

Supplementary Information includes:

Figure S1 Models used for the current distribution calculation.

Figure S2 Calculated current distribution in the free layer of the spin valve structure.

Figure S3 FMR results of the unpatterned film.

Figure S4 Magnetodynamic properties under high magnetic fields.

Table S1 Comparison of magnetic droplets in different structures.

Note S1 Description of LLG equation used in micromagnetic simulation.

Figure S5 Plots of current magnitude vs incubation time of magnetic droplet (initial: AP state).

Figure S6 Simulation of the hysteresis behavior of the droplets.

Figure S7 Simulation of the magnetic field toggling of a magnetic droplet in the DFL-MTJ.

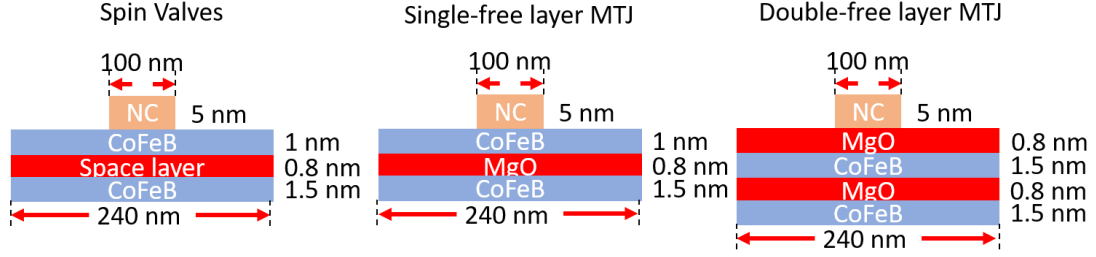


Figure S1. The models used for the current distribution calculation.

Related to Figure 4.

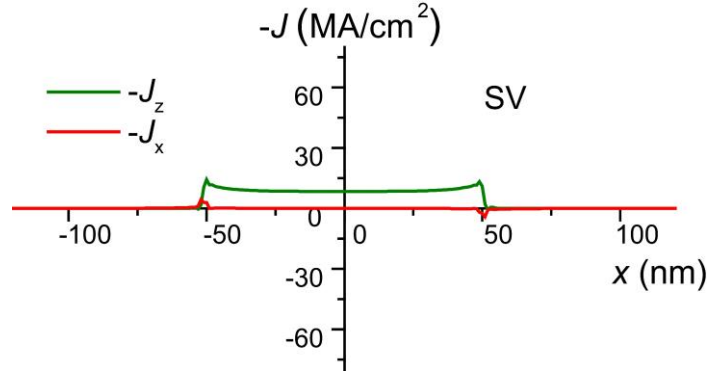


Figure S2. Calculated current distribution of nanocontact spin valve structure with a current density of 10 MA cm^{-2} . **Related to Figure 4.**

The current distributions over the MTJ are simulated by the COMSOL software. The conductivity of CoFeB is set to be $5 \times 10^5 \text{ C m}^{-1}$. For qualitative discussion in such small size (240 nm) of free-layer (simply for micromagnetic simulations), we set a $3 \times 10^3 \text{ C m}^{-1}$ conductivity of MgO (considering that we downsize the MTJ for about 50 times).

In fact, we set a spin valve model of “Nano-contact/CoFeB/Space Layer/CoFeB” with the same thickness and size of the single free layer (SFL) MTJ. The conductivity of CoFeB and Space Layer is set to be 5×10^5 and $5 \times 10^7 \text{ C m}^{-1}$, respectively. As shown in Figure S2, the

lateral current spread J_x of SV we calculated here is much smaller than that in MTJ in the main text.

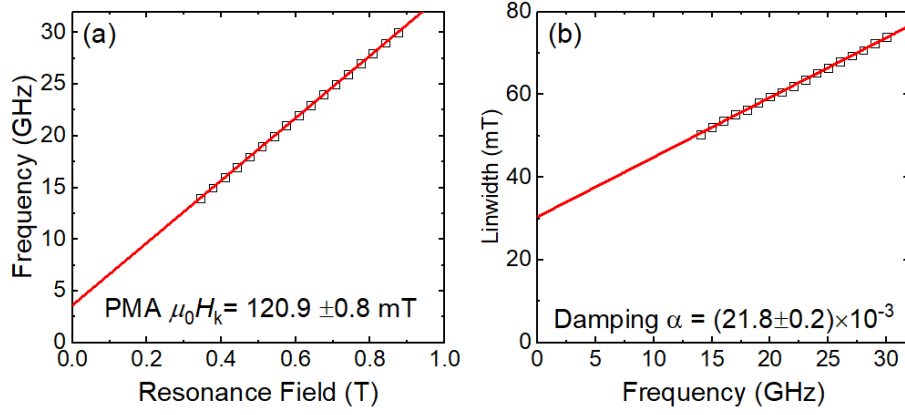


Figure S3. (a) Resonance field versus frequency (b) Frequency versus linewidth extracted from out-of-plane FMR spectra. The red line is fits based on Kittel equation. **Related to Figure 1**

To get the PMA of our double-CoFeB free layer (DFL), we conducted FMR measurements on the bulk film. The resonance field H_{res} is fitted with the out-of-plane Kittel equation

$$f = \frac{\gamma\mu_0}{2\pi} (H_{res} - H_k) \quad (\text{Equation S1})$$

where μ_0 is the permeability of free space and H_k is the perpendicular anisotropy field. $\frac{\gamma}{2\pi}$ is the gyromagnetic ratio, fitted to be 30.2 GHz/T.

The linewidth is described as

$$\Delta H = \Delta H_0 + \alpha \frac{4\pi}{\mu_0\gamma} f \quad (\text{Equation S2})$$

where ΔH_0 is the inhomogeneous broadening and $\alpha = 0.0218(2)$ is the damping constant.

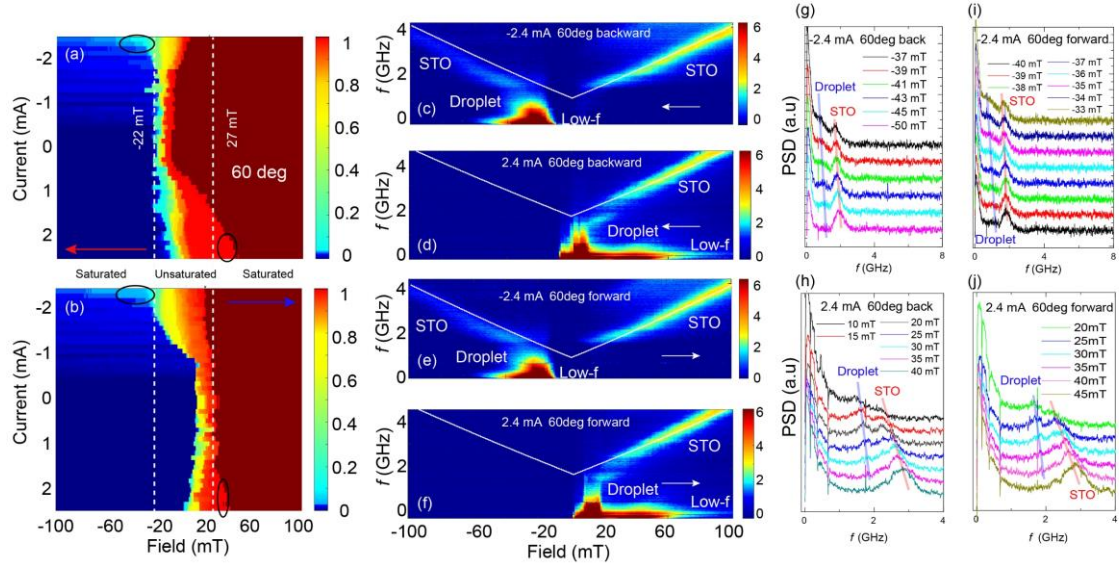


Figure S4. Current-field phase diagrams of normalized magnetoresistance with field sweep (a) backward and (b) forward at 60 degrees. The dotted lines give the boundary between the saturated and unsaturated magnetic fields for the free layer. Circles display the intermediate resistance states outside the boundary which are highly dependent on the polarized currents. (c)-(f) Frequency spectra as functions of the external perpendicular magnetic field at the angle of 60 degrees, with the current of -2.4 mA and 2.4 mA, respectively. The approximately 1 GHz droplet signal disappeared until 40 mT for 2.4 mA, -50 mT for -2.4 mA. (g)-(j) The related PSD curves which give the variations of the magnetic droplet and FMR-like mode signals along with different magnetic external fields. **Related to Figure 2.**

The high magnetic field may suppress the formation of the domain wall but not the magnetic droplet. Meanwhile, the domain wall oscillating does not show accompanied low- f noise (caused by drift resonance). We carried

out the field-sweep measurements under 60 degrees. As the normalized magneto-resistance mappings under different fields and currents are shown in Fig. S2 (a) and (b), the unsaturated magnetic fields of the free layer are between -22 and 27 mT under 60 degrees. It is noticed that there also existed the intermediate resistance states out of this magnetic range which is highly dependent on the polarized currents (spin-transfer torque: STT) as the circles shown in Fig. S2 (a) and (b). Then we focused on the microwave properties in this region. As shown in Fig. S2 (c)-(f), the ~ 1 GHz for -2.4 mA (~ 2 GHz for 2.4 mA) signal (magnetic droplet signal), low- f noise (~ 200 MHz) as well the ordinary FMR modes are observed. The plots of Fig. S2 (g)-(j) indicated that the magnetic droplet signals could exist until -50 mT for -2.4 mA and 40 mT for 2.4 mA. Meanwhile, the low- f noise roughly vanished as the intermediate resistance states disappeared. These weak droplet signals may come from the high drift resonances.

Table S1. A comparison of magnetic droplets in different structures.

Related to Figure 2.

(*o*-SV: *orthogonal spin-valve*; *p*-SV: *all perpendicular spin-valve*; SHNO: *spin hall nano-oscillator*; NC: *nano-contact*; *p*-MTJ: *all perpendicular magnetic tunnel junction*.)

Droplet in structures	NC size (D/nm)	Current density (MA cm-2)	Magnetic field	Output power	Frequency	Linewidth	Ratio compared with its FMR mode	Reference
o-SV	63	193	>0.65 T @90deg.	200 pW	24 GHz	~200 MHz	200 pW/5 pW = 40	Science 339, 15 (2013) ¹
p-SV	60	130-180	>0.3 T @30deg.	<150 pW	7-16 GHz	2-5 GHz	N/A	Phys. Rev. Lett. 120, 217204 (2018) ²
SHNO	100	153	0.06~0.12 T @82deg.	50 pW	~1 GHz	~10 MHz	50 pW/5 pW = 10	Phys. Rev. Appl. 13, 024034 (2020) ³
p-MTJ	100	5-10	0~0.1 T @30deg.	600 pW	0.9-2 GHz	~300 MHz	600 pW/2 pW = 300	This work

Note S1. The description of LLG equation used in micromagnetic simulation. **Related to Figure 4.**

Within the micromagnetic simulation, the magnetization dynamics of the free layers are determined by the following Landau–Lifshitz–Gilbert (LLG) equation with the spin-transfer torque and Zhang-Li torque included:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma\mu_0(\mathbf{m} \times \mathbf{H}_k^{eff}) + \alpha(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}) + \boldsymbol{\tau}_{STT} + \boldsymbol{\tau}_{ZL}.$$

where $\mathbf{m} = \mathbf{M}/M_S$ is the reduced magnetization at each site with saturation M_S . $\mathbf{H}_k^{eff}$ denotes the effective field, including contributions from the anisotropy field, exchange field, DMI field, interlayer coupling field, demagnetization field. α , γ and μ_0 are the damping constant, gyromagnetic ratio and vacuum magnetic permeability, respectively.

The terms of $-\gamma\mu_0(\mathbf{m} \times \mathbf{H}_k^{eff})$ and $\alpha(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t})$ describe the natural precessional damped motion of the magnetization around the effective field. The spin-transfer torque (STT) $\boldsymbol{\tau}_{STT}$ delivered by the out of plane spin-polarized current is given by:

$$\boldsymbol{\tau}_{STT} = \gamma \frac{J_z \hbar P}{2edM_S} (\mathbf{m} \times \mathbf{m}_p \times \mathbf{m})$$

where d is the thickness of the free layer, $\mathbf{m}_p$ is the polarized direction of the spin current, P is the spin-current polarization $\hbar$ is the reduced Plank constant and J_z represents the STT current density

The Zhang-Li torque is produced by the in-plane current flowing in the

free layer^{4, 5}:

$$\boldsymbol{\tau}_{ZL} = -(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{m}$$

where $\boldsymbol{u}$ is a velocity along the direction of the electron motion with magnitude $\boldsymbol{u} = \gamma j_e \hbar P / (2eM_s)$. j_e is the in-plane current density.

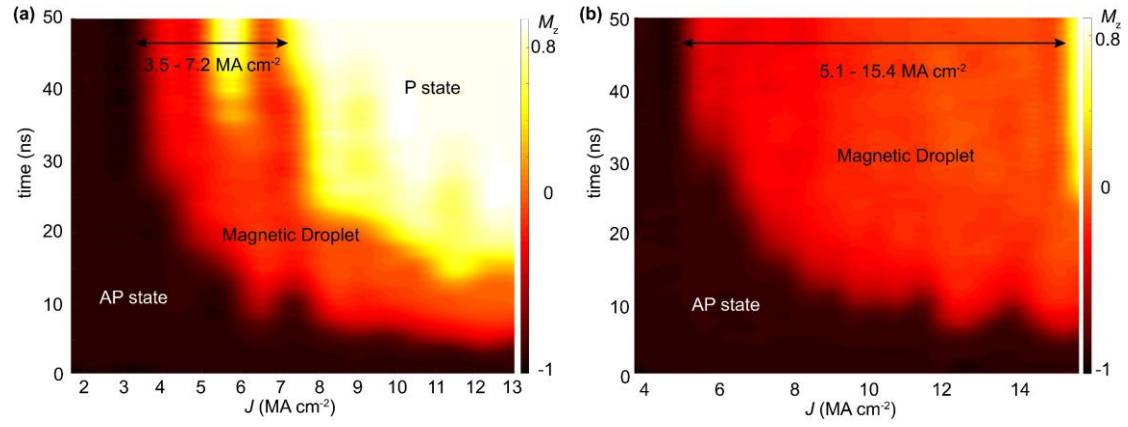


Figure S5. The plots of current magnitude vs incubation time of the nucleation magnetic droplets in SFL-MTJ, DFL-MTJ from micromagnetic simulations. The initial state is AP state. Positive current density and 10 mT were applied. **Related to Figure 4.**

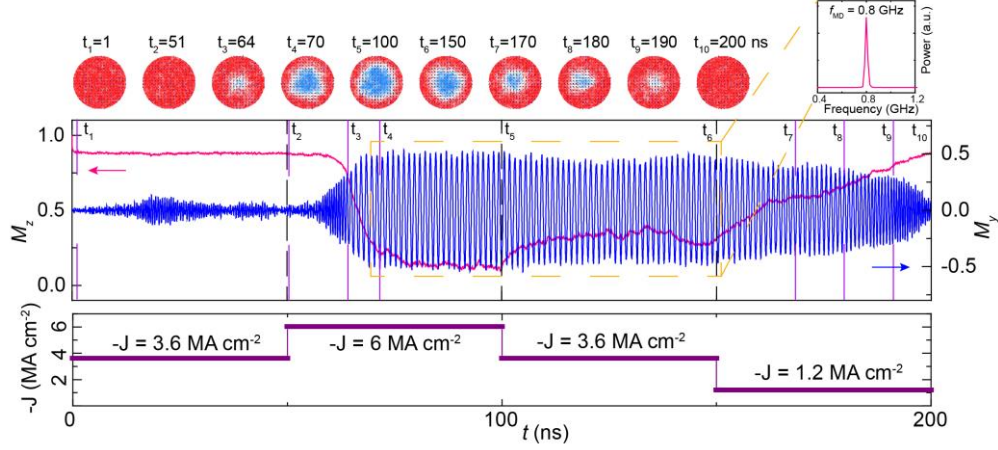


Figure S6. Nucleation and current toggling of a magnetic droplet in the DFL-MTJ. The bottom schematically gives the applied current pulses. The precession current density is less than the smallest nucleation current. As shown in the inset graph, the simulated magnetic droplet frequency is about 0.8 GHz which is similar to the experimental results. The top figures give the typical magnetization state of the selected time. **Related to Figure 4.**

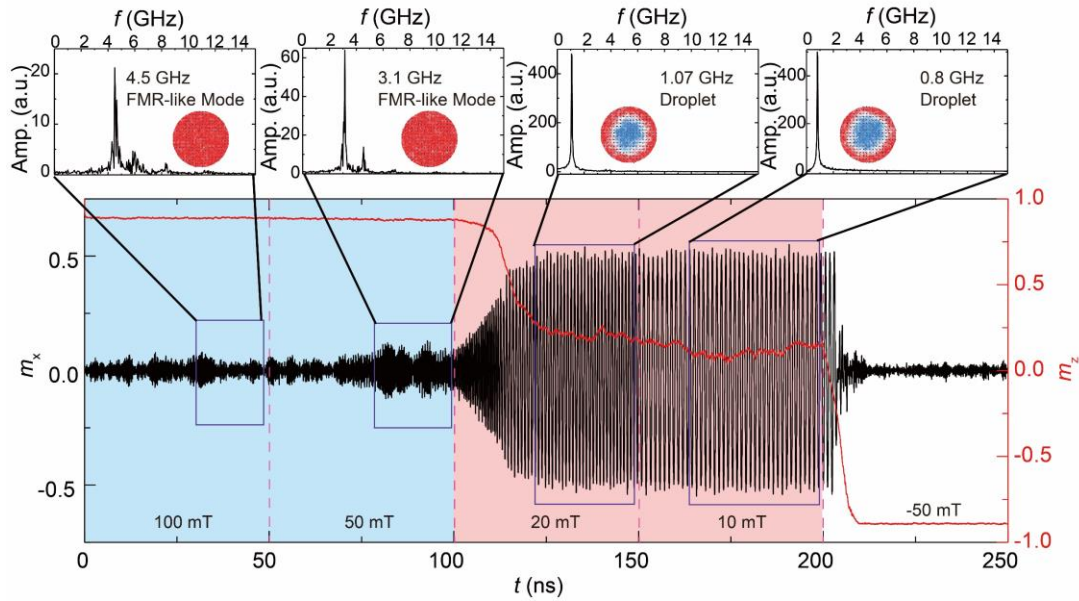


Figure S7. Nucleation and magnetic field toggling of a magnetic droplet in the DFL-MTJ. When the out-of-plane external magnetic field is large (100 mT and 50 mT), the device is under AP state and no droplet appears. The FMR mode signal is 4.5 GHz ($H = 100$ mT) and 3.1 GHz ($H = 50$ mT). When the external magnetic field is reduced, a magnetic droplet appears. Meanwhile, the precessional angle becomes much larger and the frequency is greatly decreased to 1.07 GHz ($H = 20$ mT) and 0.8 GHz ($H = 10$ mT). When the external field becomes negative, the droplet no longer exists.

Related to Figure 3.

Supplemental References:

1. M. S. M., S. S. R., P. J., A. N. T. N., C. S., P. Ye., M. P. K., I. E., E. A., D. R. K., B. S., D. A., H. M. A. and Å. J., *Science*, **2013**, 339, 1295-1298.
2. S. Chung, Q. T. Le, M. Ahlberg, A. A. Awad, M. Weigand, I. Bykova, R. Khymyn, M. Dvornik, H. Mazraati, A. Houshang, S. Jiang, T. N. A. Nguyen, E. Goering, G. Schutz, J. Grafe and J. Åkerman, *Phys. Rev. Lett.*, **2018**, 120, 217204.
3. L. Chen, S. Urazhdin, K. Zhou, Y. W. Du and R. H. Liu, *Phys Rev Appl*, **2020**, 13.
4. S. Zhang and Z. Li, *Phys. Rev. Lett.*, **2004**, 93, 127204.
5. A. Thiaville, Y. Nakatani, J. Miltat and Y. Suzuki, *Europhysics Letters (EPL)*, **2005**, 69, 990-996.