On Price Regulation in the ESG Rating Industry

Online Appendix

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0.1 Multiple ESG attributes

In the baseline analysis, we assumed that the project features only one ESG attribute, reflecting the preference of some investors for a single, aggregate measure of a project's ESG responsibility level. However, some investors may prefer disaggregated ratings. We now expand our model to incorporate two distinct attributes, labeled A and B, and assume that the lender values each separately. We continue to employ the same formulation for the lender's utility function as in the baseline model, now adjusted to account for these two dimensions. This adjustment enables us to explore how the lender's evaluation criteria and investment decisions are influenced by the additional complexity of multiple ESG factors. Let

$$\theta \triangleq \sum_{j=A,B} \theta_j,$$

with θ_A and θ_B being two i.i.d. binary random variables, each taking values 0 and 1 with equal probability. Let α_i^j be the level of precision that provider P_i ($i = 0, 1$) chooses for dimension $j = A, B$. The information policy $\wp_i \triangleq \left(p_i, \alpha_i^A, \alpha_i^B\right)$ offered by each provider P_i now specifies a service price and two levels of precision α_i^j, with $j = A, B$, each associated to a binary signal $s_i^j \in \{0, 1\}$ about each attribute. Following Azarmsa and Shapiro (2024), we assume that the policy is to sell information on both attributes for a single price — i.e., the lender either acquires both signals of a given provider or neither. Focus again on single-homing equilibria with full market coverage.

It can be shown that for every signal pair $\left(s_i^A, s_i^B\right)$, with $i = 0, 1$, the lender's optimal investment decisions feature

$$f'\left(x^\star\left(s_i^A, s_i^B, \alpha_i^A, \alpha_i^B\right)\right) = \frac{1 - \mu \sum_{j=A,B} \mathbb{E}\left[\theta^j | s_i^j, \alpha_i^j\right]}{1 - \phi},$$

with

$$x^\star\left(1, 1, \alpha_i^A, \alpha_i^B\right) \geq \max\left\{x^\star\left(0, 1, \alpha_i^A, \alpha_i^B\right), x^\star\left(1, 0, \alpha_i^A, \alpha_i^B\right)\right\},$$

$$\min\left\{x^\star\left(0, 1, \alpha_i^A, \alpha_i^B\right), x^\star\left(1, 0, \alpha_i^A, \alpha_i^B\right)\right\} \geq x^\star\left(0, 0, \alpha_i^A, \alpha_i^B\right).$$

and

$$\alpha_i^A \geq \alpha_i^B \quad \Leftrightarrow \quad x^\star\left(1,0,\alpha_i^A,\alpha_i^B\right) \geq x^\star\left(0,1,\alpha_i^A,\alpha_i^B\right).$$

The above inequalities follow immediately from the hypothesis that signals are i.i.d. and informative — i.e., $\alpha_i^j \geq \frac{1}{2}$, for every $i = 0,1$ and $j = A,B$. Using the first-order conditions to rewrite the lender's expected benefit, we can define L's state-contingent utility as

$$\lambda\left(s_i^A,s_i^B,\alpha_i^A,\alpha_i^B\right) \triangleq (1-\phi)\left[f\left(x^\star\left(s_i^A,s_i^B,\alpha_i^A,\alpha_i^B\right)\right) - x^\star\left(s_i^A,s_i^B,\alpha_i^A,\alpha_i^B\right) f'\left(x^\star\left(s_i^A,s_i^B,\alpha_i^A,\alpha_i^B\right)\right)\right],$$

so that the lender is indifferent between the two providers when it is located at

$$z\left(\wp_0,\wp_1\right) \triangleq \frac{1}{2} + \frac{p_1-p_0}{2t} + \frac{\sum_{s_0^A}\sum_{s_0^B}\lambda\left(s_0^A,s_0^B,\alpha_0^A,\alpha_0^B\right) - \sum_{s_1^A}\sum_{s_1^B}\lambda\left(s_1^A,s_1^B,\alpha_1^A,\alpha_1^B\right)}{8t}.$$

Consider a symmetric equilibrium in which every provider offers a policy $\wp^\star \triangleq \left(p^\star,\alpha^{A\star},\alpha^{B\star}\right)$. Under the hypothesis of full market coverage, provider P_0 solves

$$\max_{\wp_0\in\mathbb{R}_+\times\left[\frac{1}{2},1\right]^2} z\left(\wp_0,\wp^\star\right)p_0 - \sum_{j=A,B} c(\alpha_0^j).$$

Using the Envelope Theorem, the first-order conditions for α_0^A and α_0^B are, respectively:

$$\frac{\mu}{8t}\left(\sum_{s_0^B} x^\star\left(1,s_0^B,\alpha_0^A,\alpha_0^B\right) - \sum_{s_0^B} x^\star\left(0,s_0^B,\alpha_0^A,\alpha_0^B\right)\right)p_0 = c'\left(\alpha_0^A\right),$$

$$\frac{\mu}{8t}\left(\sum_{s_0^A} x^\star\left(s_0^A,1,\alpha_0^A,\alpha_0^B\right) - \sum_{s_0^A} x^\star\left(s_0^A,0,\alpha_0^A,\alpha_0^B\right)\right)p_0 = c'\left(\alpha_0^B\right),$$

which provide the generalization to multiple ESG attributes of the first-order condition with respect to α_i in the baseline model.

Since the information structure is symmetric along the two ESG attributes of the project, and the cost of information gathering is the same for both attributes, in equilibrium we have $\alpha_0^A = \alpha_0^B$ and similarly for provider P_1. The following then holds:

Proposition 1 *In a symmetric equilibrium with single-homing and full market coverage, the level of precision $\alpha^\star$, uniform across providers and attributes, solves the following first-order condition*

$$\mu\frac{x^\star\left(1,1,\alpha^\star,\alpha^\star\right) - x^\star\left(0,0,\alpha^\star,\alpha^\star\right)}{8} = c'\left(\alpha^\star\right),$$

while the equilibrium price is still $p^\star = t$.

The equilibrium precision hinges solely on the difference between the investment levels that the lender undertakes when it receives extreme signals — i.e., when both signals are high and when both are low. This observation immediately suggests that the findings from our baseline analysis also apply here. That is, whether the regulator wants to impose a price-cap or a price-floor depends on the sign of $\zeta'(\cdot)$.

0.2 Scale-dependent private benefit

Up to now, we assumed that the fraction $\phi \in [0,1]$ of the project revenue that cannot be seized in case of default is independent of the loan size x. While this assumption may be appropriate in instances where the law enforcement intensity is set at the industry level, and is independent of the specificities of each bankruptcy procedure, it is reasonable to imagine that in other instances, this intensity depends on the project's scale. In this section, we develop a variant of the model where the fraction of revenues that the entrepreneur can divert into private benefits, denoted $\phi(x)$, varies with the loan size. To simplify the model, we assume that the revenue function is linear — i.e., $f(x) = x$ — and define by $b(x) \triangleq \phi(x)\,x$ the private benefit of the entrepreneur: the absolute amount that obtained when diverting a fraction $\phi(x)$ of a loan of size x. We assume that the private benefit is positive and increasing in x, and distinguish two cases: a convex and a concave $b(\cdot)$.

A convex private benefit reflects the idea that entrepreneurs managing larger-scale projects have greater opportunities to divert into private benefits portions of the loans received. This could be indicative of an enforcement technology that experiences capacity constraints, often due to limited investigative and detection abilities. As the scale of the project increases, enforcement efficiency falls due to these constraints, leading to rising costs per unit of enforcement. This scenario is typical in systems where resources such as skilled personnel or technological tools are scarce and cannot be scaled up easily or cost-effectively to match the growing demands of larger projects.

A concave private benefit seems more compelling in environments where the enforcement technology benefits from economies of scale. In such cases, as the size of the loan increases, the cost of enforcement per unit of the loan decreases, making it more efficient to investigate larger projects. This could be due to factors like improved technological integration, streamlined processes, or the ability to spread fixed costs over a larger output. Another reason why the private benefit may be concave in the project size is diminishing marginal returns of theft — the entrepreneur finds it more difficult to consume or hide a larger diverted amount of resources.

We assume that μ is large enough relative to $b(\cdot), c(\cdot)$, so that L gives a positive loan amount $x^\star(0, \alpha^\star)$ to a project even after it gets a bad rating.

Convex private benefit. Suppose first that the function $b(x)$ is convex. Upon receiving signal s_i, the lender solves the following maximization problem:

$$\max_{x \geq 0} \mu \mathbb{E}[\theta | s_i, \alpha_i]\, x - b(x).$$

In an interior solution, the first-order condition is

$$\mu\mathbb{E}\left[\theta|s_i,\alpha_i\right]-b'\left(x\right)=0 \quad \Rightarrow \quad x^{\star}\left(s_i,\alpha_i\right)\triangleq\beta\left(\mu\mathbb{E}\left[\theta|s_i,\alpha_i\right]\right),$$

where $\beta\left(\cdot\right)$ is the inverse of $b'(\cdot)$, which is the entrepreneur's marginal private benefit — i.e., the additional private benefit E gains from an extra unit of loan. Importantly, since $b''\left(x\right)>0$, the second-order condition holds.

The first-order conditions of the providers' maximization problem are still as in the baseline model — i.e., the equilibrium price is equal to the unit transport cost t, and the equilibrium precision solves

$$\mu\frac{x^{\star}\left(1,\alpha^{\star}\right)-x^{\star}\left(0,\alpha^{\star}\right)}{4}=c'\left(\alpha^{\star}\right).$$

However, the derivative of the project value with respect to α is now

$$\mathcal{V}'\left(\alpha\right)=\mu\frac{x^{\star}\left(1,\alpha\right)-x^{\star}\left(0,\alpha\right)}{2}-2c'\left(\alpha\right)+\frac{\mu}{2}\left[\frac{1}{\eta\left(x^{\star}\left(1,\alpha\right)\right)}-\frac{1}{\eta\left(x^{\star}\left(0,\alpha\right)\right)}\right], \tag{1}$$

where the function $\eta\left(x\right)\triangleq b''\left(x\right)/b'\left(x\right)$ measures the relative convexity of the entrepreneur's private benefits. Alternatively, $\eta\left(\cdot\right)$ can be interpreted as the semi-elasticity of the marginal private benefit with respect to the loan size.

By the same logic as in the baseline model, we can prove the following.

Proposition 2 *Assume that $b''\left(x\right)>0$ for every $x\geq 0$. If the market is fully covered and the lender buys exactly one rating (single-homing), then the level of precision that maximizes expected value is higher (resp. lower) than the level of precision set in equilibrium if $\eta\left(x\right)$ is decreasing (resp. increasing) in x.*

Clearly, as in the baseline model, higher precision still benefits the lender. Therefore, when providers supply less precision than value maximization mandates, a price floor is optimal. By contrast, a price cap is optimal in the case of over-provision.

The interpretation of this result is as follows. When the private benefit becomes relatively more convex at larger scales, E's gain in state $\theta=1$ triggered by increased precision more than compensates for the loss in state $\theta=0$, since the former project has a larger scale, and the impact of higher precision on the marginal private benefit increases with the project scale. As the regulator internalizes the entrepreneur's profit, it prefers a higher precision than what is implemented by the providers in equilibrium. Hence, a price floor is optimal. Conversely, when private benefits become less convex at larger scales, the regulator finds it optimal to impose a price cap.

Concave private benefit. If the absolute amount that the entrepreneur can divert into private benefits is concave in the size of the loan, then the lender's maximization problem is convex and features corner solutions. To guarantee that a solution exists, suppose that the lender has a maximal loan size $x_{\max}$ (finite resources). Hence, the lender optimally chooses $x^{\star} = 0$ if

$$\mu \mathbb{E}\left[\theta | s_i, \alpha_i\right] x_{\max} - b\left(x_{\max}\right) < 0,$$

and chooses $x_{\max}$ if the reverse inequality holds. It is indifferent between 0 and $x_{\max}$ if $\mu \mathbb{E}\left[\theta | s_i\right] x_{\max} = b\left(x_{\max}\right)$. To ensure full market coverage, we assume

$$\frac{2}{5}\left[\mu \delta\left(\frac{\mu x_{\max}}{4}\right) x_{\max} - b\left(x_{\max}\right)\right] \geq t,$$

where the function $\delta\left(\cdot\right) \triangleq c'^{-1}(\cdot)$ is the inverse of the marginal cost. To make the problem interesting, we assume that

$$\frac{\mu}{2} x_{\max} < b(x_{\max}) < \mu \delta\left(\frac{\mu x_{\max}}{4}\right) x_{\max},$$

so that L prefers not to invest when ratings are pure noise, while it invests when the project type is known with certainty. Clearly, if L does not lend at its prior belief or if ratings are pure noise, then it does not lend after a bad signal $s_i = 0$ in equilibrium. We conjecture (and verify ex-post) that in equilibrium, L lends when it observes a good signal $s_i = 1$.

Consider again a symmetric equilibrium with full market coverage. Under those assumptions, L's expected benefit of buying a rating from P_i is

$$\frac{1}{2}[\mu \alpha_i x_{\max} - b\left(x_{\max}\right)],$$

and the cost at location z is $td_i + p_i$, with $d_0 = z^2$ and $d_1 = (1 - z)^2$ as in the baseline model.

Assuming again an equilibrium with single-homing, the indifferent lender is located at

$$z\left(\wp_0, \wp_1\right) = \frac{1}{2} + \frac{p_1 - p_0}{2t} - \mu \frac{\alpha_1 - \alpha_0}{4t} x_{\max},$$

which is also the demand for P_0.

Based on the first-order conditions, in a symmetric equilibrium, $p^{\star} = t$ and

$$c'\left(\alpha^{\star}\right) = \frac{\mu x_{\max}}{4} \quad \Rightarrow \quad \alpha^{\star} = \delta\left(\frac{\mu x_{\max}}{4}\right).$$

This, together with the assumption $\frac{\mu}{2} x_{\max} < b(x_{\max}) < \mu \delta\left(\frac{\mu x_{\max}}{4}\right) x_{\max}$, implies that $x^{\star}\left(1, \alpha^{\star}\right) = x_{\max}$ and $x^{\star}\left(0, \alpha^{\star}\right) = 0$.

Consider now the efficient level of precision from the value point of view. Aggregating the players' expected utilities, we have

$$\mathcal{V}\left(\alpha\right) \triangleq \frac{\alpha}{2} \mu x_{\max} - 2c\left(\alpha\right) - \frac{t}{12}.$$

The first-order condition of this implies

$$c'(\alpha^{\star\star}) = \frac{\mu x_{\max}}{4} \quad \Rightarrow \quad \alpha^{\star\star} = \alpha^{\star}.$$

Hence, the following holds.

Proposition 3 *Assume that $b''(x) \leq 0$. If the market is fully covered and the lender buys one rating only (single-homing), then the level of precision that maximizes the expected project value is equal to the level of precision that providers set in equilibrium. Hence, there is no need for price regulation.*

The intuition for this result is as follows. With a concave private benefit, the lender's expected utility features corner solutions, and the optimal investment level does not depend on the degree of information precision (over a range of precision levels strictly greater than pure noise). The investment level only depends on the signal realization, and this dependence is the same in equilibrium as in value-maximization. Even if the regulator internalizes the entrepreneur's expected profit, this profit is not affected by α. Therefore, the laissez-faire regime naturally yields an efficient outcome.

0.3 Continuum of types

Suppose now that the project type θ is a continuous random variable distributed uniformly on the unit interval $[0, 1]$. Assume that the signal structure features *replacement noise*: the signal s_i disclosed by P_i is perfectly informative with probability α_i, and is a uniform random variable independent of everything else with probability $1 - \alpha_i$. Hence, as before, $\alpha_i \in [0, 1]$ is the signal precision of the information gathered by P_i. Let $c(\alpha_i)$ be the increasing and convex cost of precision, with $c(0) = c'(0) = 0$ and $c'(1)$ sufficiently large to guarantee interior solutions. As before, the revenue function $f(\cdot)$ is increasing and concave.

The timing of the game is as in the baseline model. Conditional on observing signal s_i, the lender's expected value of θ is

$$\mathbb{E}[\theta | s_i, \alpha_i] \triangleq \mathbb{E}[\theta] + \alpha_i (s_i - \mathbb{E}[\theta]).$$

The interpretation is that the signal is either pure noise or perfectly informative, so conditional on signal s_i, the belief of the lender is that with probability $1 - \alpha_i$, the distribution of θ is uniform, and with probability α_i, the distribution is degenerate: $\theta = s_i$.

In an interior solution, the optimal investment is

$$x^{\star}(s_i, \alpha_i) \triangleq \varphi\left(\frac{1 - \mu \mathbb{E}[\theta | s_i, \alpha_i]}{1 - \phi}\right) \quad \Rightarrow \quad \frac{\partial x^{\star}(s_i, \alpha_i)}{\partial \alpha_i} = -\frac{\mu (s_i - \mathbb{E}[\theta])}{(1 - \phi) f''(x^{\star}(s_i, \alpha_i))}.$$

That is, an increase in precision expands the investment when the lender observes a signal larger than the expected project type, and vice-versa.

The lender's expected utility from buying from provider P_i is

$$u_i(\alpha_i, p_i) \triangleq \int_{s_i} \left[(1-\phi) f(x^\star(s_i,\alpha_i)) - x^\star(s_i,\alpha_i)\right] ds_i + \int_{s_i}\int_{\theta} \mu\left[\alpha_i s_i + (1-\alpha_i)\theta\right] x^\star(s_i,\alpha_i)\, d\theta ds_i - p_i - td_i,$$

where d_i is the quadratic distance to provider i as in the baseline.

We can thus state the following:

Proposition 4 *With replacement noise, in an equilibrium with full market coverage and single homing, providers charge $p^\star = t$ and set a level of precision $\alpha^\star \geq 0$ that solves*

$$\mu \frac{\int_0^1 x^\star(s_i,\alpha_i)(s_i - \mathbb{E}[\theta])\, ds_i}{2} = c'(\alpha_i),$$

The value-maximizing precision $\alpha^{\star\star}$ is higher than the equilibrium value $\alpha^\star$ if $\zeta'(\cdot) > 0$ for every $x \geq 0$, and vice-versa. Hence a price floor (resp. cap) is optimal if $\zeta'(\cdot) > 0$ (resp. ≤ 0) for every $x \geq 0$.

This result confirms that the insights of the baseline model are robust to the introduction of a continuum of types.

Proofs

Proof of Proposition 1. The proof follows immediately from evaluating the FOCs at the symmetric equilibrium precision. ■

Proof of Proposition 2. The proof follows immediately from evaluating equation (1) at the equilibrium values and adheres to the same logical steps as the main proofs of the baseline model. Hence, it is omitted for brevity. ■

Proof of Proposition 3. The proof follows immediately from the text and the fact that a corner solution is a global maximum if $b''(\cdot) \leq 0$. ■

Proof of Proposition 4. Consider again a symmetric equilibrium with full market coverage and single-homing where both providers offer a policy $\wp^\star \triangleq (p^\star, \alpha^\star)$. Focus, as before, on P_0's behavior as suppose that it offers $\wp_0$. For brevity, let

$$\Lambda(\alpha_i) \triangleq \int_{s_i} [(1-\phi) f(x^\star(s_i,\alpha_i)) - x^\star(s_i,\alpha_i)]\, ds_i + \int_{s_i}\int_\theta \mu(\alpha_i s_i + (1-\alpha_i)\theta)\, x^\star(s_i,\alpha_i)\, d\theta ds_i$$

The lender is indifferent between $\wp^\star$ and $\wp_0$ if and only if

$$\Lambda(\alpha_0) - p_0 - tz^2 = \Lambda(\alpha^\star) - p^\star - t(1-z)^2 \quad \Rightarrow \quad z = \frac{1}{2} + \frac{\Lambda(\alpha_0) - p_0 - (\Lambda(\alpha^\star) - p^\star)}{2t}.$$

with

$$\frac{\partial z}{\partial \alpha_0} = \frac{1}{2t}\int_{s_i}\int_\theta \mu(s_i - \theta)\, x(s_i,\alpha_i)\, d\theta ds_i = \frac{\mu}{2t}\int_{s_i} (s_i - \mathbb{E}[\theta])\, x^\star(s_i,\alpha_i)\, ds_i.$$

From this, we get the first-order condition with respect to α_i and, hence, the equilibrium condition. As in the baseline model, $p^\star = t$, and analogously to the baseline,

$$\mu\frac{\int_0^1 x^\star(s_i,\alpha^\star)(s_i - \mathbb{E}[\theta])\, ds_i}{2} = c'(\alpha^\star).$$

Consider now value maximization. The logic is the same as in the baseline model. If the providers' maximization problem has an interior solution, then the accuracy strictly increases in the price — i.e., the function $\alpha(p)$ that solves

$$\mu\frac{\int_0^1 x^\star(s_i,\alpha)(s_i - \mathbb{E}[\theta])\, ds_i}{2t} p = c'(\alpha)$$

is increasing in p. To find the optimal regulated price, it is enough to maximize value with respect to α. Value is

$$\mathcal{V}(\alpha) \triangleq \int_s [f(x^\star(s_i,\alpha)) - x^\star(s_i,\alpha)]\, ds + \mu\int_s (\alpha s + (1-\alpha_i)\mathbb{E}[\theta])\, x^\star(s,\alpha)\, ds - 2c(\alpha) - \frac{t}{12}.$$

Differentiating with respect to α, and evaluating the resulting condition at $\alpha^\star$, we have

$$\mathcal{V}'\left(\alpha^\star\right) = \phi \int_s f'\left(x^\star\left(s,\alpha\right)\right) \frac{\partial x^\star\left(s,\alpha^\star\right)}{\partial \alpha} ds,$$

with

$$\frac{\partial x^\star\left(s,\alpha^\star\right)}{\partial \alpha} = -\frac{\mu\left(s-\mathbb{E}\left[\theta\right]\right)}{(1-\phi)f''\left(x^\star\left(s,\alpha^\star\right)\right)}.$$

Hence

$$\mathcal{V}'\left(\alpha^\star\right) = -\frac{\mu\phi}{1-\phi}\int_s \frac{f'\left(x^\star\left(s,\alpha\right)\right)}{f''\left(x^\star\left(s,\alpha^\star\right)\right)}\left(s-\mathbb{E}\left[\theta\right]\right) ds = -\frac{\mu\phi}{1-\phi}\int_s \frac{s-\mathbb{E}\left[\theta\right]}{\zeta\left(x^\star\left(s,\alpha^\star\right)\right)} ds.$$

This is positive (i.e., the value-maximizing precision $\alpha^{\star\star}$ exceeds the equilibrium level $\alpha^\star$) if $\frac{1}{\zeta(x^\star(s,\alpha^\star))}$ decreases in s, equivalently if ζ increases in s. Similarly, $\alpha^{\star\star}$ is less than the equilibrium precision $\alpha^\star$ if ζ decreases in s. ■