

Transmission patterns that reduce population size without increasing disease prevalence: masting and chronic wasting disease in white-tailed deer

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To understand the origins of our CWD model's equilibrium behaviors, we compare it to a standard SI and SIR models with disease-induced mortality. We include the possibility of transmission that is neither purely frequency-dependent nor purely density dependent (i.e. we do not constrain the value of q), but we note that the results hold equally for the more standard frequency-dependent ($q = 1$) and density-dependent ($q = 0$) versions of this model. The SI model with is,

$$\frac{dS}{dt} = fN - \gamma S \left(\frac{I}{N^q} \right) - \mu S \quad (\text{OR3.1})$$

$$\frac{dI}{dt} = \gamma S \left(\frac{I}{N^q} \right) - \mu I - \alpha I \quad (\text{OR3.2})$$

where S and I are the population densities of the susceptible and infected categories, with total population density $N = S + I$. All parameters have the same interpretation as in the CWD model.

The equilibrium population size of the SI model is,

$$N^* = \left(\frac{\gamma(\alpha - f + \mu)}{\alpha(\mu + \alpha)} \right)^{1/(q-1)} \quad (\text{OR3.3})$$

and the equilibrium disease incidence is,

$$\frac{I^*}{N^*} = \frac{f - \mu}{\alpha} . \quad (\text{OR3.4})$$

The SIR model with disease-induced mortality is,

$$\frac{dS}{dt} = fN - \gamma S \left(\frac{I}{N^q} \right) - \mu S \quad (\text{OR3.5})$$

$$\frac{dI}{dt} = \gamma S \left(\frac{I}{N^q} \right) - \delta I - \mu I - \alpha I \quad (\text{OR3.6})$$

$$\frac{dR}{dt} = \delta I - \mu R \quad (\text{OR3.7})$$

where R is the population densities of the recovered individuals, $N = S + I + R$ now, and δ is the recovery rate. Otherwise, parameter and variable meanings are as above.

The equilibrium population size in the SIR model is,

$$N^* = \left(\frac{\gamma\mu(\delta + \mu + \alpha) - f(\mu + \delta)}{\alpha\mu(\delta + \mu + \alpha)} \right)^{1/(q-1)}. \quad (\text{OR3.8})$$

The equilibrium disease incidence is,

$$\frac{I^*}{N^*} = \frac{f - \mu}{\alpha}. \quad (\text{OR3.9})$$

Notice that for both models, the expression for equilibrium population size (equations (OR3.3) and (OR3.8)) contain all demographic and epidemiological parameters, just as in the N^* expression for the CWD model. The equilibrium disease incidences (equations (OR3.4) and (OR3.9)) not only show the key qualitative result from the CWD model (i.e. that only the demographic parameters are present in these expressions) but in fact yield the exact same expression as the one derived for CWD (eq. (10)).

The finding that equilibrium disease incidence is a function only of demographic parameters, whereas the equilibrium population size is determined by both demographic and epidemiological parameters, is thus not specific to our CWD model but in fact arises in the most basic representations of disease dynamics.