

Supplementary Material

A fast epigraph and hypograph-based approach for clustering functional data

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This document is organized as follows. In section 1 we show the results corresponding to the sensitive study related to the number of basis to be consider when applying B-splines smoothing procedure. All the possible combinations of indexes and data mentioned in Section 3 are presented here and, finally, we show three different clustering methodologies applied to the functional data scenarios S 1-2, S 1-3, S 1-5, S 1-6, S 1-7, S 1-8, S 1-9, S 10-11, S 18-19-20 and S 21-22-23, explained in Section 5 of the paper. The methodologies which appear here are the one proposed in the main paper: EHyClus, as well as functional k-means technique, proposed in Martino et al. (2019) and test based k-means, which was proposed in Zambom et al. (2019). Results from baseclust, distclust, fitfclust, distclust, modelcf and curveclust explained in Yassouridis and Leisch (2017) do not appear here as their Rand Index and execution time appear in the main manuscript.

1 Sensitive study with respect to the number of when B-splines basis.

In Table 2 the results of a sensitive analysis are presented. Its goal is to compare the values of RI obtained when performing EHyClus using different number of basis: 10, 20, 30 and 40 basis, (for the 8 first data sets 40 basis are not considered as there are not enough breaks in the interval). The scenarios considered for developing this analysis are S 1-2, S 1-3, S 1-4, S 1-5, S 1-6, S 1-7, S 1-8, S 1-9, S 10-11, S 10-12, S 15-16-17, S 18-19-20 and S 21-22-23. The results of this analysis depends on the curves analyzed and it is focused on the final goal of clustering. Thus, we may conclude that changing the number of basis when considering cubic B-spline basis does not drastically affect the final result.

Table 1: Relative RI in terms of the number of basis.

	10	20	30	40
S 1-2	100	99.2	99.4	-
S 1-3	100	97.4	98.3	-
S 1-4	100	100	99.4	-
S 1-5	100	99.2	98.7	-
S 1-6	99.1	100	99.8	-
S 1-7	99.8	100	99.8	-
S 1-8	100	97.5	98.2	-
S 1-9	100	96.9	98.1	-
S 10-11	99.1	99.5	100	98.8
S 10-12	89.9	100	98.1	92.6
S 15-16-17	100	100	100	100
S 18-19-20	99.9	99.9	100	99.6
S 21-22-23	100	100	100	100

2 Combinations of data and indexes

When implementing EHyClus, different combinations of data and indexes need to be set. In this process 18 different combinations are considered and they are all listed in Table 2.

Table 2: Representation and description of the combinations of data and indexes.

Representation	Description
$_EIHI = (EI, HI)$	The epigraph and the hypograph index on the original curves.
$d.EIHI = (dEI, dHI)$	The epigraph and the hypograph index on the first derivatives.
$d2.EIHI = (d2EI, d2HI)$	The epigraph and the hypograph index on the second derivatives.
$_d.EIHI = (EI, HI, dEI, dHI)$	The epigraph and the hypograph index on the original curves and first derivatives.
$_d2.EIHI = (EI, HI, d2EI, d2HI)$	The epigraph and the hypograph index on the original curves and second derivatives.
$dd2.EIHI = (dEI, dHI, d2EI, d2HI)$	The epigraph and the hypograph index on the first and second derivatives.
$_dd2.EIHI = (EI, HI, dEI, dHI, d2EI, d2HI)$	The epigraph and the hypograph index on the original curves, first and second derivatives.
$_d.MEI = (MEI, dMEI)$	The generalized epigraph index applied to the original curves and first derivatives.

$_d2.MEI = (MEI, d2MEI)$	The generalized epigraph index applied to the original curves and second derivatives.
$dd2.MEI = (dMEI, d2MEI)$	The generalized epigraph index applied to the first and second derivatives.
$_dd2.MEI = (MEI, dMEI, d2MEI)$	The generalized epigraph index applied to the original curves, first and second derivatives.
$_EIHIMEI = (EI, HI, MEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$d.EIHIMEI = (dEI, dHI, dMEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$d2.EIHIMEI = (d2EI, d2HI, d2MEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$_d.EIHIMEI = (EI, HI, MEI, dEI, dHI, dMEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$_d2.EIHIMEI = (EI, HI, MEI, d2EI, d2HI, d2MEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$dd2.EIHIMEI = (dEI, dHI, dMEI, d2EI, d2HI, d2MEI)$	The epigraph, the hypograph index and the generalized epigraph index applied to all the data.
$_dd2.EIHIMEI = (EI, HI, MEI, dEI, dHI, dMEI, d2EI, d2HI, d2MEI)$	The epigraph, the hypograph and the generalized version of the epigraph applied to all the data.

3 Two clusters

In this section the tables corresponding to Section 5.1 of the paper are provided. Here, results from applying EHyclus, functional k-means and model based k-means to scenarios with two clusters are shown.

Table 3: Mean results for S 1-2 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans...EIHIMEI	0.76940	0.64171	0.64378	0.00944

kmeans._.EIHIMEI	0.76760	0.64034	0.64188	0.00213
kmeans._.EIHIMEI	0.76760	0.64034	0.64188	0.00224
svc._.EIHIMEI	0.76570	0.63856	0.63988	0.00511
svc._.EIHIMEI	0.76450	0.63774	0.63872	0.00568
svc._d.EIHIMEI	0.76430	0.63724	0.63845	0.00661
kmeans._d.EIHIMEI	0.76410	0.63679	0.63828	0.00241
kmeans._d.EIHIMEI	0.76410	0.63679	0.63828	0.00344
svc._d.EIHIMEI	0.76330	0.63659	0.63741	0.00765
svc._d.MEI	0.76320	0.63467	0.63710	0.00410
svc._d.MEI	0.76270	0.63429	0.63672	0.00388
kmeans._d.MEI	0.76240	0.63393	0.63639	0.00212
kmeans._d.MEI	0.76240	0.63393	0.63639	0.00214
kkmeans._d.MEI	0.76180	0.63336	0.63592	0.01089
kkmeans._d.EIHIMEI	0.75670	0.63212	0.63343	0.01140
svc._d2.MEI	0.74880	0.62100	0.62332	0.00380
kmeans._d2.MEI	0.74810	0.62046	0.62276	0.00220
kmeans._d2.MEI	0.74810	0.62046	0.62276	0.00216
svc._d2.MEI	0.74830	0.62027	0.62261	0.00422
svc_dd2.MEI	0.74740	0.61908	0.62171	0.00550
kkmeans._d2.MEI	0.74620	0.61845	0.62086	0.01141
svc_dd2.MEI	0.74610	0.61786	0.62058	0.00456
ward.D2._.EIHIMEI	0.74610	0.63399	0.62052	0.00022
kmeans_dd2.MEI	0.74510	0.61697	0.61953	0.00228
kmeans_dd2.MEI	0.74510	0.61697	0.61953	0.00234
kmeans._d2.EIHIMEI	0.74040	0.61371	0.61553	0.00270
kmeans._d2.EIHIMEI	0.74040	0.61371	0.61553	0.00286
kkmeans_dd2.MEI	0.73860	0.61228	0.61509	0.01171
svc._d2.EIHIMEI	0.73910	0.61247	0.61429	0.00659
svc._d2.EIHIMEI	0.73850	0.61195	0.61390	0.00726
ward.D2._d.MEI	0.73900	0.62256	0.61305	0.00020
svc._dd2.EIHIMEI	0.73680	0.61066	0.61254	0.00788
kmeans._dd2.EIHIMEI	0.73650	0.60962	0.61151	0.00291
kmeans._dd2.EIHIMEI	0.73650	0.60962	0.61151	0.00304
kkmeans._d2.EIHIMEI	0.73540	0.60993	0.61149	0.01237
svc._dd2.EIHIMEI	0.73310	0.60809	0.60992	0.00901
kkmeans._dd2.EIHIMEI	0.73280	0.60802	0.60982	0.01442
ward.D2._d2.MEI	0.72970	0.61479	0.60542	0.00018
complete._d.MEI	0.72440	0.61378	0.60020	0.00035
average._d.MEI	0.71120	0.62625	0.59936	0.00027
ward.D2._d.EIHIMEI	0.72300	0.61767	0.59893	0.00037
ward.D2_dd2.MEI	0.72100	0.60659	0.59820	0.00024
average._d2.MEI	0.70730	0.61632	0.59232	0.00025
complete._d2.MEI	0.71470	0.60600	0.59230	0.00032
complete._.EIHIMEI	0.70830	0.62748	0.59175	0.00027

complete_dd2.MEI	0.71270	0.60209	0.59055	0.00037
kkmeans._d.MEI	0.69910	0.59105	0.58914	0.01620
kkmeans_dd2.MEI	0.69730	0.58434	0.58523	0.01620
ward.D2._d2.EIHIMEI	0.70370	0.59722	0.58404	0.00031
average_dd2.MEI	0.68370	0.61548	0.58112	0.00032
kkmeans._d2.MEI	0.67940	0.57740	0.57521	0.01728
kkmeans._d.EIHIMEI	0.67890	0.57421	0.57262	0.01626
complete._d.EIHIMEI	0.67470	0.61696	0.56942	0.00128
ward.D2._dd2.EIHIMEI	0.67890	0.58427	0.56494	0.00039
kkmeans._dd2.EIHIMEI	0.65980	0.55746	0.55786	0.01645
kkmeans._.EIHI	0.65890	0.62709	0.55556	0.01472
spc._d.MEI	0.62870	0.64033	0.55462	0.04852
complete._d2.EIHIMEI	0.65300	0.60211	0.55420	0.00039
kkmeans._d2.EIHIMEI	0.64950	0.55511	0.55316	0.01654
svc._.EIHI-	0.65620	0.62872	0.55295	0.00470
svc._.EIHI	0.65080	0.62698	0.54909	0.00551
kkmeans._d.EIHI	0.64070	0.61180	0.54426	0.01753
complete._dd2.EIHIMEI	0.63930	0.59730	0.54415	0.00034
spc._.EIHIMEI	0.61740	0.64219	0.54402	0.05339
spc._d2.MEI	0.61920	0.63037	0.54351	0.04687
svc._d.EIHI	0.63650	0.61441	0.54221	0.00546
spc._d.EIHIMEI	0.59620	0.64227	0.53805	0.04471
kkmeans._.EIHIMEI	0.62140	0.54562	0.53683	0.01461
kkmeans._d2.EIHI	0.61810	0.56459	0.53298	0.01442
spc._d2.EIHIMEI	0.59520	0.63158	0.53226	0.04356
svc._d.EIHI	0.61690	0.61432	0.53091	0.00724
kkmeans._dd2.EIHI	0.61150	0.56220	0.52969	0.01630
spc_dd2.MEI	0.59220	0.61984	0.52920	0.04660
kkmeans._.EIHI	0.60250	0.53293	0.52544	0.01416
svc._d2.EIHI	0.60300	0.58058	0.52510	0.00504
svc._dd2.EIHI	0.60200	0.57472	0.52396	0.00654
ward.D2._.EIHI	0.60520	0.62691	0.52301	0.00019
svc._d2.EIHI	0.60120	0.57861	0.52284	0.00569
kmeans._.EIHI	0.60900	0.62427	0.52249	0.00209
kmeans._.EIHI	0.60900	0.62427	0.52249	0.00229
kmeans._d.EIHI	0.60580	0.61818	0.52225	0.00223
kmeans._d.EIHI	0.60580	0.61818	0.52225	0.00273
svc._dd2.EIHI	0.59570	0.57188	0.52167	0.00681
ward.D2._d.EIHI	0.59640	0.62476	0.51967	0.00035
spc._.EIHI	0.58230	0.63970	0.51832	0.02904
kmeans._d2.EIHI	0.58540	0.59126	0.51502	0.00254
kmeans._d2.EIHI	0.58540	0.59126	0.51502	0.00246
kmeans._dd2.EIHI	0.58410	0.58873	0.51496	0.00257
kmeans._dd2.EIHI	0.58410	0.58873	0.51496	0.00270

spc._dd2.EIHIMEI	0.56100	0.62430	0.51487	0.04453
ward.D2._d2.EIHI	0.57660	0.60307	0.51118	0.00027
ward.D2._dd2.EIHI	0.57150	0.60113	0.50965	0.00029
complete._.EIHI	0.56830	0.63895	0.50954	0.00031
centroid._d.MEI	0.54490	0.64536	0.50903	0.00030
centroid._d2.MEI	0.54320	0.64625	0.50865	0.00116
kkmeans._d.EIHI	0.55920	0.51185	0.50847	0.01721
spc._d.EIHI	0.54080	0.64134	0.50598	0.04371
kkmeans._dd2.EIHI	0.55170	0.50501	0.50538	0.01634
complete._d.EIHI	0.55710	0.63691	0.50484	0.00041
average._.EIHIMEI	0.55030	0.64419	0.50440	0.00025
complete._d2.EIHI	0.55430	0.63116	0.50398	0.00034
complete._dd2.EIHI	0.55340	0.63204	0.50387	0.00134
kkmeans._d2.EIHI	0.54930	0.50565	0.50346	0.01613
average._d2.EIHIMEI	0.54380	0.64298	0.50312	0.00025
spc._d2.EIHI	0.54290	0.63325	0.50260	0.04530
centroid._.EIHIMEI	0.54240	0.64630	0.50203	0.00029
average._dd2.EIHIMEI	0.53820	0.64088	0.50133	0.00047
average._d.EIHIMEI	0.53780	0.64392	0.50077	0.00035
average._.EIHI	0.54020	0.64683	0.50007	0.00029
spc._dd2.EIHI	0.52550	0.63744	0.49899	0.04692
centroid._.EIHI	0.53340	0.64888	0.49861	0.00129
average._d.EIHI	0.52890	0.64913	0.49807	0.00023
average._d2.EIHI	0.52880	0.64859	0.49782	0.00028
average._dd2.EIHI	0.52480	0.64865	0.49760	0.00027
centroid._d.EIHIMEI	0.52390	0.65054	0.49713	0.00033
single._.EIHIMEI	0.52310	0.65299	0.49706	0.00030
centroid._d.EIHI	0.52150	0.65084	0.49664	0.00031
centroid._d2.EIHI	0.52020	0.65277	0.49644	0.00036
centroid._d2.EIHIMEI	0.52040	0.65250	0.49634	0.00038
single._.EIHI	0.51990	0.65393	0.49633	0.00034
centroid._dd2.EIHI	0.51610	0.65335	0.49588	0.00033
centroid._dd2.EIHIMEI	0.51450	0.65441	0.49569	0.00041
single._d2.EIHI	0.51440	0.65538	0.49569	0.00033
single._d.EIHI	0.51260	0.65528	0.49554	0.00037
single._d2.EIHIMEI	0.51260	0.65617	0.49551	0.00032
single._d2.MEI	0.51260	0.65611	0.49544	0.00031
single._d.MEI	0.51260	0.65629	0.49542	0.00133
single._d.EIHIMEI	0.51170	0.65578	0.49542	0.00038
centroid_dd2.MEI	0.51170	0.65575	0.49537	0.00033
single._dd2.EIHI	0.51050	0.65553	0.49531	0.00035
single_dd2.MEI	0.51020	0.65602	0.49528	0.00037
single._dd2.EIHIMEI	0.50980	0.65528	0.49528	0.00036
kkmeans.dd2.EIHI	0.50140	0.51306	0.49495	0.01130

kkmeans.d.EIHIMEI	0.50120	0.50372	0.49494	0.01011
kkmeans.d2.EIHIMEI	0.50120	0.49823	0.49494	0.00944
kkmeans.dd2.EIHIMEI	0.50110	0.49736	0.49493	0.01148
spc.d2.EIHI	0.50100	0.62143	0.49493	0.02034
kmeans.d2.EIHIMEI	0.50090	0.50690	0.49493	0.00213
kmeans.d2.EIHIMEI	0.50090	0.50690	0.49493	0.00203
kkmeans.d2.EIHI	0.50090	0.51917	0.49493	0.00999
kkmeans.d2.EIHI	0.50080	0.49903	0.49493	0.01158
svc.dd2.EIHIMEI	0.50090	0.50435	0.49493	0.00706
d2.EIHIMEI	0.50080	0.50382	0.49492	0.00457
svc.dd2.EIHI	0.50080	0.53400	0.49492	0.00544
kkmeans.d2.EIHIMEI	0.50080	0.49970	0.49492	0.01214
svc.d2.EIHIMEI	0.50070	0.50311	0.49492	0.00514
svc.d2.EIHI	0.50060	0.53158	0.49492	0.00493
kkmeans.dd2.EIHI	0.50070	0.49847	0.49492	0.01338
kkmeans.d.EIHI	0.50060	0.50154	0.49492	0.01123
svc.d2.EIHI	0.50060	0.52785	0.49492	0.00460
svc.dd2.EIHIMEI	0.50060	0.50424	0.49492	0.00666
kkmeans.d.EIHIMEI	0.50050	0.49999	0.49492	0.01272
svc.dd2.EIHI	0.50050	0.54237	0.49492	0.00566
kmeans.dd2.MEI	0.50040	0.49690	0.49491	0.00215
kmeans.dd2.MEI	0.50040	0.49690	0.49491	0.00210
kmeans.d2.EIHI	0.50040	0.54136	0.49491	0.00194
kmeans.d2.EIHI	0.50040	0.54136	0.49491	0.00220
kmeans.dd2.EIHI	0.50040	0.54675	0.49491	0.00229
kmeans.dd2.EIHI	0.50040	0.54675	0.49491	0.00238
spc.d2.EIHIMEI	0.50030	0.63164	0.49491	0.05080
svc.dd2.MEI	0.50040	0.49713	0.49491	0.00402
svc.d.EIHIMEI	0.50040	0.51332	0.49491	0.00577
svc.d.EIHI	0.50030	0.56913	0.49491	0.00476
kkmeans.dd2.MEI	0.50030	0.49694	0.49491	0.01059
spc.d.EIHI	0.50030	0.61837	0.49491	0.01419
pc.d.EIHIMEI	0.50030	0.63989	0.49491	0.05073
svc.dd2.MEI-kmeans	0.50030	0.49986	0.49491	0.00384
kkmeans.d.EIHI	0.50020	0.55444	0.49491	0.01213
kkmeans.dd2.MEI	0.50020	0.49877	0.49491	0.01268
kmeans.d.EIHIMEI	0.50020	0.51243	0.49491	0.00219
kmeans.d.EIHIMEI	0.50020	0.51243	0.49491	0.00230
kmeans.dd2.EIHIMEI	0.50020	0.50190	0.49491	0.00275
kmeans.dd2.EIHIMEI	0.50020	0.50187	0.49491	0.00259
spc.dd2.EIHI	0.50020	0.64149	0.49491	0.02289
svc.d.EIHIMEI	0.50020	0.50952	0.49491	0.00478
kkmeans.dd2.EIHIMEI	0.50010	0.49826	0.49490	0.01328
kmeans.d.EIHI	0.50010	0.59424	0.49490	0.00193

kmeans.d.EIHI	0.50010	0.59443	0.49490	0.00216
svc.d.EIHI	0.50010	0.56750	0.49490	0.00445
ward.D2.d2.EIHI	0.50010	0.55502	0.49490	0.00019
ward.D2.d2.EIHIMEI	0.50010	0.52661	0.49490	0.00019
average.d.EIHI	0.50000	0.63688	0.49490	0.00020
average.d.EIHIMEI	0.50000	0.62065	0.49490	0.00024
average.d2.EIHI	0.50000	0.62787	0.49490	0.00026
average.d2.EIHIMEI	0.50000	0.60485	0.49490	0.00026
average.dd2.EIHI	0.50000	0.63671	0.49490	0.00025
average.dd2.EIHIMEI	0.50000	0.62691	0.49490	0.00029
average.dd2.MEI	0.50000	0.58167	0.49490	0.00031
centroid.d.EIHI	0.50000	0.64233	0.49490	0.00027
centroid.d.EIHIMEI	0.50000	0.63227	0.49490	0.00027
centroid.d2.EIHI	0.50000	0.63435	0.49490	0.00027
centroid.d2.EIHIMEI	0.50000	0.63056	0.49490	0.00025
centroid.dd2.EIHI	0.50000	0.64674	0.49490	0.00033
centroid.dd2.EIHIMEI	0.50000	0.64851	0.49490	0.00029
centroid.dd2.MEI	0.50000	0.64313	0.49490	0.00029
complete.d.EIHI	0.50000	0.61780	0.49490	0.00031
complete.d.EIHIMEI	0.50000	0.56299	0.49490	0.00028
complete.d2.EIHI	0.50000	0.59701	0.49490	0.00032
complete.d2.EIHIMEI	0.50000	0.55063	0.49490	0.00025
complete.dd2.EIHI	0.50000	0.60505	0.49490	0.00030
complete.dd2.EIHIMEI	0.50000	0.54359	0.49490	0.00030
complete.dd2.MEI	0.50000	0.52076	0.49490	0.00031
single.d.EIHI	0.50000	0.64646	0.49490	0.00034
single.d.EIHIMEI	0.50000	0.64599	0.49490	0.00034
single.d2.EIHI	0.50000	0.64354	0.49490	0.00030
single.d2.EIHIMEI	0.50000	0.64365	0.49490	0.00038
single.dd2.EIHI	0.50000	0.64879	0.49490	0.00037
single.dd2.EIHIMEI	0.50000	0.64833	0.49490	0.00032
single.dd2.MEI	0.50000	0.64724	0.49490	0.00034
spc.dd2.EIHIMEI	0.50000	0.64489	0.49490	0.05628
pc.dd2.MEI	0.50000	0.64327	0.49490	0.04626
ward.D2.d.EIHI	0.50000	0.60465	0.49490	0.00019
ward.D2.d.EIHIMEI	0.50000	0.53226	0.49490	0.00023
ward.D2.dd2.EIHI	0.50000	0.57049	0.49490	0.00022
ward.D2.dd2.EIHIMEI	0.50000	0.52598	0.49490	0.00036
ward.D2.dd2.MEI	0.50000	0.51235	0.49490	0.00021

Table 4: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-2.

	Purity	Fmeasure	RI	Time
L^2	0.748	0.619	0.621	0.75023
dp, p=0.001	0.744	0.617	0.618	1.87242
dp, p=0.02	0.744	0.616	0.617	1.85571
dp, p=1	0.743	0.615	0.617	1.84944
dk k=2	0.669	0.571	0.571	0.83022
dp, p=100	0.636	0.556	0.548	2.00940
dp, p=1e+08	0.638	0.550	0.545	1.93400
dk, k=3	0.601	0.532	0.529	1.21892

Table 5: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-2.

	Purity	Fmeasure	RI	Time
kmeans ++	0.500	0.504	0.495	0.34436
hclust	0.500	0.500	0.495	0.22537
random	0.500	0.500	0.495	0.26233
kmeans	0.500	0.497	0.495	1.05298

Table 6: Mean results for S 1-3 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans...EIHIMEI	0.86240	0.76133	0.76269	0.00071
svc...EIHIMEI	0.86140	0.75994	0.76112	0.00031
1vc...EIHIMEI	0.86080	0.75901	0.76021	0.00032
kmeans...EIHIMEI	0.86010	0.75785	0.75918	0.00017
kmeans...EIHIMEI	0.86010	0.75785	0.75918	0.00016
kmeans._d.EIHIMEI	0.85910	0.75646	0.75776	0.00017
kmeans._d.EIHIMEI	0.85910	0.75646	0.75776	0.00019
svc._d.EIHIMEI	0.85880	0.75602	0.75733	0.00049
kkmeans._d.EIHIMEI	0.85760	0.75519	0.75648	0.00076
svc._d.EIHIMEI	0.85810	0.75524	0.75641	0.00041
kmeans._d.MEI	0.85790	0.75451	0.75606	0.00014
kmeans._d.MEI	0.85790	0.75451	0.75606	0.00018
svc._d.MEI	0.85780	0.75436	0.75595	0.00026
svc._d.MEI	0.85760	0.75409	0.75565	0.00029
kkmeans._d.MEI	0.85620	0.75186	0.75374	0.00071
svc._d2.EIHIMEI	0.84850	0.74186	0.74307	0.00041

kmeans._d2.EIHIMEI	0.84760	0.74049	0.74175	0.00019
kmeans._d2.EIHIMEI	0.84760	0.74049	0.74175	0.00019
kkmeans_dd2.MEI	0.84690	0.73901	0.74109	0.00082
svc._d2.MEI	0.84700	0.73925	0.74108	0.00027
svc._d2.EIHIMEI	0.84650	0.73906	0.74045	0.00042
kmeans_dd2.MEI	0.84620	0.73853	0.74026	0.00018
kmeans_dd2.MEI	0.84620	0.73853	0.74026	0.00016
kkmeans._d2.EIHIMEI	0.84590	0.73830	0.74005	0.00085
kmeans._d2.MEI	0.84600	0.73814	0.73989	0.00015
kmeans._d2.MEI	0.84600	0.73814	0.73989	0.00023
svc._d2.MEI	0.84580	0.73776	0.73951	0.00026
svc_dd2.MEI	0.84570	0.73770	0.73942	0.00029
svc._dd2.EIHIMEI	0.84560	0.73766	0.73895	0.00052
kkmeans._d2.MEI	0.84430	0.73681	0.73883	0.00075
kmeans._dd2.EIHIMEI	0.84500	0.73700	0.73826	0.00021
kmeans._dd2.EIHIMEI	0.84500	0.73700	0.73826	0.00020
svc_dd2.MEI	0.84240	0.73484	0.73655	0.00031
kkmeans._dd2.EIHIMEI	0.84200	0.73447	0.73636	0.00086
svc._dd2.EIHIMEI	0.84340	0.73451	0.73602	0.00050
ward.D2._d2.MEI	0.83200	0.72622	0.72214	0.00002
ward.D2._.EIHIMEI	0.82830	0.72771	0.71814	0.00002
ward.D2._d.MEI	0.82950	0.72203	0.71776	0.00002
average._d2.MEI	0.82360	0.72308	0.71559	0.00002
average._d.MEI	0.81860	0.72260	0.71188	0.00002
kkmeans_dd2.MEI	0.81570	0.71163	0.71187	0.00125
average_dd2.MEI	0.81920	0.71836	0.71051	0.00002
ward.D2_dd2.MEI	0.82180	0.71125	0.70840	0.00002
complete._d2.MEI	0.81730	0.70998	0.70524	0.00002
kkmeans._dd2.EIHIMEI	0.80680	0.70379	0.70390	0.00122
complete._d.MEI	0.81250	0.70658	0.69801	0.00009
kkmeans._.EIHI	0.80630	0.71097	0.69411	0.00081
kkmeans._dd2.EIHI	0.80640	0.69863	0.69117	0.00101
kkmeans._d.EIHI	0.80270	0.70467	0.69004	0.00092
complete_dd2.MEI	0.80320	0.69403	0.68911	0.00002
kkmeans._d2.EIHI	0.80540	0.69924	0.68863	0.00099
ward.D2._d.EIHIMEI	0.80560	0.70019	0.68862	0.00002
kkmeans._d2.MEI	0.78690	0.68555	0.68454	0.00118
ward.D2._d2.EIHIMEI	0.79810	0.69431	0.68296	0.00002
svc._.EIHI	0.79730	0.70365	0.68254	0.00035
kkmeans._d.MEI	0.77740	0.67468	0.67260	0.00117
ward.D2._dd2.EIHIMEI	0.78850	0.68101	0.66876	0.00002
svc._d2.EIHI	0.77990	0.68959	0.66491	0.00042
svc._dd2.EIHI	0.77840	0.68359	0.66267	0.00044
svc._.EIHI	0.78090	0.68727	0.66119	0.00036

complete...EIHIMEI	0.76800	0.69242	0.65789	0.00002
svc._d.EIHI	0.77250	0.68273	0.65479	0.00038
svc._d2.EIHI	0.76880	0.67854	0.65193	0.00037
svc._d.EIHI	0.77070	0.67805	0.65018	0.00036
svc._dd2.EIHI-kmeans	0.76050	0.67515	0.64716	0.00044
kkmeans._d.EIHIMEI	0.74450	0.64600	0.64412	0.00122
kkmeans._d2.EIHIMEI	0.74560	0.64440	0.64274	0.00128
complete._d2.EIHIMEI	0.75580	0.66809	0.64021	0.00003
kmeans._.EIHI	0.76110	0.67126	0.63910	0.00016
kmeans._.EIHI	0.76110	0.67126	0.63910	0.00017
kmeans._d2.EIHI	0.76040	0.66859	0.63843	0.00018
kmeans._d2.EIHI	0.76040	0.66859	0.63843	0.00017
spc._d.MEI	0.71540	0.70034	0.63841	0.00335
complete._d.EIHIMEI	0.74920	0.67688	0.63833	0.00017
kmeans._d.EIHI	0.75760	0.66894	0.63749	0.00017
kmeans._d.EIHI	0.75760	0.66894	0.63749	0.00018
kmeans._dd2.EIHI	0.75580	0.66551	0.63553	0.00018
kmeans._dd2.EIHI	0.75580	0.66551	0.63553	0.00020
complete._dd2.EIHIMEI	0.74700	0.66086	0.63068	0.00003
spc._d2.MEI	0.69370	0.69245	0.62248	0.00333
average...EIHIMEI	0.66990	0.68418	0.59755	0.00002
spc_dd2.MEI	0.66950	0.67910	0.59747	0.00364
spc._d.EIHIMEI	0.65960	0.68131	0.59609	0.00334
kkmeans._.EIHIMEI	0.69170	0.59964	0.59447	0.00100
ward.D2._.EIHI	0.71250	0.64975	0.59443	0.00002
centroid._d.MEI	0.65110	0.68320	0.59113	0.00002
ward.D2._d.EIHI	0.70310	0.64444	0.58726	0.00002
spc._d2.EIHIMEI	0.65480	0.67405	0.58548	0.00323
spc._dd2.EIHIMEI	0.65260	0.66848	0.58376	0.00334
ward.D2._d2.EIHI	0.69550	0.64003	0.57985	0.00003
kkmeans._dd2.EIHI	0.67030	0.57549	0.57379	0.00112
ward.D2._dd2.EIHI	0.67650	0.63419	0.56644	0.00002
spc...EIHIMEI	0.60280	0.66495	0.55587	0.00370
average._d2.EIHIMEI	0.60650	0.66542	0.55269	0.00003
kkmeans._.EIHI	0.63670	0.55919	0.55165	0.00101
spc...EIHI	0.61690	0.65841	0.55036	0.00272
kkmeans._d.EIHI	0.63060	0.55055	0.54657	0.00114
kkmeans._d2.EIHI	0.62820	0.54859	0.54576	0.00114
centroid._d2.MEI	0.57970	0.67481	0.54505	0.00002
spc._d.EIHI	0.59200	0.65946	0.54295	0.00336
average._d.EIHIMEI	0.58700	0.66165	0.54100	0.00003
complete._.EIHI	0.63260	0.63781	0.54041	0.00002
centroid...EIHIMEI	0.58510	0.65834	0.53237	0.00002
average._dd2.EIHIMEI	0.57660	0.65559	0.53211	0.00003

spc.dd2.MEI	0.55850	0.65284	0.53168	0.00346
complete._d.EIHI	0.60800	0.63664	0.52879	0.00002
spc._d2.EIHI	0.58210	0.64434	0.52672	0.00332
complete._dd2.EIHI	0.60830	0.63119	0.52622	0.00003
complete._d2.EIHI	0.60410	0.63439	0.52423	0.00002
spc._dd2.EIHI	0.56390	0.64525	0.52160	0.00353
average._.EIHI	0.55760	0.64451	0.50646	0.00002
average._d2.EIHI	0.54640	0.64610	0.50356	0.00002
average._d.EIHI	0.54520	0.64530	0.50286	0.00009
average._dd2.EIHI	0.53900	0.64667	0.50136	0.00002
centroid._.EIHI	0.54140	0.64704	0.50044	0.00002
spc.dd2.EIHIMEI	0.50930	0.64672	0.49972	0.00378
single._d.MEI	0.51950	0.65682	0.49956	0.00002
single._.EIHI	0.52620	0.65242	0.49806	0.00002
centroid._d.EIHIMEI	0.52320	0.65289	0.49745	0.00002
centroid._d.EIHI	0.52570	0.65094	0.49741	0.00002
single._.EIHIMEI	0.52510	0.65229	0.49734	0.00002
centroid._d2.EIHI	0.52330	0.65259	0.49714	0.00003
centroid._d2.EIHIMEI	0.52260	0.65292	0.49689	0.00003
spc.d2.EIHI	0.50580	0.62395	0.49628	0.00159
entroid._dd2.EIHI	0.51820	0.65376	0.49626	0.00003
centroid._dd2.EIHIMEI	0.51460	0.65560	0.49574	0.00011
single._d2.MEI	0.51490	0.65563	0.49560	0.00002
single._d2.EIHI	0.51260	0.65606	0.49549	0.00002
single._d.EIHIMEI	0.51260	0.65600	0.49540	0.00003
centroid_dd2.MEI	0.51270	0.65631	0.49539	0.00002
single._d2.EIHIMEI	0.51180	0.65643	0.49537	0.00002
single._d.EIHI	0.51070	0.65538	0.49535	0.00002
single_dd2.MEI	0.51130	0.65707	0.49528	0.00002
single._dd2.EIHIMEI	0.51080	0.65647	0.49528	0.00002
single._dd2.EIHI	0.50910	0.65555	0.49521	0.00002
spc.dd2.EIHI	0.50140	0.64484	0.49502	0.00180
kkmeans.dd2.EIHI	0.50100	0.51935	0.49493	0.00085
svc.d2.EIHIMEI	0.50110	0.51019	0.49493	0.00035
svc.dd2.EIHIMEI	0.50100	0.50670	0.49493	0.00044
kmeans.d2.EIHIMEI	0.50100	0.50892	0.49493	0.00015
kmeans.d2.EIHIMEI	0.50100	0.50892	0.49493	0.00016
kkmeans.d2.EIHIMEI	0.50090	0.50005	0.49493	0.00071
svc.d2.EIHIMEI	0.50090	0.50718	0.49493	0.00034
kkmeans.d.EIHIMEI	0.50080	0.50231	0.49492	0.00073
kkmeans.d.EIHIMEI	0.50080	0.49896	0.49492	0.00090
svc.d2.EIHI	0.50060	0.53944	0.49492	0.00033
svc.dd2.EIHIMEI	0.50060	0.50697	0.49492	0.00043
kkmeans.d2.EIHI	0.50070	0.52285	0.49492	0.00072

kkmeans.d2.EIHI	0.50070	0.50035	0.49492	0.00094
kkmeans.dd2.EIHI	0.50070	0.49915	0.49492	0.00093
kkmeans.d2.EIHIMEI	0.50060	0.50232	0.49492	0.00096
kkmeans.d.EIHI	0.50060	0.50076	0.49492	0.00083
svc.dd2.EIHI	0.50050	0.54407	0.49492	0.00040
svc.dd2.EIHI	0.50050	0.54171	0.49492	0.00040
kmeans.dd2.EIHIMEI	0.50040	0.50524	0.49491	0.00019
kmeans.dd2.EIHIMEI	0.50040	0.50524	0.49491	0.00020
svc.d.EIHI	0.50040	0.56418	0.49491	0.00035
kmeans.d.EIHIMEI	0.50030	0.51134	0.49491	0.00015
kmeans.d.EIHIMEI	0.50030	0.51134	0.49491	0.00017
kmeans.dd2.EIHI	0.50030	0.55599	0.49491	0.00018
kmeans.dd2.EIHI	0.50030	0.55599	0.49491	0.00018
svc.d.EIHIMEI	0.50030	0.51104	0.49491	0.00034
svc.dd2.MEI	0.50030	0.49792	0.49491	0.00027
kkmeans.d.EIHI	0.50030	0.55790	0.49491	0.00080
svc.d2.EIHI	0.50030	0.53288	0.49491	0.00031
svc.d.EIHIMEI	0.50020	0.50947	0.49491	0.00036
kkmeans.dd2.EIHIMEI	0.50020	0.49891	0.49491	0.00083
kkmeans.dd2.MEI	0.50020	0.49904	0.49491	0.00090
kmeans.d2.EIHI	0.50020	0.55098	0.49491	0.00014
kmeans.d2.EIHI	0.50020	0.55095	0.49491	0.00016
spc.d.EIHIMEI	0.50020	0.63702	0.49491	0.00359
svc.d.EIHI	0.50020	0.56550	0.49491	0.00037
svc.dd2.MEI	0.50020	0.49877	0.49491	0.00029
complete.d.EIHIMEI	0.50010	0.55230	0.49490	0.00002
complete.d2.EIHI	0.50010	0.59941	0.49490	0.00002
kkmeans.dd2.EIHIMEI	0.50010	0.49884	0.49490	0.00102
kkmeans.dd2.MEI	0.50010	0.49618	0.49490	0.00078
kmeans.d.EIHI	0.50010	0.59659	0.49490	0.00016
kmeans.d.EIHI	0.50010	0.59659	0.49490	0.00016
kmeans.dd2.MEI	0.50010	0.49728	0.49490	0.00015
kmeans.dd2.MEI	0.50010	0.49728	0.49490	0.00015
ward.D2.d2.EIHI	0.50010	0.56359	0.49490	0.00002
average.d.EIHI	0.50000	0.63726	0.49490	0.00002
average.d.EIHIMEI	0.50000	0.62616	0.49490	0.00009
average.d2.EIHI	0.50000	0.62701	0.49490	0.00001
average.d2.EIHIMEI	0.50000	0.60909	0.49490	0.00002
average.dd2.EIHI	0.50000	0.63818	0.49490	0.00002
average.dd2.EIHIMEI	0.50000	0.62848	0.49490	0.00003
average.dd2.MEI	0.50000	0.59235	0.49490	0.00002
centroid.d.EIHI	0.50000	0.64065	0.49490	0.00001
centroid.d.EIHIMEI	0.50000	0.63399	0.49490	0.00002
centroid.d2.EIHI	0.50000	0.63386	0.49490	0.00001

centroid.d2.EIHIMEI	0.50000	0.62798	0.49490	0.00002
centroid.dd2.EIHI	0.50000	0.64571	0.49490	0.00002
centroid.dd2.EIHIMEI	0.50000	0.64626	0.49490	0.00002
centroid.dd2.MEI	0.50000	0.64132	0.49490	0.00002
complete.d.EIHI	0.50000	0.62297	0.49490	0.00002
complete.d2.EIHIMEI	0.50000	0.55357	0.49490	0.00002
complete.dd2.EIHI	0.50000	0.60146	0.49490	0.00002
complete.dd2.EIHIMEI	0.50000	0.55030	0.49490	0.00003
complete.dd2.MEI	0.50000	0.51999	0.49490	0.00010
single.d.EIHI	0.50000	0.64466	0.49490	0.00010
single.d.EIHIMEI	0.50000	0.64447	0.49490	0.00003
single.d2.EIHI	0.50000	0.64504	0.49490	0.00003
single.d2.EIHIMEI	0.50000	0.64242	0.49490	0.00003
single.dd2.EIHI	0.50000	0.64786	0.49490	0.00002
single.dd2.EIHIMEI	0.50000	0.65002	0.49490	0.00002
single.dd2.MEI	0.50000	0.64926	0.49490	0.00009
spc.d.EIHI	0.50000	0.63092	0.49490	0.00104
ward.D2.d.EIHI	0.50000	0.60162	0.49490	0.00002
ward.D2.d.EIHIMEI	0.50000	0.53188	0.49490	0.00002
ward.D2.d2.EIHIMEI	0.50000	0.53110	0.49490	0.00002
ward.D2.dd2.EIHI	0.50000	0.57469	0.49490	0.00002
ward.D2.dd2.EIHIMEI	0.50000	0.52637	0.49490	0.00002
ward.D2.dd2.MEI	0.50000	0.51069	0.49490	0.00002

Table 7: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-3.

	Purity	Fmeasure	RI	Time
L^2	0.854	0.748	0.751	0.73622
$d\rho$, $\rho=0.001$	0.852	0.746	0.748	1.87491
$d\rho$, $\rho=1$	0.852	0.745	0.747	1.90657
$d\rho$, $\rho=0.02$	0.848	0.747	0.744	1.89522
$d\rho$, $\rho=100$	0.784	0.688	0.687	2.28481
$d\rho$, $\rho=1e+08$	0.776	0.659	0.660	1.94934
dk , $k=2$	0.735	0.641	0.642	0.87594
dk , $k=3$	0.672	0.590	0.587	1.21092

Table 9: Mean results for S 1-5 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k -means for initialization (green) and k -means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans._.EIHI	0.60870	0.52832	0.52319	0.01229
kkmeans.d.EIHI	0.57930	0.51886	0.51132	0.01477
kkmeans.d2.EIHI	0.56030	0.51316	0.50527	0.01483
kkmeans.dd2.MEI	0.55050	0.50814	0.50230	0.01695
kkmeans._dd2.EIHIMEI	0.54640	0.50185	0.50148	0.01937
kkmeans._d.EIHI	0.54880	0.50557	0.50142	0.01686
kkmeans._d2.MEI	0.54420	0.50678	0.50139	0.01741
kkmeans._d2.EIHIMEI	0.54800	0.50541	0.50123	0.01680
kkmeans._d.EIHIMEI	0.54690	0.50597	0.50117	0.01757
kkmeans.dd2.EIHIMEI	0.54730	0.50169	0.50109	0.01669
kkmeans._d.MEI	0.54480	0.50530	0.50075	0.01736
kkmeans.dd2.EIHI	0.53670	0.50719	0.50001	0.01754
kkmeans._d2.EIHI	0.53910	0.50530	0.49981	0.01653
kkmeans_dd2.MEI	0.53410	0.50318	0.49971	0.01825
kkmeans._.EIHIMEI	0.53940	0.50866	0.49939	0.01680
kkmeans.d2.EIHIMEI	0.53720	0.50194	0.49884	0.01600
kkmeans._dd2.EIHI	0.53500	0.50130	0.49874	0.01850
kkmeans.d.EIHIMEI	0.52940	0.50270	0.49756	0.01557
complete.d2.EIHIMEI	0.52670	0.57308	0.49727	0.00041
complete._.EIHIMEI	0.52340	0.55539	0.49715	0.00033
complete.dd2.EIHIMEI	0.52640	0.57806	0.49712	0.00034
ward.D2._.EIHIMEI	0.52210	0.52237	0.49707	0.00028
complete._d2.MEI	0.52270	0.52681	0.49705	0.00027
complete.dd2.MEI	0.52520	0.53291	0.49700	0.00030
complete.d2.EIHI	0.52650	0.62043	0.49697	0.00038
complete._d.MEI	0.52250	0.52959	0.49696	0.00032
ward.D2.d2.EIHI	0.52300	0.58562	0.49675	0.00033
kkmeans.d2.EIHI	0.52220	0.54187	0.49671	0.01570
spc.d2.EIHI	0.52490	0.61160	0.49666	0.04018
kkmeans.dd2.EIHI	0.52250	0.54072	0.49663	0.01541
svc._.EIHI	0.51640	0.62229	0.49662	0.00570
svc.d2.EIHI	0.52200	0.56124	0.49660	0.00471
svc.d2.EIHI-	0.52150	0.56090	0.49660	0.00475
average.d2.EIHIMEI	0.52380	0.63428	0.49657	0.00041
kkmeans._dd2.EIHI	0.52090	0.54313	0.49655	0.01772
svc.dd2.EIHI	0.52100	0.55934	0.49654	0.00617
svc.d.EIHI	0.52040	0.59384	0.49653	0.00563
kkmeans.d.EIHIMEI	0.52120	0.50838	0.49653	0.01314
ward.D2.d.EIHIMEI	0.51990	0.54437	0.49653	0.00035
svc.d.EIHI	0.51980	0.59950	0.49649	0.00656
kkmeans._d2.EIHI	0.52100	0.54476	0.49647	0.01557
complete.d.EIHIMEI	0.52130	0.58228	0.49646	0.00040

kmeans.d2.EIHI	0.52030	0.57541	0.49646	0.00229
kmeans.d2.EIHI	0.52030	0.57541	0.49646	0.00259
svc.d.EIHIMEI	0.51910	0.51950	0.49645	0.00526
spc.d2.EIHIMEI	0.52390	0.60373	0.49644	0.05763
svc._.EIHI	0.51670	0.61651	0.49640	0.00551
ward.D2.d2.EIHIMEI	0.52020	0.53830	0.49640	0.00029
svc._dd2.EIHI	0.51910	0.56202	0.49635	0.00711
kkmeans.d.EIHI	0.52010	0.58854	0.49633	0.01648
svc.dd2.EIHI	0.51950	0.56229	0.49632	0.00739
svc.d.EIHIMEI	0.51850	0.52153	0.49630	0.00660
svc._d.EIHI	0.51700	0.60902	0.49630	0.00613
average.d2.EIHI	0.52100	0.64687	0.49627	0.00026
kmeans.d.EIHIMEI	0.51850	0.53006	0.49625	0.00276
kmeans.d.EIHIMEI	0.51850	0.53006	0.49625	0.00243
centroid.d2.EIHI	0.52100	0.64928	0.49624	0.00029
centroid.d2.EIHIMEI	0.52160	0.64429	0.49624	0.00037
kmeans.dd2.EIHI	0.51810	0.57800	0.49623	0.00277
kmeans.dd2.EIHI	0.51810	0.57800	0.49623	0.00273
svc._d2.EIHI	0.51860	0.57583	0.49622	0.00571
average.dd2.MEI	0.51870	0.62783	0.49621	0.00028
complete.dd2.EIHI	0.52000	0.62853	0.49619	0.00022
svc.d2.EIHIMEI	0.51920	0.51136	0.49618	0.00504
complete._d2.EIHIMEI	0.51750	0.55762	0.49617	0.00036
svc.dd2.EIHIMEI	0.51770	0.50759	0.49615	0.00637
ward.D2.dd2.EIHIMEI	0.51740	0.53420	0.49614	0.00030
svc._dd2.EIHI	0.51780	0.56037	0.49611	0.00723
complete_dd2.MEI	0.51450	0.51853	0.49610	0.00032
spc.d.EIHIMEI	0.51910	0.59483	0.49610	0.05436
svc.d2.EIHIMEI	0.51850	0.51119	0.49609	0.00526
ward.D2.dd2.EIHI	0.51720	0.58616	0.49609	0.00036
svc._d.EIHI	0.51750	0.61091	0.49608	0.00590
ward.D2._d2.EIHI	0.51680	0.60155	0.49605	0.00036
spc.dd2.EIHI	0.51990	0.62361	0.49605	0.04915
spc._.EIHIMEI	0.51720	0.56957	0.49605	0.05192
kkmeans.d2.EIHIMEI	0.51660	0.50248	0.49605	0.01160
average._d.MEI	0.51520	0.58603	0.49602	0.00027
svc._d2.EIHI	0.51690	0.57301	0.49602	0.00617
kkmeans.dd2.EIHIMEI	0.51690	0.50101	0.49602	0.01435
average._d2.MEI	0.51750	0.58334	0.49601	0.00033
spc.d.EIHI	0.51720	0.61973	0.49601	0.02705
ward.D2._dd2.EIHI	0.51680	0.59555	0.49601	0.00040
ward.D2.dd2.MEI	0.51550	0.51419	0.49600	0.00022
ward.D2.d.EIHI	0.51770	0.61143	0.49599	0.00043
kkmeans._d2.MEI	0.51440	0.49811	0.49599	0.01343

kmeans.d2.EIHIMEI	0.51640	0.51307	0.49597	0.00260
kmeans.d2.EIHIMEI	0.51640	0.51307	0.49597	0.00274
kkmeans._.EIHIMEI	0.51550	0.49931	0.49597	0.01083
kkmeans.dd2.MEI	0.51530	0.49560	0.49595	0.01296
svc._d.EIHIMEI	0.51360	0.50474	0.49595	0.00691
spc._d2.EIHI	0.51720	0.61276	0.49594	0.05063
complete._dd2.EIHI	0.51590	0.63539	0.49593	0.00038
kmeans.d.EIHI	0.51680	0.61387	0.49592	0.00244
kmeans.d.EIHI	0.51680	0.61387	0.49592	0.00244
spc.dd2.MEI	0.51650	0.59549	0.49592	0.05587
kkmeans._d.EIHIMEI	0.51310	0.49983	0.49591	0.01446
kkmeans._d.EIHI	0.51630	0.59207	0.49590	0.01895
complete.d.EIHI	0.51700	0.63670	0.49590	0.00034
kmeans._.EIHIMEI	0.51440	0.50344	0.49590	0.00248
kmeans._.EIHIMEI	0.51440	0.50344	0.49590	0.00250
kkmeans._.EIHI	0.51340	0.62128	0.49588	0.01681
centroid.d.EIHIMEI	0.51700	0.64805	0.49587	0.00171
spc._dd2.EIHI	0.51650	0.62058	0.49587	0.04987
complete._d2.EIHI	0.51600	0.63741	0.49587	0.00022
kmeans._dd2.EIHI	0.51540	0.58696	0.49586	0.00290
kmeans._dd2.EIHI	0.51540	0.58696	0.49586	0.00311
spc.dd2.EIHIMEI	0.51600	0.60438	0.49585	0.04876
kkmeans._d2.EIHIMEI	0.51490	0.49970	0.49582	0.01496
svc._d.EIHIMEI	0.51230	0.50359	0.49582	0.00658
ward.D2._d.MEI	0.51220	0.51803	0.49582	0.00029
svc._d.MEI	0.51320	0.49826	0.49582	0.00448
complete._d.EIHIMEI	0.51330	0.57012	0.49582	0.00035
kmeans._d2.EIHI	0.51550	0.59107	0.49580	0.00266
kmeans._d2.EIHI	0.51550	0.59107	0.49580	0.00268
svc.dd2.EIHIMEI	0.51500	0.50703	0.49580	0.00635
kmeans.dd2.EIHIMEI	0.51510	0.50873	0.49579	0.00298
kmeans.dd2.EIHIMEI	0.51510	0.50873	0.49579	0.00325
svc._.EIHIMEI	0.51420	0.50387	0.49579	0.00500
kmeans._d.MEI	0.51310	0.49930	0.49579	0.00243
kmeans._d.MEI	0.51310	0.49930	0.49579	0.00269
average.dd2.EIHIMEI	0.51670	0.64775	0.49579	0.00033
ward.D2._d2.MEI	0.51310	0.51422	0.49579	0.00023
kmeans._d2.MEI	0.51280	0.49794	0.49578	0.00229
kmeans._d2.MEI	0.51280	0.49794	0.49578	0.00261
spc._.EIHI	0.51440	0.62894	0.49578	0.01942
spc._d2.MEI	0.51360	0.58531	0.49577	0.05049
svc._d2.EIHIMEI	0.51290	0.50347	0.49576	0.00688
kmeans._d.EIHI	0.51360	0.62534	0.49576	0.00253
kmeans._d.EIHI	0.51360	0.62534	0.49576	0.00272

svc._d2.EIHIMEI	0.51330	0.50170	0.49575	0.00660
svc._d.MEI	0.51300	0.49871	0.49575	0.00464
svc._.EIHIMEI	0.51370	0.50200	0.49575	0.00513
svc._d2.MEI	0.51220	0.49754	0.49575	0.00530
kkmeans_dd2.MEI	0.51190	0.49602	0.49574	0.01382
ward.D2._d.EIHI	0.51370	0.62497	0.49574	0.00039
kkmeans._dd2.EIHIMEI	0.51370	0.49955	0.49574	0.01676
spc._d.EIHIMEI	0.51290	0.57735	0.49574	0.04958
svc._d2.MEI	0.51250	0.49816	0.49573	0.00419
average.dd2.EIHI	0.51650	0.65209	0.49572	0.00034
single.d2.EIHI	0.51650	0.65488	0.49572	0.00031
centroid.dd2.MEI	0.51550	0.65073	0.49572	0.00026
ward.D2._d2.EIHIMEI	0.51240	0.51979	0.49570	0.00031
kmeans._d.EIHIMEI	0.51160	0.50121	0.49570	0.00289
kmeans._d.EIHIMEI	0.51160	0.50121	0.49570	0.00285
complete._dd2.EIHIMEI	0.51130	0.56272	0.49569	0.00042
single.d2.EIHIMEI	0.51630	0.65449	0.49569	0.00026
kkmeans._d.MEI	0.51270	0.49659	0.49568	0.01237
average.d.EIHIMEI	0.51600	0.64531	0.49568	0.00033
average.d.EIHI	0.51560	0.65024	0.49567	0.00031
centroid._d2.MEI	0.51400	0.64973	0.49566	0.00026
kmeans_dd2.MEI	0.51110	0.49854	0.49566	0.00258
kmeans_dd2.MEI	0.51110	0.49854	0.49566	0.00248
spc._d.EIHI	0.51350	0.62744	0.49566	0.04367
svc.dd2.MEI	0.51310	0.49732	0.49565	0.00424
kmeans.dd2.MEI	0.51300	0.49723	0.49565	0.00246
kmeans.dd2.MEI	0.51300	0.49723	0.49565	0.00244
svc_dd2.MEI	0.51190	0.49793	0.49564	0.00593
centroid.d.EIHI	0.51520	0.65235	0.49561	0.00141
spc._d2.EIHIMEI	0.51330	0.57892	0.49561	0.05329
kmeans._d2.EIHIMEI	0.51240	0.50405	0.49561	0.00309
kmeans._d2.EIHIMEI	0.51240	0.50405	0.49561	0.00308
svc._dd2.EIHIMEI	0.51250	0.50044	0.49560	0.00848
centroid.dd2.EIHI	0.51530	0.65479	0.49560	0.00036
svc.dd2.MEI	0.51270	0.49618	0.49558	0.00439
centroid.dd2.EIHIMEI	0.51530	0.65422	0.49558	0.00040
ward.D2._.EIHI	0.51250	0.63381	0.49558	0.00029
svc_dd2.MEI	0.51150	0.49887	0.49558	0.00451
centroid._d.MEI	0.51320	0.65197	0.49552	0.00156
centroid._.EIHIMEI	0.51270	0.64739	0.49551	0.00146
kmeans._dd2.EIHIMEI	0.51150	0.50386	0.49551	0.00348
kmeans._dd2.EIHIMEI	0.51150	0.50386	0.49551	0.00350
average._.EIHIMEI	0.51180	0.64185	0.49550	0.00032
average_dd2.MEI	0.51030	0.58360	0.49550	0.00029

average._d2.EIHIMEI	0.51210	0.64970	0.49549	0.00033
svc._dd2.EIHIMEI	0.51100	0.50428	0.49548	0.00877
complete._.EIHI	0.51150	0.64606	0.49546	0.00036
spc._d.MEI	0.50990	0.57514	0.49544	0.05247
complete._d.EIHI	0.51130	0.64343	0.49544	0.00030
single._.EIHIMEI	0.51310	0.65606	0.49543	0.00043
single.d.EIHIMEI	0.51330	0.65587	0.49542	0.00036
single._.EIHI	0.51280	0.65573	0.49542	0.00022
kmeans._.EIHI	0.51080	0.63391	0.49542	0.00229
kmeans._.EIHI	0.51080	0.63391	0.49542	0.00241
average._.EIHI	0.51180	0.65222	0.49541	0.00038
average._d2.EIHI	0.51210	0.65364	0.49540	0.00032
average._d.EIHIMEI	0.51170	0.65085	0.49540	0.00038
centroid._.EIHI	0.51160	0.65359	0.49539	0.00032
centroid._d2.EIHI	0.51230	0.65585	0.49538	0.00042
spc._dd2.EIHIMEI	0.50840	0.58702	0.49537	0.05077
single.d.EIHI	0.51220	0.65615	0.49535	0.00031
average._d.EIHI	0.51140	0.65440	0.49535	0.00027
average._dd2.EIHIMEI	0.51080	0.65264	0.49534	0.00038
single.dd2.EIHIMEI	0.51210	0.65674	0.49534	0.00034
centroid._d2.EIHIMEI	0.51110	0.65582	0.49533	0.00039
average._dd2.EIHI	0.51100	0.65457	0.49532	0.00032
ward.D2._d.EIHIMEI	0.50800	0.51804	0.49531	0.00031
single.dd2.MEI	0.51130	0.65672	0.49531	0.00030
centroid._d.EIHI	0.51120	0.65584	0.49530	0.00030
single._d2.EIHI	0.51130	0.65662	0.49530	0.00039
ward.D2._dd2.EIHIMEI	0.50750	0.52422	0.49529	0.00039
centroid._dd2.EIHI	0.51120	0.65657	0.49529	0.00036
centroid_dd2.MEI	0.51040	0.65575	0.49529	0.00027
single.dd2.EIHI	0.51130	0.65716	0.49528	0.00039
centroid._d.EIHIMEI	0.51080	0.65592	0.49527	0.00045
centroid._dd2.EIHIMEI	0.51060	0.65673	0.49526	0.00040
single._d.EIHI	0.51060	0.65673	0.49525	0.00034
single._d2.MEI	0.51010	0.65604	0.49525	0.00034
single._dd2.EIHI	0.51040	0.65708	0.49524	0.00052
single._d2.EIHIMEI	0.51000	0.65689	0.49522	0.00027
single._d.MEI	0.50960	0.65672	0.49522	0.00156
single._d.EIHIMEI	0.50970	0.65693	0.49521	0.00145
single._dd2.EIHIMEI	0.50910	0.65656	0.49519	0.00051
ward.D2_dd2.MEI	0.50460	0.51584	0.49510	0.00025
single_dd2.MEI	0.50500	0.65498	0.49505	0.00034
spc_dd2.MEI	0.50360	0.58456	0.49505	0.04957

Table 8: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-3.

	Purity	Fmeasure	RI	Time
kmeans ++	0.500	0.504	0.495	0.33267
random	0.500	0.502	0.495	0.27125
hclust	0.500	0.498	0.495	0.23871
kmeans	0.500	0.498	0.495	0.18432

Table 10: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-5.

	Purity	Fmeasure	RI	Time
$d\rho, \rho=1e+08$	0.514	0.541	0.496	1.28681
$dk, k= 3,$	0.521	0.509	0.496	0.27404
$d\rho, \rho=1$	0.516	0.508	0.496	1.42959
$d\rho, \rho=0.001$	0.517	0.507	0.496	1.38306
$dk, k = 2$	0.519	0.506	0.496	0.54214
$d\rho, \rho=0.02$	0.518	0.506	0.496	1.46735
L^2	0.517	0.505	0.496	0.49132
$d\rho, \rho=100$	0.510	0.549	0.495	1.21699

Table 11: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-5.

	Purity	Fmeasure	RI	Time
kmeans ++	0.523	0.511	0.497	0.38773
random	0.523	0.505	0.497	0.35540
hclust	0.522	0.505	0.497	0.27504
kmeans	0.520	0.501	0.497	0.25441

Table 12: Mean results for S 1-6 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans._.EIHI	0.64430	0.55576	0.55089	0.01063
kkmeans.d.EIHI	0.62780	0.54717	0.53791	0.01349
kkmeans_dd2.MEI	0.61340	0.53460	0.53274	0.01667
kkmeans._d.EIHI	0.61350	0.53379	0.52812	0.01580
kkmeans._dd2.EIHIMEI	0.61360	0.52772	0.52804	0.01559
kkmeans.d2.EIHI	0.60560	0.53386	0.52712	0.01437
kkmeans._d.MEI	0.60880	0.52914	0.52550	0.01538
kkmeans._d2.MEI	0.60660	0.52769	0.52241	0.01568
kkmeans.dd2.MEI	0.60310	0.52577	0.52067	0.01688
kkmeans._d.EIHIMEI	0.59690	0.52043	0.51860	0.01556
kkmeans._d2.EIHIMEI	0.59430	0.52199	0.51806	0.01517
kkmeans.dd2.EIHI	0.58600	0.52534	0.51803	0.01649
kkmeans.dd2.EIHIMEI	0.58770	0.51521	0.51400	0.01479
kkmeans._d2.EIHI	0.57630	0.51769	0.51341	0.01594
kkmeans._dd2.EIHI	0.56380	0.51472	0.51150	0.01662
kkmeans._.EIHIMEI	0.57200	0.51967	0.50800	0.01447
kkmeans.d2.EIHIMEI	0.56770	0.51152	0.50795	0.01394
kkmeans.d.EIHIMEI	0.55840	0.50867	0.50440	0.01532
complete._d2.MEI	0.54900	0.53619	0.50300	0.00030
complete._d.MEI	0.54760	0.53479	0.50228	0.00034
complete.dd2.MEI	0.54690	0.53661	0.50189	0.00137
complete._.EIHIMEI	0.54390	0.56321	0.50156	0.00029
ward.D2._d.MEI	0.54180	0.52209	0.50117	0.00031
ward.D2._.EIHIMEI	0.53980	0.52377	0.50095	0.00036
kkmeans._dd2.EIHI	0.54510	0.58550	0.50092	0.01824
1vc.dd2.EIHI	0.54700	0.59996	0.50069	0.00552
complete_dd2.MEI	0.53960	0.52561	0.50047	0.00032
average._d2.MEI	0.54270	0.63018	0.50046	0.00029
complete.dd2.EIHIMEI	0.54310	0.60997	0.50038	0.00028
kkmeans.dd2.EIHI	0.54290	0.58062	0.50026	0.01867
average._d.MEI	0.53950	0.63889	0.50019	0.00029
complete._d2.EIHIMEI	0.53960	0.57848	0.50013	0.00135
complete.d2.EIHIMEI	0.54470	0.59798	0.50012	0.00034
ward.D2.d2.EIHIMEI	0.54200	0.55870	0.50006	0.00022
complete._d.EIHIMEI	0.53820	0.58375	0.49988	0.00047
svc._dd2.EIHI	0.54090	0.60484	0.49982	0.00785
complete._dd2.EIHIMEI	0.53810	0.57588	0.49971	0.00035

kkmeans.d2.EIHI	0.54020	0.58195	0.49970	0.01603
svc._.EIHI	0.53070	0.63281	0.49969	0.00541
kkmeans._d2.EIHI	0.53930	0.58471	0.49963	0.01720
kmeans.dd2.EIHI	0.54270	0.62275	0.49960	0.00233
kmeans.dd2.EIHI	0.54270	0.62275	0.49960	0.00264
svc._dd2.EIHI	0.54150	0.60575	0.49959	0.00699
ward.D2.dd2.EIHI	0.54250	0.62771	0.49948	0.00025
ward.D2._d2.MEI	0.53190	0.52139	0.49941	0.00036
svc.dd2.EIHI	0.54030	0.59966	0.49938	0.00550
svc._.EIHIMEI	0.53660	0.50984	0.49934	0.00470
kmeans.d2.EIHI	0.54100	0.61803	0.49931	0.00216
kmeans.d2.EIHI	0.54090	0.61788	0.49930	0.00225
kmeans._.EIHIMEI	0.53620	0.50671	0.49929	0.00225
kmeans._.EIHIMEI	0.53620	0.50671	0.49929	0.00234
svc.d2.EIHIMEI	0.53750	0.52981	0.49928	0.00560
ward.D2.d.EIHIMEI	0.53780	0.56090	0.49926	0.00031
kkmeans._d.EIHI	0.53880	0.62158	0.49924	0.01772
svc._dd2.EIHIMEI	0.53590	0.50919	0.49922	0.00768
kkmeans_dd2.MEI	0.53490	0.50116	0.49916	0.01328
svc._.EIHIMEI	0.53590	0.50760	0.49912	0.00450
kmeans.d2.EIHIMEI	0.53770	0.53440	0.49912	0.00234
kmeans.d2.EIHIMEI	0.53770	0.53440	0.49912	0.00237
ward.D2._d2.EIHIMEI	0.53350	0.53000	0.49912	0.00030
kmeans.dd2.EIHIMEI	0.53650	0.53368	0.49912	0.00266
kmeans.dd2.EIHIMEI	0.53650	0.53368	0.49912	0.00293
kmeans._d2.MEI	0.53400	0.50363	0.49911	0.00233
kmeans._d2.MEI	0.53400	0.50363	0.49911	0.00239
svc.d2.EIHIMEI	0.53690	0.52769	0.49908	0.00487
svc.d2.EIHI	0.53780	0.59848	0.49906	0.00407
spc._d.MEI	0.53760	0.59361	0.49904	0.04825
ward.D2._d.EIHIMEI	0.53310	0.53257	0.49904	0.00026
kmeans._dd2.EIHI	0.53950	0.62661	0.49903	0.00283
kmeans._dd2.EIHI	0.53950	0.62661	0.49903	0.00271
kkmeans._d2.MEI	0.53510	0.50020	0.49902	0.01260
svc.dd2.MEI	0.53200	0.50500	0.49902	0.00381
kkmeans.dd2.EIHIMEI	0.53590	0.51267	0.49900	0.01519
svc._d2.MEI	0.53390	0.50267	0.49900	0.00383
kmeans._d.EIHIMEI	0.53490	0.51007	0.49900	0.00271
kmeans._d.EIHIMEI	0.53490	0.51007	0.49900	0.00272
kkmeans._d.EIHIMEI	0.53390	0.50509	0.49898	0.01409
svc._d2.EIHI	0.53850	0.60636	0.49897	0.00558
kmeans.d.EIHIMEI	0.53750	0.54985	0.49897	0.00261
kmeans.d.EIHIMEI	0.53750	0.54985	0.49897	0.00242
ward.D2.d2.EIHI	0.53970	0.62125	0.49892	0.00018

spc.dd2.MEI	0.53860	0.63428	0.49891	0.04852
svc._d.MEI	0.53400	0.50156	0.49889	0.00400
spc.d2.EIHIMEI	0.53930	0.62302	0.49889	0.05003
svc._d.EIHIMEI	0.53540	0.51448	0.49888	0.00630
kmeans._d.MEI	0.53410	0.50201	0.49887	0.00210
kmeans._d.MEI	0.53410	0.50201	0.49887	0.00234
svc._d2.MEI	0.53310	0.50213	0.49887	0.00382
svc._.EIHI	0.52900	0.63475	0.49885	0.00547
svc.d.EIHIMEI	0.53690	0.54653	0.49883	0.00440
svc._d.EIHIMEI	0.53430	0.51451	0.49882	0.00648
ward.D2._d2.EIHI	0.53860	0.62950	0.49881	0.00021
spc._d2.MEI	0.53710	0.61375	0.49878	0.04960
svc._dd2.EIHIMEI	0.53310	0.50826	0.49876	0.00866
svc.d2.EIHI	0.53570	0.59696	0.49874	0.00478
kkmeans._.EIHIMEI	0.53530	0.50161	0.49873	0.01006
ward.D2._dd2.EIHIMEI	0.53200	0.53145	0.49871	0.00045
kmeans._d2.EIHI	0.53770	0.62372	0.49868	0.00241
kmeans._d2.EIHI	0.53770	0.62372	0.49868	0.00263
ward.D2.dd2.MEI	0.53160	0.52196	0.49867	0.00036
svc_dd2.MEI	0.53280	0.50099	0.49864	0.00433
svc.dd2.MEI	0.53010	0.50380	0.49863	0.00364
svc._d2.EIHI	0.53580	0.60410	0.49862	0.00554
svc.dd2.EIHIMEI	0.53490	0.52434	0.49861	0.00591
kkmeans._d.MEI	0.53360	0.49912	0.49857	0.01213
kkmeans.d.EIHIMEI	0.53490	0.52077	0.49854	0.01204
kkmeans._d2.EIHIMEI	0.53270	0.50389	0.49853	0.01293
svc.dd2.EIHIMEI	0.53300	0.52889	0.49853	0.00578
kkmeans.d2.EIHIMEI	0.53330	0.51564	0.49852	0.01126
svc.d.EIHIMEI	0.53400	0.53912	0.49852	0.00463
kkmeans.d.EIHI	0.53390	0.62088	0.49851	0.01826
ward.D2._dd2.EIHI	0.53710	0.63179	0.49846	0.00035
svc_dd2.MEI	0.53230	0.50111	0.49846	0.00408
svc._d.MEI	0.53190	0.50171	0.49845	0.00451
kmeans._d2.EIHIMEI	0.53060	0.50825	0.49835	0.00294
kmeans._d2.EIHIMEI	0.53060	0.50825	0.49835	0.00287
kkmeans._dd2.EIHIMEI	0.52920	0.50357	0.49834	0.01415
spc_dd2.MEI	0.53400	0.61526	0.49832	0.04418
kmeans_dd2.MEI	0.53130	0.50149	0.49832	0.00228
kmeans_dd2.MEI	0.53130	0.50149	0.49832	0.00231
kmeans._dd2.EIHIMEI	0.53000	0.50752	0.49830	0.00337
kmeans._dd2.EIHIMEI	0.53000	0.50752	0.49830	0.00326
average_dd2.MEI	0.53080	0.64652	0.49826	0.00031
kkmeans.dd2.MEI	0.53220	0.50003	0.49826	0.01190
complete.d2.EIHI	0.53590	0.63993	0.49821	0.00035

svc._d2.EIHIMEI	0.53030	0.50967	0.49821	0.00687
complete.d.EIHIMEI	0.53180	0.59579	0.49818	0.00033
spc.d2.EIHI	0.53650	0.63398	0.49816	0.03595
svc.d.EIHI	0.53060	0.62734	0.49814	0.00485
ward.D2.dd2.EIHIMEI	0.53040	0.55227	0.49814	0.00032
kkmeans._.EIHI	0.52760	0.63140	0.49810	0.01501
average.dd2.MEI	0.53340	0.64746	0.49810	0.00027
spc._.EIHIMEI	0.53200	0.59081	0.49803	0.05157
spc._d.EIHIMEI	0.53220	0.60854	0.49803	0.04640
spc.d.EIHIMEI	0.53370	0.60958	0.49801	0.04476
svc.d.EIHI	0.53230	0.62451	0.49799	0.00523
ward.D2_dd2.MEI	0.52690	0.52000	0.49798	0.00043
svc._d2.EIHIMEI	0.52840	0.51275	0.49793	0.00562
spc.dd2.EIHIMEI	0.53340	0.63004	0.49785	0.04696
svc._d.EIHI	0.53180	0.63368	0.49772	0.00570
average.d2.EIHIMEI	0.53070	0.64751	0.49771	0.00031
spc._d2.EIHI	0.53330	0.63193	0.49769	0.04190
spc._d2.EIHIMEI	0.53210	0.61520	0.49768	0.04450
spc._dd2.EIHIMEI	0.53230	0.61991	0.49765	0.04429
spc.dd2.EIHI	0.53350	0.64091	0.49765	0.04300
complete._dd2.EIHI	0.52820	0.64575	0.49763	0.00037
complete.dd2.EIHI	0.53020	0.64077	0.49761	0.00026
spc._dd2.EIHI	0.53170	0.64006	0.49754	0.04540
kmeans.dd2.MEI	0.52580	0.50115	0.49753	0.00216
kmeans.dd2.MEI	0.52580	0.50115	0.49753	0.00278
kmeans._d.EIHI	0.53090	0.64219	0.49745	0.00244
kmeans._d.EIHI	0.53090	0.64219	0.49745	0.00239
kmeans.d.EIHI	0.53010	0.64098	0.49735	0.00218
kmeans.d.EIHI	0.53010	0.64098	0.49735	0.00219
svc._d.EIHI	0.52880	0.63092	0.49732	0.00519
complete._d2.EIHI	0.52740	0.64612	0.49717	0.00127
average.d2.EIHI	0.52780	0.64995	0.49716	0.00030
centroid.d2.EIHIMEI	0.52700	0.65050	0.49715	0.00137
ward.D2._d.EIHI	0.52830	0.64356	0.49704	0.00135
spc.d.EIHI	0.52780	0.62757	0.49694	0.02547
ward.D2.d.EIHI	0.52760	0.64163	0.49693	0.00022
spc._d.EIHI	0.52690	0.64232	0.49674	0.04231
average._.EIHIMEI	0.52300	0.64683	0.49669	0.00031
centroid.d2.EIHI	0.52450	0.65166	0.49665	0.00024
average.d.EIHIMEI	0.52320	0.65199	0.49652	0.00031
complete.d.EIHI	0.52360	0.64975	0.49652	0.00033
complete._d.EIHI	0.52260	0.65068	0.49651	0.00027
centroid._d2.MEI	0.52080	0.65295	0.49644	0.00025
spc._.EIHI	0.52220	0.62647	0.49641	0.01610

centroid...EIHIMEI	0.52040	0.65114	0.49633	0.00027
centroid.d.EIHIMEI	0.52120	0.65275	0.49629	0.00035
single.d2.EIHIMEI	0.51900	0.65422	0.49615	0.00025
average.dd2.EIHIMEI	0.52020	0.65310	0.49614	0.00028
kmeans...EIHI	0.52020	0.64666	0.49611	0.00217
kmeans...EIHI	0.52020	0.64666	0.49611	0.00228
centroid._d.MEI	0.51790	0.65413	0.49610	0.00029
centroid.dd2.MEI	0.51910	0.65402	0.49607	0.00024
ward.D2...EIHI	0.51980	0.64716	0.49606	0.00023
average.d.EIHI	0.51950	0.65361	0.49601	0.00026
centroid.d.EIHI	0.51920	0.65391	0.49597	0.00027
average.dd2.EIHI	0.51860	0.65369	0.49593	0.00026
average._d.EIHIMEI	0.51720	0.65424	0.49591	0.00035
average._d2.EIHIMEI	0.51810	0.65371	0.49590	0.00133
complete...EIHI	0.51830	0.65322	0.49586	0.00035
average._dd2.EIHIMEI	0.51760	0.65410	0.49586	0.00044
average._d2.EIHI	0.51710	0.65441	0.49584	0.00026
single.d.EIHIMEI	0.51700	0.65485	0.49574	0.00032
single.d2.EIHI	0.51670	0.65498	0.49573	0.00021
single...EIHIMEI	0.51600	0.65520	0.49571	0.00032
centroid.dd2.EIHI	0.51600	0.65491	0.49570	0.00027
average._dd2.EIHI	0.51550	0.65532	0.49568	0.00032
centroid.dd2.EIHIMEI	0.51560	0.65518	0.49568	0.00038
single.d.EIHI	0.51630	0.65504	0.49568	0.00029
average...EIHI	0.51610	0.65462	0.49563	0.00027
single.dd2.EIHI	0.51480	0.65568	0.49560	0.00033
centroid...EIHI	0.51560	0.65520	0.49558	0.00027
average._d.EIHI	0.51540	0.65519	0.49558	0.00036
centroid._d2.EIHIMEI	0.51460	0.65565	0.49558	0.00139
centroid._d.EIHI	0.51450	0.65566	0.49551	0.00029
centroid._d2.EIHI	0.51420	0.65588	0.49551	0.00027
single...EIHI	0.51440	0.65588	0.49550	0.00033
single.dd2.EIHIMEI	0.51380	0.65615	0.49550	0.00033
single._d.EIHI	0.51350	0.65626	0.49545	0.00028
single._d2.EIHI	0.51320	0.65638	0.49543	0.00131
centroid_dd2.MEI	0.51280	0.65648	0.49543	0.00026
single._dd2.EIHI	0.51300	0.65647	0.49542	0.00037
single._d.EIHIMEI	0.51300	0.65646	0.49541	0.00035
centroid._d.EIHIMEI	0.51310	0.65613	0.49541	0.00037
single.dd2.MEI	0.51270	0.65659	0.49539	0.00029
single._d2.MEI	0.51250	0.65659	0.49539	0.00139
centroid._dd2.EIHI	0.51240	0.65662	0.49538	0.00031
single._d2.EIHIMEI	0.51220	0.65679	0.49535	0.00031
single._dd2.EIHIMEI	0.51180	0.65696	0.49532	0.00035

single._d.MEI	0.51170	0.65691	0.49532	0.00025
centroid._dd2.EIHIMEI	0.51150	0.65708	0.49530	0.00044
single_dd2.MEI	0.51090	0.65732	0.49525	0.00030

Table 13: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-6.

	Purity	Fmeasure	RI	Time
dk , $k=3$	0.625	0.594	0.532	0.23964
L^2	0.615	0.581	0.528	0.20434
$d\rho$, $\rho=0.02$	0.614	0.588	0.526	0.77332
$d\rho$, $\rho=1$	0.612	0.585	0.526	0.77657
$d\rho$, $\rho=0.001$	0.610	0.581	0.525	0.79802
dk , $k=2$	0.603	0.568	0.523	0.26130
$d\rho$, $\rho=1e+08$	0.598	0.585	0.522	0.69797
$d\rho$, $\rho=100$	0.597	0.598	0.520	0.69434

Table 14: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-6.

	Purity	Fmeasure	RI	Time
kmeans ++	0.617	0.577	0.529	0.42781
hclust	0.616	0.575	0.528	0.350
random	0.617	0.573	0.528	0.38683
kmeans	0.601	0.554	0.521	0.34597

Table 15: Mean results for S 1-7 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k -means for initialization (green) and k -means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans.dd2.EIHIMEI-	0.54490	0.50305	0.50110	0.01748
kkmeans.dd2.EIHI	0.54240	0.50526	0.50098	0.01776
kkmeans._d.MEI	0.54210	0.49903	0.50080	0.01253
complete._d.MEI	0.54310	0.51707	0.50062	0.00036
kkmeans._d.EIHI	0.54190	0.50578	0.50053	0.01720
svc._d.EIHIMEI	0.54130	0.50065	0.50050	0.00732
kkmeans._d.EIHIMEI	0.54180	0.50231	0.50045	0.01702
kkmeans._dd2.EIHIMEI	0.53990	0.50090	0.50037	0.01592
svc._d2.MEI	0.53920	0.49966	0.50034	0.00417
ward.D2.dd2.MEI	0.54040	0.51392	0.50024	0.00041

kmeans._d2.MEI	0.53940	0.49951	0.50023	0.00245
kmeans._d2.MEI	0.53940	0.49951	0.50023	0.00243
kkmeans._dd2.EIHI	0.54170	0.50386	0.50019	0.01717
kmeans._d2.EIHIMEI	0.53940	0.50080	0.50019	0.00298
kmeans._d2.EIHIMEI	0.53940	0.50080	0.50019	0.00284
kmeans._d.EIHIMEI	0.53970	0.50113	0.50017	0.00297
kmeans._d.EIHIMEI	0.53970	0.50113	0.50017	0.00320
svc._d.EIHIMEI	0.53990	0.50231	0.50010	0.00752
svc._d2.MEI	0.53840	0.49933	0.50007	0.00527
kkmeans._.EIHI	0.54060	0.50793	0.50004	0.01279
kkmeans.dd2.MEI	0.54240	0.50329	0.50003	0.01686
svc._d.MEI	0.53930	0.49973	0.50003	0.00465
svc._d2.EIHIMEI	0.53870	0.50134	0.49998	0.00637
svc._.EIHIMEI	0.53960	0.50111	0.49996	0.00503
svc._.EIHIMEI	0.53920	0.50132	0.49996	0.00508
svc._dd2.EIHIMEI	0.53800	0.50043	0.49995	0.00790
kkmeans._d2.MEI	0.54110	0.50121	0.49994	0.01728
kkmeans_dd2.MEI	0.53840	0.49776	0.49993	0.01487
svc._d.MEI	0.53840	0.49925	0.49991	0.00488
kkmeans.d.EIHI	0.53920	0.50717	0.49991	0.01539
kkmeans.d.EIHIMEI	0.54020	0.50283	0.49990	0.01113
kmeans._d.MEI	0.53850	0.49907	0.49988	0.00228
kmeans._d.MEI	0.53850	0.49907	0.49988	0.00241
kkmeans._.EIHIMEI	0.53900	0.49864	0.49987	0.01067
kkmeans.dd2.MEI	0.54090	0.49885	0.49984	0.01241
kkmeans.d.EIHIMEI	0.53780	0.50319	0.49980	0.01608
kmeans._.EIHIMEI	0.53830	0.50088	0.49976	0.00247
kmeans._.EIHIMEI	0.53830	0.50088	0.49976	0.00268
kkmeans.d2.EIHI	0.53820	0.50507	0.49975	0.01726
svc._dd2.EIHIMEI	0.53700	0.50083	0.49975	0.00785
kkmeans._d2.EIHI	0.53930	0.50759	0.49970	0.01699
svc.dd2.MEI	0.53900	0.49895	0.49968	0.00411
ward.D2._d2.EIHIMEI	0.53860	0.51458	0.49967	0.00030
kkmeans_dd2.MEI	0.53780	0.50101	0.49967	0.01848
kmeans._dd2.EIHIMEI	0.53790	0.49941	0.49966	0.00338
kmeans._dd2.EIHIMEI	0.53790	0.49941	0.49966	0.00330
svc._d2.EIHIMEI	0.53680	0.50156	0.49965	0.00649
kkmeans.d2.EIHIMEI	0.54010	0.50347	0.49965	0.01502
kkmeans._d.EIHIMEI	0.53730	0.49775	0.49963	0.01442
kkmeans._d2.EIHIMEI	0.53560	0.49935	0.49961	0.01598
kkmeans._d.MEI	0.53810	0.50495	0.49956	0.01745
kkmeans._d2.MEI	0.53630	0.49830	0.49954	0.01413
svc.d.EIHIMEI	0.53810	0.50728	0.49953	0.00493
svc.d.EIHIMEI	0.53800	0.50817	0.49952	0.00489

svc.dd2.MEI	0.53880	0.49887	0.49949	0.00413
complete.dd2.MEI	0.53890	0.52582	0.49947	0.00036
kmeans.dd2.MEI	0.53910	0.49863	0.49947	0.00250
kmeans.dd2.MEI	0.53910	0.49863	0.49947	0.00240
svc_dd2.MEI	0.53640	0.49797	0.49938	0.00453
kmeans.d.EIHIMEI	0.53770	0.50735	0.49935	0.00259
kmeans.d.EIHIMEI	0.53770	0.50735	0.49935	0.00249
svc_dd2.MEI	0.53600	0.49851	0.49934	0.00448
kmeans_dd2.MEI	0.53560	0.49823	0.49933	0.00251
kmeans_dd2.MEI	0.53560	0.49823	0.49933	0.00256
complete_dd2.MEI	0.53750	0.52040	0.49931	0.00041
ward.D2.d.EIHIMEI	0.53900	0.52946	0.49929	0.00035
kkmeans._d2.EIHIMEI	0.53640	0.50280	0.49928	0.01714
svc.d.EIHI	0.53640	0.57913	0.49928	0.00544
svc.dd2.EIHI	0.53670	0.54227	0.49927	0.00578
svc.d.EIHI	0.53730	0.57857	0.49925	0.00578
ward.D2._.EIHIMEI	0.53760	0.51784	0.49922	0.00045
complete._d2.MEI	0.53700	0.52012	0.49917	0.00040
ward.D2._d.MEI	0.53620	0.51519	0.49910	0.00034
complete._d.EIHIMEI	0.53400	0.55425	0.49906	0.00035
ward.D2.dd2.EIHIMEI	0.53690	0.52276	0.49906	0.00036
complete._.EIHIMEI	0.53690	0.53789	0.49905	0.00033
svc.dd2.EIHI	0.53490	0.53723	0.49904	0.00585
ward.D2._d2.MEI	0.53530	0.51335	0.49904	0.00037
svc.d2.EIHIMEI	0.53490	0.50338	0.49903	0.00490
ward.D2_dd2.MEI	0.53530	0.51494	0.49901	0.00040
kkmeans.dd2.EIHIMEI	0.53590	0.50036	0.49896	0.01442
kkmeans._.EIHIMEI	0.53400	0.51160	0.49888	0.01519
svc._dd2.EIHI	0.53360	0.54284	0.49887	0.00696
kmeans.dd2.EIHIMEI	0.53700	0.50181	0.49885	0.00307
kmeans.dd2.EIHIMEI	0.53700	0.50181	0.49885	0.00329
kkmeans._dd2.EIHIMEI	0.53440	0.49964	0.49883	0.01832
svc.dd2.EIHIMEI	0.53590	0.50080	0.49883	0.00769
kmeans.d2.EIHIMEI	0.53450	0.50409	0.49882	0.00258
kmeans.d2.EIHIMEI	0.53450	0.50409	0.49882	0.00244
kkmeans._d.EIHI	0.53250	0.57021	0.49882	0.01893
kkmeans.d2.EIHI	0.53460	0.52514	0.49882	0.01307
kkmeans._d2.EIHI	0.53470	0.52655	0.49880	0.01523
kkmeans.d2.EIHIMEI	0.53470	0.49925	0.49878	0.01132
complete._dd2.EIHIMEI	0.53210	0.55556	0.49877	0.00038
complete.dd2.EIHIMEI	0.53360	0.55207	0.49874	0.00042
kkmeans.dd2.EIHI	0.53300	0.51576	0.49872	0.01371
svc.d2.EIHIMEI	0.53360	0.50356	0.49870	0.00502
ward.D2._dd2.EIHIMEI	0.53270	0.51856	0.49865	0.00034

svc.dd2.EIHIMEI	0.53470	0.50159	0.49865	0.00636
average._d2.MEI	0.53120	0.56382	0.49862	0.00142
kkmeans._dd2.EIHI	0.53230	0.51993	0.49857	0.01708
complete.d2.EIHIMEI	0.53350	0.56426	0.49854	0.00038
svc.d2.EIHI	0.53330	0.53553	0.49853	0.00453
svc.d2.EIHI	0.53400	0.53511	0.49853	0.00500
svc._d2.EIHI	0.53310	0.54548	0.49851	0.00593
kkmeans.d.EIHI	0.53390	0.57280	0.49849	0.01558
kmeans.d.EIHI	0.53400	0.60140	0.49848	0.00222
kmeans.d.EIHI	0.53400	0.60140	0.49848	0.00221
svc._dd2.EIHI	0.53110	0.54515	0.49843	0.00668
ward.D2.d2.EIHIMEI	0.53340	0.52404	0.49843	0.00029
svc._d.EIHI	0.53280	0.58507	0.49841	0.00595
svc._d.EIHI	0.53160	0.58562	0.49835	0.00683
ward.D2._d.EIHIMEI	0.53150	0.52129	0.49830	0.00042
kmeans.dd2.EIHI	0.53210	0.54723	0.49827	0.00273
kmeans.dd2.EIHI	0.53190	0.54714	0.49826	0.00289
kmeans.d2.EIHI	0.53280	0.54497	0.49824	0.00226
kmeans.d2.EIHI	0.53280	0.54497	0.49824	0.00256
spc._d.MEI	0.52930	0.58528	0.49817	0.05601
svc._d2.EIHI	0.53020	0.54704	0.49815	0.00571
spc._dd2.EIHIMEI	0.52770	0.58030	0.49812	0.04448
complete._d2.EIHIMEI	0.53080	0.53884	0.49810	0.00040
kmeans._dd2.EIHI	0.53070	0.55541	0.49803	0.00308
kmeans._dd2.EIHI	0.53050	0.55532	0.49799	0.00332
ward.D2.d.EIHI	0.52930	0.60528	0.49792	0.00032
ward.D2.dd2.EIHI	0.53120	0.56584	0.49791	0.00037
complete.d.EIHIMEI	0.52990	0.57470	0.49789	0.00039
spc.d.EIHIMEI	0.52790	0.60229	0.49785	0.05388
ward.D2.d2.EIHI	0.52930	0.56249	0.49776	0.00038
kmeans._d2.EIHI	0.52890	0.55955	0.49765	0.00267
kmeans._d2.EIHI	0.52890	0.55955	0.49765	0.00276
kmeans._d.EIHI	0.52810	0.61261	0.49757	0.00265
kmeans._d.EIHI	0.52810	0.61261	0.49757	0.00275
spc._.EIHIMEI	0.52620	0.57897	0.49754	0.05463
spc._d.EIHIMEI	0.52560	0.58268	0.49754	0.05491
spc._d2.EIHIMEI	0.52360	0.57627	0.49751	0.04726
average_dd2.MEI	0.52510	0.57703	0.49744	0.00027
ward.D2._d2.EIHI	0.52750	0.57180	0.49738	0.00026
average.dd2.MEI	0.52310	0.61321	0.49737	0.00033
spc.dd2.EIHIMEI	0.52690	0.59517	0.49730	0.04801
spc.dd2.MEI	0.52450	0.58814	0.49722	0.04930
ward.D2._dd2.EIHI	0.52600	0.58049	0.49709	0.00040
complete.d2.EIHI	0.52490	0.60078	0.49706	0.00027

spc.d2.EIHIMEI	0.52340	0.58903	0.49705	0.05545
spc_dd2.MEI	0.52410	0.59069	0.49700	0.04610
spc.dd2.EIHI	0.52530	0.60234	0.49693	0.04896
spc.d.EIHI	0.52350	0.61966	0.49693	0.02738
spc._dd2.EIHI	0.52340	0.60876	0.49692	0.04607
svc._.EIHI	0.52360	0.61193	0.49689	0.00685
spc._d2.MEI	0.52270	0.58706	0.49689	0.04750
svc._.EIHI	0.52340	0.61450	0.49688	0.00521
average._d.MEI	0.52290	0.57910	0.49687	0.00040
complete.dd2.EIHI	0.52220	0.61443	0.49676	0.00033
ward.D2._d.EIHI	0.52230	0.61436	0.49665	0.00035
kkmeans._.EIHI	0.52290	0.60874	0.49661	0.01609
complete._d2.EIHI	0.51980	0.61840	0.49650	0.00024
spc.d2.EIHI	0.52040	0.60375	0.49649	0.04185
complete.d.EIHI	0.52110	0.63139	0.49646	0.00033
average.d2.EIHIMEI	0.51930	0.62563	0.49640	0.00027
spc._d2.EIHI	0.51790	0.60325	0.49624	0.04683
spc._d.EIHI	0.51820	0.62603	0.49616	0.04587
ward.D2._.EIHI	0.51840	0.62739	0.49612	0.00031
average.d.EIHIMEI	0.51840	0.64067	0.49610	0.00026
complete._dd2.EIHI	0.51680	0.62798	0.49607	0.00156
kmeans._.EIHI	0.51790	0.63035	0.49602	0.00245
kmeans._.EIHI	0.51790	0.63035	0.49602	0.00257
average._.EIHIMEI	0.51670	0.62967	0.49601	0.00028
spc._.EIHI	0.51720	0.62710	0.49599	0.01906
centroid.d2.EIHIMEI	0.51730	0.63774	0.49596	0.00032
complete._d.EIHI	0.51640	0.64252	0.49593	0.00027
average.d2.EIHI	0.51580	0.63864	0.49586	0.00030
centroid.d.EIHIMEI	0.51600	0.64535	0.49584	0.00048
average.dd2.EIHIMEI	0.51490	0.64502	0.49582	0.00028
centroid._.EIHIMEI	0.51460	0.64398	0.49572	0.00036
average.d.EIHI	0.51470	0.64819	0.49572	0.00036
complete._.EIHI	0.51450	0.64434	0.49569	0.00028
centroid.d2.EIHI	0.51380	0.64403	0.49568	0.00021
average._d.EIHIMEI	0.51430	0.64706	0.49566	0.00034
centroid.d.EIHI	0.51370	0.64971	0.49563	0.00028
average._d2.EIHIMEI	0.51290	0.63749	0.49555	0.00036
average.dd2.EIHI	0.51290	0.65093	0.49552	0.00032
average._.EIHI	0.51320	0.65051	0.49551	0.00029
average._dd2.EIHIMEI	0.51260	0.64909	0.49550	0.00047
centroid._d2.MEI	0.51300	0.65150	0.49548	0.00037
average._d.EIHI	0.51240	0.65327	0.49546	0.00035
average._dd2.EIHI	0.51230	0.65311	0.49544	0.00040
centroid._d.EIHIMEI	0.51130	0.65540	0.49540	0.00038

centroid._d.MEI	0.51100	0.65055	0.49539	0.00040
centroid._.EIHI	0.51200	0.65246	0.49539	0.00022
single.d.EIHIMEI	0.51220	0.65449	0.49538	0.00153
single.d.EIHI	0.51250	0.65529	0.49538	0.00026
average._d2.EIHI	0.51170	0.65120	0.49537	0.00034
single.d2.EIHIMEI	0.51130	0.65260	0.49537	0.00029
centroid._d2.EIHI	0.51170	0.65489	0.49534	0.00030
centroid.dd2.EIHIMEI	0.51130	0.65480	0.49533	0.00024
centroid.dd2.MEI	0.51050	0.65387	0.49532	0.00040
centroid.dd2.EIHI	0.51130	0.65487	0.49532	0.00035
centroid._d.EIHI	0.51090	0.65507	0.49532	0.00038
single._.EIHIMEI	0.51090	0.65478	0.49530	0.00028
single.d2.EIHI	0.51070	0.65329	0.49530	0.00026
centroid._d2.EIHIMEI	0.51070	0.65477	0.49530	0.00028
single._.EIHI	0.51090	0.65551	0.49530	0.00033
single.dd2.EIHIMEI	0.51110	0.65652	0.49528	0.00040
centroid._dd2.EIHI	0.51060	0.65637	0.49527	0.00036
single._d.EIHI	0.51050	0.65632	0.49526	0.00034
single.dd2.EIHI	0.51050	0.65658	0.49525	0.00161
single._d.MEI	0.51020	0.65617	0.49525	0.00034
single._d.EIHIMEI	0.51050	0.65640	0.49524	0.00026
single._dd2.EIHI	0.51030	0.65677	0.49524	0.00039
single.dd2.MEI	0.51000	0.65598	0.49524	0.00035
single._d2.MEI	0.51000	0.65644	0.49523	0.00040
centroid_dd2.MEI	0.51010	0.65685	0.49522	0.00044
single._d2.EIHI	0.50980	0.65625	0.49522	0.00028
single_dd2.MEI	0.50990	0.65729	0.49521	0.00039
centroid._dd2.EIHIMEI	0.50970	0.65711	0.49520	0.00038
single._d2.EIHIMEI	0.50950	0.65702	0.49520	0.00036
single._dd2.EIHIMEI	0.50960	0.65715	0.49520	0.00046

Table 16: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-7.

	Purity	Fmeasure	RI	Time
$d\rho$, $\rho=100$	0.541	0.504	0.500	0.81032
L^2	0.540	0.501	0.500	0.19487
dk , $k=2$	0.541	0.502	0.500	0.24496
$d\rho$, $\rho=0.02$	0.538	0.501	0.500	0.77945
dk , $k=3$	0.537	0.504	0.499	0.28691
$d\rho$, $\rho=1$	0.537	0.501	0.499	0.75827
$d\rho$, $\rho=0.001$	0.537	0.500	0.499	0.76092
$d\rho$, $\rho=1e+08$	0.537	0.508	0.499	0.76512

Table 17: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-7.

	Purity	Fmeasure	RI	Time
hclust	0.541	0.501	0.501	0.24218
random	0.539	0.502	0.500	0.32934
kmeans	0.540	0.500	0.500	0.22335
kmeans ++	0.536	0.501	0.499	0.43285

Table 18: Mean results for S 1-8 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
kkmeans_dd2.MEI	0.94960	0.90349	0.90429	0.00998
kmeans_dd2.MEI	0.94900	0.90243	0.90319	0.00237
kmeans_dd2.MEI	0.94900	0.90243	0.90319	0.00247
svc_dd2.MEI	0.94890	0.90226	0.90303	0.00368
svc_dd2.MEI	0.94870	0.90188	0.90264	0.00406
kkmeans._dd2.EIHIMEI	0.93690	0.88067	0.88161	0.01118
svc._dd2.EIHIMEI	0.93660	0.88058	0.88139	0.00631
kmeans._dd2.EIHIMEI	0.93640	0.88028	0.88108	0.00304
kmeans._dd2.EIHIMEI	0.93640	0.88028	0.88108	0.00324
kmeans._d.MEI	0.93630	0.87969	0.88054	0.00243
kmeans._d.MEI	0.93630	0.87969	0.88054	0.00253
kkmeans._d.MEI	0.93620	0.87947	0.88040	0.00968
svc._dd2.EIHIMEI	0.93600	0.87942	0.88025	0.00612
svc._d.MEI	0.93610	0.87938	0.88022	0.00386
svc._d.MEI	0.93550	0.87836	0.87918	0.00379
svc._d2.MEI	0.93520	0.87796	0.87884	0.00366
kmeans._d2.MEI	0.93480	0.87747	0.87827	0.00233
kmeans._d2.MEI	0.93480	0.87747	0.87827	0.00233
svc._d2.MEI	0.93440	0.87679	0.87757	0.00384
kkmeans._d2.MEI	0.93190	0.87581	0.87682	0.00970
average_dd2.MEI	0.93210	0.87598	0.87529	0.00038
ward.D2_dd2.MEI	0.93190	0.87462	0.87425	0.00025
kkmeans_dd2.MEI	0.91610	0.86487	0.86480	0.01552
kkmeans._d2.EIHIMEI	0.92410	0.85871	0.85964	0.01044
kkmeans._d.EIHIMEI	0.92390	0.85826	0.85922	0.01059
svc._d.EIHIMEI	0.92270	0.85642	0.85722	0.00528
svc._d.EIHIMEI	0.92240	0.85601	0.85676	0.00560
kmeans._d2.EIHIMEI	0.92200	0.85553	0.85630	0.00292

kmeans._d2.EIHIMEI	0.92200	0.85553	0.85630	0.00300
svc._d2.EIHIMEI	0.92200	0.85551	0.85629	0.00532
svc._d2.EIHIMEI	0.92200	0.85541	0.85625	0.00552
kmeans._d.EIHIMEI	0.92200	0.85527	0.85605	0.00257
kmeans._d.EIHIMEI	0.92200	0.85527	0.85605	0.00311
ward.D2._dd2.EIHIMEI	0.91920	0.85518	0.85393	0.00046
average._d.MEI	0.91650	0.85024	0.84885	0.00039
kkmeans._dd2.EIHIMEI	0.91050	0.84659	0.84663	0.01851
centroid._d.MEI	0.90180	0.85005	0.84087	0.00027
ward.D2._d.MEI	0.91070	0.84174	0.83980	0.00019
complete_dd2.MEI	0.90840	0.84209	0.83862	0.00033
ward.D2._d.EIHIMEI	0.90800	0.83835	0.83598	0.00041
kkmeans._.EIHIMEI	0.90940	0.83390	0.83488	0.01029
centroid_dd2.MEI	0.88190	0.85481	0.83354	0.00029
average._d2.MEI	0.90550	0.83217	0.83028	0.00029
kmeans._.EIHIMEI	0.90650	0.82927	0.82999	0.00269
kmeans._.EIHIMEI	0.90650	0.82927	0.82999	0.00237
svc._.EIHIMEI	0.90610	0.82853	0.82931	0.00441
svc._.EIHIMEI	0.90550	0.82766	0.82837	0.00424
ward.D2._d2.MEI	0.90150	0.82686	0.82432	0.00024
centroid._d2.MEI	0.88780	0.82317	0.81435	0.00024
complete._d.MEI	0.89140	0.81528	0.81068	0.00026
spc_dd2.MEI	0.85900	0.83675	0.80807	0.05090
complete._d2.MEI	0.88720	0.80833	0.80338	0.00034
ward.D2._d2.EIHIMEI	0.88670	0.80490	0.80136	0.00042
kkmeans._d.MEI	0.87540	0.80121	0.79959	0.01737
ward.D2._.EIHIMEI	0.88440	0.80003	0.79668	0.00027
complete._dd2.EIHIMEI	0.86630	0.80477	0.78937	0.00045
spc._d.MEI	0.84560	0.81472	0.78546	0.05385
kkmeans._d2.MEI	0.86090	0.78718	0.78542	0.01639
kkmeans._d.EIHIMEI	0.86440	0.78320	0.78279	0.01669
spc._dd2.EIHIMEI	0.83080	0.81944	0.78032	0.04888
complete._d2.EIHIMEI	0.85930	0.79334	0.77789	0.00034
complete._d.EIHIMEI	0.84830	0.79092	0.76977	0.00032
kkmeans._d2.EIHIMEI	0.84950	0.76452	0.76427	0.01686
complete._.EIHIMEI	0.83930	0.76818	0.74910	0.00032
spc._d2.MEI	0.80520	0.78994	0.74619	0.05317
spc._d.EIHIMEI	0.77810	0.78095	0.72438	0.05092
kmeans.dd2.MEI	0.83400	0.72123	0.72247	0.00222
kmeans.dd2.MEI	0.83400	0.72123	0.72247	0.00232
svc.dd2.MEI	0.83270	0.72064	0.72179	0.00419
svc.dd2.MEI	0.83340	0.72054	0.72145	0.00429
kkmeans.dd2.MEI	0.82700	0.71562	0.71739	0.01180
average._dd2.EIHIMEI	0.73800	0.77089	0.69647	0.00039

ward.D2.dd2.MEI	0.80320	0.69330	0.68437	0.00146
svc.dd2.EIHIMEI	0.80250	0.68101	0.68224	0.00627
svc.dd2.EIHIMEI	0.80210	0.68095	0.68180	0.00755
kmeans.dd2.EIHIMEI	0.80080	0.67922	0.68024	0.00285
kmeans.dd2.EIHIMEI	0.80080	0.67922	0.68024	0.00316
spc._d2.EIHIMEI	0.73580	0.74979	0.67937	0.04633
kkmeans.dd2.EIHIMEI	0.79720	0.67574	0.67729	0.01366
kkmeans._.EIHIMEI	0.75220	0.65827	0.65327	0.01564
complete.dd2.MEI	0.77300	0.67195	0.65247	0.00033
average.dd2.MEI	0.75690	0.68510	0.65122	0.00028
ward.D2.dd2.EIHIMEI	0.76890	0.66122	0.64739	0.00043
kkmeans.dd2.EIHIMEI	0.75270	0.64016	0.64073	0.01730
average._d.EIHIMEI	0.68110	0.73027	0.63666	0.00153
average._d2.EIHIMEI	0.68190	0.72453	0.63317	0.00025
kkmeans.d2.EIHIMEI	0.75740	0.62987	0.63140	0.00981
spc._.EIHIMEI	0.69060	0.71689	0.63133	0.05180
svc.d2.EIHIMEI	0.75660	0.63052	0.63052	0.00469
kmeans.d2.EIHIMEI	0.75550	0.63051	0.62957	0.00246
kmeans.d2.EIHIMEI	0.75550	0.63051	0.62957	0.00241
svc.d2.EIHIMEI	0.75520	0.62999	0.62925	0.00458
complete.dd2.EIHIMEI	0.73740	0.64304	0.61826	0.00054
ward.D2.d2.EIHIMEI	0.73450	0.62995	0.61299	0.00030
kkmeans.d.EIHIMEI	0.73570	0.60828	0.60962	0.01112
kkmeans.dd2.MEI	0.70910	0.61103	0.60872	0.01817
kmeans.d.EIHIMEI	0.73440	0.60881	0.60853	0.00234
kmeans.d.EIHIMEI	0.73440	0.60881	0.60853	0.00242
svc.d.EIHIMEI	0.73460	0.60867	0.60851	0.00508
svc.d.EIHIMEI	0.73380	0.60774	0.60770	0.00494
spc.dd2.EIHIMEI	0.67680	0.66505	0.59238	0.05046
ward.D2.d.EIHIMEI	0.71030	0.60615	0.58854	0.00025
spc.dd2.MEI	0.66590	0.66784	0.58571	0.04981
complete.d2.EIHIMEI	0.70220	0.62352	0.58570	0.00029
complete.d.EIHIMEI	0.69130	0.60829	0.57672	0.00026
average._.EIHIMEI	0.61870	0.68849	0.57321	0.00041
kkmeans.d.EIHIMEI	0.66520	0.56714	0.56304	0.01627
kkmeans.d2.EIHIMEI	0.65000	0.56018	0.55773	0.01607
spc.d2.EIHIMEI	0.63550	0.63514	0.55358	0.05693
kkmeans._dd2.EIHI	0.65230	0.62141	0.55095	0.01858
kkmeans._d2.EIHI	0.64780	0.61669	0.54873	0.01833
kkmeans._d.EIHI	0.65210	0.62176	0.54779	0.01633
centroid._.EIHIMEI	0.58690	0.67610	0.54737	0.00029
kkmeans._.EIHI	0.64960	0.62407	0.54702	0.01553
svc._.EIHI	0.64740	0.62561	0.54695	0.00556
svc._dd2.EIHI	0.64290	0.62455	0.54376	0.00737

svc._.EIHI	0.64350	0.62615	0.54303	0.00545
svc._d2.EIHI	0.64090	0.62329	0.54170	0.00613
svc._d.EIHI	0.63900	0.62625	0.54150	0.00677
svc._dd2.EIHI	0.63880	0.62035	0.54080	0.00710
svc._d2.EIHI	0.63640	0.61959	0.53881	0.00619
spc.d.EIHIMEI	0.61010	0.62772	0.53820	0.05070
svc._d.EIHI	0.63390	0.62579	0.53694	0.00616
average.dd2.EIHIMEI	0.59160	0.65135	0.53539	0.00023
kmeans._d2.EIHI	0.62080	0.62680	0.52848	0.00282
kmeans._d2.EIHI	0.62080	0.62680	0.52848	0.00277
kmeans._dd2.EIHI	0.62070	0.62680	0.52837	0.00269
kmeans._dd2.EIHI	0.62070	0.62680	0.52837	0.00300
kmeans._.EIHI	0.61750	0.62694	0.52657	0.00372
kmeans._.EIHI	0.61740	0.62707	0.52655	0.00231
ward.D2._d.EIHI	0.61410	0.62728	0.52650	0.00031
kmeans._d.EIHI	0.61730	0.62685	0.52640	0.00247
kmeans._d.EIHI	0.61730	0.62685	0.52640	0.00262
ward.D2._.EIHI	0.61440	0.62768	0.52631	0.00027
ward.D2._dd2.EIHI	0.61040	0.62897	0.52497	0.00031
ward.D2._d2.EIHI	0.60890	0.62963	0.52406	0.00038
centroid._d.EIHIMEI	0.55030	0.66624	0.52109	0.00036
average.d2.EIHIMEI	0.57690	0.64103	0.51926	0.00045
kkmeans._dd2.EIHI	0.58260	0.51967	0.51897	0.01804
kkmeans.d2.EIHI	0.59170	0.53473	0.51488	0.01202
kkmeans._d2.EIHI	0.57550	0.51687	0.51401	0.01791
spc._d2.EIHI	0.57600	0.63873	0.51387	0.05258
kkmeans._.EIHI	0.57910	0.52100	0.51386	0.01512
spc._d.EIHI	0.57670	0.63719	0.51366	0.05082
svc.dd2.EIHI	0.58490	0.55162	0.51355	0.00558
svc.d2.EIHI	0.58610	0.54800	0.51354	0.00555
svc.d2.EIHI	0.58780	0.54854	0.51350	0.00504
spc._dd2.EIHI	0.57780	0.63489	0.51329	0.04843
spc._.EIHI	0.57670	0.63675	0.51279	0.02929
kmeans.dd2.EIHI	0.58330	0.55528	0.51224	0.00268
kmeans.dd2.EIHI	0.58330	0.55528	0.51224	0.00287
kkmeans.dd2.EIHI	0.58250	0.53089	0.51223	0.01439
kmeans.d2.EIHI	0.58280	0.55329	0.51212	0.00232
kmeans.d2.EIHI	0.58280	0.55329	0.51212	0.00246
svc.dd2.EIHI	0.57810	0.55259	0.51118	0.00588
ward.D2.d2.EIHI	0.58110	0.56450	0.51117	0.00027
complete._dd2.EIHI	0.57210	0.63857	0.51054	0.00037
complete._d.EIHI	0.57060	0.63925	0.51018	0.00037
complete._d2.EIHI	0.57130	0.63854	0.50970	0.00038
ward.D2.dd2.EIHI	0.57360	0.57257	0.50940	0.00036

complete...EIHI	0.56830	0.63881	0.50868	0.00024
average.d.EIHIMEI	0.54540	0.64430	0.50773	0.00040
kkmeans._d.EIHI	0.56110	0.50830	0.50718	0.01620
kkmeans.d2.EIHI	0.56300	0.51188	0.50615	0.01507
centroid.d2.EIHIMEI	0.54790	0.64595	0.50597	0.00033
kkmeans.dd2.EIHI	0.55950	0.50748	0.50514	0.01667
centroid.dd2.MEI	0.53560	0.65386	0.50475	0.00026
centroid._d2.EIHIMEI	0.52990	0.65828	0.50461	0.00036
spc.d2.EIHI	0.55250	0.60340	0.50379	0.03142
kkmeans.d.EIHI	0.55130	0.50711	0.50306	0.01194
spc.dd2.EIHI	0.54420	0.61039	0.50159	0.04870
complete.d2.EIHI	0.54410	0.61905	0.50135	0.00029
centroid.d.EIHIMEI	0.53060	0.64882	0.50132	0.00032
average._dd2.EIHI	0.53720	0.64913	0.50091	0.00155
average._d2.EIHI	0.53720	0.64853	0.50056	0.00039
complete.dd2.EIHI	0.54050	0.62645	0.50052	0.00156
average...EIHI	0.53800	0.64829	0.50022	0.00022
centroid._dd2.EIHIMEI	0.52290	0.65642	0.49983	0.00040
single_dd2.MEI	0.51620	0.65963	0.49958	0.00038
average._d.EIHI	0.53520	0.64880	0.49941	0.00039
single._d.MEI	0.51640	0.65901	0.49925	0.00027
centroid...EIHI	0.53260	0.64952	0.49878	0.00031
svc.d.EIHI	0.53110	0.57105	0.49812	0.00474
centroid._d.EIHI	0.52850	0.65120	0.49810	0.00035
kkmeans.d.EIHI	0.52940	0.55959	0.49784	0.01326
average.d2.EIHI	0.52480	0.64691	0.49768	0.00140
svc.d.EIHI	0.52770	0.56997	0.49729	0.00460
centroid._d2.EIHI	0.52360	0.65268	0.49708	0.00036
spc.d.EIHI	0.52210	0.60452	0.49675	0.01975
kmeans.d.EIHI	0.52280	0.59343	0.49670	0.00224
kmeans.d.EIHI	0.52280	0.59343	0.49670	0.00236
centroid.d2.EIHI	0.52160	0.64775	0.49669	0.00026
ward.D2.d.EIHI	0.52320	0.59791	0.49663	0.00024
centroid._dd2.EIHI	0.52070	0.65365	0.49656	0.00040
average.dd2.EIHI	0.52060	0.64924	0.49645	0.00037
complete.d.EIHI	0.51990	0.62997	0.49622	0.00144
centroid.dd2.EIHIMEI	0.51590	0.65498	0.49598	0.00042
single...EIHIMEI	0.51610	0.65531	0.49580	0.00039
single.d2.EIHIMEI	0.51430	0.65586	0.49577	0.00029
centroid.dd2.EIHI	0.51570	0.65343	0.49576	0.00033
single...EIHI	0.51580	0.65538	0.49570	0.00144
average.d.EIHI	0.51370	0.64861	0.49564	0.00024
single.d.EIHIMEI	0.51430	0.65534	0.49559	0.00037
centroid.d.EIHI	0.51330	0.65060	0.49557	0.00023

single._d.EIHIMEI	0.51360	0.65630	0.49556	0.00043
single._d.EIHI	0.51410	0.65605	0.49555	0.00037
single.d2.EIHI	0.51330	0.65475	0.49553	0.00047
single._d2.EIHI	0.51350	0.65627	0.49547	0.00034
single._dd2.EIHI	0.51300	0.65648	0.49543	0.00030
single._d2.EIHIMEI	0.51290	0.65641	0.49540	0.00032
single._d2.MEI	0.51240	0.65663	0.49539	0.00034
single.dd2.MEI	0.51170	0.65672	0.49532	0.00032
single._dd2.EIHIMEI	0.51170	0.65699	0.49531	0.00043
single.d.EIHI	0.51080	0.65445	0.49530	0.00025
single.dd2.EIHIMEI	0.51140	0.65703	0.49529	0.00039
single.dd2.EIHI	0.51070	0.65697	0.49527	0.00042

Table 19: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-8.

	Purity	Fmeasure	RI	Time
$d\rho, \rho=1$	0.996	0.993	0.993	0.50043
$d\rho, \rho=0.001$	0.996	0.993	0.993	0.49838
L^2	0.996	0.993	0.993	0.13231
$d\rho, \rho=0.02$	0.996	0.993	0.993	0.49600
$dk, k=2$	0.897	0.875	0.874	0.23107
$dk, k=3$	0.869	0.841	0.839	0.26480
$d\rho, \rho=100$	0.839	0.800	0.799	0.83068
$d\rho, \rho=1e+08$	0.549	0.512	0.502	0.84295

Table 20: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means (Zamboni et al. (2019)) procedure with four different initialization to simulated data from S 1-8.

	Purity	Fmeasure	RI	Time
kmeans ++	0.934	0.877	0.877	0.30939
kmeans	0.933	0.875	0.876	0.14881
random	0.930	0.870	0.871	0.25801
hclust	0.930	0.869	0.870	0.15243

Table 21: Mean results for S 1-9 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
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kmeans_dd2.MEI	0.94790	0.90026	0.90107	0.00231
kmeans_dd2.MEI	0.94790	0.90026	0.90107	0.00229
svc_dd2.MEI	0.94790	0.90023	0.90105	0.00415
svc_dd2.MEI	0.94770	0.89989	0.90071	0.00386
kkmeans_dd2.MEI	0.94720	0.89903	0.89987	0.01032
kkmeans._d.MEI	0.93670	0.88007	0.88106	0.00940
svc._dd2.EIHIMEI	0.93600	0.87920	0.88014	0.00651
svc._d.MEI	0.93570	0.87853	0.87941	0.00386
kmeans._d.MEI	0.93570	0.87838	0.87929	0.00247
kmeans._d.MEI	0.93570	0.87838	0.87929	0.00243
kmeans._dd2.EIHIMEI	0.93530	0.87818	0.87905	0.00301
kmeans._dd2.EIHIMEI	0.93530	0.87818	0.87905	0.00305
kkmeans._dd2.EIHIMEI	0.93520	0.87775	0.87877	0.01097
kkmeans._d2.MEI	0.93520	0.87752	0.87854	0.00965
svc._d.MEI	0.93520	0.87753	0.87845	0.00390
svc._dd2.EIHIMEI	0.93250	0.87781	0.87750	0.00640
kmeans._d2.MEI	0.93420	0.87605	0.87693	0.00213
kmeans._d2.MEI	0.93420	0.87605	0.87693	0.00228
svc._d2.MEI	0.93370	0.87512	0.87603	0.00356
svc._d2.MEI	0.93340	0.87460	0.87553	0.00369
kkmeans_dd2.MEI	0.92970	0.87515	0.87500	0.01465
svc._d.EIHIMEI	0.92440	0.85911	0.86015	0.00560
kmeans._d.EIHIMEI	0.92370	0.85787	0.85892	0.00276
kmeans._d.EIHIMEI	0.92370	0.85787	0.85892	0.00297
kkmeans._d.EIHIMEI	0.92360	0.85766	0.85883	0.01066
kmeans._d2.EIHIMEI	0.92350	0.85767	0.85869	0.00277
kmeans._d2.EIHIMEI	0.92350	0.85767	0.85869	0.00271
kkmeans._d2.EIHIMEI	0.92340	0.85743	0.85856	0.01043
svc._d.EIHIMEI	0.92330	0.85716	0.85826	0.00563
svc._d2.EIHIMEI	0.92240	0.85574	0.85677	0.00690
svc._d2.EIHIMEI	0.92230	0.85573	0.85672	0.00560
kkmeans._dd2.EIHIMEI	0.91780	0.85349	0.85399	0.01791
average_dd2.MEI	0.91820	0.85491	0.85287	0.00029
average._d.MEI	0.91620	0.85000	0.84843	0.00027
ward.D2_dd2.MEI	0.91430	0.84788	0.84586	0.00035
ward.D2._d.MEI	0.90830	0.83810	0.83586	0.00024
complete_dd2.MEI	0.90670	0.83702	0.83414	0.00067
ward.D2._d.EIHIMEI	0.90730	0.83608	0.83405	0.00042
kkmeans._.EIHIMEI	0.90840	0.83237	0.83359	0.00928
svc._.EIHIMEI	0.90800	0.83178	0.83298	0.00475
kmeans._.EIHIMEI	0.90740	0.83090	0.83199	0.00260
kmeans._.EIHIMEI	0.90740	0.83090	0.83199	0.00263
svc._.EIHIMEI	0.90680	0.83026	0.83122	0.00443

centroid._d.MEI	0.90050	0.83708	0.83070	0.00026
centroid_dd2.MEI	0.88670	0.84450	0.82862	0.00034
ward.D2._dd2.EIHIMEI	0.90290	0.83097	0.82808	0.00038
spc_dd2.MEI	0.88000	0.84114	0.82262	0.05158
centroid._d2.MEI	0.88900	0.82271	0.81452	0.00024
complete._d.MEI	0.89260	0.81979	0.81424	0.00028
average._d2.MEI	0.89440	0.81678	0.81333	0.00036
ward.D2._d2.MEI	0.89230	0.81442	0.81054	0.00028
ward.D2._d2.EIHIMEI	0.89040	0.81074	0.80728	0.00030
complete._dd2.EIHIMEI	0.88340	0.81279	0.80450	0.00043
complete._d2.MEI	0.88640	0.80788	0.80250	0.00035
spc._d.MEI	0.85870	0.82154	0.79732	0.05004
ward.D2._.EIHIMEI	0.87560	0.79086	0.78528	0.00043
kkmeans._d.MEI	0.85820	0.78589	0.78462	0.01644
kkmeans._d.EIHIMEI	0.86660	0.78215	0.78222	0.01726
complete._d2.EIHIMEI	0.85350	0.78185	0.76626	0.00035
kkmeans._d2.MEI	0.83930	0.76604	0.76485	0.01652
spc._dd2.EIHIMEI	0.81430	0.80965	0.76414	0.05046
complete._.EIHIMEI	0.84500	0.77409	0.75668	0.00040
complete._d.EIHIMEI	0.83410	0.78140	0.75529	0.00029
kkmeans._d2.EIHIMEI	0.83770	0.75348	0.75226	0.01889
spc._d2.MEI	0.80090	0.78234	0.73817	0.05150
svc.dd2.MEI	0.82140	0.70446	0.70625	0.00409
kmeans.dd2.MEI	0.82130	0.70442	0.70613	0.00243
kmeans.dd2.MEI	0.82130	0.70442	0.70613	0.00250
svc.dd2.MEI	0.82110	0.70393	0.70577	0.00412
kkmeans.dd2.MEI	0.81470	0.69916	0.70125	0.01221
spc._d.EIHIMEI	0.73510	0.75851	0.68471	0.04872
spc._d2.EIHIMEI	0.73690	0.75638	0.68425	0.04703
ward.D2.dd2.MEI	0.79220	0.68215	0.67156	0.00031
svc.dd2.EIHIMEI	0.79000	0.66654	0.66813	0.00640
kmeans.dd2.EIHIMEI	0.78860	0.66494	0.66664	0.00288
kmeans.dd2.EIHIMEI	0.78860	0.66494	0.66664	0.00318
kkmeans.dd2.EIHIMEI	0.78750	0.66292	0.66485	0.01341
kkmeans._.EIHIMEI	0.76830	0.66910	0.66438	0.01775
svc.dd2.EIHIMEI	0.78380	0.66256	0.66251	0.00743
average._dd2.EIHIMEI	0.70490	0.74238	0.65856	0.00040
complete.dd2.MEI	0.77440	0.66908	0.65333	0.00047
average._d2.EIHIMEI	0.69350	0.72203	0.63776	0.00025
ward.D2.dd2.EIHIMEI	0.75710	0.65062	0.63635	0.00035
average.dd2.MEI	0.74340	0.67040	0.63472	0.00030
spc._.EIHIMEI	0.68400	0.71719	0.62742	0.05299
average._d.EIHIMEI	0.67210	0.72139	0.62550	0.00028
kkmeans.dd2.MEI	0.72160	0.62211	0.62052	0.01820

svc.d2.EIHIMEI	0.74270	0.61561	0.61668	0.00485
kkmeans.d2.EIHIMEI	0.74210	0.61413	0.61592	0.01041
svc.d2.EIHIMEI	0.74180	0.61452	0.61541	0.00449
kmeans.d2.EIHIMEI	0.74030	0.61349	0.61413	0.00256
kmeans.d2.EIHIMEI	0.74030	0.61349	0.61413	0.00259
kkmeans.dd2.EIHIMEI	0.71930	0.61378	0.61345	0.01705
kkmeans.d.EIHIMEI	0.73470	0.60725	0.60939	0.00990
kmeans.d.EIHIMEI	0.73310	0.60667	0.60803	0.00250
kmeans.d.EIHIMEI	0.73310	0.60667	0.60803	0.00365
svc.d.EIHIMEI	0.73320	0.60696	0.60797	0.00486
svc.d.EIHIMEI	0.73240	0.60655	0.60748	0.00476
complete.dd2.EIHIMEI	0.72130	0.63456	0.60643	0.00031
average...EIHIMEI	0.65240	0.70389	0.60221	0.00031
ward.D2.d2.EIHIMEI	0.72370	0.61771	0.60197	0.00037
ward.D2.d.EIHIMEI	0.72350	0.61306	0.60066	0.00051
spc.dd2.EIHIMEI	0.65180	0.66382	0.58094	0.04778
spc.dd2.MEI	0.66210	0.66196	0.58022	0.04799
complete.d2.EIHIMEI	0.69340	0.61885	0.57972	0.00036
kkmeans._d2.EIHI	0.68030	0.63038	0.57492	0.01778
complete.d.EIHIMEI	0.68720	0.60787	0.57426	0.00033
svc...EIHI	0.67960	0.63684	0.57097	0.00729
kkmeans._d.EIHI	0.67650	0.63045	0.56990	0.01610
kkmeans._dd2.EIHI	0.67750	0.62667	0.56975	0.01811
svc._dd2.EIHI	0.67410	0.62971	0.56673	0.00703
kkmeans.d2.EIHIMEI	0.66610	0.56833	0.56639	0.01599
kkmeans...EIHI	0.67600	0.62957	0.56599	0.01515
kkmeans.d.EIHIMEI	0.66220	0.56589	0.56119	0.01660
svc._dd2.EIHI	0.66640	0.62920	0.56027	0.00705
svc...EIHI	0.66870	0.62974	0.55957	0.00560
svc._d.EIHI	0.66790	0.62828	0.55872	0.00630
svc._d2.EIHI	0.66000	0.62770	0.55530	0.00605
svc._d.EIHI	0.66140	0.62758	0.55410	0.00629
svc._d2.EIHI	0.65700	0.62589	0.55380	0.00577
average.dd2.EIHIMEI	0.60910	0.64620	0.54601	0.00030
spc.d.EIHIMEI	0.60020	0.63226	0.53648	0.05292
spc.d2.EIHIMEI	0.60380	0.63159	0.53406	0.05507
kmeans._d2.EIHI	0.63080	0.62257	0.53255	0.00261
kmeans._d2.EIHI	0.63080	0.62257	0.53255	0.00268
kmeans...EIHI	0.63060	0.62317	0.53240	0.00236
kmeans...EIHI	0.63060	0.62317	0.53240	0.00253
kmeans._d.EIHI	0.63040	0.62322	0.53226	0.00252
kmeans._d.EIHI	0.63040	0.62322	0.53226	0.00250
kmeans._dd2.EIHI	0.62820	0.62221	0.53150	0.00265
kmeans._dd2.EIHI	0.62820	0.62221	0.53150	0.00301

ward.D2...EIHI	0.62300	0.62676	0.53141	0.00033
ward.D2._d.EIHI	0.62090	0.62787	0.53042	0.00149
ward.D2._d2.EIHI	0.61070	0.62697	0.52436	0.00025
ward.D2._dd2.EIHI	0.60880	0.62573	0.52339	0.00144
kkmeans._d2.EIHI	0.59500	0.52464	0.52306	0.01811
kkmeans...EIHI	0.59700	0.52921	0.52172	0.01555
kkmeans._dd2.EIHI	0.58800	0.52235	0.52148	0.01715
average.d2.EIHIMEI	0.58760	0.62756	0.52108	0.00145
spc...EIHI	0.58090	0.64321	0.52007	0.03322
kkmeans._d.EIHI	0.58590	0.52214	0.51949	0.01697
centroid._d.EIHIMEI	0.54680	0.66298	0.51663	0.00034
centroid...EIHIMEI	0.55040	0.65911	0.51509	0.00037
spc._d2.EIHI	0.57130	0.64042	0.51255	0.05191
complete._d.EIHI	0.57580	0.63721	0.51193	0.00036
spc._dd2.EIHI	0.56750	0.63970	0.51099	0.04980
complete...EIHI	0.57280	0.63821	0.50972	0.00029
spc._d.EIHI	0.56390	0.63984	0.50948	0.04827
average.d.EIHIMEI	0.55300	0.63018	0.50821	0.00039
complete._dd2.EIHI	0.56690	0.63806	0.50767	0.00032
complete._d2.EIHI	0.56400	0.63940	0.50660	0.00036
kkmeans.d2.EIHI	0.57010	0.52875	0.50634	0.01243
svc.dd2.EIHI	0.56700	0.54531	0.50624	0.00612
svc.d2.EIHI	0.56880	0.54155	0.50619	0.00520
svc.d2.EIHI	0.56810	0.54517	0.50613	0.00483
kkmeans.dd2.EIHI	0.56700	0.52878	0.50553	0.01483
svc.dd2.EIHI	0.56610	0.54035	0.50549	0.00581
kmeans.d2.EIHI	0.56410	0.54746	0.50475	0.00237
kmeans.d2.EIHI	0.56400	0.54731	0.50472	0.00257
kmeans.dd2.EIHI	0.56280	0.54969	0.50453	0.00264
kmeans.dd2.EIHI	0.56280	0.54969	0.50453	0.00433
centroid._dd2.EIHIMEI	0.52750	0.65901	0.50410	0.00049
single._d.MEI	0.52290	0.66078	0.50393	0.00039
centroid.dd2.MEI	0.53410	0.65334	0.50356	0.00033
ward.D2.d2.EIHI	0.55900	0.56491	0.50341	0.00042
centroid.d2.EIHIMEI	0.54720	0.64251	0.50325	0.00035
ward.D2.dd2.EIHI	0.54990	0.56867	0.50147	0.00019
centroid._d2.EIHIMEI	0.52770	0.65488	0.50064	0.00028
spc.d2.EIHI	0.54040	0.60512	0.50035	0.03063
average...EIHI	0.53900	0.64734	0.49976	0.00038
complete.d2.EIHI	0.54100	0.61177	0.49975	0.00029
centroid.d.EIHIMEI	0.53100	0.64067	0.49942	0.00039
average._d.EIHI	0.53700	0.64799	0.49928	0.00029
kkmeans.d2.EIHI	0.53670	0.50431	0.49925	0.01432
kkmeans.dd2.EIHI	0.53740	0.50464	0.49913	0.01681

single._d2.MEI	0.51820	0.65749	0.49878	0.00039
average._dd2.EIHI	0.53500	0.64817	0.49853	0.00034
complete.dd2.EIHI	0.53360	0.62026	0.49852	0.00032
average._d2.EIHI	0.53520	0.64801	0.49845	0.00028
centroid._.EIHI	0.53160	0.64984	0.49840	0.00028
centroid._d.EIHI	0.52910	0.65068	0.49784	0.00031
centroid._d2.EIHI	0.52700	0.65111	0.49723	0.00037
centroid._dd2.EIHI	0.52580	0.65163	0.49718	0.00031
centroid.dd2.EIHIMEI	0.52120	0.65205	0.49682	0.00030
single._.EIHIMEI	0.52300	0.65281	0.49676	0.00052
single._.EIHI	0.52110	0.65362	0.49666	0.00036
average.d2.EIHI	0.52270	0.63649	0.49662	0.00029
average.dd2.EIHI	0.51900	0.64014	0.49642	0.00030
spc.dd2.EIHI	0.51230	0.63781	0.49635	0.04553
centroid.d2.EIHI	0.52080	0.64109	0.49629	0.00144
single.d2.EIHIMEI	0.51720	0.65427	0.49606	0.00036
single._dd2.EIHI	0.51540	0.65490	0.49580	0.00043
single._d.EIHIMEI	0.51620	0.65526	0.49580	0.00139
centroid.dd2.EIHI	0.51360	0.64868	0.49578	0.00027
single._d2.EIHI	0.51470	0.65561	0.49569	0.00034
single._d2.EIHIMEI	0.51450	0.65594	0.49564	0.00031
single._d.EIHI	0.51480	0.65506	0.49564	0.00036
single.d2.EIHI	0.51410	0.65192	0.49561	0.00024
single.d.EIHIMEI	0.51300	0.65479	0.49550	0.00033
single._dd2.EIHIMEI	0.51320	0.65639	0.49544	0.00045
single.dd2.MEI	0.51280	0.65655	0.49541	0.00044
single.dd2.EIHIMEI	0.51130	0.65536	0.49535	0.00039
single_dd2.MEI	0.51160	0.65695	0.49531	0.00038
single.dd2.EIHI	0.50630	0.65230	0.49515	0.00052
kkmeans.d.EIHI	0.50200	0.49928	0.49497	0.01099
svc.d.EIHI	0.50150	0.57165	0.49496	0.00492
kkmeans.d.EIHI	0.50130	0.55993	0.49495	0.01203
svc.d.EIHI	0.50080	0.57256	0.49492	0.00488
spc.d.EIHI	0.50070	0.60971	0.49492	0.01609
kmeans.d.EIHI	0.50040	0.60081	0.49491	0.00223
kmeans.d.EIHI	0.50040	0.60081	0.49491	0.00238
complete.d.EIHI	0.50030	0.62224	0.49491	0.00026
ward.D2.d.EIHI	0.50030	0.60092	0.49491	0.00041
average.d.EIHI	0.50010	0.63797	0.49490	0.00040
centroid.d.EIHI	0.50010	0.64295	0.49490	0.00026
single.d.EIHI	0.50010	0.64417	0.49490	0.00027

Table 22: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 1-9.

	Purity	Fmeasure	RI	Time
$d\rho$, $\rho=0.02$	0.996	0.991	0.992	0.50788
$d\rho$, $\rho=1$	0.996	0.991	0.992	0.50479
L^2	0.995	0.991	0.991	0.13043
$d\rho$, $\rho=0.001$	0.995	0.991	0.991	0.50067
dk , $k=2$	0.871	0.856	0.854	0.22058
dk , $k=3$	0.8384	0.818497	0.817222	
$d\rho$, $\rho=100$	0.783	0.777	0.767	0.73592
$d\rho$, $\rho=1e+08$	0.501	0.507	0.494	0.64659

Table 23: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 1-9.

	Purity	Fmeasure	RI	Time
random	0.935	0.877	0.878	0.26316
kmeans	0.935	0.877	0.878	0.14693
hclust	0.934	0.876	0.877	0.14506
kmeans ++	0.934	0.875	0.876	0.32626

Table 24: Mean results for S 10-11 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k -means for initialization (green) and k -means for initialization (orange).

	Purity	Fmeasure	RI	Time
svc.d.EIHIMEI	0.72760	0.60262	0.60335	0.00483
kmeans._dd2.EIHIMEI	0.72760	0.60352	0.60323	0.00314
kmeans._dd2.EIHIMEI	0.72760	0.60352	0.60323	0.00338
svc.d.EIHIMEI	0.72720	0.60223	0.60305	0.00481
kkmeans.d.EIHIMEI	0.72650	0.60057	0.60261	0.00913
kmeans.d.EIHIMEI	0.72630	0.60133	0.60226	0.00254
kmeans.d.EIHIMEI	0.72630	0.60133	0.60226	0.00239
svc._dd2.EIHIMEI	0.71810	0.59816	0.59779	0.00805
kmeans._d2.EIHIMEI	0.72030	0.59678	0.59651	0.00296
kmeans._d2.EIHIMEI	0.72030	0.59678	0.59651	0.00335
svc._dd2.EIHIMEI	0.71200	0.59379	0.59357	0.00779
kmeans._dd2.MEI	0.71450	0.59194	0.59322	0.00251
kmeans._dd2.MEI	0.71450	0.59194	0.59322	0.00253
svc._dd2.MEI	0.71480	0.59172	0.59288	0.00564
svc._d2.EIHIMEI	0.71130	0.59198	0.59186	0.00626
svc._dd2.MEI	0.71160	0.58995	0.59122	0.00472

ward.D2.d.EIHIMEI	0.71450	0.60259	0.59082	0.00032
svc._d2.EIHIMEI	0.70890	0.59018	0.58979	0.00668
kkmeans_dd2.MEI	0.70520	0.58648	0.58808	0.01375
kkmeans._dd2.EIHIMEI	0.70160	0.58766	0.58758	0.01460
svc._d2.MEI	0.70770	0.58625	0.58739	0.00397
kkmeans._d2.EIHIMEI	0.70220	0.58550	0.58667	0.01413
svc._d2.MEI	0.70660	0.58515	0.58615	0.00433
kmeans._d2.MEI	0.70590	0.58454	0.58554	0.00221
kmeans._d2.MEI	0.70590	0.58454	0.58554	0.00263
complete.d.EIHIMEI	0.70550	0.60189	0.58447	0.00032
kkmeans._d2.MEI	0.70250	0.58260	0.58428	0.01278
ward.D2._dd2.EIHIMEI	0.68790	0.59010	0.57424	0.00042
ward.D2_dd2.MEI	0.68520	0.58313	0.57213	0.00036
kkmeans_dd2.MEI	0.67870	0.57340	0.57191	0.01690
kmeans._d.MEI	0.68700	0.57007	0.57056	0.00236
kmeans._d.MEI	0.68700	0.57007	0.57056	0.00246
kkmeans._d.MEI	0.68730	0.56862	0.57039	0.01027
kkmeans._d.EIHIMEI	0.68690	0.56902	0.57037	0.01073
svc._d.MEI	0.68630	0.56975	0.57003	0.00401
svc._d.MEI	0.68570	0.56883	0.56956	0.00397
svc._d.EIHIMEI	0.68490	0.56878	0.56872	0.00637
kmeans._d.EIHIMEI	0.68470	0.56989	0.56867	0.00265
kmeans._d.EIHIMEI	0.68470	0.56989	0.56867	0.00285
svc._d.EIHIMEI	0.68420	0.56989	0.56841	0.00665
ward.D2._d2.MEI	0.68050	0.57788	0.56829	0.00028
ward.D2._d2.EIHIMEI	0.67990	0.58285	0.56690	0.00033
svc.dd2.MEI	0.68070	0.56564	0.56653	0.00415
svc.dd2.MEI	0.68040	0.56475	0.56598	0.00411
kkmeans.dd2.MEI	0.67990	0.56340	0.56587	0.01072
kmeans.dd2.MEI	0.67940	0.56454	0.56542	0.00219
kmeans.dd2.MEI	0.67940	0.56454	0.56542	0.00236
kkmeans._.EIHIMEI	0.67890	0.56320	0.56384	0.00987
kmeans._.EIHIMEI	0.67850	0.56547	0.56370	0.00227
kmeans._.EIHIMEI	0.67850	0.56547	0.56370	0.00232
svc._.EIHIMEI	0.67720	0.56484	0.56283	0.00504
svc._.EIHIMEI	0.67710	0.56338	0.56268	0.00602
average.d.EIHIMEI	0.65130	0.62011	0.56035	0.00028
ward.D2.dd2.MEI	0.66900	0.57341	0.56012	0.00034
svc.dd2.EIHIMEI	0.67240	0.56249	0.55966	0.00674
ward.D2._d.MEI	0.66950	0.57059	0.55950	0.00033
kkmeans.dd2.EIHIMEI	0.66990	0.55896	0.55889	0.01218
complete_dd2.MEI	0.66370	0.58107	0.55845	0.00033
kmeans.dd2.EIHIMEI	0.67030	0.56084	0.55823	0.00266
kmeans.dd2.EIHIMEI	0.67030	0.56084	0.55823	0.00287

svc.dd2.EIHIMEI	0.66910	0.56082	0.55737	0.00658
kkmeans._dd2.EIHIMEI	0.65890	0.55544	0.55616	0.01736
kkmeans.d2.EIHIMEI	0.66560	0.55480	0.55493	0.01105
spc.d.EIHIMEI	0.64280	0.61808	0.55443	0.05179
ward.D2._d.EIHIMEI	0.66380	0.57560	0.55441	0.00035
svc.d2.EIHIMEI	0.66540	0.55732	0.55440	0.00482
svc.d2.EIHIMEI	0.66410	0.55699	0.55363	0.00470
ward.D2._.EIHIMEI	0.66360	0.56908	0.55345	0.00033
kmeans.d2.EIHIMEI	0.66370	0.55612	0.55328	0.00242
kmeans.d2.EIHIMEI	0.66370	0.55612	0.55328	0.00250
complete._d2.MEI	0.65570	0.57221	0.55124	0.00035
complete._d.MEI	0.65600	0.57653	0.55040	0.00034
average_dd2.MEI	0.63950	0.61048	0.55036	0.00030
complete._dd2.EIHIMEI	0.64610	0.59157	0.54944	0.00039
complete.dd2.MEI	0.65460	0.57649	0.54901	0.00034
kkmeans.d.EIHIMEI	0.64390	0.55598	0.54853	0.01604
complete._.EIHIMEI	0.64500	0.58591	0.54617	0.00028
average._d.MEI	0.64440	0.59380	0.54613	0.00034
kkmeans._d2.EIHIMEI	0.63330	0.54637	0.54383	0.01851
kkmeans._d.MEI	0.63700	0.54580	0.54335	0.01726
kkmeans._d2.MEI	0.63220	0.54514	0.54196	0.01618
ward.D2.dd2.EIHIMEI	0.64200	0.56772	0.54132	0.00029
kkmeans.dd2.MEI	0.63330	0.54346	0.54117	0.01621
kkmeans._d2.EIHI	0.64060	0.56175	0.54085	0.01441
complete._d2.EIHIMEI	0.63290	0.59044	0.54056	0.00035
ward.D2.d2.EIHIMEI	0.64120	0.56490	0.53982	0.00023
kkmeans._d.EIHIMEI	0.63230	0.54248	0.53960	0.01721
kkmeans.dd2.EIHI	0.64000	0.55724	0.53889	0.01200
svc._d2.EIHI	0.63560	0.57631	0.53864	0.00611
kkmeans._dd2.EIHI	0.63610	0.56245	0.53848	0.01361
complete._d.EIHIMEI	0.63370	0.58821	0.53840	0.00041
average._d2.MEI	0.61590	0.61177	0.53782	0.00035
kkmeans.dd2.EIHIMEI	0.62900	0.53991	0.53765	0.01713
kkmeans.d2.EIHI	0.63370	0.55274	0.53626	0.01205
spc_dd2.MEI	0.60570	0.62199	0.53550	0.04887
svc._d2.EIHI	0.63040	0.56696	0.53522	0.00575
svc._dd2.EIHI	0.62710	0.56938	0.53421	0.00719
average.dd2.MEI	0.62370	0.58318	0.53416	0.00029
svc.d2.EIHI	0.62830	0.56031	0.53329	0.00492
kmeans._dd2.EIHI	0.62760	0.57538	0.53314	0.00279
kmeans._dd2.EIHI	0.62760	0.57538	0.53314	0.00271
kmeans._d2.EIHI	0.62710	0.57577	0.53296	0.00278
kmeans._d2.EIHI	0.62710	0.57577	0.53296	0.00265
complete.dd2.EIHIMEI	0.62570	0.58800	0.53272	0.00033

spc._dd2.EIHIMEI	0.59930	0.62663	0.53214	0.04488
svc.dd2.EIHI	0.62700	0.55844	0.53209	0.00557
svc.d2.EIHI	0.62820	0.55965	0.53209	0.00465
kmeans.dd2.EIHI	0.62750	0.56762	0.53194	0.00229
kmeans.dd2.EIHI	0.62750	0.56762	0.53194	0.00269
kmeans.d2.EIHI	0.62740	0.56757	0.53191	0.00222
1 kmeans.d2.EIHI	0.62740	0.56757	0.53191	0.00227
svc.dd2.EIHI	0.62660	0.56414	0.53166	0.00556
svc._dd2.EIHI	0.62290	0.57416	0.53158	0.00724
complete.d2.EIHIMEI	0.62500	0.58402	0.53144	0.00031
spc._d.MEI	0.60570	0.61373	0.53007	0.05129
centroid.d.EIHIMEI	0.58470	0.63553	0.52753	0.00026
ward.D2._d2.EIHI	0.61600	0.58775	0.52641	0.00032
spc._d2.MEI	0.59010	0.62261	0.52581	0.04896
ward.D2.dd2.EIHI	0.61590	0.57807	0.52563	0.00027
ward.D2.d2.EIHI	0.61360	0.57641	0.52441	0.00039
spc._d.EIHIMEI	0.58850	0.62224	0.52411	0.05049
centroid._d.MEI	0.59340	0.61727	0.52408	0.00027
ward.D2._dd2.EIHI	0.61100	0.58428	0.52379	0.00144
spc.dd2.MEI	0.59450	0.60761	0.52331	0.05078
spc._.EIHIMEI	0.58700	0.61956	0.52206	0.05359
spc.dd2.EIHIMEI	0.59110	0.60397	0.52130	0.05315
spc._d2.EIHIMEI	0.58130	0.62800	0.52067	0.04778
kkmeans.d2.EIHIMEI	0.58760	0.52026	0.51662	0.01468
spc.d2.EIHIMEI	0.57790	0.60595	0.51400	0.05436
kkmeans._.EIHIMEI	0.57640	0.52319	0.51326	0.01474
kkmeans.dd2.EIHI	0.58000	0.51821	0.51308	0.01525
complete.d2.EIHI	0.57960	0.60641	0.51172	0.00022
kkmeans.d2.EIHI	0.56980	0.51446	0.51013	0.01523
kkmeans._d2.EIHI	0.57150	0.51614	0.50998	0.01539
kkmeans._dd2.EIHI	0.56910	0.51237	0.50961	0.01680
complete.dd2.EIHI	0.57130	0.61352	0.50943	0.00030
spc._dd2.EIHI	0.56340	0.62034	0.50935	0.05110
spc.d2.EIHI	0.56470	0.61375	0.50877	0.03772
centroid.dd2.MEI	0.55800	0.62952	0.50875	0.00026
spc.dd2.EIHI	0.56550	0.61414	0.50874	0.04582
svc._.EIHI	0.56850	0.61967	0.50832	0.00537
average.d2.EIHIMEI	0.56220	0.62597	0.50816	0.00143
average.dd2.EIHIMEI	0.56210	0.62640	0.50797	0.00030
svc._d.EIHI	0.57060	0.61668	0.50777	0.00769
kkmeans._.EIHI	0.56370	0.61754	0.50725	0.01603
7kkmeans.d.EIHI	0.56690	0.56149	0.50684	0.00894
svc._d.EIHI	0.56580	0.61988	0.50680	0.00601
svc._.EIHI	0.56440	0.61947	0.50670	0.00556

svc.d.EIHI	0.56680	0.58830	0.50665	0.00579
spc.d.EIHI	0.56790	0.59368	0.50656	0.01216
kkmeans._d.EIHI	0.56390	0.61876	0.50593	0.01637
svc.d.EIHI	0.56300	0.58764	0.50571	0.00474
kkmeans._.EIHI	0.55990	0.51184	0.50567	0.01195
spc._d2.EIHI	0.55550	0.62652	0.50564	0.04681
complete._d2.EIHI	0.55760	0.62409	0.50560	0.00032
complete._dd2.EIHI	0.55640	0.62608	0.50524	0.00047
kkmeans._d.EIHI	0.55560	0.50844	0.50434	0.01357
spc._.EIHI	0.54830	0.63592	0.50289	0.01843
ward.D2.d.EIHI	0.55080	0.61425	0.50224	0.00037
average._.EIHIMEI	0.53970	0.64141	0.50211	0.00027
ward.D2._d.EIHI	0.54860	0.63415	0.50204	0.00033
centroid.d2.EIHIMEI	0.54280	0.63764	0.50195	0.00028
ward.D2._.EIHI	0.54780	0.63479	0.50181	0.00028
average._d.EIHIMEI	0.53830	0.64225	0.50151	0.00031
kmeans.d.EIHI	0.54960	0.61775	0.50144	0.00220
kmeans.d.EIHI	0.54960	0.61775	0.50144	0.00217
kkmeans.d.EIHI	0.54380	0.52625	0.50126	0.00862
kmeans._.EIHI	0.54840	0.63471	0.50119	0.00214
kmeans._.EIHI	0.54840	0.63471	0.50119	0.00242
kmeans._d.EIHI	0.54830	0.63466	0.50117	0.00230
kmeans._d.EIHI	0.54830	0.63466	0.50117	0.00264
centroid.dd2.EIHIMEI	0.53760	0.64205	0.50113	0.00035
spc._d.EIHI	0.54420	0.63358	0.50079	0.02936
average._d2.EIHIMEI	0.53460	0.64511	0.50011	0.00029
average.dd2.EIHI	0.53840	0.64066	0.50001	0.00028
average.d2.EIHI	0.53770	0.64205	0.49977	0.00144
average._dd2.EIHIMEI	0.52960	0.64837	0.49953	0.00040
complete.d.EIHI	0.53210	0.63868	0.49855	0.00014
centroid.dd2.EIHI	0.53140	0.64484	0.49849	0.00145
centroid.d2.EIHI	0.52930	0.64605	0.49829	0.00033
complete._d.EIHI	0.53270	0.64492	0.49808	0.00027
centroid._d.EIHIMEI	0.52730	0.64855	0.49800	0.00048
centroid._d2.MEI	0.52370	0.64952	0.49799	0.00024
centroid._.EIHIMEI	0.52740	0.64688	0.49790	0.00023
complete._.EIHI	0.52970	0.64693	0.49765	0.00027
centroid_dd2.MEI	0.52100	0.65250	0.49737	0.00024
average._dd2.EIHI	0.52160	0.65127	0.49674	0.00034
average._d2.EIHI	0.52150	0.65170	0.49657	0.00041
average._.EIHI	0.52250	0.65145	0.49649	0.00036
centroid._d2.EIHIMEI	0.51930	0.65387	0.49646	0.00041
average._d.EIHI	0.52200	0.65142	0.49638	0.00037
centroid._.EIHI	0.52090	0.65176	0.49627	0.00031

centroid._d.EIHI	0.51980	0.65218	0.49615	0.00033
centroid.d.EIHI	0.51740	0.65217	0.49611	0.00028
average.d.EIHI	0.51760	0.65195	0.49607	0.00029
single.dd2.EIHIMEI	0.51600	0.65466	0.49598	0.00057
single.d2.EIHIMEI	0.51670	0.65376	0.49597	0.00029
single._.EIHIMEI	0.51670	0.65400	0.49590	0.00137
centroid._dd2.EIHIMEI	0.51680	0.65437	0.49585	0.00042
single.d2.EIHI	0.51490	0.65476	0.49579	0.00031
single.dd2.EIHI	0.51490	0.65494	0.49578	0.00035
single._d.EIHI	0.51540	0.65467	0.49574	0.00037
single._.EIHI	0.51530	0.65452	0.49573	0.00040
single._d.MEI	0.51500	0.65512	0.49572	0.00030
centroid._dd2.EIHI	0.51560	0.65483	0.49571	0.00033
centroid._d2.EIHI	0.51570	0.65503	0.49567	0.00038
single._d.EIHIMEI	0.51500	0.65479	0.49566	0.00150
single._d2.EIHI	0.51360	0.65583	0.49555	0.00045
single._dd2.EIHI	0.51360	0.65583	0.49555	0.00038
single._dd2.EIHIMEI	0.51320	0.65600	0.49553	0.00047
single_dd2.MEI	0.51270	0.65595	0.49553	0.00141
single.dd2.MEI	0.51330	0.65573	0.49547	0.00029
single.d.EIHI	0.51280	0.65541	0.49547	0.00025
single.d.EIHIMEI	0.51320	0.65604	0.49546	0.00029
single._d2.EIHIMEI	0.51240	0.65647	0.49542	0.00040
single._d2.MEI	0.51230	0.65611	0.49537	0.00027

Table 25: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 10-11.

	Purity	Fmeasure	RI	Time
L^2	0.758	0.636	0.637	0.9562
$d\rho, \rho=0.02$	0.757	0.637	0.637	9.6435
$d\rho, \rho=0.001$	0.756	0.635	0.635	9.2899
$d\rho, \rho=1$	0.749	0.631	0.631	9.9870
$d\rho, \rho=1e+08$	0.734	0.609	0.609	7.5003
$d\rho, \rho=100$	0.703	0.606	0.596	9.3033
$dk, k=2$	0.687	0.588	0.588	0.78738
$dk, k=3$	0.653	0.562	0.561	0.6807

	Purity	Fmeasure	RI	Time
kmeans ++	0.735	0.614	0.612	0.61413
random	0.731	0.610	0.610	0.55656
hclust	0.726	0.603	0.602	0.36616
kmeans	0.721	0.598	0.599	0.29926

Table 26: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 10-11.

4 Three clusters

In this section the tables corresponding to Section 5.2 of the paper are presented. Here, results from applying EHyclus, functional k -means and model based k -means to scenarios with three clusters are shown.

Table 27: Mean results for S 18-19-20 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k -means for initialization (green) and k -means for initialization (orange).

	Purity	Fmeasure	RI	Time
kmeans._d2.MEI	0.68873	0.57816	0.71941	0.00221
kmeans._d2.MEI	0.68873	0.57816	0.71941	0.00244
kmeans_dd2.MEI	0.68920	0.57805	0.71937	0.00232
kmeans_dd2.MEI	0.68920	0.57805	0.71937	0.00258
svc._d2.MEI	0.68780	0.57704	0.71847	0.00611
svc._d2.EIHIMEI	0.68680	0.57658	0.71814	0.01167
complete._d2.MEI	0.67679	0.58880	0.71806	0.00115
svc_dd2.MEI	0.68753	0.57535	0.71773	0.00669
svc._d2.EIHIMEI	0.68660	0.57608	0.71770	0.01274
kmeans._dd2.EIHIMEI	0.68693	0.57598	0.71762	0.00352
kmeans._dd2.EIHIMEI	0.68693	0.57598	0.71762	0.00346
kmeans._d2.EIHIMEI	0.68654	0.57594	0.71759	0.00319
kmeans._d2.EIHIMEI	0.68654	0.57594	0.71759	0.00298
svc._dd2.EIHIMEI	0.68653	0.57471	0.71712	0.01514
svc._d2.MEI	0.68607	0.57469	0.71686	0.00638
svc_dd2.MEI	0.68620	0.57455	0.71664	0.00694
svc._dd2.EIHIMEI	0.68507	0.57424	0.71640	0.01660
ward.D2._d2.MEI	0.67260	0.57958	0.71316	0.00034
kkmeans_dd2.MEI	0.66920	0.57594	0.71309	0.01818
complete_dd2.MEI	0.67234	0.58100	0.71171	0.00032
kkmeans._dd2.EIHIMEI	0.67060	0.57331	0.71159	0.01917
ward.D2._d2.EIHIMEI	0.66933	0.57788	0.71094	0.00039
ward.D2_dd2.MEI	0.66914	0.57606	0.71052	0.00030

kkmeans._d2.MEI	0.67047	0.57090	0.71023	0.01850
ward.D2._dd2.EIHIMEI	0.66607	0.57781	0.71002	0.00045
kkmeans._d2.EIHIMEI	0.66700	0.57118	0.70974	0.01962
kkmeans._dd2.EIHIMEI	0.66173	0.56637	0.70878	0.02454
average_dd2.MEI	0.66233	0.57826	0.70629	0.00038
complete._d2.EIHIMEI	0.66367	0.57554	0.70625	0.00043
average._d2.MEI	0.66366	0.57509	0.70510	0.00031
complete._dd2.EIHIMEI	0.66166	0.57240	0.70249	0.00129
kkmeans_dd2.MEI	0.64873	0.55124	0.69717	0.02217
kkmeans._d2.MEI	0.64654	0.54871	0.69586	0.02217
average._d2.EIHIMEI	0.65080	0.58723	0.69522	0.00044
spc._dd2.EIHIMEI	0.65073	0.57777	0.69441	0.07912
kkmeans._d2.EIHIMEI	0.64387	0.54148	0.69159	0.02516
average._dd2.EIHIMEI	0.64513	0.58570	0.69079	0.00050
svc.dd2.MEI	0.64227	0.52564	0.68268	0.00647
spc._d2.EIHIMEI	0.63986	0.57042	0.68112	0.08122
kkmeans._.EIHIMEI	0.62766	0.52831	0.68108	0.01923
kmeans.dd2.MEI	0.64014	0.52211	0.68017	0.00238
kmeans.dd2.MEI	0.64014	0.52211	0.68017	0.00268
kkmeans._d.MEI	0.62600	0.52324	0.67989	0.01738
centroid._d2.MEI	0.63747	0.58886	0.67983	0.00036
svc.dd2.MEI	0.63667	0.52151	0.67936	0.00663
kkmeans._d.EIHIMEI	0.62347	0.52549	0.67858	0.01804
svc._d.MEI	0.63387	0.51814	0.67668	0.00668
svc._d.MEI	0.63333	0.51786	0.67646	0.00636
kmeans._d.MEI	0.63347	0.51713	0.67594	0.00211
kmeans._d.MEI	0.63347	0.51713	0.67594	0.00220
ward.D2.dd2.MEI	0.62813	0.52977	0.67444	0.00040
svc._d.EIHIMEI	0.63447	0.51708	0.67439	0.01299
svc._.EIHIMEI	0.63247	0.51664	0.67411	0.01020
kkmeans.dd2.MEI	0.62707	0.52158	0.67388	0.02012
svc._.EIHIMEI	0.63220	0.51660	0.67366	0.00866
ward.D2._.EIHIMEI	0.62407	0.52816	0.67318	0.00036
kmeans._d.EIHIMEI	0.63200	0.51584	0.67284	0.00285
kmeans._d.EIHIMEI	0.63200	0.51584	0.67284	0.00311
svc._d.EIHIMEI	0.63166	0.51584	0.67284	0.01208
kmeans._.EIHIMEI	0.63180	0.51587	0.67279	0.00238
kmeans._.EIHIMEI	0.63180	0.51587	0.67279	0.00259
spc_dd2.MEI	0.63193	0.56919	0.67191	0.08599
complete._d.MEI	0.62760	0.52503	0.67151	0.00034
centroid_dd2.MEI	0.62267	0.60933	0.67004	0.00033
ward.D2._d.MEI	0.62460	0.52100	0.66975	0.00028
ward.D2._d.EIHIMEI	0.62373	0.52416	0.66966	0.00045
complete._.EIHIMEI	0.61974	0.52617	0.66728	0.00034

average._d.MEI	0.62300	0.52138	0.66559	0.00032
complete._d.EIHIMEI	0.61960	0.52022	0.66277	0.00042
ward.D2.d2.EIHIMEI	0.60920	0.52092	0.66257	0.00030
centroid._.EIHIMEI	0.62014	0.52731	0.66151	0.00113
centroid._d.MEI	0.61914	0.52823	0.66047	0.00032
average._.EIHIMEI	0.61760	0.52049	0.65893	0.00044
kkmeans.d2.EIHIMEI	0.59706	0.49841	0.65884	0.01883
kkmeans.dd2.EIHIMEI	0.59027	0.49159	0.65757	0.02484
kkmeans.dd2.MEI	0.59260	0.49257	0.65707	0.02366
average._d.EIHIMEI	0.61521	0.51950	0.65683	0.00038
average.dd2.MEI	0.61080	0.55790	0.65607	0.00027
kkmeans.dd2.EIHIMEI	0.59567	0.49831	0.65513	0.02119
spc._d2.MEI	0.62000	0.55884	0.65478	0.08369
kkmeans._d.MEI	0.58487	0.48463	0.65199	0.02212
complete.dd2.MEI	0.61306	0.51990	0.65195	0.00040
ward.D2.dd2.EIHIMEI	0.59467	0.50655	0.65161	0.00116
kkmeans._d.EIHIMEI	0.58660	0.48450	0.65133	0.02352
svc.d2.EIHIMEI	0.58667	0.48619	0.64880	0.00999
svc.dd2.EIHIMEI	0.58613	0.48411	0.64821	0.01264
svc.d2.EIHIMEI	0.58660	0.48808	0.64795	0.01015
svc.dd2.EIHIMEI	0.58513	0.48405	0.64788	0.01240
kmeans.d2.EIHIMEI	0.58573	0.48641	0.64772	0.00255
kmeans.d2.EIHIMEI	0.58573	0.48641	0.64772	0.00267
kkmeans._.EIHI	0.56553	0.51093	0.64771	0.01359
kmeans.dd2.EIHIMEI	0.58520	0.48283	0.64765	0.00348
kmeans.dd2.EIHIMEI	0.58520	0.48283	0.64765	0.00375
spc._d.MEI	0.60600	0.51255	0.64665	0.08728
kkmeans.d2.EIHIMEI	0.58413	0.47712	0.64629	0.02366
centroid._d.EIHIMEI	0.60513	0.53234	0.64514	0.00046
spc.dd2.MEI	0.60307	0.54102	0.64402	0.08391
centroid._d2.EIHIMEI	0.59513	0.59706	0.63417	0.00115
spc._d.EIHIMEI	0.59393	0.51447	0.63132	0.08205
complete.dd2.EIHIMEI	0.57347	0.50321	0.62403	0.00040
centroid._dd2.EIHIMEI	0.58207	0.60173	0.62178	0.00049
complete.d2.EIHIMEI	0.57547	0.50215	0.62055	0.00113
kkmeans._.EIHIMEI	0.53246	0.43830	0.61715	0.02125
spc.dd2.EIHIMEI	0.56899	0.53186	0.61017	0.08115
spc.d2.EIHIMEI	0.57107	0.54214	0.60881	0.08833
average.d2.EIHIMEI	0.57199	0.54204	0.60651	0.00103
average.dd2.EIHIMEI	0.55514	0.53394	0.59077	0.00040
kkmeans._d.EIHI	0.46820	0.40144	0.58506	0.01922
spc._.EIHIMEI	0.55573	0.50910	0.57231	0.09555
kkmeans._dd2.EIHI	0.44460	0.36249	0.56681	0.02300
kkmeans.d2.EIHI	0.43980	0.36091	0.56577	0.02106

kkmeans.dd2.EIHI	0.43774	0.35949	0.56446	0.02327
kkmeans._d2.EIHI	0.44080	0.36280	0.56440	0.02284
kkmeans.d.EIHIMEI	0.39467	0.33983	0.55309	0.02241
kkmeans.d.EIHIMEI	0.40960	0.35099	0.55067	0.02030
kkmeans._d2.EIHI	0.49853	0.42841	0.55013	0.02347
kkmeans._dd2.EIHI	0.50167	0.42922	0.54866	0.02382
kkmeans.dd2.EIHI	0.47800	0.40226	0.54715	0.02054
svc.d.EIHIMEI	0.40380	0.35402	0.54550	0.00964
svc.d.EIHIMEI	0.40420	0.35533	0.54466	0.00922
kmeans.d.EIHIMEI	0.40287	0.35639	0.54336	0.00253
kmeans.d.EIHIMEI	0.40287	0.35639	0.54336	0.00270
kkmeans.d2.EIHI	0.47814	0.40736	0.54259	0.01896
ward.D2.d.EIHIMEI	0.40160	0.36114	0.53990	0.00109
kkmeans.d.EIHI	0.33946	0.34066	0.53807	0.01228
centroid.d2.EIHIMEI	0.51720	0.53186	0.53799	0.00034
svc._d2.EIHI	0.49327	0.43664	0.52966	0.01260
svc._dd2.EIHI	0.49360	0.43683	0.52958	0.01600
kkmeans._.EIHI	0.49427	0.48392	0.52901	0.01647
centroid.dd2.MEI	0.51086	0.53888	0.52845	0.00032
svc._d2.EIHI	0.49293	0.43640	0.52803	0.01162
vc._dd2.EIHI	0.49067	0.43372	0.52710	0.01620
svc.d2.EIHI	0.47393	0.41951	0.52605	0.00813
svc.d2.EIHI	0.47207	0.41886	0.52541	0.00826
svc.dd2.EIHI	0.46887	0.41864	0.52464	0.01191
ward.D2.d2.EIHI	0.47240	0.41942	0.52417	0.00040
svc.dd2.EIHI	0.47080	0.41999	0.52402	0.01254
kmeans.dd2.EIHI	0.47067	0.41996	0.52334	0.00281
kmeans.dd2.EIHI	0.47067	0.41996	0.52334	0.00295
kmeans.d2.EIHI	0.47187	0.42090	0.52265	0.00233
kmeans.d2.EIHI	0.47187	0.42095	0.52263	0.00247
kmeans._d2.EIHI	0.48620	0.43760	0.51950	0.00295
kmeans._d2.EIHI	0.48620	0.43760	0.51950	0.00292
kmeans._dd2.EIHI	0.48420	0.43716	0.51833	0.00367
kmeans._dd2.EIHI	0.48393	0.43709	0.51812	0.00357
complete.d.EIHIMEI	0.39847	0.38878	0.51409	0.00035
ward.D2.dd2.EIHI	0.45940	0.42005	0.51349	0.00035
ward.D2._d2.EIHI	0.48713	0.44612	0.51242	0.00032
ward.D2._dd2.EIHI	0.48113	0.44153	0.51235	0.00047
svc._.EIHI	0.48780	0.49422	0.51075	0.01052
kkmeans.d.EIHI	0.33752	0.37264	0.51048	0.01450
kkmeans._d.EIHI	0.47340	0.45091	0.50996	0.01957
svc._.EIHI	0.47374	0.47738	0.49309	0.01145
svc.d.EIHI	0.33685	0.39352	0.48966	0.00939
svc.d.EIHI	0.33685	0.39478	0.48763	0.00892

svc._d.EIHI	0.45647	0.46294	0.48745	0.01170
svc._d.EIHI	0.45127	0.45649	0.48186	0.01206
ward.D2._.EIHI	0.45947	0.45372	0.47823	0.00031
kmeans.d.EIHI	0.33611	0.40559	0.47524	0.00220
kmeans.d.EIHI	0.33605	0.40561	0.47522	0.00200
ward.D2.d.EIHI	0.33564	0.40518	0.47398	0.00021
ward.D2._d.EIHI	0.44713	0.45399	0.47096	0.00038
kmeans._.EIHI	0.45227	0.45227	0.46994	0.00205
kmeans._.EIHI	0.45227	0.45227	0.46994	0.00215
spc.d.EIHI	0.33631	0.40919	0.46548	0.01534
kmeans._d.EIHI	0.44360	0.45272	0.46377	0.00272
kmeans._d.EIHI	0.44360	0.45272	0.46377	0.00255
average.d.EIHIMEI	0.37320	0.42149	0.46309	0.00030
spc.d.EIHIMEI	0.36946	0.42071	0.45787	0.08672
spc.d2.EIHI	0.43160	0.44725	0.45685	0.04121
spc._.EIHI	0.45213	0.48600	0.45613	0.01539
complete.d2.EIHI	0.43373	0.45291	0.45131	0.00036
spc.dd2.EIHI	0.41373	0.44493	0.45127	0.07387
omplete.dd2.EIHI	0.43147	0.45355	0.44976	0.00040
spc._dd2.EIHI	0.43507	0.45949	0.44908	0.07587
spc._d2.EIHI	0.44394	0.46536	0.44669	0.06918
centroid.d.EIHIMEI	0.36646	0.43260	0.44564	0.00034
complete._dd2.EIHI	0.43800	0.47000	0.43752	0.00042
complete._d2.EIHI	0.43306	0.47158	0.43200	0.00035
spc._d.EIHI	0.42800	0.47317	0.43097	0.03850
complete.d.EIHI	0.33497	0.44432	0.42484	0.00035
complete._d.EIHI	0.42186	0.47238	0.42045	0.00033
complete._.EIHI	0.42400	0.47566	0.41856	0.00028
average.d2.EIHI	0.40673	0.47128	0.40641	0.00032
average.dd2.EIHI	0.40267	0.47122	0.40440	0.00037
average.d.EIHI	0.33424	0.45792	0.40281	0.00028
centroid.dd2.EIHIMEI	0.40240	0.49283	0.39973	0.00043
centroid.d.EIHI	0.33397	0.46460	0.39219	0.00030
average._.EIHI	0.40553	0.48062	0.39139	0.00025
centroid.d2.EIHI	0.39400	0.47629	0.38908	0.00116
centroid._.EIHI	0.40187	0.48201	0.38707	0.00030
average._d.EIHI	0.39920	0.48293	0.38647	0.00111
single._.EIHIMEI	0.39826	0.49241	0.38433	0.00035
centroid.dd2.EIHI	0.38474	0.47949	0.37989	0.00036
average._dd2.EIHI	0.38540	0.48219	0.37637	0.00038
average._d2.EIHI	0.38440	0.48182	0.37612	0.00033
centroid._d.EIHI	0.38500	0.48457	0.37313	0.00036
single.d.EIHI	0.33377	0.47662	0.37063	0.00037
single._d.MEI	0.36868	0.49467	0.36043	0.00037

single._d2.MEI	0.36762	0.49560	0.35923	0.00034
centroid._d2.EIHI	0.36913	0.48641	0.35849	0.00032
single._.EIHI	0.37173	0.48790	0.35807	0.00183
centroid._dd2.EIHI	0.36494	0.48670	0.35554	0.00114
single.d.EIHIMEI	0.34080	0.48593	0.35114	0.00033
single.d2.EIHI	0.35813	0.48870	0.34778	0.00035
single.dd2.EIHI	0.35047	0.48815	0.34762	0.00037
single.d2.EIHIMEI	0.35767	0.48911	0.34762	0.00040
single._d.EIHI	0.35301	0.48916	0.34677	0.00035
single_dd2.MEI	0.35308	0.49147	0.34383	0.00038
single.dd2.MEI	0.35221	0.49038	0.34314	0.00030
single._d2.EIHI	0.35221	0.49041	0.34259	0.00116
single._dd2.EIHI	0.35068	0.49034	0.34257	0.00041
single._d.EIHIMEI	0.35221	0.49049	0.34255	0.00032
single.dd2.EIHIMEI	0.34848	0.49060	0.34101	0.00043
single._d2.EIHIMEI	0.35068	0.49078	0.34091	0.00057
single._dd2.EIHIMEI	0.34988	0.49075	0.34074	0.00057

Table 28: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 18-19-20.

	Purity	Fmeasure	RI	Time
$d\rho, \rho=1e+08$	0.758	0.633	0.754	7.76938
$d\rho, \rho=100$	0.642	0.615	0.727	6.69048
$dk, k=2$	0.652	0.573	0.715	1.66545
$d\rho, \rho=1$	0.672	0.556	0.699	6.93797
$d\rho, \rho=0.001$	0.673	0.554	0.698	7.45932
L^2	0.671	0.554	0.698	1.56638
$d\rho, \rho=0.02$	0.669	0.550	0.695	7.21358
$dk, k=3$	0.543	0.468	0.642	1.75887

Table 29: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 18-19-20.

	Purity	Fmeasure	RI	Time
kmeans	0.818	0.7512	0.821	1.33809
hclust	0.813	0.744	0.816	1.30426
kmeans ++	0.801	0.739	0.808	1.55496
random	0.772	0.721	0.790	1.31402

Table 30: Mean results for S 21-22-23 considering Euclidean distance (gray), Mahalanobis distance (pink), a gaussian kernel (yellow), a polynomial kernel (blue), kernel k-means for initialization (green) and k-means for initialization (orange).

	Purity	Fmeasure	RI	Time
kmeans._.EIHI	0.99660	0.99718	0.99759	0.00143
kmeans._.EIHI	0.99660	0.99718	0.99759	0.00173
ward.D2._.EIHI	0.99473	0.99342	0.99512	0.00024
kmeans_dd2.MEI	0.99066	0.98137	0.98775	0.00219
kmeans_dd2.MEI	0.99066	0.98137	0.98775	0.00216
svc_dd2.MEI	0.99053	0.98111	0.98758	0.00527
kmeans.dd2.MEI	0.98979	0.97962	0.98659	0.00193
kmeans.dd2.MEI	0.98979	0.97962	0.98659	0.00195
svc_dd2.MEI	0.98739	0.97778	0.98495	0.00652
ward.D2_dd2.MEI	0.98720	0.97507	0.98354	0.00036
kkmeans._.EIHI	0.97380	0.97602	0.98014	0.00748
svc.dd2.MEI	0.98000	0.96944	0.97836	0.00447
svc.dd2.MEI	0.98013	0.96918	0.97834	0.00553
average_dd2.MEI	0.97660	0.96935	0.97766	0.00029
kkmeans_dd2.MEI	0.97433	0.96239	0.97346	0.01846
ward.D2.dd2.MEI	0.97853	0.95847	0.97256	0.00024
ward.D2._dd2.EIHIMEI	0.97627	0.95424	0.96975	0.00040
svc._d.MEI	0.97547	0.95201	0.96842	0.00509
kmeans._d.MEI	0.97520	0.95150	0.96809	0.00211
kmeans._d.MEI	0.97520	0.95150	0.96809	0.00205
average._d.MEI	0.96994	0.94540	0.96337	0.00113
svc._d.MEI	0.96847	0.94314	0.96211	0.00610
ward.D2._d.MEI	0.96427	0.93247	0.95527	0.00033
average.dd2.MEI	0.94933	0.93932	0.95450	0.00092
kmeans._dd2.EIHIMEI	0.96447	0.93025	0.95399	0.00317
kmeans._dd2.EIHIMEI	0.96447	0.93025	0.95399	0.00347
complete_dd2.MEI	0.95453	0.93137	0.95260	0.00039
centroid._d.MEI	0.94980	0.93178	0.95049	0.00101
complete._d.MEI	0.95947	0.92370	0.94946	0.00030
svc._dd2.EIHIMEI	0.95827	0.92379	0.94888	0.00968
svc._dd2.EIHIMEI	0.95727	0.92156	0.94747	0.01115
average._dd2.EIHIMEI	0.93167	0.92387	0.94144	0.00039
spc.dd2.MEI	0.92793	0.92546	0.94046	0.07889
kkmeans._.EIHI	0.91680	0.92774	0.93905	0.00813
spc_dd2.MEI	0.92267	0.92482	0.93879	0.07598
ward.D2.dd2.EIHIMEI	0.94966	0.90860	0.93862	0.00037
spc._dd2.EIHIMEI	0.91547	0.90851	0.92805	0.06987

kkmeans.dd2.MEI	0.92067	0.88688	0.92190	0.01947
kmeans.dd2.EIHIMEI	0.93527	0.87602	0.91783	0.00290
kmeans.dd2.EIHIMEI	0.93527	0.87602	0.91783	0.00388
spc._d.MEI	0.90681	0.89275	0.91737	0.08136
spc._.EIHI	0.88941	0.90011	0.91719	0.01046
svc.dd2.EIHIMEI	0.93320	0.87462	0.91693	0.00889
kkmeans._dd2.EIHIMEI	0.91907	0.87795	0.91614	0.02143
svc.dd2.EIHIMEI	0.93273	0.87391	0.91600	0.00899
spc.dd2.EIHIMEI	0.89880	0.88510	0.91045	0.07526
kkmeans_dd2.MEI	0.91753	0.87005	0.91028	0.01630
average.dd2.EIHIMEI	0.90247	0.87711	0.90823	0.00117
kkmeans.dd2.MEI	0.90953	0.86162	0.90364	0.01645
complete._dd2.EIHIMEI	0.90020	0.85944	0.90174	0.00052
complete.dd2.MEI	0.89354	0.85376	0.89787	0.00033
kkmeans._d.MEI	0.90607	0.84851	0.89618	0.01561
svc._.EIHI	0.84101	0.86919	0.88724	0.00608
kkmeans._d.MEI	0.88347	0.83556	0.88703	0.02054
average._d.EIHIMEI	0.89200	0.82860	0.88234	0.00039
single._.EIHI	0.83874	0.87279	0.87611	0.00033
spc._d.EIHIMEI	0.86360	0.83388	0.87439	0.07371
kmeans._d.EIHIMEI	0.89487	0.80943	0.87323	0.00304
kmeans._d.EIHIMEI	0.89487	0.80943	0.87323	0.00311
svc._d.EIHIMEI	0.89420	0.80882	0.87276	0.00883
svc._d.EIHIMEI	0.89187	0.80566	0.87062	0.00891
kkmeans.dd2.EIHIMEI	0.87313	0.81111	0.87039	0.02182
kkmeans._dd2.EIHIMEI	0.87240	0.81141	0.86894	0.01728
ward.D2._d.EIHIMEI	0.87786	0.80676	0.86788	0.00035
centroid.dd2.MEI	0.80133	0.82959	0.85512	0.00029
svc._.EIHI	0.79294	0.83044	0.85004	0.00701
complete._.EIHI	0.78660	0.82539	0.84592	0.00030
centroid_dd2.MEI	0.77966	0.82088	0.84520	0.00098
kkmeans._d.EIHIMEI	0.84347	0.76339	0.83834	0.02165
kkmeans.dd2.EIHIMEI	0.83474	0.76534	0.83664	0.01791
complete.dd2.EIHIMEI	0.83114	0.76814	0.83599	0.00110
spc.d.EIHIMEI	0.80594	0.78368	0.82904	0.08230
kkmeans._d.EIHIMEI	0.82727	0.74627	0.82607	0.01718
kmeans.d.EIHIMEI	0.83927	0.72236	0.81362	0.00232
kmeans.d.EIHIMEI	0.83927	0.72236	0.81362	0.00236
svc.d.EIHIMEI	0.83820	0.72192	0.81344	0.00626
svc.d.EIHIMEI	0.83100	0.71384	0.80776	0.00643
complete._d.EIHIMEI	0.80340	0.71589	0.80292	0.00034
centroid._d.EIHIMEI	0.78560	0.74332	0.80248	0.00039
kmeans._d2.MEI	0.80994	0.68959	0.79421	0.00226
kmeans._d2.MEI	0.80994	0.68959	0.79421	0.00235

svc._d2.MEI	0.80294	0.68427	0.79007	0.00571
svc._d2.MEI	0.79760	0.68060	0.78745	0.00640
kkmeans.d.EIHIMEI	0.78107	0.68359	0.78206	0.01705
centroid._dd2.EIHIMEI	0.68500	0.74367	0.77615	0.00049
kmeans._d2.EIHIMEI	0.78713	0.66334	0.77529	0.00316
kmeans._d2.EIHIMEI	0.78713	0.66334	0.77529	0.00315
average.d.EIHIMEI	0.77674	0.67842	0.77364	0.00025
svc._d2.EIHIMEI	0.78213	0.66065	0.77301	0.01158
ward.D2.d.EIHIMEI	0.76994	0.67154	0.77078	0.00028
average._.EIHI	0.70966	0.76891	0.76818	0.00022
svc._d2.EIHIMEI	0.77293	0.65185	0.76661	0.01183
svc._dd2.EIHI	0.79260	0.65765	0.76634	0.01061
kmeans._dd2.EIHI	0.79373	0.65753	0.76603	0.00286
kmeans._dd2.EIHI	0.79373	0.65753	0.76603	0.00291
svc.dd2.EIHI	0.79233	0.65647	0.76595	0.00885
svc.dd2.EIHI	0.79366	0.65717	0.76589	0.00896
kmeans.dd2.EIHI	0.79326	0.65695	0.76564	0.00262
kmeans.dd2.EIHI	0.79326	0.65695	0.76564	0.00278
svc._dd2.EIHI	0.79080	0.65399	0.76400	0.01029
kkmeans.d.EIHIMEI	0.74667	0.65080	0.76144	0.01926
ward.D2._dd2.EIHI	0.77653	0.66255	0.76110	0.00035
ward.D2.dd2.EIHI	0.77520	0.66095	0.76002	0.00109
ward.D2._d2.MEI	0.75420	0.64989	0.75965	0.00026
spc._dd2.EIHI	0.75394	0.69919	0.75871	0.07454
centroid.dd2.EIHIMEI	0.68274	0.72080	0.75626	0.00103
kkmeans._d2.MEI	0.74893	0.64539	0.75586	0.01777
spc.dd2.EIHI	0.74607	0.69627	0.75475	0.07743
kkmeans._d2.EIHIMEI	0.74087	0.63261	0.74932	0.01713
complete.d.EIHIMEI	0.74388	0.64801	0.74900	0.00032
ward.D2._d2.EIHIMEI	0.73620	0.63206	0.74631	0.00038
kkmeans.dd2.EIHI	0.73907	0.62855	0.74601	0.02294
kkmeans._dd2.EIHI	0.74274	0.62933	0.74371	0.02466
centroid.d.EIHIMEI	0.75774	0.65104	0.74365	0.00040
average._d2.MEI	0.71334	0.65171	0.74000	0.00036
kkmeans._dd2.EIHI	0.72034	0.61916	0.73814	0.01836
average._dd2.EIHI	0.73687	0.65920	0.73524	0.00034
kkmeans.dd2.EIHI	0.70927	0.61804	0.73480	0.01750
average.dd2.EIHI	0.73693	0.65755	0.73373	0.00031
kkmeans._d2.MEI	0.70360	0.59873	0.72840	0.02141
complete._d2.MEI	0.71193	0.61389	0.72816	0.00097
kkmeans._d2.EIHIMEI	0.70567	0.59684	0.72762	0.02303
svc._d.EIHI	0.73467	0.58744	0.71682	0.00913
svc.d.EIHI	0.73340	0.58605	0.71636	0.00739
svc._d.EIHI	0.73320	0.58612	0.71621	0.00914

svc.d.EIHI	0.73287	0.58558	0.71572	0.00688
kmeans._d.EIHI	0.73254	0.58508	0.71539	0.00236
kmeans._d.EIHI	0.73254	0.58508	0.71539	0.00263
complete._dd2.EIHI	0.71633	0.61527	0.71525	0.00048
kmeans.d.EIHI	0.73240	0.58485	0.71519	0.00211
kmeans.d.EIHI	0.73240	0.58485	0.71519	0.00224
complete.dd2.EIHI	0.71493	0.61460	0.71472	0.00032
spc._d2.MEI	0.69307	0.61711	0.71383	0.07462
average._d2.EIHIMEI	0.67347	0.63347	0.71255	0.00111
complete._d2.EIHIMEI	0.68720	0.60073	0.71193	0.00044
ward.D2._.EIHIMEI	0.68046	0.57701	0.70824	0.00028
kkmeans._d.EIHI	0.67947	0.56659	0.70583	0.01657
kkmeans.d.EIHI	0.67313	0.56659	0.70441	0.01615
spc._d2.EIHIMEI	0.68873	0.62019	0.70414	0.07647
svc._.EIHIMEI	0.68560	0.55533	0.70394	0.00797
kmeans._.EIHIMEI	0.68567	0.55440	0.70378	0.00209
kmeans._.EIHIMEI	0.68567	0.55440	0.70378	0.00218
svc._.EIHIMEI	0.68380	0.55306	0.70250	0.00846
centroid._.EIHIMEI	0.66914	0.58191	0.70227	0.00039
average._.EIHIMEI	0.67114	0.57634	0.70223	0.00040
kkmeans._.EIHIMEI	0.67134	0.55958	0.70193	0.01676
ward.D2.d.EIHI	0.70366	0.57755	0.69965	0.00030
ward.D2._d.EIHI	0.70320	0.57553	0.69813	0.00031
complete._.EIHIMEI	0.67427	0.55932	0.69141	0.00038
complete._d.EIHI	0.70667	0.56904	0.68644	0.00032
spc.d.EIHI	0.66327	0.58515	0.68630	0.07937
centroid._d2.MEI	0.64520	0.63887	0.68604	0.00033
complete.d.EIHI	0.70593	0.56694	0.68457	0.00126
spc._d.EIHI	0.65194	0.58348	0.67751	0.07721
average._d.EIHI	0.69093	0.56543	0.67679	0.00027
average.d.EIHI	0.69079	0.56304	0.67511	0.00037
kkmeans.d2.EIHIMEI	0.64173	0.51571	0.67082	0.01732
kkmeans._d.EIHI	0.62060	0.51523	0.66763	0.02101
kkmeans.d.EIHI	0.61493	0.50947	0.66581	0.01996
kmeans.d2.EIHIMEI	0.65466	0.51169	0.66528	0.00255
kmeans.d2.EIHIMEI	0.65466	0.51169	0.66528	0.00253
svc.d2.EIHIMEI	0.65213	0.51080	0.66441	0.00891
svc.d2.EIHIMEI	0.65293	0.51098	0.66433	0.00836
ward.D2.d2.EIHIMEI	0.64466	0.52515	0.66343	0.00035
centroid._.EIHI	0.62621	0.70453	0.65804	0.00027
single._d.MEI	0.60394	0.67851	0.64667	0.00031
centroid._d.EIHI	0.67740	0.55333	0.64125	0.00036
centroid.d.EIHI	0.67720	0.55345	0.64121	0.00027
single.dd2.MEI	0.59014	0.67540	0.63783	0.00035

kkmeans.d2.EIHIMEI	0.57514	0.46154	0.63616	0.02049
kkmeans._d2.EIHI	0.59873	0.47576	0.63438	0.01678
spc._.EIHIMEI	0.61673	0.53950	0.63070	0.09195
kkmeans.d2.EIHI	0.59500	0.46907	0.62920	0.01686
centroid.dd2.EIHI	0.60527	0.60209	0.62886	0.00036
centroid._dd2.EIHI	0.60460	0.60166	0.62810	0.00036
kkmeans._.EIHIMEI	0.54127	0.44649	0.62517	0.01853
complete.d2.EIHIMEI	0.60560	0.51320	0.62514	0.00033
1svc._d2.EIHI	0.61114	0.47697	0.62155	0.01031
kmeans._d2.EIHI	0.61120	0.47791	0.62119	0.00233
kmeans._d2.EIHI	0.61120	0.47791	0.62119	0.00257
ward.D2.d2.EIHI	0.60100	0.48746	0.62113	0.00030
svc._d2.EIHI	0.61134	0.47724	0.62084	0.01053
svc.d2.EIHI	0.60894	0.47695	0.62058	0.00762
svc.d2.EIHI	0.60960	0.47601	0.62031	0.00766
kmeans.d2.EIHI	0.60927	0.47706	0.61997	0.00231
kmeans.d2.EIHI	0.60927	0.47706	0.61997	0.00229
ward.D2._d2.EIHI	0.60067	0.48853	0.61898	0.00033
kkmeans.d2.EIHI	0.54060	0.43205	0.61416	0.02087
single_dd2.MEI	0.55973	0.65349	0.60932	0.00037
kkmeans._d2.EIHI	0.52734	0.42173	0.60741	0.02184
spc.d2.EIHIMEI	0.58893	0.51758	0.60672	0.08415
single.d.EIHIMEI	0.55181	0.62274	0.59171	0.00030
average.d2.EIHIMEI	0.56946	0.51644	0.58363	0.00031
single.dd2.EIHIMEI	0.52901	0.62170	0.56840	0.00039
single._dd2.EIHIMEI	0.51334	0.61593	0.55300	0.00127
centroid._d2.EIHIMEI	0.52560	0.57449	0.54800	0.00038
complete.d2.EIHI	0.54373	0.47653	0.54751	0.00028
complete._d2.EIHI	0.54440	0.47632	0.54496	0.00037
spc._d2.EIHI	0.53120	0.48999	0.54152	0.08383
spc.d2.EIHI	0.51793	0.48562	0.52776	0.07260
centroid.d2.EIHIMEI	0.52526	0.49742	0.52190	0.00027
average.d2.EIHI	0.49706	0.48076	0.49323	0.00105
average._d2.EIHI	0.49960	0.48080	0.49230	0.00040
single._d.EIHI	0.48027	0.52637	0.48666	0.00106
single._d.EIHIMEI	0.46421	0.57364	0.48645	0.00048
single.d.EIHI	0.47440	0.52322	0.48104	0.00027
single._.EIHIMEI	0.48747	0.50940	0.47662	0.00044
centroid._d2.EIHI	0.46187	0.48112	0.44775	0.00031
centroid.d2.EIHI	0.46047	0.48070	0.44741	0.00030
single._dd2.EIHI	0.42347	0.52904	0.42820	0.00108
single.dd2.EIHI	0.42327	0.52892	0.42796	0.00044
single.d2.EIHIMEI	0.36914	0.48780	0.35539	0.00035
single._d2.EIHI	0.36814	0.48785	0.35446	0.00103

single.d2.EIHI	0.36814	0.48784	0.35445	0.00107
single._d2.MEI	0.35415	0.48993	0.34384	0.00035
single._d2.EIHIMEI	0.35334	0.49031	0.34318	0.00040

Table 31: Mean values of Purity, F-measure, Rand Index and execution time for the functional k -means procedure (Martino et al. (2019)) with truncated Mahalanobis distance, generalized Mahalanobis distance and L^2 distance to simulated data from S 21-22-23.

	Purity	Fmeasure	RI	Time
L^2	0.805	0.706	0.800	1.54671
$d\rho$, $\rho=0.001$	0.802	0.705	0.799	6.26749
$d\rho$, $\rho=0.02$	0.798	0.701	0.796	6.31080
$d\rho$, $\rho=1$	0.796	0.696	0.793	6.50564
$d\rho$, $\rho=1e+08$	0.789	0.666	0.778	8.02051
$d\rho$, $\rho=100$	0.677	0.619	0.732	6.04808
dk , $k=2$	0.655	0.588	0.726	1.75120
dk , $k=3$	0.578	0.510	0.672	1.80598

Table 32: Mean values of Purity, F-measure, Rand Index and execution time for the test based k -means procedure (Zambom et al. (2019)) with four different initialization to simulated data from S 21-22-23.

	Purity	Fmeasure	RI	Time
kmeans ++	0.949	0.906	0.937	0.83632
hclust	0.948	0.903	0.936	0.93619
kmeans	0.946	0.902	0.935	0.68308
random	0.921	0.878	0.915	1.14316

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