

Online Appendix: Proofs of Lemmas 1 and 2

Proof of Lemma 1

The optimization problem of the principal in the first-best solution under team production is as follows:

$$\max_{\{f_i, v_i, e_{i1}, e_{i2}\}_{i=1,2}} \Pi = E(x) - \sum_{i=1}^2 E(S_i) \quad (35)$$

subject to

$$E(S_i + R_i) - C_i \geq 0. \quad (36)$$

As the effort is contractible, we can set $v_i = 0$ without loss of generality. Furthermore, as the principal observes the effort in the first-best solution, we set $\hat{e} = e$, so that $E(R_i) = 0$ follows immediately. With binding participation constraints (36) for both agents, the objective function of the principal becomes:

$$\begin{aligned} \max_{\{e_{i1}, e_{i2}\}_{i=1,2}} \Pi &= E(x) - \sum_{i=1}^2 C_i(e_{i1}, e_{i2}) \\ &= b_T(e_{11} + e_{21}) - \left(\frac{e_{11}^2}{2} + \lambda_1 \frac{e_{12}^2}{2} + \frac{e_{21}^2}{2} + \lambda_2 \frac{e_{22}^2}{2} \right). \end{aligned} \quad (37)$$

Optimizing for agent effort leads to the following first-best effort levels:

$$e_{i1}^{FB,T} = b_T, \quad e_{i2}^{FB,T} = 0, \quad i = 1, 2. \quad (38)$$

Inserting (38) into (37), we obtain the first-best surplus of the principal as

$$\Pi^{FB,T} = b_T^2. \quad (39)$$

Under individual production, setting again $v_i = 0$, with the binding participation constraints for both agents, the principal's objective function in the first-best-solution is very similar to that under team production in (37), and given by

$$\begin{aligned} \max_{\{e_{i1}, e_{i2}\}_{i=1,2}} \Pi &= E(x_1 + x_2) - \sum_{i=1}^2 C_i(e_{i1}, e_{i2}) \\ &= b_I(e_{11} + e_{21}) - \left(\frac{e_{11}^2}{2} + \lambda_1 \frac{e_{12}^2}{2} + \frac{e_{21}^2}{2} + \lambda_2 \frac{e_{22}^2}{2} \right). \end{aligned} \quad (40)$$

As the only difference to the first-best solution under team production is the productivity b_I instead of b_T , the solution to (40) follows immediately:

$$e_{i1}^{FB,I} = b_I, \quad e_{i2}^{FB,I} = 0, \quad i = 1, 2, \quad \Pi^{FB,I} = b_I^2. \quad (41)$$

Proof of Lemma 2

In the second-best solution, the principal's optimization problem under team production is

$$\max_{\{f_i, v_i\}_{i=1,2}} \Pi = E(x) - \sum_{i=1}^2 E(S_i) \quad (42)$$

subject to (with $i = 1, 2$)

$$EU_i = E(S_i + R_i) - C_i \geq 0, \quad (43)$$

$$(e_{i1}, e_{i2}) \in \operatorname{argmax}_{e_{i1}', e_{i2}'} EU_i. \quad (44)$$

The principal maximizes expected output minus expected compensation, subject to two constraints. The first constraint (43) is the participation constraint that ensures that the agents receive at least their reservation wages as equilibrium payoffs. The second constraint (44) is the incentive compatibility constraint for the agents' efforts.

We first derive the impression management effort of agent i under team production by solving the first-order condition $\partial EU_i / \partial e_{i2} = 0$. With

$$\begin{aligned} EU_i &= E(S_i + R_i) - C_i = E(f_i + v_i x + \gamma_i E(a_i | I, \hat{e})) - \frac{e_{i1}^2}{2} - \lambda_i \frac{e_{i2}^2}{2} \\ &= f_i + \gamma_i \beta_{i0} + (v_i + \beta_{ix} \gamma_i) b_T (e_{i1} + e_{i2}) + \gamma_i (\beta_{ii} e_{i2} + \beta_{ij} e_{j2}) - \frac{e_{i1}^2}{2} - \lambda_i \frac{e_{i2}^2}{2}, \end{aligned} \quad (45)$$

we obtain $\partial EU_i / \partial e_{i2} = \gamma_i \beta_{ii} - \lambda_i e_{i2} = 0$, so that the following level of impression management results

$$e_{i2}^T = \frac{\beta_{ii} \gamma_i}{\lambda_i}, \quad (46)$$

which is independent of the contract terms.

We now derive the optimal incentive rates and related productive efforts. From $\partial EU_i / \partial e_{i1} = 0$ (see (45)), we can operationalize the incentive compatibility constraint for e_{i1} as $e_{i1} = b_T(v_i + \beta_{ix}\gamma_i)$. Jointly with $e_{i2} = \frac{\beta_{ii}\gamma_i}{\lambda_i}$ and with the binding participation constraint (43) (with $E(R_i) = 0$)¹⁷ for both agents, the principal's objective function $\Pi(v_1, v_2)$ in the second-best solution under team production as a function of the incentive rates is given by

$$\Pi(v_1, v_2) = \sum_{i=1}^2 \frac{b_T^2 \lambda_i (2 - v_i - \beta_{ix}\gamma_i) (v_i + \gamma_i \beta_{ix}) - \beta_{ii}^2 \gamma_i^2}{2\lambda_i}. \quad (47)$$

From the first-order condition with respect to v_i , $\frac{\partial \Pi(v_1, v_2)}{\partial v_i} = b_T^2 (1 - v_i - \gamma_i \beta_{ix}) = 0$, we derive the optimal incentive rates $v_i^T = 1 - \beta_{ix}\gamma_i$ for $i = 1, 2$. Inserting the incentive rate v_i^T into equation (47) for $i = 1, 2$, leads to the principal's equilibrium surplus $\Pi^T = b_T^2 - \sum_{i=1}^2 \left(\frac{\beta_{ii}^2 \gamma_i^2}{2\lambda_i} \right)$ and inserting it into $e_{i1} = b_T(v_i + \beta_{ix}\gamma_i)$ yields the equilibrium productive effort $e_{i1}^T = b_T$.

Now consider individual production and the principal's second-best optimization problem:

$$\max_{\{f_i, v_i\}_{i=1,2}} \Pi = E(x_1 + x_2) - \sum_{i=1}^2 E(S_i) \quad (48)$$

subject to (with $i = 1, 2$)

$$EU_i = E(S_i + R_i) - C_i \geq 0, \quad (49)$$

$$(e_{i1}, e_{i2}) \in \operatorname{argmax}_{e_{i1}', e_{i2}'} EU_i. \quad (50)$$

The principal's optimization problem under individual production is very similar to that under team production, with two key differences: First, instead of the expected team output $E(x)$, the principal now earns the sum of expected individual outputs $E(x_1 + x_2)$ (and total compensation depends on x_1 and x_2 instead of x) and the agents' productivity is now b_I (instead of b_T). Second, the reputational payoff of agent i is now given by $R_i = \gamma_i E(a_i | x_i, y_i, \hat{e})$ and the corresponding weight on impression management signal y_i is ψ_{ii} (instead of β_{ii}). Similar reasoning as under team production yields the impression management effort under individual production as

$$e_{i2}^I = \frac{\psi_{ii}\gamma_i}{\lambda_i}.$$

¹⁷ The agents' reputational payoffs do not affect their participation constraints in equilibrium, since under correct conjectures ($\hat{e} = e$), we have $E(R_i) = 0$.

The incentive compatibility constraint for the productive effort, similar to the team setting, can be operationalized as $e_{i1} = b_I(v_i + \psi_{ix_i}\gamma_i)$. Inserting this expression for e_{i1} , $e_{i2} = \frac{\psi_{ii}\gamma_i}{\lambda_i}$, and the binding participation constraint (49) (with $E(R_i) = 0$) for $i = 1, 2$, into objective function (48) yields

$$\Pi(v_1, v_2) = \sum_{i=1}^2 \frac{b_I^2 \lambda_i (2 - v_i - \psi_{ix_i} \gamma_i) (v_i + \gamma_i \psi_{ix_i}) - \gamma_i^2 \psi_{ii}^2}{2 \lambda_i}. \quad (51)$$

From the first-order condition with respect to v_i , $\frac{\partial \Pi(v_1, v_2)}{\partial v_i} = b_I^2 (1 - v_i - \gamma_i \psi_{ix_i}) = 0$, we obtain the optimal incentive rates under individual production as $v_i^I = 1 - \psi_{ix_i} \gamma_i$ for $i = 1, 2$.

Inserting the incentive rate v_i^I into equation (51) for $i = 1, 2$, leads to the principal's equilibrium surplus $\Pi^I = b_I^2 - \sum_{i=1}^2 \left(\frac{\gamma_i^2 \psi_{ii}^2}{2 \lambda_i} \right)$ and inserting it into $e_{i1} = b_I(v_i + \psi_{ix_i} \gamma_i)$ yields the equilibrium productive effort $e_{i1}^I = b_I$.