Pricing levered warrants under the CEV diffusion model

Abstract

This supplementary file contains further details and results of our paper "Pricing levered warrants under the CEV diffusion model". The material provided here is meant to be read strictly in conjunction with this paper.

Keywords: CEV model; Warrants; Dilution; Debt; Volatility.
JEL Classification: C63; G13; G32.

A. Proof of equation (15)

Itô's lemma implies that

$$dS_t = \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial V}dV_t + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}(dV_t)^2, \tag{A.1}$$

where dV_t is given by equation (1). Therefore,

$$\begin{aligned} dS_t &= \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial V}\left[rV_t dt + \sigma(V_t)V_t dW_t^{\mathbb{Q}}\right] + \frac{1}{2}\frac{\partial^2 S}{\partial V^2}\left[rV_t dt + \sigma(V_t)V_t dW_t^{\mathbb{Q}}\right]^2 \\ &= \left[\frac{\partial S}{\partial t} + rV_t\frac{\partial S}{\partial V} + \frac{1}{2}\sigma(V_t)^2 V_t^2 \frac{\partial^2 S}{\partial V^2}\right]dt + \sigma(V_t)V_t\frac{\partial S}{\partial V}dW_t^{\mathbb{Q}}, \end{aligned} \tag{A.2}$$

where $(dV_t)^2$ has been computed through the usual rules $dtdt = dtdW_t^{\mathbb{Q}} = dW_t^{\mathbb{Q}}dt = 0$ and $dW_t^{\mathbb{Q}}dW_t^{\mathbb{Q}} = dt$.

Dividing both sides of equation (A.2) by S_t, we obtain

$$\frac{dS_t}{S_t} = \frac{\left[\frac{\partial S}{\partial t} + rV_t\frac{\partial S}{\partial V} + \frac{1}{2}\sigma(V_t)^2 V_t^2 \frac{\partial^2 S}{\partial V^2}\right]}{S_t}dt + \sigma(V_t)\frac{V_t}{S_t}\frac{\partial S}{\partial V}dW_t^{\mathbb{Q}}. \tag{A.3}$$

Equation (A.3) reveals that the stock volatility is the term attached to $dW_t^{\mathbb{Q}}$, that is $\sigma_S = \sigma(V_t)\frac{V_t}{S_t}\frac{\partial S}{\partial V}$. Since we are valuing warrants at time $t = 0$, the local volatility, $\sigma(V_t)$, is equal to the firm's initial volatility, σ_V, due to the scale parameter δ. Thus, we have that

$$\sigma_S = \sigma_V \frac{V_t}{S_t}\frac{\partial S}{\partial V}, \tag{A.4}$$

which results in equation (15) of the paper.

B. Warrants prices for the case with $T = T_D$

Table B.1: Warrants prices under the CEV model with $\beta = 1$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM1V	$F = 1000$ LWM1S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	7.9881	7.6157	7.9294	11.9284	8.8584	8442.71	23.08	7.8990
100	23.7026	23.1761	23.7475	29.3893	24.3922	11101.83	24.01	23.8106
110	31.5080	30.9895	31.5737	37.6369	32.0888	12180.33	24.24	31.6606
Panel A2: Medium dilution, $M = 50$								
75	7.9881	6.6806	7.7222	10.0680	7.6160	8748.11	25.35	7.6923
100	23.7026	22.0120	23.8600	26.8537	22.8293	12060.04	27.41	23.9345
110	31.5080	29.9040	31.7487	35.0379	30.6000	13455.77	27.79	31.8493
Panel A3: High dilution, $M = 100$								
75	7.9881	6.1305	7.5096	8.8640	6.8447	9111.25	27.64	7.4828
100	23.7026	21.5306	23.9192	25.3543	22.0660	13262.86	30.70	24.0071
110	31.5080	29.5233	31.8594	33.5317	29.9488	15059.48	31.15	31.9739
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	15.2617	14.7264	15.1966	19.8709	16.7439	8503.17	37.18	15.2682
100	32.7120	32.0525	32.7337	38.3547	34.0311	11179.59	38.27	32.9350
110	40.6338	39.9646	40.6792	46.5517	41.8642	12259.39	38.55	40.9137
Panel B2: Medium dilution, $M = 50$								
75	15.2617	13.4267	14.9571	17.4482	14.9235	9094.48	40.74	15.0787
100	32.7120	30.5750	32.7582	35.4433	31.9543	12488.46	42.37	33.0037
110	40.6338	38.5071	40.7750	43.5665	39.7960	13890.46	42.68	41.0459
Panel B3: High dilution, $M = 100$								
75	15.2617	12.7223	14.6999	15.9334	13.8424	9815.09	44.18	14.8739
100	32.7120	29.9248	32.7231	33.7397	30.8797	14123.65	46.10	33.0027
110	40.6338	37.9120	40.7993	41.8500	38.7803	15932.78	46.35	41.0934

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 1$ when warrants and debt have the same maturity. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrant prices are obtained using six different models: 1) SHLW is computed via the Schroder (1989) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM1V is the extension of the model proposed by Abínzano and Navas (2013) based on unobservable variables (equation (32)); 6) LWM1S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (36). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 1$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 3$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table B.2: Warrants prices under the lognormal model ($\beta = 2$)

	BSM	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM1V	$F = 1000$ LWM1S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + F\mathrm{e}^{-rTD}$	$V = \widetilde{E} + F\mathrm{e}^{-rTD}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	8.8572	8.4238	8.8124	12.4879	9.7497	8,451.20	23.16	8.7392
100	23.6712	23.0678	23.6834	29.0265	24.3697	11,101.79	24.10	23.7982
110	31.1412	30.5471	31.1540	37.0044	31.7276	12,176.94	24.33	31.3129
Panel A2: Medium dilution, $M = 50$								
75	8.8572	7.3306	8.6528	10.5388	8.3024	8,792.52	25.79	8.5741
100	23.6712	21.7231	23.6766	26.4201	22.5677	12,054.22	27.88	23.8082
110	31.1412	29.3052	31.1385	34.3519	30.0140	13,429.78	28.30	31.3195
Panel A3: High dilution, $M = 100$								
75	8.8572	6.6809	8.4880	9.2876	7.4183	9,204.22	28.54	8.4041
100	23.6712	21.1659	23.6065	24.9203	21.7085	13,239.21	31.74	23.7540
110	31.1412	28.8899	31.0467	32.8720	29.3033	14,988.54	32.28	31.2473
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	16.6081	15.9481	16.5598	20.6566	18.1815	8,528.68	37.07	16.4939
100	32.5992	31.7675	32.5672	37.6269	34.0112	11,191.79	38.24	32.8007
110	39.9462	39.0913	39.9089	45.2799	41.2695	12,266.05	38.55	40.2254
Panel B2: Medium dilution, $M = 50$								
75	16.6081	14.3207	16.3742	18.0228	15.8659	9,179.16	41.22	16.3116
100	32.5992	29.8605	32.3964	34.4448	31.3243	12,496.58	43.07	32.6573
110	39.9462	37.1907	39.7088	42.0115	38.5565	13,866.60	43.47	40.0572
Panel B3: High dilution, $M = 100$								
75	16.6081	13.4056	16.1638	16.3873	14.5768	9,973.96	45.43	16.1071
100	32.5992	28.9817	32.1504	32.6266	29.9781	14,107.09	47.81	32.4353
110	39.9462	36.3897	39.4214	40.1972	37.2826	15,843.24	48.24	39.7961

Note: This table prices European-style call warrants under the lognormal model when warrants and debt have the same maturity. The results are obtained replicating the paper of Abínzano and Navas (2013) under the lognormal assumption and using the same parameters configuration. The table indicates if the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns, σ_S, is used in all calculations. Warrant prices are obtained using six different models: 1) BSM is the Black and Scholes (1973) and Merton (1973) stock option model; the price of the warrant obtained using the BSM model, w_{BSM}, is used to compute the approximate value of firm's equity $\widetilde{E} = NS_t + Mw_{BSM}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the classical valuation model taking $V = \widetilde{E}$; 3) STU is the model proposed by Schulz and Trautmann (1994) and Ukhov (2004); 4) CWMD is the classical warrant model taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM1V is the model proposed by Abínzano and Navas (2013) based on unobservable variables; 6) LWM1S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations. In all models, the number of shares of stock is $N = 100$, the maturity of debt is $T_D = 3$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond is $F = 1000$ and warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table B.3: Warrants prices under the CEV model with $\beta = 3$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM1V	$F = 1000$ LWM1S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	9.8491	9.3400	9.8219	13.1270	10.7317	8460.89	23.26	9.6986
100	23.6976	23.0137	23.6691	28.7168	24.3617	11102.51	24.17	23.8421
110	30.8278	30.1606	30.7787	36.4415	31.3638	12174.35	24.41	31.0163
Panel A2: Medium dilution, $M = 50$								
75	9.8491	8.0511	9.6718	11.0634	9.0944	8844.64	26.33	9.5933
100	23.6976	21.4857	23.3680	26.0476	22.3817	12054.20	28.30	23.7020
110	30.8278	28.7803	30.3665	33.7625	29.5113	13411.04	28.70	30.7983
Panel A3: High dilution, $M = 100$								
75	9.8491	7.2795	9.3036	9.7522	8.0726	9313.27	29.53	9.4176
100	23.6976	20.8639	22.4270	24.5663	21.4286	13224.72	32.29	23.2606
110	30.8278	28.3595	29.2124	32.3396	28.7642	14934.90	32.74	30.2473
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	16.6858	15.9047	16.1292	19.9750	18.1443	8625.34	36.75	17.3442
100	30.6243	29.6174	29.7455	34.8640	31.8669	11330.50	37.74	32.0053
110	37.1853	36.1369	36.1818	41.7727	38.3130	12422.70	37.99	38.8403
Panel B2: Medium dilution, $M = 50$								
75	16.6858	13.9421	13.9186	17.1653	15.6099	9364.04	40.06	16.0516
100	30.6243	27.2464	26.2047	31.4188	28.8322	12701.57	41.21	30.1235
110	37.1853	33.7457	32.1526	38.2175	35.2333	14087.83	41.44	36.8154
Panel B3: High dilution, $M = 100$								
75	16.6858	12.7877	11.6862	15.4144	14.0447	10130.45	42.41	14.4223
100	30.6243	26.0860	22.5740	29.4707	27.1889	14187.64	43.51	27.8462
110	37.1853	32.6824	28.0258	36.2870	33.6722	15925.99	43.66	34.4344

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 3$ when warrants and debt have the same maturity. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrant prices are obtained using six different models: 1) SHLW is computed via the Heston et al. (2007) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM1V is the extension of the model proposed by Abínzano and Navas (2013) based on unobservable variables (equation (32)); 6) LWM1S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (36). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 3$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 3$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

C. Warrants prices for the case with $T < T_D$

Table C.1: Warrants prices under the CEV diffusion model with $\beta = 0$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ CG	$F = 1000$ LWM2S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	1.2416	1.1500	1.1901	3.3356	1.6061	8370.79	22.80	1.2109
100	12.3013	11.9454	12.3304	17.7869	12.6947	10979.72	24.28	12.3767
110	19.6699	19.3153	19.7450	25.9413	19.9575	12053.16	24.61	19.7606
Panel A2: Medium dilution, $M = 50$								
75	1.2416	0.9077	1.0258	2.5705	1.2533	8410.65	24.03	1.0982
100	12.3013	11.1296	12.4109	15.7116	11.6464	11475.55	28.89	12.7175
110	19.6699	18.5767	19.9747	23.7081	18.9740	12848.32	29.71	20.1945
Panel A3: High dilution, $M = 100$								
75	1.2416	0.7445	0.8803	2.0483	1.0140	8451.60	25.23	0.9950
100	12.3013	10.7592	12.4661	14.4138	11.1079	12125.24	34.00	13.1081
110	19.6699	18.3276	20.1669	22.3435	18.5629	13883.71	35.15	20.6874
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	4.2473	4.0045	4.1554	7.5075	5.0298	8321.48	37.30	4.0789
100	18.0733	17.5990	18.1023	23.5147	18.8807	10949.78	39.04	17.8889
110	25.5175	25.0291	25.6034	31.4758	26.2747	12023.14	39.41	25.3425
Panel B2: Medium dilution, $M = 50$								
75	4.2473	3.3776	3.8457	6.1013	4.1547	8452.93	41.33	4.0567
100	18.0733	16.5120	18.1745	21.1275	17.3810	11661.08	46.15	18.6578
110	25.5175	23.9669	25.8576	28.9532	24.7560	13046.40	46.77	26.3115
Panel B3: High dilution, $M = 100$								
75	4.2473	2.9807	3.5470	5.1631	3.5880	8625.45	45.95	4.1111
100	18.0733	16.0104	18.2109	19.6693	16.6104	12666.90	54.00	19.8027
110	25.5175	23.5431	26.0578	27.4489	24.0530	14462.51	54.54	27.6773

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 0$ assuming that the warrants have shorter maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrants prices are obtained using six different models: 1) SHLW is computed via the Schroder (1989) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) CG is the extension of the model proposed by Crouhy and Galai (1994) (expression (29)); 6) LWM2S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (40). In all models, the number of shares of stock is $N = 100$, the maturity of warrants is $T = 1$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 0$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 3$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table C.2: Warrants prices under the CEV diffusion model with $\beta = 1$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ CG		$F = 1000$ LWM2S	
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	1.5497	1.4367	1.4949	3.6249	1.9336	8379.47	22.79	1.4933
100	12.2819	11.9003	12.3034	17.5787	12.6599	10988.52	24.21	12.3721
110	19.4401	19.0599	19.4933	25.5886	19.7044	12060.55	24.55	19.5640
Panel A2: Medium dilution, $M = 50$								
75	1.5497	1.1383	1.3172	2.8100	1.5151	8432.154	24.05	1.3536
100	12.2819	11.0205	12.3561	15.5037	11.5451	11493.81	28.56	12.5931
110	19.4401	18.2701	19.6438	23.3620	18.6651	12856.89	29.47	19.8553
Panel A3: High dilution, $M = 100$								
75	1.5497	0.9378	1.1556	2.2550	1.2318	8486.812	25.27	1.2247
100	12.2819	10.6175	12.3809	14.2199	10.9694	12142.92	33.28	12.8171
110	19.4401	18.0135	19.7514	22.0227	18.2396	13873.39	34.67	20.1290
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	4.9171	4.6321	4.8329	7.9861	5.6926	8400.212	36.98	4.8496
100	17.9943	17.4584	18.0086	23.0814	18.6982	11031.55	38.66	18.1113
110	25.0626	24.5031	25.1128	30.7008	25.6662	12102.59	39.06	25.2222
Panel B2: Medium dilution, $M = 50$								
75	4.9171	3.8955	4.5472	6.5106	4.7025	8578.895	40.70	4.7613
100	17.9943	16.2122	18.0281	20.6328	17.0834	11752.49	45.06	18.4625
110	25.0626	23.2693	25.2412	28.1071	24.0422	13110.79	45.85	25.6469
Panel B3: High dilution, $M = 100$								
75	4.9171	3.4278	4.2682	5.5306	4.0571	8794.378	44.75	4.6995
100	17.9943	15.6174	18.0090	19.1543	16.2236	12680.09	51.88	18.8264
110	25.0626	22.7647	25.3126	26.5904	23.2656	14402.89	52.86	26.0575

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 1$ assuming that the warrants have shorter maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrant prices are obtained using six different models: 1) SHLW is computed via the Schroder (1989) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) CG is the extension of the model proposed by Crouhy and Galai (1994) (expression (29)); 6) LWM2S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (40). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 1$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 1$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 3$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table C.3: Warrants prices under the lognormal model ($\beta = 2$)

	BSM	CWM	STU	$F = 1000$ CWMD	$F = 1000$ CG	$F = 1000$ LWM2S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	1.9052	1.7675	1.8501	3.9362	2.2883	8,378.46	22.79	1.7924
100	12.2756	11.8680	12.2902	17.3875	12.6142	10,985.13	24.15	12.3254
110	19.2280	18.8245	19.2604	25.2690	19.4486	12,052.57	24.52	19.3126
Panel A2 :Medium dilution, $M = 50$								
75	1.9052	1.4038	1.6686	3.0678	1.7983	8,443.12	24.04	1.6064
100	12.2756	10.9246	12.3173	15.3152	11.4374	11,478.65	28.27	12.3364
110	19.2280	17.9917	19.3349	23.0581	18.3636	12,832.34	29.32	19.3965
Panel A3: High dilution, $M = 100$								
75	1.9052	1.1600	1.4996	2.4780	1.4675	8,507.24	25.23	1.4413
100	12.2756	10.4913	12.3135	14.0494	10.8318	12,098.83	32.68	12.3482
110	19.2280	17.7368	19.3592	21.7516	17.9348	13,806.11	34.37	19.4286
Painel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	5.6669	5.3316	5.5974	8.5135	6.3958	8,409.23	36.81	5.4961
100	17.9693	17.3726	17.9718	22.7159	18.5827	11,035.43	38.45	18.0477
110	24.6768	24.0526	24.6948	30.0258	25.1767	12,100.45	38.88	24.8078
Panel B2: Medium dilution, $M = 50$								
75	5.6669	4.4636	5.3613	6.9540	5.2719	8,627.24	40.07	5.2360
100	17.9693	15.9717	17.9468	20.2132	16.8211	11,758.84	44.21	17.9921
110	24.6768	22.6697	24.7058	27.3856	23.4087	13,105.38	45.17	24.8330
Panel B3: High dilution, $M = 100$								
75	5.6669	3.9111	5.1311	5.9234	4.5394	8,861.23	43.42	4.9947
100	17.9693	15.2929	17.8776	18.7260	15.8770	12,657.72	50.20	17.9520
110	24.6768	22.1097	24.6502	25.8828	22.5723	14,341.54	51.62	24.7841

Note: This table prices European-style call warrants under the lognormal model when the maturity of warrants is shorter than the maturity of the debt. The results are obtained replicating the paper of Abínzano and Navas (2013) under the lognormal assumption and adopting the same parameters configuration. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of the stock returns, σ_S, is used in all calculations. Warrant prices are obtained using six different models: 1) BSM is the Black and Scholes (1973) and Merton (1973) stock option model; the price of the warrant obtained using the BSM model, w_{BSM}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{BSM}$, where N is the number of shares of stock and M is the number of the outstanding warrants; 2) CWM is the classical valuation model taking $V = \widetilde{E}$; 3) STU is the model proposed by Schulz and Trautmann (1994) and Ukhov (2004); 4) CWMD is the classical warrant model taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) CG is the Crouhy and Galai (1994) model; 6) LWM2S is the same model but now based on observable variables S_t and σ_S. V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations. In all models, the number of shares of stock is $N = 100$, the maturity of debt is $T_D = 3$, the maturity of warrant is $T = 1$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond is $F = 1000$ and warrants prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table C.4: Warrants prices under the CEV diffusion model with $\beta = 3$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ CG	$F = 1000$ LWM2S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	2.3140	2.1472	2.2626	4.2729	2.7154	8391.50	24.99	2.7223
100	12.2819	11.8479	12.2904	17.2103	12.6307	10990.93	25.01	12.6573
110	19.0305	18.6054	19.0431	24.9777	19.2588	12057.20	25.00	19.2809
Panel A2: Medium dilution, $M = 50$								
75	2.3140	1.7071	2.0923	3.3459	2.1214	8471.86	24.96	2.1024
100	12.2819	10.8402	12.2945	15.1432	11.3639	11364.39	21.73	10.0069
110	19.0305	17.7368	19.0443	22.7904	18.1088	12755.51	25.01	17.7820
Panel A3: High dilution, $M = 100$								
75	2.3140	1.4129	1.9322	2.7184	1.6768	8542.93	25.17	1.6293
100	12.2819	10.3784	12.2646	13.8995	10.6744	12194.07	31.64	11.3735
110	19.0305	17.4918	18.9866	21.5226	17.6440	13821.98	31.35	17.8928
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	6.5163	6.1197	6.4716	9.1009	5.5060	8661.57	40.58	6.1999
100	17.9938	17.3342	17.9862	22.4016	16.1965	11277.21	39.28	17.4872
110	24.3436	23.6578	24.3302	29.4218	22.2036	12331.72	38.72	23.7239
Panel B2: Medium dilution, $M = 50$								
75	6.5163	5.0918	6.3190	7.4378	3.4553	8905.18	39.09	4.0299
100	17.9938	15.7773	17.8891	19.8488	13.2230	12370.99	42.31	15.0089
110	24.3436	22.1401	24.1790	26.7545	19.2341	13944.28	43.80	21.6259
Panel B3: High dilution, $M = 100$								
75	6.5163	4.4358	6.1414	6.3452	1.7040	9093.13	38.51	2.0659
100	17.9938	15.0198	17.5096	18.3631	10.5235	12996.52	39.31	12.0469
110	24.3436	21.5445	23.6323	25.2882	16.6818	16271.58	47.31	18.6631

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 3$ assuming that the warrants have shorter maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrants prices are obtained using six different models: 1) SHLW is computed via the Heston et al. (2007) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) CG is the extension of the model proposed by Crouhy and Galai (1994) (expression (29)); 6) LWM2S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (40). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 1$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 3$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 3$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

D. Warrants prices for the case with $T > T_D$

Table D.1: Warrants prices under the CEV diffusion model with $\beta = 0$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM3V		$F = 1000$ LWM3S	
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	7.2170	6.8948	7.1494	11.9550	8.5257	8529.91	23.08	7.6341
100	23.8024	23.3528	23.8819	30.5142	25.1688	11200.74	24.06	24.6734
110	31.9532	31.5169	32.0749	39.0941	33.2720	12283.31	24.27	32.9130
Panel A2: Medium dilution, $M = 50$								
75	7.2170	6.0903	6.9127	10.0453	7.3336	8856.15	26.47	8.0502
100	23.8024	22.3756	24.1165	27.9016	23.6689	12246.97	28.49	25.8570
110	31.9532	30.6154	32.4460	36.4093	31.8343	13663.66	28.67	34.1860
Panel A3: High dilution, $M = 100$								
75	7.2170	5.6230	6.6721	8.7950	6.5885	9323.83	30.49	8.7071
100	23.8024	21.9856	24.3008	26.3060	22.9087	13689.95	33.11	27.3709
110	31.9532	30.3044	32.7536	34.7880	31.1550	15529.41	33.07	35.7616
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	14.1365	13.7049	14.0622	19.8909	16.2428	8596.04	37.16	14.8544
100	33.0790	32.5989	33.1514	40.0819	35.2385	11289.00	38.37	34.2526
110	41.6110	41.1475	41.7314	48.9030	43.7449	12375.23	38.64	42.8955
Panel B2: Medium dilution, $M = 50$								
75	14.1365	12.6819	13.7942	17.4974	14.5918	9256.13	43.46	16.4253
100	33.0790	31.5811	33.3443	37.3040	33.4774	12771.44	44.78	36.7920
110	41.6110	40.2004	42.0706	46.0920	42.0417	14211.27	44.80	45.5852
Panel B3: High dilution, $M = 100$								
75	14.1365	12.1623	13.5163	15.9834	13.6167	10276.95	50.81	18.6768
100	33.0790	31.1988	33.4785	35.6334	32.5729	14869.41	51.03	39.6121
110	41.6110	39.8806	42.3276	44.4146	41.2006	16750.43	50.55	48.4013

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 0$ assuming that the warrants have longer maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrants prices are obtained using six different models: 1) SHLW is computed via the Schroder (1989) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM3V is the extension of the model proposed by Abínzano and Navas (2013) based on unobservable variables (equation (49)); 6) LWM3S is the same model but now based on observable variables S_t and σ_S. V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (50). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 0$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 1$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table D.2: Warrants prices under the CEV diffusion model with $\beta = 1$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM3V	$F = 1000$ LWM3S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	7.9881	7.6157	7.9294	12.4172	9.2741	8536.08	23.01	8.2885
100	23.7026	23.1761	23.7475	30.0482	25.0014	11198.06	23.99	24.4586
110	31.5080	30.9895	31.5737	38.3340	32.7481	12277.23	24.21	32.3639
Panel A2: Medium dilution, $M = 50$								
75	7.9881	6.6806	7.7222	10.4469	7.9423	8874.71	26.03	8.4286
100	23.7026	22.0120	23.8600	27.3621	23.3088	12203.51	28.07	24.9972
110	31.5080	29.9040	31.7487	35.5720	31.1153	13599.64	28.36	32.9167
Panel A3: High dilution, $M = 100$								
75	7.9881	6.1305	7.5096	9.1656	7.1083	9317.50	29.36	8.6414
100	23.7026	21.5306	23.9192	25.7526	22.4492	13511.46	32.25	25.5763
110	31.5080	29.5233	31.8594	33.9463	30.3550	15302.68	32.49	33.4865
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	15.2617	14.7264	15.1966	20.4264	17.2759	8610.63	36.91	15.7385
100	32.7120	32.0525	32.7337	39.0148	34.6926	11289.89	38.13	33.6349
110	40.6338	39.9646	40.6792	47.2378	42.5590	12370.35	38.43	41.6698
Panel B2: Medium dilution, $M = 50$								
75	15.2617	13.4267	14.9571	17.8781	15.3415	9273.72	42.04	16.4141
100	32.7120	30.5750	32.7582	35.9499	32.4684	12689.44	43.68	34.7215
110	40.6338	38.5071	40.7750	44.0906	40.3359	14093.55	43.91	42.8006
Panel B3: High dilution, $M = 100$								
75	15.2617	12.7223	14.6999	16.2734	14.1785	10181.47	47.57	17.2934
100	32.7120	29.9248	32.7231	34.1347	31.2874	14545.60	49.13	35.9299
110	40.6338	37.9120	40.7993	42.2564	39.2052	16354.48	49.15	44.0159

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 1$ assuming that the warrants have longer maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrants prices are obtained using six different models: 1) SHLW is computed via the Schroder (1989) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM3V is the extension of the model proposed by Abínzano and Navas (2013) based on unobservable variables (equation (49)); 6) LWM3S is the same model but now based on observable variables S_t and σ_S. V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (50). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 1$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 1$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table D.3: Warrants prices under the lognormal model ($\beta = 2$)

	BSM	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM3V	$F = 1000$ LWM3S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + Fe^{-rT_D}$	$V = \widetilde{E} + Fe^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	8.8572	8.4238	8.8124	12.9474	10.1038	8541.60	22.93	9.0056
100	23.6712	23.0678	23.6834	29.6622	24.9009	11194.30	23.91	24.3047
110	31.1412	30.5471	31.1540	37.6851	32.3075	12270.08	24.15	31.8955
Panel A2: Medium dilution, $M = 50$								
75	8.8572	7.3306	8.6528	10.8974	8.6068	8893.88	25.59	8.8493
100	23.6712	21.7231	23.6766	26.9158	23.0220	12166.86	27.70	24.3179
110	31.1412	29.3052	31.1385	34.8790	30.5062	13545.89	28.12	31.9019
Panel A3: High dilution, $M = 100$								
75	8.8572	6.6809	8.4880	9.5754	7.6685	9319.95	28.37	8.6868
100	23.6712	21.1659	23.6065	25.3129	22.0777	13377.17	31.56	24.2654
110	31.1412	28.8899	31.0467	33.2849	29.6983	15133.29	32.09	31.8291
Painel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	16.6081	15.9481	16.5598	21.1650	18.5087	8618.68	36.69	16.7596
100	32.5992	31.7675	32.5672	38.2435	34.4078	11282.93	37.94	33.2297
110	39.9462	39.0913	39.9089	45.9275	41.6810	12357.54	38.27	40.7086
Panel B2: Medium dilution, $M = 50$								
75	16.6081	14.3207	16.3742	18.4191	16.2408	9279.91	40.84	16.5813
100	32.5992	29.8605	32.3964	34.9241	31.7490	12604.63	42.76	33.0884
110	39.9462	37.1907	39.7088	42.5129	39.0028	13977.03	43.18	40.5414
Panel B3: High dilution, $M = 100$								
75	16.6081	13.4056	16.1638	16.7038	14.8635	10088.37	45.04	16.3781
100	32.5992	28.9817	32.1504	33.0054	30.3283	14236.39	47.48	32.8669
110	39.9462	36.3897	39.4214	40.5914	37.6485	15977.38	47.93	40.2803

Note: This table prices European-style call warrants under the lognormal model when warrants have longer maturity than debt. The results are obtained replicating the paper of Abínzano and Navas (2013) under the lognormal assumption and using the same parameters configuration. The table indicates if the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns, σ_S, is used in all calculations. Warrant prices are obtained using six different models: 1) BSM is the Black and Scholes (1973) and Merton (1973) stock option model; the price of the warrant obtained using the BSM model, w_{BSM}, is used to approximate the value of firm's equity, $\widetilde{E} = NS_t + Mw_{BSM}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the classical valuation model taking $V = \widetilde{E}$; 3) STU is the model proposed by Schulz and Trautmann (1994) and Ukhov (2004); 4) CWMD is the classical warrant model taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM3V is the model proposed by Abínzano and Navas (2013) based on unobservable variables; 6) LWM3S is the same model but now based on observable variables S_t and σ_S; V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations. In all models, the number of shares of stock is $N = 100$, the maturity of debt is $T_D = 1$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond is $F = 1000$ and warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

Table D.4: Warrants prices under the CEV diffusion model with $\beta = 3$

	SHLW	CWM	STU	$F = 1000$ CWMD	$F = 1000$ LWM3V	$F = 1000$ LWM3S		
	S	$V = \widetilde{E}$	S	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$	$V = \widetilde{E} + F\mathrm{e}^{-rT_D}$		S	
S	w	w	w	w	w	V^*	$\sigma_V^*(\%)$	w
Panel A: Low volatility, $\sigma_S = 25\%$								
Panel A1: Low dilution, $M = 10$								
75	9.8491	9.3310	9.8219	13.5550	11.0642	8550.60	22.85	9.8267
100	23.6976	23.0137	23.6691	29.3297	24.8783	11194.50	23.83	24.2254
110	30.8278	30.1606	30.7787	37.1080	31.9423	12267.27	24.10	31.5062
Panel A2: Medium dilution, $M = 50$								
75	9.8491	8.0511	9.6718	11.4007	9.3594	8918.88	25.14	9.3341
100	23.6976	21.4857	23.3680	26.5323	22.8012	12139.45	27.35	23.7513
110	30.82775	28.7803	30.3665	34.2854	29.9794	13503.11	27.90	31.0269
Panel A3: High dilution, $M = 100$								
75	9.8491	7.2795	9.3036	10.0260	8.2908	9332.87	27.43	8.8087
100	23.6976	20.8639	22.4270	24.9554	21.7756	13259.24	30.93	23.0784
110	30.8278	28.3595	29.2124	32.7534	29.1449	14977.56	31.78	30.2640
Panel B: High volatility, $\sigma_S = 40\%$								
Panel B1: Low dilution, $M = 10$								
75	16.6858	15.9047	16.1292	20.4164	18.3539	8623.71	36.44	17.2418
100	30.6243	29.6174	29.7455	35.4205	32.1476	11267.40	37.74	31.6553
110	37.1853	36.1369	36.1818	42.3668	38.6121	12333.92	38.10	38.3240
Panel B2: Medium dilution, $M = 50$								
75	16.6858	13.9421	13.9186	17.5124	15.7787	9225.34	39.61	15.4863
100	30.6243	27.2464	26.2047	31.8584	29.0531	12386.05	41.80	28.7103
110	37.1853	33.7457	32.1526	38.6857	35.4638	13692.67	42.41	34.8461
Panel B3: High dilution, $M = 100$								
75	16.6858	12.7877	11.6862	15.6952	14.1839	9801.72	42.52	13.5078
100	30.6243	26.0860	22.5740	29.8248	27.3625	13450.80	45.53	24.9982
110	37.1853	32.6824	28.0258	36.6621	33.8498	14983.95	46.36	30.3275

Note: This table prices European-style call warrants under the CEV diffusion model with $\beta = 3$ assuming that the warrants have longer maturity than the debt. The table indicates whether the stock price S or the firm value V is used in the calculation and if the model considers debt F (whether correctly or incorrectly). The volatility of stock returns is used in all calculations. Warrant prices are obtained using six different models: 1) SHLW is computed via Heston et al. (2007) stock option formula; the price of the warrant obtained using the SHLW, w_{SHLW}, is used to compute the approximate value of the equity $\widetilde{E} = NS_t + Mw_{SHLW}$, where N is the number of shares of stock and M is the number of outstanding warrants; 2) CWM is the extension of the classical valuation model taking $V = \widetilde{E}$; 3) STU is the extension of the Schulz and Trautmann (1994) and Ukhov (2004) model to the CEV process; 4) CWMD is the extension of the classical warrant model but now taking $V = \widetilde{E} + Fe^{-rT_D}$; 5) LWM3V is the extension of the model proposed by Abínzano and Navas (2013) based on unobservable variables (equation (49)); 6) LWM3S is the same model but now based on observable variables S_t and σ_S. V^* and σ_V^* denote the firm value and firm volatility which solve the system of equations (50). In all models, the number of shares of stock is $N = 100$, the maturity of warrant is $T = 3$, the risk-free interest rate is $r = 0.0488$, the strike price is $X = 100$, the elasticity parameter is $\beta = 3$ and each warrant gives its owner the right to buy $k = 1$ share. The face value of the zero-coupon bond as well as its maturity are $F = 1000$ and $T_D = 1$, respectively. Warrant prices are calculated for three stock prices, $S \in \{75, 100, 110\}$, three levels of dilution, $M \in \{10, 50, 100\}$, and two levels of stock volatility, $\sigma_S \in \{0.25, 0.4\}$.

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