

Optimal Control of Boolean Control Networks: A Data-Driven Perspective

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Appendix A Example 1: λ switch

Table A1 reports the complete dataset collected from the experiments associated with Example 1. Each row corresponds to a distinct experiment, and each column represents a specific time instant. The entries of the table contain the values of both the state and the input recorded at the corresponding time.

Table A2 presents the solutions to the finite-horizon data-optimal control problem for all possible initial states. The entries in the third row correspond to the data-optimal control sequence, i.e., the values of $\mathbf{u}(0) \times \mathbf{u}(1) \times \dots \times \mathbf{u}(T-1)$.

Experiment	$\frac{\mathbf{x}_d^i(0)}{\mathbf{u}_d^i(0)}$	$\frac{\mathbf{x}_d^i(1)}{\mathbf{u}_d^i(1)}$	$\frac{\mathbf{x}_d^i(2)}{\mathbf{u}_d^i(2)}$	$\frac{\mathbf{x}_d^i(3)}{\mathbf{u}_d^i(3)}$	$\frac{\mathbf{x}_d^i(4)}{\mathbf{u}_d^i(4)}$	$\frac{\mathbf{x}_d^i(5)}{\mathbf{u}_d^i(5)}$	$\frac{\mathbf{x}_d^i(6)}{\mathbf{u}_d^i(6)}$	$\frac{\mathbf{x}_d^i(7)}{\mathbf{u}_d^i(7)}$
1	$\frac{\delta_{32}^3}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{15}}{\delta_2^1}$	$\frac{\delta_{32}^{25}}{\delta_2^1}$	$\frac{\delta_{32}^{28}}{\delta_2^2}$	$\frac{\delta_{32}^8}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\text{—}}$
2	$\frac{\delta_{32}^2}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\text{—}}$	—	—	—	—	—
3	$\frac{\delta_{32}^4}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\text{—}}$	—	—	—	—	—
4	$\frac{\delta_{32}^{17}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{15}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\text{—}}$	—	—	—
5	$\frac{\delta_{32}^{22}}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	—	—	—	—	—	—
6	$\frac{\delta_{32}^{30}}{\delta_2^1}$	$\frac{\delta_{32}^{11}}{\delta_2^1}$	$\frac{\delta_{32}^{26}}{\delta_2^1}$	$\frac{\delta_{32}^4}{\delta_2^1}$	—	—	—	—
7	$\frac{\delta_{32}^{12}}{\delta_2^1}$	$\frac{\delta_{32}^2}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	—	—	—	—	—
8	$\frac{\delta_{32}^7}{\delta_2^2}$	$\frac{\delta_{32}^{32}}{\delta_2^2}$	—	—	—	—	—	—
9	$\frac{\delta_{32}^{29}}{\delta_2^1}$	$\frac{\delta_{32}^{27}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	—	—	—	—	—
10	$\frac{\delta_{32}^1}{\delta_2^2}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{15}}{\delta_2^1}$	$\frac{\delta_{32}^{25}}{\delta_2^1}$	—	—	—	—
11	$\frac{\delta_{32}^6}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	—	—	—	—	—	—

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Experiment	$\frac{\mathbf{x}_d^i(0)}{\mathbf{u}_d^i(0)}$	$\frac{\mathbf{x}_d^i(1)}{\mathbf{u}_d^i(1)}$	$\frac{\mathbf{x}_d^i(2)}{\mathbf{u}_d^i(2)}$	$\frac{\mathbf{x}_d^i(3)}{\mathbf{u}_d^i(3)}$	$\frac{\mathbf{x}_d^i(4)}{\mathbf{u}_d^i(4)}$	$\frac{\mathbf{x}_d^i(5)}{\mathbf{u}_d^i(5)}$	$\frac{\mathbf{x}_d^i(6)}{\mathbf{u}_d^i(6)}$	$\frac{\mathbf{x}_d^i(7)}{\mathbf{u}_d^i(7)}$
12	$\frac{\delta_{32}^5}{\delta_2^2}$	$\frac{\delta_{32}^{32}}{\delta_2^2}$	$\frac{\delta_{32}^{15}}{\delta_2^2}$	$\frac{\delta_{32}^4}{\delta_2^2}$	$\frac{\delta_{32}^{15}}{\delta_2^2}$	$\frac{\delta_{32}^4}{\delta_2^2}$	$\frac{\delta_{32}^{15}}{\delta_2^2}$	$\frac{\delta_{32}^4}{\delta_2^2}$
13	$\frac{\delta_{32}^{16}}{\delta_2^1}$	$\frac{\delta_{32}^9}{\delta_2^1}$	$\frac{\delta_{32}^{26}}{\delta_2^1}$	$\frac{\delta_{32}^4}{\delta_2^1}$	$\frac{\delta_{32}^{16}}{\delta_2^1}$	$\frac{\delta_{32}^9}{\delta_2^1}$	$\frac{\delta_{32}^{26}}{\delta_2^1}$	$\frac{\delta_{32}^4}{\delta_2^1}$
14	$\frac{\delta_{32}^{20}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{20}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{20}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{20}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$
15	$\frac{\delta_{32}^{18}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{18}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{18}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$	$\frac{\delta_{32}^{18}}{\delta_2^2}$	$\frac{\delta_{32}^{24}}{\delta_2^2}$
16	$\frac{\delta_{32}^{19}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{19}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{19}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{19}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$
17	$\frac{\delta_{32}^{10}}{\delta_2^1}$	$\frac{\delta_{32}^2}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	$\frac{\delta_{32}^{10}}{\delta_2^1}$	$\frac{\delta_{32}^2}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	$\frac{\delta_{32}^{10}}{\delta_2^1}$	$\frac{\delta_{32}^2}{\delta_2^1}$
18	$\frac{\delta_{32}^{21}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{15}}{\delta_2^1}$	$\frac{\delta_{32}^{21}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{15}}{\delta_2^1}$	$\frac{\delta_{32}^{21}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$
19	$\frac{\delta_{32}^{23}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{23}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{23}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$	$\frac{\delta_{32}^{23}}{\delta_2^1}$	$\frac{\delta_{32}^{32}}{\delta_2^1}$
20	$\frac{\delta_{32}^{13}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\delta_2^2}$	$\frac{\delta_{32}^{13}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\delta_2^2}$	$\frac{\delta_{32}^{13}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\delta_2^2}$	$\frac{\delta_{32}^{13}}{\delta_2^2}$	$\frac{\delta_{32}^{31}}{\delta_2^2}$
21	$\frac{\delta_{32}^{14}}{\delta_2^1}$	$\frac{\delta_{32}^9}{\delta_2^1}$	$\frac{\delta_{32}^{26}}{\delta_2^1}$	$\frac{\delta_{32}^8}{\delta_2^1}$	$\frac{\delta_{32}^{24}}{\delta_2^1}$	$\frac{\delta_{32}^{14}}{\delta_2^1}$	$\frac{\delta_{32}^9}{\delta_2^1}$	$\frac{\delta_{32}^{26}}{\delta_2^1}$

Table A1 Data collected in the offline experiments

Initial state	δ_{32}^1	δ_{32}^2	δ_{32}^3	δ_{32}^4	δ_{32}^5	δ_{32}^6	δ_{32}^7	δ_{32}^8
Optimal cost	3.61	19.85	6.46	14.15	9.31	20.80	7.41	16.05
Optimal control	δ_{64}^{64}	δ_{64}^{33}	δ_{64}^{32}	δ_{64}^{33}	δ_{64}^{64}	δ_{64}^{33}	δ_{64}^{64}	δ_{64}^{33}

Initial state	δ_{32}^9	δ_{32}^{10}	δ_{32}^{11}	δ_{32}^{12}	δ_{32}^{13}	δ_{32}^{14}	δ_{32}^{15}	δ_{32}^{16}
Optimal cost	12.39	20.56	15.24	21.51	2.85	16.33	0.95	10.63
Optimal control	δ_{64}^9	δ_{64}^{17}	δ_{64}^9	δ_{64}^{17}	δ_{64}^{64}	δ_{64}^5	δ_{64}^{64}	δ_{64}^5

Initial state	δ_{32}^{17}	δ_{32}^{18}	δ_{32}^{19}	δ_{32}^{20}	δ_{32}^{21}	δ_{32}^{22}	δ_{32}^{23}	δ_{32}^{24}
Optimal cost	7.41	19.85	9.31	16.05	3.61	16.05	4.56	15.10
Optimal control	δ_{64}^{32}	δ_{64}^{33}	δ_{64}^{32}	δ_{64}^{33}	δ_{64}^{32}	δ_{64}^1	δ_{64}^{32}	δ_{64}^1

Initial state	δ_{32}^{25}	δ_{32}^{26}	δ_{32}^{27}	δ_{32}^{28}	δ_{32}^{29}	δ_{32}^{30}	δ_{32}^{31}	δ_{32}^{32}
Optimal cost	19.52	13.25	3.61	20.75	11.03	17.13	0.00	2.80
Optimal control	δ_{64}^{25}	δ_{64}^{17}	δ_{64}^{32}	δ_{64}^{49}	δ_{64}^{16}	δ_{64}^5	δ_{64}^{64}	δ_{64}^{64}

Table A2 Solution to the finite-horizon data-optimal control problem