

Appendix:

“Tighter Reformulations using Classical Dawson and Sankoff Bounds for Approximating Two-Stage Chance-Constrained Programs”

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Below is the complete reformulation of model (1) using the approximation presented by equation (3), valid for all $\alpha \in (1 - \frac{1}{N|T|}, 1)$.

$$\max_{x,y} \quad \sum_{t \in T} (R_t x_t - \mathbb{E}[B_t y_t^\omega]) \tag{1a}$$

$$\text{s.t.} \quad x_t - y_t^\omega - w_t^\omega \leq M_t^\omega u_t^\omega, \quad \forall t \in T, \omega \in \Omega \tag{1b}$$

$$\left(\sum_{t \in T} (t^2 + t) \kappa_t \right) \varepsilon \geq 2 \left(\sum_{t \in T} (t + 1) \delta_t \right) - 2(S_1 + S_2) \tag{1c}$$

$$v_{tt'}^\omega \leq u_t^\omega, \quad \forall t, t' \in T, t' > t, \omega \in \Omega \tag{2a}$$

$$v_{t,t'}^\omega \leq u_{t'}^\omega, \quad \forall t, t' \in T, t' > t, \omega \in \Omega \tag{2b}$$

$$v_{tt'}^\omega \geq u_t^\omega + u_{t'}^\omega - 1, \quad \forall t, t' \in T, t' > t, \omega \in \Omega \tag{2c}$$

$$S_1 = \sum_{t \in T} \frac{1}{N} \sum_{\omega \in \Omega} u_t^\omega, \tag{2d}$$

$$S_2 = \sum_{t, t' \in T, t' > t} \frac{1}{N} \sum_{\omega \in \Omega} v_{tt'}^\omega. \tag{2e}$$

$$\delta_t \leq |T|\kappa_t, \quad t = 1, \dots, |T| - 1 \quad (3a)$$

$$\delta_t \geq S_1 - |T|(1 - \kappa_t), \quad t = 1, \dots, |T| - 1 \quad (3b)$$

$$\sum_{t \in T} \kappa_t = 1 \quad (3c)$$

$$\sum_{t \in T} (t + 1)\delta_t \leq 2(S_1 + S_2) \quad (3d)$$

$$\sum_{t \in T} (t + 1)\delta_t \geq 2(S_1 + S_2) - \alpha S_1. \quad (3e)$$

$$\kappa_t \in \{0, 1\}, \quad \forall t \in T \quad (4a)$$

$$0 \leq \delta_t \leq S_1 \leq |T|, \quad \forall t \in T \quad (4b)$$

$$0 \leq y_t^\omega \leq \Delta, \quad \forall t \in T, \omega \in \Omega \quad (4c)$$

$$x_t \geq 0, \quad \forall t \in T. \quad (4d)$$