		
HGIT	1/2018	1/2018	1/2018	1/2016	5/2017	1/2018	1/2016	1/2018				
RREEF			9/2017	1/2016	1/2016	11/2020	6/2021		6/2021			
BREIT	5/2017	1/2017	6/2017					1/2017				
CFIT	10/2020	11/2020	9/2020	6/2017	6/2017			9/2020	6/2017			
AIREIT	7/2018		11/2017					11/2017				
NGCREIT	6/2018	12/2019	1/2019					7/2018				
SREIT	1/2019	1/2019	1/2019					1/2019				
CPREIF	10/2019	10/2019	10/2019					10/2019				
BFRI	12/2019	6/2022						12/2019				
CWREIT	5/2023		12/2021	4/2020				12/2023	1/2019			
INREIT	6/2021	6/2021	6/2021					6/2021				
KREST	10/2023	10/2022		3/2022	7/2021			7/2021				
PGIMRT	11/2022	11/2022	11/2022					11/2022				
JPMREIT	1/2023	8/2023						11/2022				
APOLLO	10/2024	5/2024						2/2024	4/2023	1/2024	12/2024	1/2023
IPCALT	12/2024		5/2024					12/2023				

Table IA.2

Average Annual Fee Differentials Across Share Classes of each NAV REIT

This table reports average fee differentials (annualized) for each share class of each NAV REIT relative to the lowest fee share class of the respective NAV REIT. Specifically, for a given NAV REIT, we start by calculating the average return of each of its share classes for the period over which all of its share classes are available. Then, we define the lowest fee share class for this NAV REIT as the share class with the highest average return over this period. Finally, the average annual fee differential for a given share class is the average return difference between the lowest fee share class and the given share class (multiplied by twelve). Since both share classes come from the same NAV REIT over the same period, the difference reflects fee differentials and not variation in fund performance. We provide columns for each of the common share classes (D, S, T, I) as well as generic columns that contain information on other share classes (which are organized in alphabetic order within each NAV REIT). See Table 1 for a detailed description of the NAV REITs in this table and Table 2 for basic information on each NAV REIT.

NAV REIT	Retail Share Classes (Min Inv < \$1M)							Institutional Share Classes (Min Inv ≥ \$1M)				
	D	S	T	Other1	Other2	Other3	Other4	I	Other1	Other2	Other3	Other4
AREIT	0.3%	0.9%	0.9%	0.0%				0.0%				
JLLIPT				0.8%	0.3%				0.2%	0.0%		
HGIT	0.2%	0.8%	1.0%	0.0%	0.1%	0.0%	0.5%	0.0%				
RREEF			0.8%	1.0%	0.5%	0.0%	1.0%		0.1%			
BREIT	0.5%	0.9%	0.8%					0.0%				
CFIT	0.2%	0.7%	0.8%	0.0%	1.0%			0.0%	0.0%			
AIREIT	0.2%		0.8%					0.0%				
NGCREIT	0.2%	1.0%	0.6%					0.0%				
SREIT	0.4%	0.7%	0.7%					0.0%				
CPREIF	0.3%	0.9%	0.9%					0.0%				
BFRI	0.0%	1.5%						0.5%				
CWREIT	0.0%		1.5%	0.6%				0.6%	0.6%			
INREIT	0.2%	0.3%	0.4%					0.0%				
KREST	0.2%	0.8%		0.0%	0.8%			0.0%				
PGIMRT	0.3%	0.9%	0.9%					0.0%				
JPMREIT	0.0%	0.5%						0.0%				
APOLLO	1.6%	2.2%						1.4%	0.1%	0.0%	0.0%	0.9%
IPCALT	0.2%		0.7%					0.0%				

Table IA.3

Annualized Return Difference Between Institutional and Retail Share Classes

This table provides, for each NAV REIT, the median value for the difference in returns between (value-weighted) institutional share classes and (value-weighted) retail share classes. In this analysis, the sample for each NAV REIT is restricted to the period over which at least one share class of each type is available. All returns are annualized (i.e., multiplied by twelve). See Table 1 for a detailed description of the NAV REITs in this table and Table 2 for basic information on each NAV REIT.

NAV REIT	Number of Months	Median Return Differential
AREIT	108	0.27%
JLLIPT	108	0.58%
HGIT	84	0.60%
RREEF	43	0.50%
BREIT	96	0.81%
CFIT	91	0.30%
AIREIT	86	0.85%
NGCREIT	78	0.67%
SREIT	72	0.76%
CPREIF	63	0.80%
BFRI	61	0.92%
CWREIT	57	0.80%
INREIT	43	0.24%
KREST	42	0.86%
PGIMRT	26	0.64%
JPMREIT	24	0.05%
APOLLO	8	1.72%
IPCALT	8	1.04%

Table IA.4

Historical Performance Analysis: Information Ratios with Fixed Weights

This table presents our analysis of the performance of NAV REITs using the information ratios (α/σ_e) obtained from the alternative alpha method. Specifically, it provides the implied annualized information ratios (i.e., monthly information ratios multiplied by $\sqrt{12}$) from the increase in Sharpe ratio as we add prespecified NAV REIT index and fund allocations to a public market portfolio based on public bonds (b), public equities (e), and public REITs (r). The first two rows represent the weights of the public indices on the public market portfolio. The third row represents the reallocation weight going to the NAV REIT investment. Mechanically, each of the other weights similarly decrease by their same relative proportions in columns one through three of each panel in the updated portfolios. For example, in the last three columns of the table, the 40%, 30%, and 30% shown would be 32%, 24%, and 24%, respectively, after updating the portfolio to include NAV REITs. Rows four and six through sixteen provide the alphas of the respective portfolios with NAV REIT index and funds (based on Equation 14). Row five provides the t-statistic (based on bootstrap simulations) for values in row four. All of the results are based on the unsmoothed returns. All returns are in excess of the returns on the 1-month Treasury bill. See Table 1 for a detailed description of the VW index and NAV REIT funds in this table.

$w_b =$	$w_b = 0\%$						$w_b = 40\%$					
$(w_e, w_r) =$	$(w_e, w_r) = (100\%, 0\%)$			$(w_e, w_r) = (50\%, 50\%)$			$(w_e, w_r) = (60\%, 0\%)$			$(w_e, w_r) = (30\%, 30\%)$		
$\pi =$	5%	10%	20%	5%	10%	20%	5%	10%	20%	5%	10%	20%
VW	0.12	0.17	0.24	0.11	0.16	0.23	0.15	0.21	0.30	0.13	0.19	0.28
	(1.08)	(1.06)	(1.00)	(1.23)	(1.21)	(1.17)	(1.21)	(1.17)	(1.08)	(1.33)	(1.30)	(1.24)
AREIT	0.11	0.16	0.23	0.10	0.14	0.21	0.14	0.20	0.28	0.12	0.17	0.25
JLLIPT	0.08	0.12	0.16	0.07	0.11	0.15	0.11	0.15	0.20	0.09	0.13	0.18
HGIT	0.15	0.22	0.32	0.13	0.19	0.28	0.18	0.26	0.38	0.15	0.22	0.34
RREEF	0.10	0.14	0.19	0.09	0.13	0.18	0.13	0.18	0.24	0.11	0.16	0.22
BREIT	0.14	0.20	0.28	0.13	0.18	0.26	0.17	0.24	0.34	0.15	0.22	0.31
CFIT	0.08	0.11	0.15	0.07	0.10	0.15	0.10	0.14	0.19	0.09	0.12	0.18
AIREIT	0.12	0.16	0.21	0.11	0.15	0.20	0.14	0.20	0.25	0.13	0.18	0.24
NGCREIT	0.11	0.16	0.24	0.10	0.14	0.21	0.14	0.20	0.29	0.12	0.17	0.25
SREIT	0.10	0.14	0.19	0.10	0.14	0.19	0.13	0.18	0.24	0.12	0.16	0.23
CPREIF	0.15	0.21	0.32	0.12	0.17	0.26	0.17	0.24	0.36	0.13	0.19	0.29
BFRI	0.11	0.16	0.24	0.09	0.13	0.20	0.13	0.18	0.27	0.10	0.14	0.21

Table IA.5
Historical Performance Analysis: Alphas with Fixed Weights
(Observed Returns)

This table replicates Table 8 in the main text using observed returns (instead of unsmoothed returns) of NAV REITs. Specifically, it provides the implied annualized alphas (i.e., monthly alphas multiplied by twelve) from the increase in Sharpe ratio as we add prespecified NAV REIT index and fund allocations to a public market portfolio based on public bonds (b), public equities (e), and public REITs (r). The first two rows represent the weights of the public indices on the public market portfolio. The third row represents the reallocation weight going to the NAV REIT investment. Mechanically, each of the other weights similarly decrease by their same relative proportions in columns one through three of each panel in the updated portfolios. For example, in the last three columns of the table, the 40%, 30%, and 30% shown would be 32%, 24%, and 24%, respectively, after updating the portfolio to include NAV REITs. Rows four and six through sixteen provide the alphas of the respective portfolios with NAV REIT index and funds (based on Equation 14). Row five provides the t-statistic (based on bootstrap simulations) for values in row four. All of the results are based on the unsmoothed returns. All returns are in excess of the returns on the 1-month Treasury bill. See Table 1 for a detailed description of the VW index and NAV REIT funds in this table.

$w_b =$	$w_b = 0\%$						$w_b = 40\%$					
$(w_e, w_r) =$	$(w_e, w_r) = (100\%, 0\%)$			$(w_e, w_r) = (50\%, 50\%)$			$(w_e, w_r) = (60\%, 0\%)$			$(w_e, w_r) = (30\%, 30\%)$		
$\pi =$	5%	10%	20%	5%	10%	20%	5%	10%	20%	5%	10%	20%
VW	0.5%	0.8%	1.1%	0.4%	0.7%	1.0%	0.6%	0.9%	1.4%	0.5%	0.8%	1.2%
AREIT	0.4%	0.6%	0.8%	0.3%	0.5%	0.7%	0.4%	0.7%	1.0%	0.4%	0.5%	0.8%
JLLIPT	0.4%	0.6%	0.9%	0.3%	0.5%	0.7%	0.5%	0.7%	1.0%	0.4%	0.6%	0.8%
HGIT	0.6%	0.9%	1.3%	0.5%	0.7%	1.1%	0.7%	1.0%	1.5%	0.6%	0.9%	1.3%
RREEF	0.5%	0.7%	1.1%	0.4%	0.6%	0.9%	0.6%	0.9%	1.3%	0.5%	0.7%	1.1%
BREIT	0.7%	1.0%	1.5%	0.6%	0.9%	1.3%	0.8%	1.2%	1.8%	0.7%	1.0%	1.6%
CFIT	0.5%	0.7%	1.0%	0.4%	0.6%	0.9%	0.6%	0.8%	1.2%	0.5%	0.7%	1.0%
AIREIT	0.8%	1.2%	1.8%	0.7%	1.0%	1.5%	1.0%	1.4%	2.1%	0.8%	1.1%	1.8%
NGCREIT	0.4%	0.6%	0.9%	0.3%	0.5%	0.7%	0.5%	0.7%	1.0%	0.4%	0.6%	0.9%
SREIT	0.6%	0.9%	1.4%	0.5%	0.8%	1.2%	0.7%	1.1%	1.6%	0.6%	0.9%	1.4%
CPREIF	0.7%	1.1%	1.6%	0.6%	0.8%	1.2%	0.8%	1.2%	1.8%	0.6%	0.9%	1.4%
BFRI	0.7%	1.1%	1.7%	0.5%	0.8%	1.2%	0.8%	1.2%	1.8%	0.6%	0.8%	1.3%

Table IA.6

January 2016 to September 2023 Performance Analysis: Factor Regression Alphas (Alternative Unsmoothed Returns)

This table replicates Table 7 in the main text using data over the period from January 2016 to September 2023 (instead of December 2024). Specifically, it provides annualized alphas (i.e., monthly alphas multiplied by twelve) obtained by regressing the unsmoothed returns of the value-weighted NAV REIT index (VW) and NAV REIT funds (with at least five years of returns) on those of indices based on public bonds (*b*), public equities (*e*), and public REITs (*r*). More specifically, it provides alphas under many alternative unsmoothing processes. Panel A considers 3-step AR(L) unsmoothing processes with L from 1 to 6 (AIC and BIC select L=2 and L=1 as the optimal L values, respectively). Panel B considers 3-step MA(L) unsmoothing processes with L=3,6,9,12,15,18 (AIC and BIC select L=5 and L=2 as the optimal L values, respectively). All returns are in excess of the returns on the 1-month Treasury bill. We provide alpha t-statistics under two different methods to estimate standard errors. The first (t_W) is from White (1980) and is robust to residual heteroskedasticity, but not autocorrelation. The second (t_{NW}) is from Newey and West (1987, 1994) with pre prewhitening as suggested in Andrews and Monahan (1992), and is robust to residual heteroskedasticity and autocorrelation. See Table for a detailed description of the VW index and NAV REIT funds in this table.

	L =	PANEL A: Autoregressive Unsmoothing - AR(L)								PANEL B: Moving Average Unsmoothing - MA(L)							
		1	2	3	4	5	6	AIC=2	BIC=1	3	6	9	12	15	18	AIC=5	BIC=2
VW	α	5.9%	5.5%	5.4%	5.1%	5.1%	5.1%	5.5%	5.9%	6.1%	5.6%	5.5%	5.3%	4.9%	3.7%	5.7%	6.2%
	$[t_W]$	[2.97]	[2.21]	[1.88]	[1.61]	[1.56]	[1.69]	[2.21]	[2.97]	[3.36]	[2.39]	[2.16]	[1.90]	[1.46]	[0.83]	[2.60]	[3.66]
	(t_{NW})	(3.44)	(2.89)	(2.43)	(2.08)	(2.00)	(2.18)	(2.89)	(3.44)	(3.62)	(3.10)	(2.82)	(2.47)	(1.86)	(1.07)	(3.32)	(3.81)
AREIT	α	4.3%	3.9%	3.4%	3.6%	3.5%	2.9%	3.9%	4.3%	4.3%	4.2%	4.2%	4.0%	3.6%	3.5%	4.2%	4.5%
	$[t_W]$	[2.37]	[1.58]	[0.83]	[1.08]	[0.98]	[0.46]	[1.58]	[2.37]	[2.27]	[1.85]	[1.54]	[1.28]	[0.86]	[0.91]	[2.02]	[2.68]
	(t_{NW})	(2.28)	(1.91)	(0.90)	(1.18)	(1.05)	(0.49)	(1.91)	(2.28)	(1.96)	(1.75)	(1.68)	(1.32)	(0.92)	(0.94)	(1.79)	(1.87)
JLLIPT	α	3.9%	3.7%	3.1%	3.3%	3.6%	3.0%	3.7%	3.9%	4.1%	4.1%	4.0%	3.6%	3.3%	3.3%	4.1%	4.3%
	$[t_W]$	[1.59]	[1.47]	[0.71]	[0.91]	[1.44]	[0.60]	[1.47]	[1.59]	[2.13]	[1.89]	[1.46]	[1.14]	[0.79]	[0.83]	[2.01]	[2.63]
	(t_{NW})	(1.75)	(1.70)	(0.75)	(0.97)	(1.53)	(0.62)	(1.70)	(1.75)	(2.06)	(1.86)	(1.55)	(1.15)	(0.86)	(0.87)	(1.95)	(1.79)
HGIT	α	6.0%	5.9%	5.7%	5.8%	5.7%	5.6%	5.9%	6.0%	6.1%	6.1%	6.1%	5.7%	5.4%	5.1%	6.1%	6.2%
	$[t_W]$	[2.90]	[2.53]	[2.01]	[2.25]	[2.17]	[1.87]	[2.53]	[2.90]	[2.85]	[2.42]	[2.06]	[1.72]	[1.23]	[1.27]	[2.53]	[2.92]
	(t_{NW})	(3.42)	(3.04)	(2.16)	(2.47)	(2.41)	(2.03)	(3.04)	(3.42)	(2.99)	(2.54)	(2.30)	(1.80)	(1.36)	(1.37)	(2.80)	(3.09)
RREEF	α	3.9%	3.4%	2.3%	2.1%	2.5%	1.9%	3.4%	3.9%	4.0%	4.0%	3.7%	3.4%	3.1%	3.4%	3.9%	4.3%
	$[t_W]$	[1.55]	[1.04]	[0.36]	[0.30]	[0.43]	[0.24]	[1.04]	[1.55]	[1.73]	[1.63]	[1.20]	[0.95]	[0.68]	[0.75]	[1.69]	[2.25]
	(t_{NW})	(1.54)	(1.11)	(0.37)	(0.31)	(0.45)	(0.24)	(1.11)	(1.54)	(1.63)	(1.55)	(1.20)	(0.96)	(0.74)	(0.74)	(1.64)	(2.01)
BREIT	α	7.7%	7.2%	7.1%	6.9%	7.1%	7.3%	7.2%	7.7%	7.9%	8.4%	8.3%	8.0%	7.5%	7.4%	8.1%	8.0%
	$[t_W]$	[2.59]	[2.00]	[1.78]	[1.62]	[1.81]	[2.06]	[2.00]	[2.59]	[3.23]	[3.46]	[2.74]	[2.32]	[1.62]	[1.74]	[3.16]	[3.55]
	(t_{NW})	(3.13)	(2.33)	(2.00)	(1.82)	(2.03)	(2.27)	(2.33)	(3.13)	(3.33)	(3.42)	(3.00)	(2.37)	(1.74)	(1.78)	(3.34)	(3.63)
CFIT	α	3.9%	3.3%	3.2%	3.2%	3.4%	3.4%	3.3%	3.9%	3.7%	3.7%	3.6%	3.4%	2.9%	2.7%	3.8%	4.0%
	$[t_W]$	[2.39]	[1.49]	[1.17]	[1.26]	[1.58]	[1.32]	[1.49]	[2.39]	[1.71]	[1.48]	[1.14]	[0.96]	[0.61]	[0.61]	[1.66]	[2.14]
	(t_{NW})	(2.17)	(1.46)	(1.12)	(1.21)	(1.51)	(1.24)	(1.46)	(2.17)	(1.50)	(1.41)	(1.19)	(0.95)	(0.64)	(0.60)	(1.51)	(1.40)
AIREIT	α	7.6%	6.0%	5.4%	6.1%	5.7%	5.6%	6.0%	7.6%	8.2%	7.7%	7.4%	7.1%	6.6%	6.6%	7.8%	8.2%
	$[t_W]$	[1.20]	[0.86]	[0.60]	[0.90]	[0.71]	[0.63]	[0.86]	[1.20]	[2.51]	[2.32]	[2.21]	[1.86]	[1.28]	[1.42]	[2.91]	[3.18]
	(t_{NW})	(1.10)	(1.05)	(0.65)	(0.98)	(0.76)	(0.67)	(1.05)	(1.10)	(2.13)	(2.13)	(2.37)	(1.84)	(1.38)	(1.39)	(2.95)	(1.94)
NGCREIT	α	6.1%	5.8%	5.1%	5.2%	5.5%	5.2%	5.8%	6.1%	6.2%	6.1%	5.9%	5.6%	5.3%	5.0%	6.2%	6.5%
	$[t_W]$	[2.30]	[1.87]	[0.80]	[0.89]	[1.09]	[0.72]	[1.87]	[2.30]	[2.52]	[2.10]	[1.64]	[1.37]	[0.97]	[0.99]	[2.32]	[3.10]
	(t_{NW})	(2.21)	(2.12)	(0.84)	(0.95)	(1.14)	(0.73)	(2.12)	(2.21)	(2.39)	(1.92)	(1.77)	(1.33)	(1.01)	(0.97)	(2.17)	(1.79)
SREIT	α	6.5%	5.7%	4.3%	3.7%	4.7%	4.5%	5.7%	6.5%	6.6%	6.7%	6.5%	6.3%	5.4%	5.1%	7.0%	7.2%
	$[t_W]$	[1.68]	[1.15]	[0.51]	[0.37]	[0.64]	[0.54]	[1.15]	[1.68]	[2.14]	[1.85]	[1.50]	[1.34]	[0.83]	[0.86]	[2.19]	[2.71]
	$[t_{NW}]$	(1.88)	(1.22)	(0.51)	(0.37)	(0.64)	(0.53)	(1.22)	(1.88)	(1.92)	(1.61)	(1.48)	(1.25)	(0.82)	(0.81)	(2.10)	(2.15)
CPREIF	α	8.0%	7.7%	7.3%	7.0%	7.5%	7.9%	7.7%	8.0%	8.3%	8.4%	8.3%	8.2%	6.6%	6.5%	8.6%	9.0%
	$[t_W]$	[2.02]	[2.21]	[1.43]	[1.06]	[1.83]	[3.14]	[2.21]	[2.02]	[2.08]	[2.15]	[1.72]	[1.46]	[0.90]	[0.95]	[2.42]	[3.15]
	(t_{NW})	(2.50)	(2.60)	(1.56)	(1.15)	(1.95)	(3.23)	(2.60)	(2.50)	(2.31)	(2.09)	(1.90)	(1.37)	(0.94)	(0.91)	(2.78)	(2.61)
BFRI	α	7.8%	7.1%	6.5%	6.6%	6.4%	6.4%	7.1%	7.8%	8.1%	8.0%	8.4%	7.8%	6.4%	6.3%	8.9%	8.7%
	$[t_W]$	[1.78]	[1.67]	[1.07]	[1.08]	[0.94]	[0.76]	[1.67]	[1.78]	[2.02]	[1.99]	[1.38]	[1.36]	[0.84]	[0.89]	[2.07]	[2.47]
	(t_{NW})	(2.41)	(2.08)	(1.18)	(1.19)	(1.04)	(0.83)	(2.08)	(2.41)	(2.34)	(1.94)	(1.69)	(1.28)	(0.87)	(0.86)	(2.36)	(2.34)

Table IA.7

January 2016 to September 2023 Performance Analysis: Alphas with Fixed Weights

This table replicates Table 8 in the main text using data over the period from January 2016 to September 2023 (instead of December 2024). Specifically, it provides the implied annualized alphas (i.e., monthly alphas multiplied by twelve) from the increase in Sharpe ratio as we add prespecified NAV REIT index and fund allocations to a public market portfolio based on public bonds (b), public equities (e), and public REITs (r). The first two rows represent the weights of the public indices on the public market portfolio. The third row represents the reallocation weight going to the NAV REIT investment. Mechanically, each of the other weights similarly decrease by their same relative proportions in columns one through three of each panel in the updated portfolios. For example, in the last three columns of the table, the 40%, 30%, and 30% shown would be 32%, 24%, and 24%, respectively, after updating the portfolio to include NAV REITs. Rows four and six through sixteen provide the alphas of the respective portfolios with NAV REIT index and funds (based on Equation 14). Row five provides the t-statistic (based on bootstrap simulations) for values in row four. All of the results are based on the unsmoothed returns. All returns are in excess of the returns on the 1-month Treasury bill. See Table 1 for a detailed description of the VW index and NAV REIT funds in this table.

$w_b =$ $(w_e, w_r) =$ $\pi =$	$w_b = 0\%$						$w_b = 40\%$					
	$(w_e, w_r) = (100\%, 0\%)$			$(w_e, w_r) = (50\%, 50\%)$			$(w_e, w_r) = (60\%, 0\%)$			$(w_e, w_r) = (30\%, 30\%)$		
	5%	10%	20%	5%	10%	20%	5%	10%	20%	5%	10%	20%
VW	0.9%	1.3%	1.9%	0.8%	1.2%	1.7%	1.1%	1.6%	2.3%	0.9%	1.4%	2.1%
	(2.01)	(1.99)	(1.95)	(2.17)	(2.17)	(2.16)	(2.09)	(2.08)	(2.03)	(2.16)	(2.17)	(2.18)
AREIT	0.9%	1.2%	1.8%	0.7%	1.1%	1.6%	1.0%	1.5%	2.2%	0.9%	1.3%	1.9%
JLLIPT	0.8%	1.1%	1.7%	0.7%	1.0%	1.5%	1.0%	1.4%	2.0%	0.8%	1.2%	1.7%
HGIT	0.9%	1.4%	2.0%	0.8%	1.2%	1.8%	1.1%	1.6%	2.4%	0.9%	1.4%	2.1%
RREEF	1.0%	1.4%	1.9%	0.8%	1.2%	1.7%	1.2%	1.7%	2.3%	1.0%	1.4%	2.1%
BREIT	1.3%	1.9%	2.9%	1.2%	1.7%	2.5%	1.6%	2.3%	3.4%	1.3%	2.0%	3.0%
CFIT	0.6%	0.9%	1.3%	0.5%	0.7%	1.1%	0.7%	1.0%	1.5%	0.6%	0.8%	1.3%
AIREIT	2.2%	3.1%	4.4%	1.9%	2.7%	3.9%	2.5%	3.6%	5.0%	2.1%	3.0%	4.4%
NGCREIT	0.8%	1.2%	1.8%	0.7%	1.0%	1.5%	1.0%	1.4%	2.1%	0.8%	1.1%	1.8%
SREIT	1.5%	2.1%	3.1%	1.3%	1.8%	2.7%	1.7%	2.4%	3.6%	1.4%	2.1%	3.1%
CPREIF	0.9%	1.3%	2.0%	0.7%	1.0%	1.5%	1.0%	1.4%	2.2%	0.6%	0.9%	1.5%
BFRI	1.0%	1.5%	2.3%	0.7%	1.1%	1.7%	1.0%	1.6%	2.4%	0.6%	0.9%	1.6%