

Electronic supplementary material

Online Resource 1: Proof that our region of study is invariant

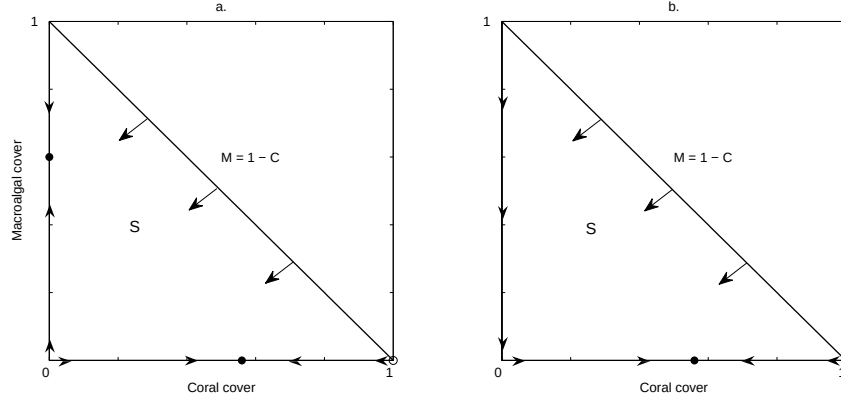


Figure 1: Phase plane analysis of the original model (Equations 1-3) in the text for $g < \gamma$ (a.) and $g > \gamma$ (b.). The solid dots correspond to the equilibria $(0, \frac{r-d}{r})$ and $(1 - \frac{g}{\gamma}, 0)$ and the open dot corresponds the point $(1, 0)$ which is not well-defined (but not included) in S . In both (a.) and (b.), trajectories near the line $M = 1 - C$ are directed into the region S , $\frac{dM}{dt} = 0$ on the coral axis, and $\frac{dC}{dt} = 0$ on the macroalgae axis. Additionally, at the point $(0, 0)$ $\frac{dM}{dt} = \frac{dC}{dt} = 0$ and any initial condition beginning near $(0, 1)$ or $(1, 0)$ is directed away from these points. This demonstrates that S is an invariant set and therefore trajectories beginning within region S remain the region for all t .

Here, we show that the original model (Equations 1-3 in the main text) is invariant on the region $0 < M + T < 1$ and $C, M, T \geq 0$. Notice that our region requires an initial state containing both algae and coral. First, recall that the model is constructed such that $T = 1 - C - M$ so given that $C, M \geq 0$ and $0 < M + C \leq 1$, then $0 \leq T < 1$. Thus, we only need to consider the region defined by $C, M \geq 0$ and $0 < M + T < 1$ (or equivalently, $0 < 1 - C < 1$). Hereafter, we will refer to this region as S .

To demonstrate that S is an invariant set, we provide a phase plane analysis of C versus M on the border of our region (with the results displayed in Fig. 1). While we do not provide an exhaustive analysis that shows the dynamics near the equilibria lying in the interior of the region, this will suffice to show that S is invariant.

The system is not well defined when $M = T = 0$ (or equivalently $C = 1$), which lies on the border of (but is not included in) S . We show that any trajectory beginning in S , however, will never obtain the point $C = 1$. Consider the dynamics of coral cover in the limit as $C \rightarrow 1$:

$$\frac{dC}{dt} \rightarrow -d < 0. \quad (1)$$

Thus, any point beginning sufficiently close to $C = 1$ will be forced to move away from this value. Now, we consider the dynamics on the line $M + C = 1$ given that $M, C < 1$ (implying that $T = 0$) by considering $\frac{d(M+C)}{dt}$. Here,

$$\frac{dM}{dt} = aMC - g, \text{ and} \quad (2)$$

$$\frac{dC}{dt} = -dC - aMC \quad (3)$$

so that

$$\frac{d(M+C)}{dt} = -g - dC < 0 \quad (4)$$

$\forall C$, demonstrating that points beginning by this portion of the border of S are directed inward. When $C = 0$,

$$\frac{dC}{dt} = 0, \text{ and} \quad (5)$$

$$\frac{dM}{dt} = -gM + \gamma MT. \quad (6)$$

Here, we have $\frac{dM}{dt} = 0$ if $M = 0$ and when $M \neq 0$, $\frac{dM}{dt} > 0$ if $T > \frac{g}{\gamma}$ (or $M < 1 - \frac{g}{\gamma}$), and $\frac{dM}{dt} < 0$ otherwise. ESM Fig. 1 demonstrates both qualitatively different states (i.e $g < \gamma$ versus $g > \gamma$). Also, notice that as $M \rightarrow 1$ and $C = 0$, $\frac{dM}{dt} < 0$, so the value of $M = 1$ will never be attained. Additionally, for $M = C = 0$,

$$\frac{dC}{dt} = 0, \text{ and} \quad (7)$$

$$\frac{dM}{dt} = 0 \quad (8)$$

showing that M will never become negative-valued.

Finally, we consider $M = 0$:

$$\frac{dC}{dt} = rTC - dC, \text{ and} \quad (9)$$

$$\frac{dM}{dt} = 0. \quad (10)$$

If $C = 0$ then $\frac{dC}{dt} = 0$, otherwise, $\frac{dC}{dt} > 0$ if $T > \frac{d}{r}$ (or $C < 1 - \frac{d}{r}$) and $\frac{dC}{dt} < 0$ otherwise.

Fig. 1 demonstrates the above results; Figure 1a assumes $g < \gamma$ and Fig. 1b assumes $g > \gamma$. Notice that Fig. 1a has two equilibria and depending on the value of g these equilibria are either stable or saddle points. Fig. 1b has one globally stable equilibrium.

We note that this invariance extends to the model given by Equations 4-6 and 8, and can be demonstrated using an identical proof but replacing parameter g with grazing function $g(P) = P$. The same results are obtained using the fact that $0 \leq P \leq 1$.

Online Resource 2: Parameter sensitivity

The parameter selection for the model prior to the addition of explicit parrotfish dynamics are identical to the parameterisation provided in the original model (Mumby et al 2007). However, given the complexity of finding precise parameter values, we provide a sensitivity analysis to demonstrate that the parameter set we selected can be considered a “typical” parameter set for our problem. In Figure 1 of the text we give the bifurcation diagram of coral cover versus the grazing intensity g . We can find the bifurcation values analytically; it is defined by a parabola given by

$$C(g) = \frac{a(a+r) - (2a+d)\gamma \pm \sqrt{a(a-4g)(a+r)^2 + 2a(d+2g)(a+r)\gamma + d^2\gamma^2}}{2a(a+r-\gamma)} \quad (11)$$

and a horizontal line given by

$$C(g) = \frac{r-d}{r}. \quad (12)$$

Additionally, there is a line of equilibria at $C(g) = 0$. It is clear that variations in parameter values simply correspond to shifts in the parabola structure and in the horizontal line. Thus, it is important for the parameters to fall within reasonable magnitudes but precision is of lesser importance as our goal is to determine qualitative, rather than quantitative, results. In other words, small changes in parameter value still lead to qualitatively similar results and our observations still apply. We do, however, require that $r > d$ to ensure that the natural mortality of coral does not exceed the rate at which it can recruit to and overgrow

coral. For these reasons, we have chosen a “typical” set of parameters that provide a general understanding of hysteresis in our system and capture all interesting behavior. Furthermore, as stated in the main text the parameter values that we have chosen exhibit similar behavior to a more complex simulation model which had been tested against an independent 20-year data set from Jamaica and was found to follow the observed trends (Mumby et al 2005).

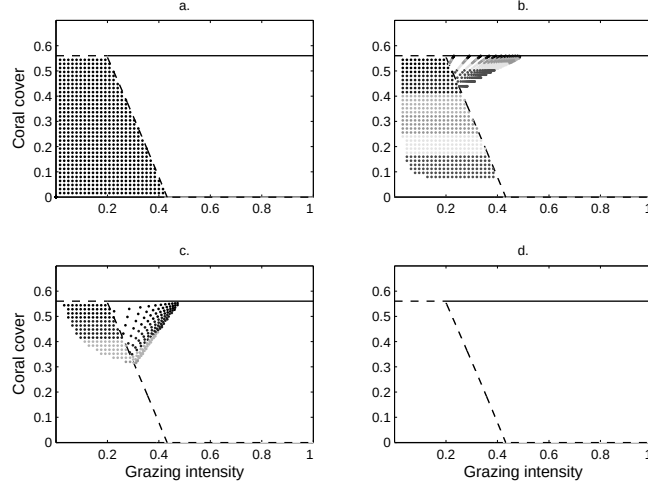


Figure 2: The same simulation was performed as in Fig. 2 in the main text; however, a , d , γ , and r are each increased by 10% to demonstrate the qualitative similarity in the results obtained by variations in parameters.

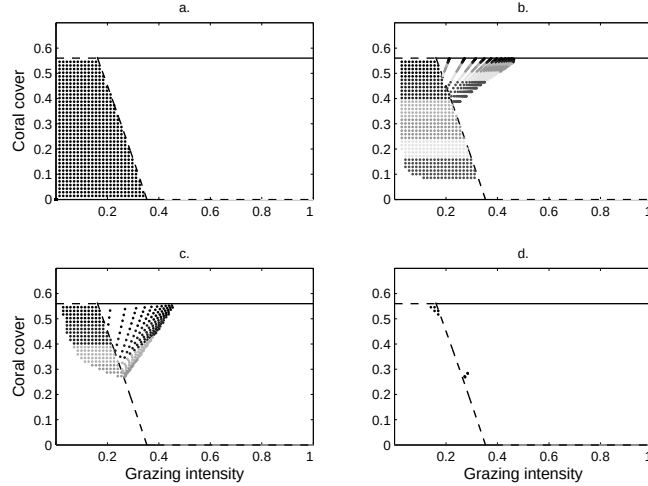


Figure 3: The same simulation was performed as in Fig. 2 in the main text; however, a , d , γ , and r are each decreased by 10% to demonstrate the qualitative similarity in the results obtained by variations in parameters.

Furthermore, we demonstrate that increasing and decreasing each parameter (a , d , γ , and r) by 10% in Fig. 2 in the main text results in qualitatively similar results, further justifying our conclusions given this parameter set (see ESM Figures 2 and 3, respectively). As evidenced by these figures, variations in parameter values result in qualitatively similar results. Testing this range of parameter values reflects reasonable variation in the parameter values. Furthermore, the structure of the bifurcation diagram as well

as our results indicate that the output is not very sensitive to parameter changes.

The above sensitivity analysis provides justification for the original model parameters. We now provide a similar analysis of the parameter used in the definition of $K(C)$ given that food availability is the primary limiting factor of parrotfish abundance, or

$$K(C) = 1 - zC \quad (13)$$

where $0 \leq z < 1$. Figures 3b and 4 in the main text use the value $z = 0.5$; however, the time integrated behavior is qualitatively similar for all feasible values of z . To demonstrate this, ESM Fig. 4 shows the bifurcation diagrams of fishing effort versus coral cover for several values of z . Similar to that demonstrated in above, the equilibria are given by a parabola and a horizontal line; by varying the value of z we simply vary the shape of the parabola. In addition, we see that hysteresis is still possible through variations in fishing effort levels (f) as seen in Figures 4a-d. In addition, we demonstrate that the transient behavior of the coral, macroalgae, and parrotfish dynamics is qualitatively similar to those demonstrated by Fig. 4 in the main text (see ESM Fig. 5).

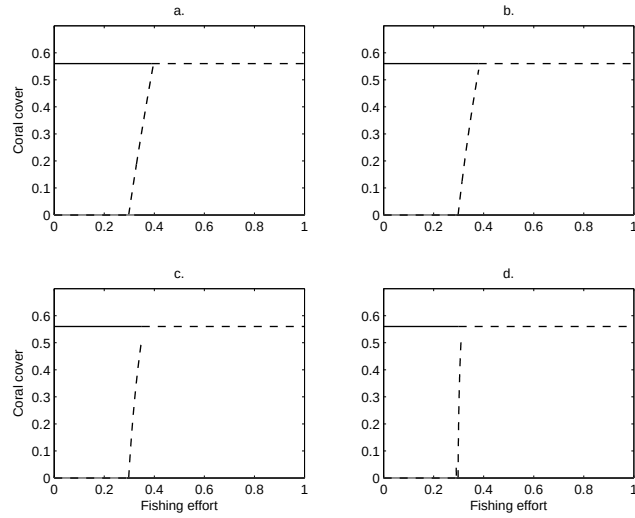


Figure 4: Bifurcation diagram of fishing effort f versus coral cover C for $z = 0.1$ (a.), $z = 0.3$ (b.), $z = 0.6$ (c.), and $z = 0.9$ (d.). Solid lines represent stable equilibria and dashed lines represent unstable equilibria. Similar to Fig. 3b. in the main text, hysteresis is possible for each value of z .

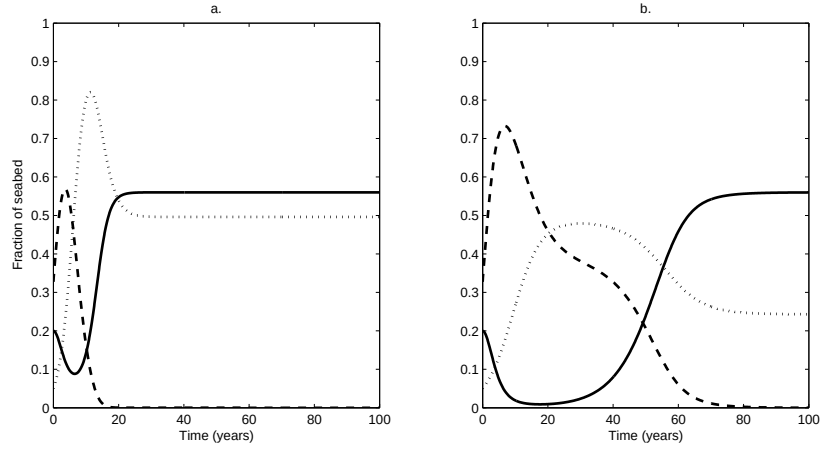


Figure 5: Time integrated dynamics with food availability as the primary limiting factor of parrotfish abundance. Solid line is coral cover, dashed line is macroalgal cover, and the dotted line is parrotfish abundance. Here, $z = 0.9$ with $f = 0$ (a.) and $f = 0.25$ (b.). Again, $C_0 = 0.2$ and M_0 is at its equilibrium value. The time dynamics are qualitatively similar to those shown in Fig. 4 in the main text