Supplementary material for "A (dual) network model for heat transfer in porous media"

S1 Permeability comparison Berea sandstone sample

We estimate the intrinsic Darcy permeability $\mathbf{K} = \mathrm{diag}(K_x, K_y, K_z)$ of the 200^3 voxel sample of Berea sandstone used in the numerical analysis in this work. Firstly, we estimated $\mathbf{K}$ using `OpenFOAM`, as described in [2, 3]. Considering only the void space voxels as computational grid, which we refined once such that each voxel is discretized with eight grid cells, we prescribed a fixed pressure difference between the cube-shaped sample's inlet and outlet in x, y, and z-direction. We enforced no-flow, no-slip boundary conditions on the lateral sides. The mean Darcy velocity is computed in the main flow direction to evaluate $\mathbf{K}$. Secondly, using the network model with all solid nodes and connections removed, we apply a pressure drop between the pores at the inlet and the outlet and close all remaining pores at the domain boundary. The total fluid flow is computed at the outlet pores. Dividing the total flow by the CT-image's side area of $1.14 \times 10^{-6}\,\mathrm{m}^2$ yields an averaged Darcy velocity from which the intrinsic permeability is estimated using Darcy's law. The results of both simulations are given in Table 1. For comparison, we also provide the results of a Lattice-Boltzmann (LB) simulation for the full 400^3 Berea sandstone sample reported in [1]. The `OpenFOAM` calculations slightly overestimate the LB values by around 7 % on average but this

Table 1: **Permeability estimates for Berea sandstone sample.** Intrinsic Darcy permeability for the three principal directions estimated by `OpenFOAM` and the pore-network model (PNM) for a 200^3 voxel sample and by [1] for the full 400^3 voxel sample using a Lattice-Boltzmann method.

direction	`OpenFOAM` (200^3)	PNM (200^3)	LB (400^3)	error PNM-`OpenFOAM`
	$1 \times 10^{-12}\,\mathrm{m}^2$			%
x	1.41	1.09	1.34	−22.40
y	1.41	1.32	1.29	−6.86
z	1.24	1.18	1.18	−4.73

could be related to the smaller sample size (200^3 vs. 400^3 voxels) and the sample's heterogeneity. Table 1 shows that the PNM's permeability values agree well with the results of `OpenFOAM`. For the z-direction, the difference is less than 5 %.

References

[1] H. Dong and M. J. Blunt. Pore-network extraction from micro-computerized-tomography images. Physical Review E, 80(3), 2009.

[2] R. Guibert, M. Nazarova, P. Horgue, G. Hamon, P. Creux, and G. Debenest. Computational Permeability Determination from Pore-Scale Imaging: Sample Size, Mesh and Method Sensitivities. Transport in Porous Media, 107(3):641–656, 2015.

[3] B. P. Muljadi, M. J. Blunt, A. Q. Raeini, and B. Bijeljic. The impact of porous media heterogeneity on non-darcy flow behaviour from pore-scale simulation. Advances in Water Resources, 95:329–340, 2016.