Supplementary Material
Time Series Analysis of Rankings: A GARCH-Type Approach

Luiza S.C. Piancastelli*, Wagner Barreto-Souza †

1 Monte Carlo EM Algorithm

Figure 1 summarizes the simulation study comparing informed (`Imputed`) and random (`Random`) initializations for the MCEM, as described in section 4.1. The left panel shows the distribution of iterations until convergence, and the right panel shows the corresponding final parameter estimates. This study employs R-GARCH(1,0) simulated data with 20% randomly introduced missingness and one hundred Monte Carlo replicates.

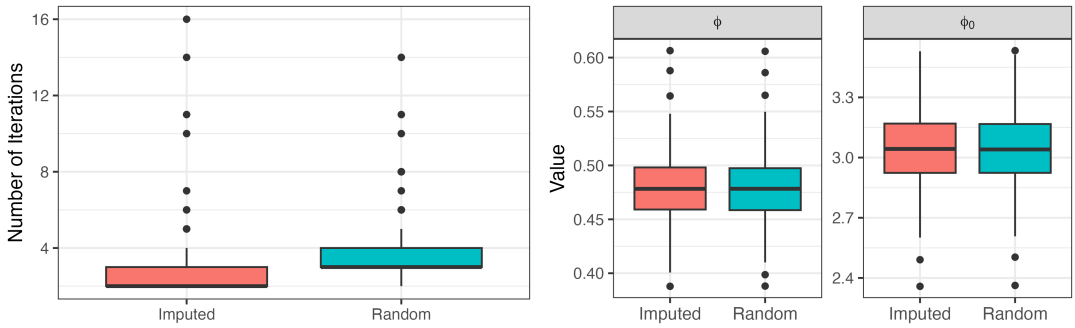

Figure 1: Comparison of MCEM initialisation strategies. Boxplots show the distribution of the number of iterations until convergence (left), and parameter estimates (right) for each method. `Imputed`: uniform completion of missing ranks followed by MLE, `Random`: random initial guesses.

2 Complete Tennis Ranking Analysis

Table 1 includes the MLEs and standard errors of Kendall and Hamming models of order $p = 3, q = 0$ fitted to the data application of section 5.1.

References

Holý, V. and Zouhar, J. (2022). Modelling time-varying rankings with autoregressive and score-driven dynamics. *Journal of the Royal Statistical Society Series C: Applied Statistics*, 71(5):1427–1450.

*Email: luiza.piancastelli@ucd.ie
†Email: wagner.barreto-souza@ucd.ie

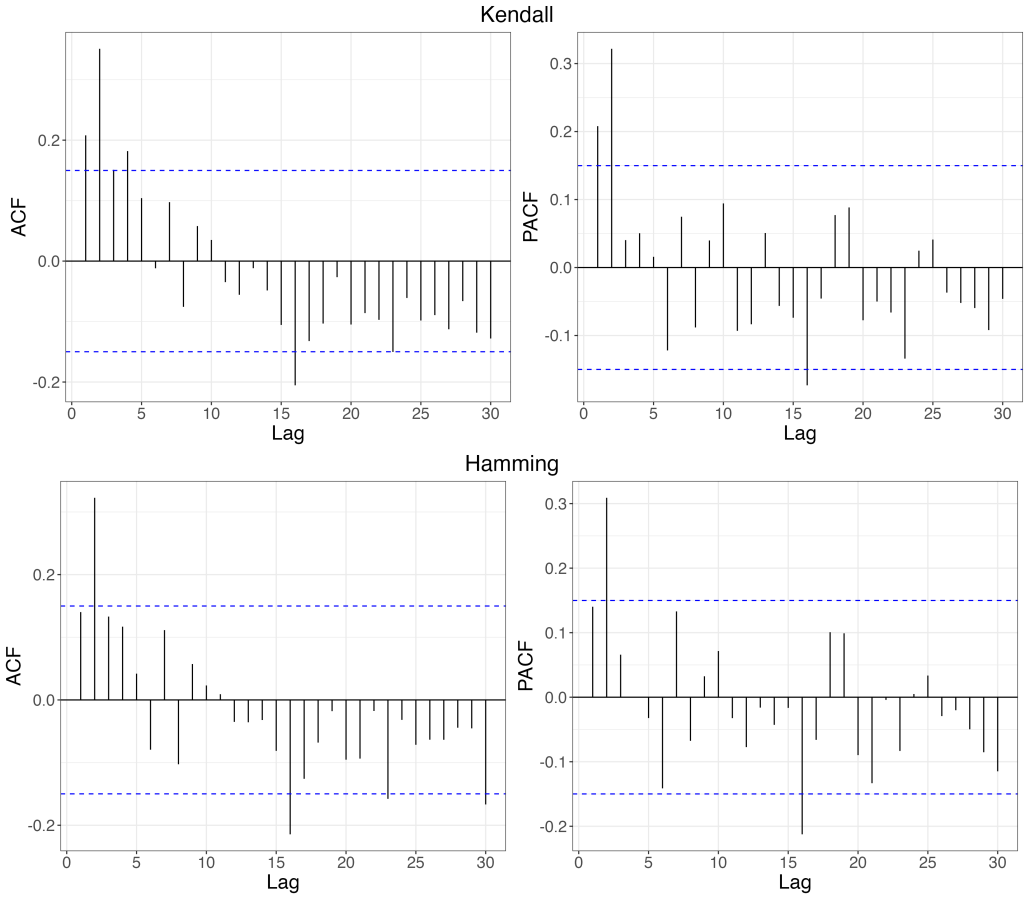

Figure 2: Autocorrelation function (ACF) and partial autocorrelation function (PACF) of the lag-one Kendall and Hamming distance trajectories $d(\pi_t, \pi_{t-1}), t = 2, \ldots, 172$, computed for the tennis rankings data.

	ϕ_0	ϕ_1	ϕ_2	ϕ_3
Kendall	32.080 (2.345)	0.274 (0.021)	0.377 (0.020)	0.063 (0.018)
Hamming	6.955 (0.847)	0.070 (0.038)	0.320 (0.038)	0.061 (0.041)

Table 1: Maximum likelihood estimates and their respective standard errors (in parentheses) of Kendall and Hamming R-GARCH(3,0) models fitted to the tennis rankings data.

Liu, M., Li, Q., and Zhu, F. (2022a). Modeling air quality level with a flexible categorical autoregression. *Stochastic Environmental Research and Risk Assessment*, 36(9):2835–2845.

Liu, M., Zhu, F., and Zhu, K. (2022b). Modeling normalcy-dominant ordinal time series: an application to air quality level. *Journal of Time Series Analysis*, 43(3):460–478.

Vitelli, V., Sørensen, Ø., Crispino, M., Frigessi, A., and Arjas, E. (2018). Probabilistic preference learning with the mallows rank model. *Journal of Machine Learning Research*, 18(158):1–49.