

**Electronical supplementary information to
Does More Teaching Experience Mean More Expertise? A Nonlinear Perspective From
the COACTIV Study**

Stefan Krauss*, Stefanie Hohenleitner, Nicole Steib, Alfred Lindl, & Sven Hilbert
University of Regensburg

Authors

Stefan Krauss *corresponding author*

Faculty of Mathematics, University of Regensburg, Germany

E-Mail: stefan.krauss@ur.de

ORCID: 0000-0002-7440-9154

Stefanie Hohenleitner

Faculty of Mathematics, University of Regensburg, Germany

Nicole Steib

Faculty of Mathematics, University of Regensburg, Germany

ORCID: 0000-0002-4178-5030

Alfred Lindl

Faculty of Human Sciences, University of Regensburg, Germany

ORCID: 0000-0002-6969-8385

Sven Hilbert

Faculty of Human Sciences, University of Regensburg, Germany

ORCID: 0000-0001-5808-8357

Appendix A: Historical research paradigms

Table A1. History of Paradigms in Research on Teachers

Variable	Paradigm				
	Personality	Process-product	Process-mediation-product	Expertise	Professional competence
Time	From ca. 1900 (increasingly empirical from about 1940)	From ca. 1960	From ca. 1975	From ca. 1985 (internationally predominant until today)	From ca. 2005 (today predominant in German-speaking countries)
Influenced by	Trait-orientated personality theories (from ca. 1940 also personality tests)	Behaviorism (behavior of the teacher)	Cognitively extended behaviorism (including students' behavior and thinking)	Cognitivism (focus on thinking and knowledge of the teacher)	General concept of competence (conceptual reference to student competencies)
Research method	Tests and questionnaires (laboratory), personality of the teacher in the foreground	Lesson observation (later also with video technology), action of the teacher in the foreground	Lesson observation, also student questionnaires	Integration of research methods; professional knowledge and perception scales; also qualitatively focusing on single teachers	Quantitative: tests, video vignettes, or questionnaires for forming reflective scales; focusing on samples and populations
Remarks	Initially a humanities focus; few empirically reliable findings; today Renaissance based on more modern personality theories	First robust and stable findings; teaching aspects objectively, reliably, and validly measurable	Addition of student cognitions and affects as mediators (motivation, emotion, learning, etc.)	Systemic view; emphasis again on the person of the teacher; professional knowledge decisive; also qualitative research on individual expertise development	Integration of also "noncognitive" characteristics (e.g., motivational orientations, self-regulatory abilities); later inclusion of teaching action in "competence as continuum"

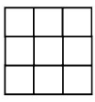
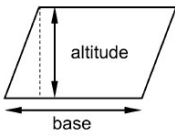
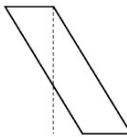
Note. Year dates are rough approximations (adapted from Krauss, 2020; Krauss & Bruckmaier, 2014; Lindl et al., 2023).

Appendix B: Conceptualizations, operationalizations, scales or example items for each of the 13 expertise indicators

Appendix B1: Pedagogical content knowledge (PCK) and content knowledge (CK)

Conceptualization. Two paper-and-pencil knowledge tests assessing the subject-specific knowledge domains (CK and PCK) were newly developed in the framework of the COACTIV study. The focus of the CK test is on background knowledge of the secondary-school curriculum. The test comprises 13 items, ranging from tasks at a lower secondary level (e.g., simple proofs in elementary geometry or algebra) to tasks requiring an understanding of infinitesimal concepts. The PCK test (22 items) assesses three subfacets: *knowledge of explaining and representing content* (11 items), *expertise on typical student errors and difficulties* (7 items), and *familiarity with multiple solutions to tasks* (4 items). Two trained raters independently evaluated the participants' open answers to both the PCK and CK tests using a proven coding scheme (for details, see Krauss et al., 2017; example items with possible answers are shown in Fig. B.1).

Fig. B.1 Sample items and sample solutions from the COACTIV tests on mathematics teachers' pedagogical content knowledge (PCK) and content knowledge (CK) as assessed by paper-and-pencil instruments

Knowledge Category (Subscale)	Sample Item	Sample Response (score = 1)
PCK (Task)	How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem (together with underlying reasoning) as possible.	<u>Algebraic response</u> Area of original square: a^2 Area of new square is then $(3a)^2 = 9a^2$ i.e., 9 times the area of the original square. <u>Geometric response</u> Nine times the area of the original square a {  } $3a$ {  }
PCK (StCog)	The area of a parallelogram can be calculated by multiplying the length of its base by its altitude.  Please sketch an example of a parallelogram to which students might fail to apply the above formula.	 Note: The crucial aspect to be covered in this teacher response is that students might run into problems if the foot of the altitude is outside of the given parallelogram.
PCK (E&R)	A student says: I don't understand why $(-1) \cdot (-1) = 1$ Please outline as many different ways of explaining this mathematical fact to your student as possible.	The "permanence principle," although it does not prove the statement, can be used to illustrate the logic behind the multiplication of two negative numbers and thus foster conceptual understanding: $\begin{array}{rcl} 3 \cdot (-1) & = & -3 \\ 2 \cdot (-1) & = & -2 \\ 1 \cdot (-1) & = & -1 \\ 0 \cdot (-1) & = & 0 \\ (-1) \cdot (-1) & = & 1 \\ (-2) \cdot (-1) & = & 2 \end{array}$ $-1 \leftarrow$ $\rightarrow +1$
CK (no subscales)	Is it true that $0.999999... = 1$? Please give detailed reasons for your answer.	Let $0.999... = a$ Then $10a = 9.99...$, hence, $10a - a = 9.99... - 0.999...$ $\quad \quad \quad \underline{9a} \quad \quad \quad \underline{9}$ Therefore $a = 1$; hence, the statement is true.

Note. See chapter 8 in the COACTIV compendium (Krauss et al., 2013; 2020).

COACTIV = Cognitive Activation in the Classroom (2003/2004) study;
Task = familiarity with multiple solutions to tasks;
StCog = knowledge of typical student errors and difficulties,
E&R = knowledge of explaining

Appendix B2: Constructivist beliefs (CB)

Conceptualization. Constructivist beliefs center on the idea that individuals actively build their own knowledge and understanding of the world through experiences and interactions. This contrasts with passive absorption of information, suggesting that learning is an active, reflective process where learners integrate new knowledge with existing understandings (Voss et al., 2013). In COACTIV, the sum scores of the teacher scales *Mathematics as Process*, *Independent and Discursive Learning*, and *Confidence in Students' Autonomous Learning* were combined to form the Constructivist Beliefs scale (Tables B2.1–B2.3).

Table B2.1 Mathematics as a process

Variable	Details
Number of items	4
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	In mathematics, there are lots of things you can find out and try out by yourself. Mathematics exercises and problems can be solved in various different ways. The mathematical laws are mental constructions of the mathematicians. I really enjoy teaching mathematics in this class. When working on mathematical problems, you can often discover new things (relationships, rules, concepts).

Table B2.2 Independent and insightful discursive learning

Variable	Details
Number of items	12
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	Teachers should allow students to find their own ways of solving simple mathematics problems. It helps students to understand mathematics if they are allowed to discuss their own ideas for solving problems. Teachers should allow students who have difficulty solving a word problem to carry on with their own attempts at solving it. Students should be allowed to think up their own ways of solving simple problems before the teacher shows how to solve them. Teachers should encourage students to think up their own solutions to simple problems. It is important to keep changing the structure of the problem, even in the same topic, to encourage the students to engage in mathematical thinking. Students should have the opportunity to explain their methods in detail, even if their approach is wrong. In mathematics, teaching goals are achieved best if students find their own methods for solving problems. In applied problems, students should be given the opportunity to explain their approach in detail. Teachers should encourage students to look for their own ways of solving mathematics problems, even if their approaches are inefficient.

	Students learn mathematics best by discovering their own ways to solve relatively simple problems. It is important for students to find out how to solve word problems and applied problems themselves.
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Table B2.3 Confidence in the mathematical independence of students

Variable	Details
Number of items	5
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	Students can find solutions to many mathematics problems without guidance. Most students can find their own solutions to simple applied problems. Students can usually find out how to solve simple problems by themselves. Given suitable material, students can develop computation procedures by themselves. Most students can find solutions to many mathematics problems without the help of adults.

Note. See chapter 12 in the COACTIV compendium (Voss et al., 2013).

COACTIV = Cognitive Activation in the Classroom (2003/2004) study.

Appendix B3: Enthusiasm (Ent)

Conceptualization. Enthusiasm, as an aspect of motivation, can be understood as an individual orientation reflecting a habitual positive affective experience of one's professional activity (Kunter et al., 2008). A theoretical distinction is drawn between two dimensions of enthusiasm: a subject-related dimension, referring to enthusiasm for the subject (Items 1–3), and an activity-related dimension, referring to enthusiasm for teaching (Items 4 and 5). For the present analyses, we merged the two scales (for details see Kunter, 2013; all closed items are shown in Table B.3).

Table B.3 Enthusiasm for math and math instruction

Variable	Details
Question/instruction	How do you evaluate your relationship to your PISA class? How strongly do you agree with the following statements about your relationship with this class?
Number of items	5
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	I find mathematics exciting and try to convey my enthusiasm to the students. In lessons, I try to get the students in this class enthusiastic about the subject of mathematics. Even now, I am still enthusiastic about the subject of mathematics. I really enjoy teaching mathematics in this class. I teach mathematics in this class with great enthusiasm.

Note. See chapter 13 in the COACTIV compendium (Kunter, 2013).

COACTIV = Cognitive Activation in the Classroom (2003/2004) study.

Appendix B4: Job satisfaction (JS)

Conceptualization. Job satisfaction is understood as a cognitive evaluation of the professional situation. For instance, job satisfaction depends on self-regulatory skills, the absence of stress, or self-efficacy. Thus, it can be seen as a consequence of a teacher's expertise and was assessed using the Job Diagnostic Survey (Hackman & Oldham, 1975). The six closed items are displayed in Table B.4.

Table B.4 Job satisfaction

Variable	Details
Question/instruction	How satisfied are you with your job?
Number of items	6
Scoring/codes	1 = strongly disagree 2 = disagree 3 = agree 4 = strongly agree
Items	It is hard to be happy in our job. For me, the advantages of the job far outweigh the disadvantages. Given the choice, I would definitely become a teacher again. I sometimes wonder if it would have been better for me to pursue another career. For me, there is no better job. I sometimes regret having become a teacher.

Note. See chapter 14 in the COACTIV compendium (Klusmann, 2013).

COACTIV = Cognitive Activation in the Classroom (2003/2004) study.

Appendix B5: Methodological action competence (MAC) and subject-specific action competence (SAC)

Conceptualization. As an implementation of perception, interpretation, and decision making, teachers were shown three short video clips (re-enacted lesson excerpts stopping at critical moments) and asked to propose didactically appropriate continuations for each situation. Teachers' open-ended responses were evaluated on five dimensions by trained raters using a detailed coding scheme (see Bruckmaier et al., 2016; Krauss et al., 2020). While Dimensions 1 and 2 were subject independent (MAC), Dimensions 3, 4, and 5 were subject dependent. All dimensions are summarized in Table B.5.

MAC: Dimension 1 (student orientation) reflects a constructivist approach, assessing whether *teachers promote student-centered, cognitively demanding activities*. Dimension 2 (methodological accuracy) *evaluates the use of appropriate, non-subject-specific methods* (e.g., social forms) including clarity of description.

SAC: Dimension 3 (insight orientation) measures the *emphasis on conceptual understanding in teachers' lesson continuation rather than on formal techniques*. Dimension 4 (content-specific accuracy) *evaluates the correctness and precision of the mathematical content* described. Dimension 5 (specific instructional opportunity) identifies whether *a unique didactic opportunity embedded in each of the three situations was recognized and seized on* by the teacher.

Table B.5 Situation-specific competencies and corresponding dimensions

Situation-specific competence	Dimension	Meaning of dimension
Methodological action competence	1. Student orientation	Who is the focus of the teaching and does the “mathematical work”—the teacher or the students?
	2. Methodological accuracy	How precisely is the methodological procedure (e.g., social forms or media) depicted? Are the chosen methods appropriate for the given situation?
Subject-specific action competence	3. Insight orientation	Which competency is mainly addressed—calculation skills or profound mathematical understanding?
	4. Content-specific accuracy	How precisely and correctly are mathematical facts and the content-related procedure depicted?
	5. Specific instructional opportunity	In Video 1: Let the students realize the mistake on their own. In Video 2: Take into account several solution strategies. In Video 3: Recognize potential for mathematical modeling.

Note. Captured using video vignettes. Responses were coded using a trichotomous rating scale (codes 0, 1, 2; for details see Bruckmaier et al., 2016; Krauss et al., 2020).

Appendix B6: Instructional quality (CA, CS, CM)

Conceptualization. Various methodological approaches are available to capture the complexity of the teaching process (e.g., Charalambous & Praetorius, 2018; Praetorius et al., 2018). To assess instructional quality in PISA classes, a parsimonious model with three dimensions—cognitive activation (CA), constructive individual learning support (CS), and classroom management (CM)—was adopted. Each dimension was represented by multiple indicators (see Kunter et al., 2013).

B.6.1 Cognitive activation (CA): Several scales for both teachers and students were implemented to assess the potential of a teacher’s lessons for cognitive activation, that is, for students’ independent, in-depth mathematical thought processes. These included scales on cognitively activating tasks and on teachers’ support for cognitive autonomy. All five scales are provided in Tables B6.1.1–B6.1.5.

B6.2 Constructive individual learning support (CS): Individual learning support describes the teacher’s individual scaffolding and support for student learning, including the personal-social level (Kunter & Voss, 2013). Individual learning support was assessed using five scales from the PISA student questionnaire: *Constructive Responses to Student Errors*, *Respectful Treatment of Students*, *Adaptive Support and Explanations for Difficult Tasks*, *Patience in Dealing with Comprehension Difficulties*, and a *Caring Ethos* (see Tables B6.2.1–B6.2.5).

B6.3 Classroom management (CM): Classroom management refers to the teacher’s ability to respond promptly to various simultaneous stimuli, coordinating complex classroom occurrences to maximize learning time and minimize noninstructional activities (e.g., Evertson & Weinstein, 2006). This dimension was assessed using scales from both the PISA student and the COACTIV teacher questionnaires, addressing aspects such as disruption prevention and effective use of lesson time (for the three scales, see Tables B6.3.1–B6.3.3).

Appendix B6.1 Cognitive activation (CA)

Table B6.1.1 Cognitively activating tasks

Variable	Details
Question/instruction	In your PISA class, how often do you use the following approaches and problems that are sometimes recommended in the literature?
Number of items	8
Scoring/codes	1 = rarely or never 2 = sometimes 3 = often 4 = usually
Items	I set problems where it is not just computation that is important, but choosing the right approach. I set problems where it is not just the correct solution that is important, but understanding the mathematical thought process. I set problems where students have to take time to think. I set problems for which there is no immediately visible solution. I set problems that do not have a clear-cut solution and ask students for explanations. I set questions that cannot be answered spontaneously, but that force students to think. I sometimes ask students to demonstrate several different ways of solving a problem. I ask students to compare and evaluate different ways of solving problems.

Table B6.1.2 Supporting cognitive autonomy

Variable	Details
Number of items	4
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	I let students solve mathematics problems according to their personal preferences. I let students apply their own strategies to solve math problems. When working on problems in mathematics lessons, I wait to see if the students come up with the right answer by themselves first. When I help students with a problem, I encourage them to find the right answer themselves first.

Table B6.1.3 Cognitively challenging approach to students' ideas and errors

Variable	Details
Question/instruction	How do you see your role when it comes to covering a new topic?
Number of items	5
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	I work on the basis of the students' suggestions and carry on with that work until the students notice that something doesn't add up. I sometimes deliberately let the students go astray until they realize that something must be wrong. If a student makes a mistake when a new topic is being covered, I initially accept the suggestion without comment and carry on working through the problem with the students until the mistake becomes obvious.

	If a student makes a mistake when a new topic is being covered, I ask the class for their opinion without commenting myself. I work on the basis of the students' ideas and carry on with that work until the students see whether their approach will lead to the goal or whether incongruities are becoming apparent.
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Table B6.1.4 Independence and obligation to give reasons when working on tasks

Variable	Details
Question/instruction	How do you develop a topic in mathematics?
Number of items	8
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	Our mathematics teacher gets us to compare and evaluate different ways of solving problems. In mathematics lessons our teacher sometimes lets us go astray in our work until we notice that something is wrong. In mathematics lessons our teacher sometimes accepts mistakes and lets us carry on until we see that something must be wrong. Our mathematics teacher sometimes asks us to demonstrate several different ways of solving a problem. Our mathematics teacher asks us to explain our thought processes thoroughly. Our mathematics teacher often expects us to explain our work in detail. Our mathematics teacher lets us solve problems according to our personal preferences. Our mathematics teacher lets us use our own strategies to solve difficult problems.

Table B6.1.5 Discursive treatment of different student solutions

Variable	Details
Question/instruction	How do you develop a topic in mathematics?
Number of items	5
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	The teacher often asks if anyone has found a different way of solving a problem. We often discuss the different ways we have found to solve a problem. Different students often present their ways of solving a problem. We often come up with different ways of solving problems in groups and then discuss these with the whole class. The teacher often does not immediately say whether or not an answer is correct.

Appendix B6.2: Constructive individual learning support (CS)

Table B6.2.1 Constructive Responses to Student Errors

Variable	Details
Question/instruction	How do you develop a topic in mathematics?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Our mathematics teacher is patient if someone makes a mistake in the lesson. Our mathematics teacher does not mind if we make mistakes. Our mathematics teacher makes sure that people don't laugh if someone makes a mistake in class.

Table B6.2.2 Respectful Treatment of Students

Variable	Details (scale inverted for analyses)
Question/instruction	How happy are you with your mathematics teacher?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Our mathematics teacher hurts students' feelings. Our mathematics teacher sometimes upsets students. Our mathematics teacher belittles students.

Table B6.2.3 Adaptive Support and Explanation for Difficult Tasks

Variable	Details
Question/instruction	How does your mathematics teacher explain?
Number of items	4
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Our mathematics teacher gives good examples that make the problems understandable. Our mathematics teacher sets problems that all students can work on to the best of their ability. Our mathematics teacher makes formulas so clear to me that I can apply them to new problems as well. Our mathematics teacher explains difficult points, in particular, very slowly and carefully.

Table B6.2.4 Patience in Dealing with Comprehension Difficulties

Variable	Details
Question/instruction	At what pace does your mathematics teacher teach?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Our mathematics teacher remains patient even if he / she has to explain things several times. Our mathematics teacher takes time to help those who need longer to understand things. Our mathematics teacher remains patient even if a student cannot keep up.

Table B6.2.5 Caring Ethos

Variable	Details
Question/instruction	How well does your mathematics teacher know your class?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Our mathematics teacher always takes time to talk if students want to discuss something with him / her. Our mathematics teacher always addresses students' problems. Our mathematics teacher does his / her best to respond to students' requests as far as possible.

*Appendix B6.3: Classroom management (CM)***Table B6.3.1** Disruptions in math lessons

Variable	Details
Question/instruction	How are your math lessons?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	In mathematics, teaching is very often interrupted. In mathematics, students talk among themselves the whole time. In mathematics, students mess around the whole time.

Table B6.3.2 Time wasting in math lessons

Variable	Details
Question/instruction	How are your math lessons?
Number of items	3
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	In mathematics, it takes a very long time at the start of the lesson until the students have settled down and started working. In mathematics, a lot of lesson time is wasted. In mathematics, the lesson often starts late.

Table B6.3.3 Disruptions and time wasting

Variable	Details
Question/instruction	Individual classes can differ greatly. How do you try to manage your PISA class and how well do you succeed in this class?
Number of items	8
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	I have to reprimand this class a lot to get them to quiet down. The students in this class mess around a lot. Lessons are very often disrupted in this class. It is difficult to start the lesson on time in this class. In this class, it takes a long time at the beginning of the mathematics lesson until the students are quiet and begin working. The students in this class talk among themselves a lot. I often have the impression that a lot of time is wasted in mathematics lessons in this class. Somebody usually has some kind of problem when I want to start work.

Note. See chapter 5 in the COACTIV compendium (Kunter & Voss, 2013). COACTIV = Cognitive Activation in the Classroom (2003/2004) study.

Appendix B7: Experienced enjoyment (S-JOY) and experienced anxiety (S-ANX)

Conceptualization. Enjoyment of mathematics and corresponding anxiety were assessed using the PISA student questionnaire (Pekrun et al., 2002). One scale measured students' experienced joy and interest (Appendices B7.1 and B7.2) and a second scale measured experienced anxiety (Appendices B7.3 and B7.4). In Fig. 1b, the minus sign (–) indicates that lower values correspond to lower levels of anxiety and that the scale had to be inverted accordingly.

Table B7.1 Joy

Variable	Details
Question/instruction	To what extent do the following statements apply to you?
Number of items	6
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	Mathematics is fun. I look forward to mathematics lessons. I find the material so exciting that mathematics lessons are real fun. For me, mathematics class is such fun that I really feel like participating. When I do math homework, I am in a good mood. I enjoy mathematics exams.

Table B7.2 Interest

Variable	Details
Question/instruction	Please read each of the following statements about math and decide how much they apply to you.
Number of items	5
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	When I do mathematics, I sometimes get totally absorbed. Mathematics is important to me personally. Because doing mathematics is fun, I wouldn't want to give it up. I am interested in mathematics. Doing mathematics is one of my favorite activities.

Table B7.3 Anxiety

Variable	Details (scale inverted for analyses)
Question/instruction	To what extent do the following statements apply to you?
Number of items	9
Scoring/codes	1 = strongly agree 2 = agree 3 = disagree 4 = strongly disagree
Items	I'm scared of mathematics. I'm so scared of mathematics that I'd prefer not to go to school at all. In mathematics I worry that I understand less than my classmates. When I do my math homework, I worry whether I'll be able to do it all. I get really nervous before mathematics exams. I worry about failing mathematics exams before they have even started. I'm so scared of mathematics exams that I'd prefer not to have to sit them at all. I get really worried when a mathematics exam is coming up. I get so worked up during mathematics exams that I can't concentrate properly.

Table B7.4 Anxiousness

Variable	Details (scale inverted for analyses)
Question/instruction	How do you feel when learning math? Please indicate how much you agree with the following statements.
Number of items	5
Scoring/codes	1 = disagree 2 = disagree somewhat 3 = agree somewhat 4 = agree
Items	I often worry that it will be difficult for me in math lessons. I get very tense when I have to do math homework. I get very anxious when solving math problems. I feel helpless when solving math problems. I am worried that I will get bad grades in math.

Note. See chapter 6 in the COACTIV compendium (Kunter & Voss, 2013). COACTIV = Cognitive Activation in the Classroom (2003/2004) study.

Appendix B8: Mathematical learning gains (S-ΔM)

The learning gains of students in previous COACTIV analyses were typically calculated by measuring students' mathematical performance at the end of grade 10 with a curricular-valid subtest of PISA 2004, while statistically controlling for individual performance at the end of the previous year (i.e., PISA 2003). Sample items from both PISA tests are provided in Organisation for Economic Co-operation and Development (2009).

We were interested in attributing this learning gain directly to a specific teacher, so we calculated the average student performance of each 10th-grade teacher's class in both PISA tests and *z*-transformed both variables with respect to all teachers whose classes took part in PISA 2004. Note that it was irrelevant whether these teachers were already teaching these classes in the 9th grade (this was the case for about 85% of the classes). Since structural equation models neither estimate such a difference nor assign it to a specific teacher, the modeling of the learning gain of specific classes within one year as the difference between the two measurement points deviates from previous COACTIV analyses.

Appendix B9: Remark regarding the COACTIV-study

After the German “PISA shock” following the first PISA study in 2000—when German students scored below average in all three PISA domains (Baumert et al., 2001)—a unique situation developed in Germany: Since the topic was broadly discussed in the media, there was strong public pressure on schools and the educational system. Furthermore, at that time, teachers were not yet overloaded with requests to participate in empirical studies, and some were even interested in stating their views and contributing to potential improvements. For these reasons, in 2003 (the next PISA round), it was possible to implement an additional measurement point for students in 2004. Therefore, aspects of teacher competence and lesson features that predicted student learning *gains* in mathematics (i.e., from 2003 to 2004) could be identified empirically. Moreover, mathematics teachers of PISA classes at that time were willing to make about 12 hours (!) of their time available—distributed across one school year—to participate in the satellite teacher study COACTIV. To this day, a comparable study, including a longitudinal student PISA survey, has not been conducted. In the CECO (Classroom Experience, Characteristics, and Outcome) study accompanying PISA 2022, for example, only one student measurement point could be realized (Schiepe-Tiska et al., 2025).

Appendix C: Correlations of the overall expertise index and the 13 expertise indicators with years of professional experience

Table C1 Bivariate correlations (Pearson's product-moment) of the overall expertise (OE) index and the 13 expertise indicators with years of professional experience in the whole sample and in five selected teacher groups (Gy, NGy, 0–8, 9–27, and 28–42 years of experience)

Correlation <i>r</i> with years of professional experience (<i>p</i> value)		Professional competence					Situation-specific skills		Instructional quality			Student mediation		Teaching goals	
		Professional knowledge		Affective beliefs and motivational orientations			Perception, interpretation, decision making		Basic dimensions			Affects			
Sample	OE-index	CK	PCK	CB	ENT	JS	MAC	SAC	CA	CS	CM	S-ANX	S-JOY	S-ΔM	
Total (<i>N</i> = 217)	-.18* (.01)	-.18* (.01)	-.15* (.05)	-.12 (.08)	-.15* (.03)	-.08 (.25)	-.24** (<i><.01</i>)	-.10 (.25)	-.05 (.47)	-.25** (<i><.01</i>)	.10 (.14)	.02 (.76)	-.04 (.55)	.05 (.53)	
School type	Gy (<i>N</i> =87)	-.24* (.03)	-.12 (.29)	-.27* (.02)	-.15 (.19)	-.24* (.03)	.00 (.99)	-.24 (.09)	-.11 (.43)	-.11 (.34)	-.30* (.01)	.21* (.06)	.04 (.76)	-.08 (.49)	-.06 (.61)
	NGy (<i>N</i> =130)	-.09 (.34)	-.16 (.09)	-.02 (.85)	-.08 (.40)	-.08 (.41)	-.14 (.14)	-.23* (.03)	-.07 (.52)	.03 (.73)	-.23* (.02)	.03 (.72)	-.02 (.79)	-.05 (.60)	.09 (.35)
Professional experience (years)	0–8 (<i>N</i> =32)	.15 (.41)	-.19 (.31)	-.50* (.01)	.01 (.97)	.43* (.01)	-.08 (.66)	.25 (.35)	.09 (.73)	.19 (.30)	.25 (.17)	.50** (<i><.01</i>)	.11 (.56)	.32 (.08)	.19 (.30)
	9–27 (<i>N</i> =106)	-.21* (.03)	-.02 (.87)	-.03 (.75)	.02 (.85)	-.19* (.05)	-.24* (.01)	-.15 (.19)	-.21* (.07)	.01 (.90)	-.36** (<i><.01</i>)	-.07 (.50)	.13 (.19)	-.15 (.12)	.24* (.02)
	28–42 (<i>N</i> =63)	.30* (.02)	.11 (.43)	.12 (.41)	-.03 (.83)	.14 (.27)	.22 (.09)	-.09 (.53)	.18 (.21)	.16 (.22)	.17 (.20)	.26 (.04)	-.09 (.49)	.17 (.20)	.18 (.18)

Note. Gy = Academic track; NGy = nonacademic track; CK = content knowledge; PCK= pedagogical content knowledge; CB = constructivist beliefs; ENT = enthusiasm; JS = job satisfaction; MAC = methodological action competence; SAC = subject-specific action competence; CA = cognitive activation; CS = constructive support; CM = classroom management; S-ANX = students' anxiety (negatively poled); S-JOY = students' joy and interest; S-ΔM = students' competency gain; * = $p < .05$; ** = $p < .01$.

Appendix D: Individual expertise profiles of four exemplary “expert teachers”

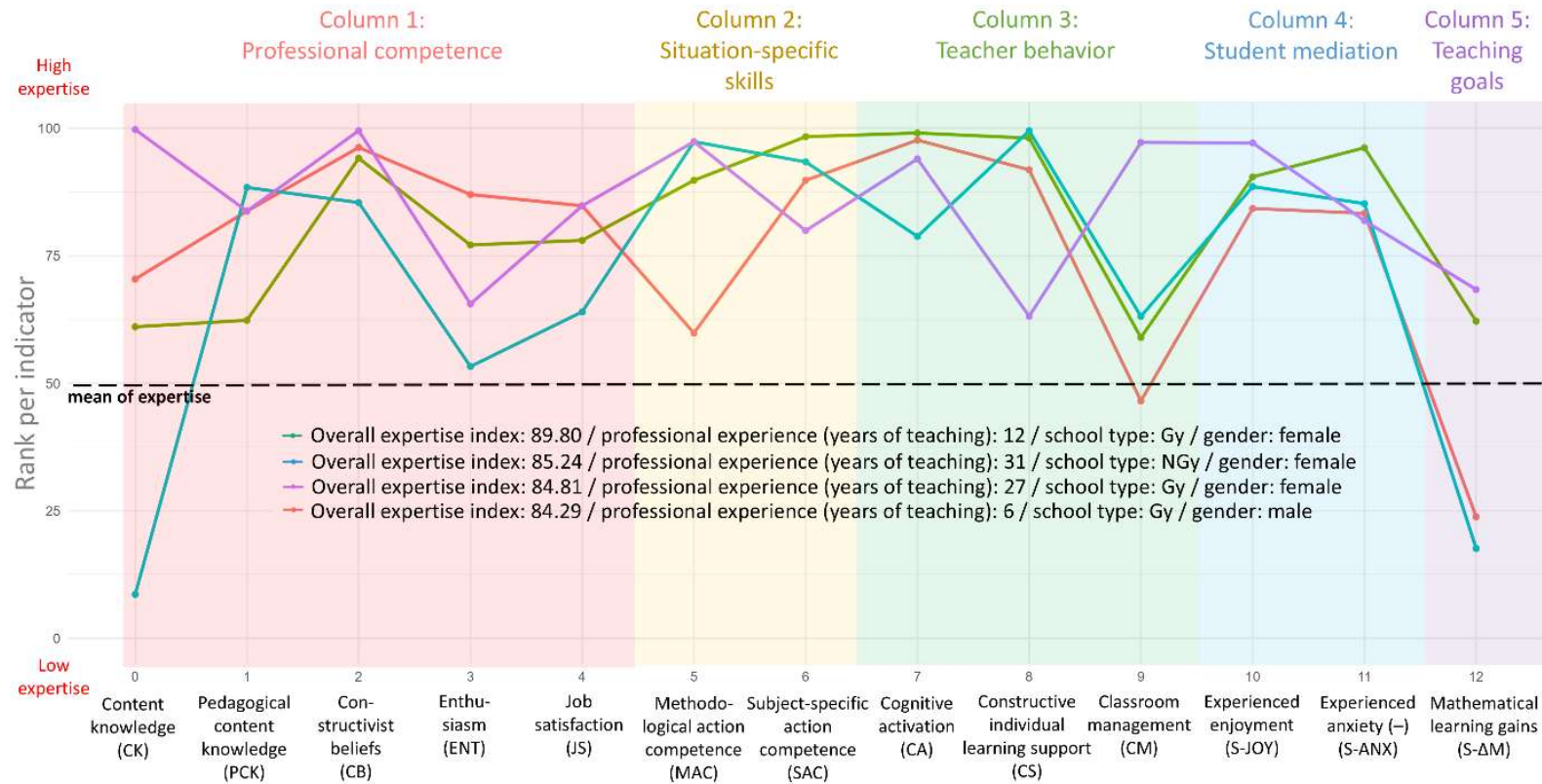


Fig. D1 Profiles of four exemplary teachers from the “top 10” in terms of OE index ranking (the facets, i.e., expertise indicators, are ordered according to the cascade model). It becomes clear that these teachers scored above average in almost all facets. First, note that CK is known to be particularly high in academic-track (Gy) teachers (in Table 1, CK shows the largest school-type differences between Gy and nonacademic, NGy, tracks); second, note that the learning gains of students are influenced by many aspects beyond the teacher or classroom characteristics alone (e.g., parental support, school climate, or culture-specific aspects). Consequently, in Table 1, both facets (CK and S-ΔM) display the descriptively weakest part-whole corrected item-total correlations r_{it} .

References

- Baumert, J., Klieme, E., Neubrand, M., Prenzel, M., Schiefele, U., Schneider, W., Stanat, P., Tillmann, K.-J., & Weiß, M. (2001). *PISA 2000. Basiskompetenzen von Schülerinnen und Schülern im internationalen Vergleich*. Leske + Budrich. <https://doi.org/10.1007/978-3-322-83412-6>
- Bruckmaier, G., Krauss, S., Blum, W., & Leiss, D. (2016). Measuring mathematical teachers' professional competence by using video clips (COACTIV video). *ZDM – The International Journal on Mathematics Education*, 48(1–2), 111–124. <https://doi.org/10.1007/s11858-016-0772-1>
- Evertson, C. M., & Weinstein, C. S. (Eds.). (2006). *Handbook of classroom management: Research, practice, and contemporary issues*. Erlbaum.
- Hackman, J. R., & Oldham, G. R. (1975). Development of the job diagnostic survey. *Journal of Applied Psychology*, 60(2), 159–170. <https://doi.org/10.1037/h0076546>
- Klusmann, U. (2013). *Occupational self-regulation*. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 291–308). Springer. https://doi.org/10.1007/978-1-4614-5149-5_14
- Krauss, S. (2020). Expertise-Paradigma in der Lehrerinnen- und Lehrerbildung. In C. Cramer, J. König, M. Rothland, & S. Blömeke (Eds.), *Handbuch Lehrerinnen- und Lehrerbildung* (pp. 154–162). Klinkhardt. <https://doi.org/10.35468/hblb2020-018>
- Krauss, S., & Bruckmaier, G. (2014). Das Experten-Paradigma in der Forschung zum Lehrerberuf. In E. Terhart, H. Bennewitz, & M. Rothland (Eds.), *Handbuch der Forschung zum Lehrerberuf* (2nd ed., pp. 241–261). Waxmann.
- Krauss, S., Blum, W., Brunner, M., Neubrand, M., Baumert, J., Kunter, M., Besser, M., & Elsner, J. (2013). Mathematics teachers' domain-specific professional knowledge: Conceptualization and test construction in COACTIV. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 147–174). Springer.
- Krauss, S., Bruckmaier, G., Lindl, A., Hilbert, S., Binder, K., Steib, N., & Blum, W. (2020). Competence as a continuum in the COACTIV study: The “cascade model.” *ZDM – Mathematics Education*, 52(3), 311–327. <https://doi.org/10.1007/s11858-020-01151-z>
- Kunter, M. (2013). Motivation as an aspect of professional competence: Research findings on teacher enthusiasm. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 273–289). Springer. https://doi.org/10.1007/978-1-4614-5149-5_13
- Kunter, M., & Voss, T. (2013). The model of instructional quality in COACTIV: A multicriteria analysis. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 97–124). Springer. https://doi.org/10.1007/978-1-4614-5149-5_6
- Kunter, M., Tsai, Y. M., Klusmann, U., Brunner, M., Krauss, S., & Baumert, J. (2008). Students' and mathematics teachers' perceptions of teacher enthusiasm and instruction. *Learning and Instruction*, 18(5), 468–482. <https://doi.org/10.1016/j.learninstruc.2008.06.008>

- Lindl, A., Krauss, S., & Buchholtz, N. (2023). Professionswissen von Mathematiklehrkräften – eine einleitende Übersicht. In S. Krauss & A. Lindl (Eds.), *Professionswissen von Mathematiklehrkräften – Implikationen aus der Forschung für die Praxis* (pp. 1–35). Springer. https://doi.org/10.1007/978-3-662-64381-5_1
- Organisation for Economic Co-operation and Development. (2009). *Learning Mathematics for Life: A Perspective from PISA*. Paris. <https://doi.org/10.1787/9789264075009-en>
- Pekrun, R., Götz, J. S., Zirngibl, A., vom Hofe, R., & Blum, W. (2002). *Skalenhandbuch PALMA 1. Messzeitpunkt (5. Klassenstufe)*. University of Munich, Institute of Educational Psychology.
- Schiepe-Tiska, A., Krauss, S., Reiss, K., & Lewalter, D. (Eds.). (2025). *PISA-Ceco–Dokumentation der Erhebungsinstrumente der Begleitstudie zu PISA 2022: Einblicke in den mathematisch-naturwissenschaftlichen Unterricht*. Waxmann.
- Voss, T., Kleickmann, T., Kunter, M., & Hachfeld, A. (2013). Mathematics teacher’ beliefs. In M. Kunter, J. Baumert, W. Blum, U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 235–258). Springer. https://doi.org/10.1007/978-1-4614-5149-5_12