Appendix 1: Content Structure of the Course *Geometry for Student Teachers*

Part 1: Plane geometry using methods of linear algebra	
1	basic definitions, congruence, Euclidean isometries
2	line reflections, orthographic projections, rotations
3	perpendicular bisectors of line segments, congruence theorem SSS, three reflections theorem
4	Exploration of the Euclidean isometry group
5	*interface-week congruence*
Part 2: Axiomatic plane geometry in metric spaces (absolute geometry)	
6	circles, straight lines, isometries in metric spaces
7	*Saccheri planes*: incidence axiom + reflection axiom, orthogonality, orthographic projections
8	point reflections, perpendicular bisectors of line segments
9	congruence theorems, three reflections theorem, angles, theorems about angles which does not presuppose the parallel axiom
10	symmetry
11	*interface-week symmetry*
Part 3: Axiomatic plane geometry with the parallel axiom	
12	parallelism, the parallel axiom, Poincaré half-plane, digressions on the idea of axiomatization, history of the parallel axiom
13	the vector space of translations
14	Saccheri planes + parallel axiom is isometric isomorphic to $(\mathbb{R}^2, d_2)$

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Appendix 2: Example: Competence Grid for Self-Assessment

Classify yourself in the following competence grid. In the gray column, write "+" for "I can," "o" for "I can to a certain extent," and "-" for "I cannot.

1. I can define typical terms used in teaching geometry at the lower secondary school level in a mathematically precise way:
    a. Circle
    b. Line segment
    c. Straight line
    d. Angle and its size
    e. Orthogonality
    f. Congruence
    g. Symmetry
    h. Similarity
    i. Perpendicular bisector
    j. Bisecting angle
    k. Orthogonal projection
2. I know justifications for typical theorems used in teaching geometry in lower secondary school:
    a. Sum of angles of a triangle
    b. Thales' theorem
    c. Base angle theorem
    d. Vertical and supplementing angles theorem
    e. Theorems about angles at intersected lines
    f. Pythagorean theorem
    g. Intercept theorem
3. I know typical geometric mappings including their definition and typical properties (e.g. fixed points or inversions)
    a. Reflection
    b. Rotation
    c. Translation
    d. Glide reflection
    e. Centric extension
4. I can systematically describe the different results that occur when the same or different geometric mappings are combined.
5. I know a consistent and rigorous mathematical background for all topics mentioned up to this point.
6. I can relate intermediate geometry to high school analytic geometry.

7. I have the necessary mathematical background knowledge to act in a geometry classroom professionally.
    a. Responding to student comments and mistakes in a manner that promotes learning
    b. Moderating productive discussions
    c. Analyze and evaluate approaches in instructional materials (e.g., textbooks)

Appendix 3: Example: Interface Task on the Topic Congruence

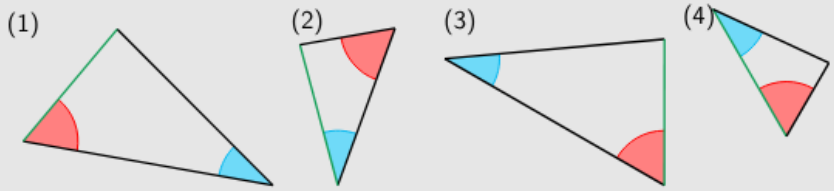
ePortfolioaufgabe 5 (Two Angles and One Side)

Imagine that your students are working in class on the following task from the textbook *Neue Wege 7* (Körner et al., 2014, S. 193).

The triangles all have one side of length 2.5 cm and angles of 35° and 60°.

a) Which triangles are congruent to each other? Give reasons.

b) Construct the triangles in your notebook. For which triangles do you have to determine an additional size?



a) Solve the textbook task.

Hint: For a) in the textbook task, remember in particular to justify both congruence and non-congruence.

Hint: In b), ‚construct' means using a compass and ruler.

Your student Emma is confused and asks: „How does this fit in, that according to the congruence theorems two triangles are congruent if they match at two angles and one side?"

b) Clarify Emma's question from a mathematical perspective.

c) How would you respond to Emma's question?

A translated version of the ePortfolio task ePF5 (see Fig. 1), which is an example of an *interface task*.

Appendix 4: Complete set of guiding questions for ePF3 and ePF 20

ePF3	1) What positive, negative, or generally special memories do you associate with the geometry lessons in your school days?
	2) Describe the role geometry has played in your studies to this point.
	3) At this point, how prepared do you feel to teach geometry in school?
	4) Why do you think geometry is an important topic in school mathematics?
ePF20	1) Choose two contents of the lecture (e.g., a theorem, a proof, an example) that particularly stuck in your mind. Introduce them briefly and describe what exactly was special for you in each case.
	2) At this point, how prepared do you feel to teach geometry in school? How did this course contribute to that? What else would you have liked to see in this math course?
	3) *How did this ePortfolio accompany your learning process? Was it useful for your learning of mathematics? To what extent has it contributed to your professionalization as a prospective mathematics teacher?*
	4) Why do you think geometry is an important topic in school mathematics?

Guiding questions in the activities ePF3 and ePF20. The answers to the questions in *italics* were evaluated as part of this study.

Appendix 5: Additional Category Frequencies

	pre (ePF3) | N = 40		post (ePF20) | N = 34	
	(+) available	(-) not available	(+) available	(-) not available
(2) PCK of … | pre: 37 (25) | post: 9 (8)				
(2.1) typical learning difficulties	4 (4)	1 (1)	-	1 (1)
(2.2) different approaches to mathematical topics	4 (4)	6 (4)	3 (3)	-
(2.3) *different methods for teaching/learning*	4 (4)	3(3)	-	-
(2.4) using digital tools for doing mathematics	1 (1)	1 (1)	1 (1)	-
(2.5) didactically reduced definitions, theorems, and proofs	-	-	-	3 (3)
(2.6) no specific focus	3 (3)	10 (9)	-	1 (1)

The QCA's category system regarding the pedagogical content knowledge PSTs mention when reflecting on their self-efficacy in teaching geometry.