Supplementary material for "Many weak and few strong links: The importance of link strength distributions for stabilising patterns in competition networks"

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1 Supplementary Note 1: Derivation of link strengths

The link strengths were derived from an observational data set of overgrowth competition between bryozoan colonies (Koch et al., 2023). For clarity and background, we describe below the methods described in Koch et al., 2023. The data sets originate from 3 different polar regions: Signy Island and Adelaide Island in the Antarctic as well as Spitzbergen, Svalbard in the Arctic. For each sampling location, the data set includes an abundance table, containing the number of colonies per species, as well as a species-contact-matrix that summarises the outcomes of all spatial contests between each pair of competing species. For details on data collection and preparation, see Barnes and Kuklinski (2004) and Koch et al. (2023), respectively.

The observed competitive outcomes are first translated into biomass loss rates f_{ij}, describing the loss in biomass of a species i as a result of the interference competition with species j. The total biomass loss rate [biomass/time] is calculated by summing up the number of wins W_{ij}, losses L_{ij} and draws D_{ij}, each weighted with a cost value, which describes a fixed proportion of biomass loss per colony per time:

$$f_{ij} = -0.1W_{ij} - 0.9L_{ij} - 0.2D_{ij} \tag{1}$$

Note, that the cost-values are based on assumptions, as it is impossible to measure the amount of biomass loss. Koch et al., 2023 performed robustness analysis that showed that the choice of those values did not affect the results of their analysis.

Then, the biomass loss rates are used to calculate link strengths, which are defined as the elements of the Jacobian matrices. These elements a_{ij} describe the per-capita effect of a change in the biomass of species j on the biomass of species i. This step is based on an assumption of equilibrium dynamics, which is necessary to be able to test network stability (see Koch et al. (2023) for details). Furthermore, it is assumed that the dynamics around this equilibrium are described by the following Lotka-Volterra equations:

$$\frac{dX_i}{dt} = r_i X_i - \sum_{j=1}^{n} c_{ij} X_i X_j \tag{2}$$

where X_i is the population density of species i, r_i its intrinsic growth rate, and c_{ij} a constant which describes competition intensity between species i and j. The link strengths are the partial derivatives of these equations, evaluated at equilibrium. For interspecificinteractions, the link strength is thus $a_{ij} = -c_{ij}X_i^*$. For intraspecific interactions, they are $a_{ii} = r_i - \sum_{ij}^{n} c_{ij}X_j^* - 2c_{ii}X_i^*$, which simplifies into $a_{ii} = -c_{ii}X_i^*$ at equilibrium where $r_i - \sum_{ij}^{n} c_{ij}X_j^* - c_{ii}X_i^* = 0$.

Biomass loss rates f_{ij} are assumed to correspond to the loss term due to competition $-c_{ij}X_iX_j$ and the observed abundance B_i of species i is assumed to correspond to its equilibrium density. Therefore, the link strengths can be calculated as:

$$a_{ij} = -c_{ij}X_i^* = \frac{-c_{ij}X_i^*X_j^*}{X_j^*} = \frac{f_{ij}}{B_j} \tag{3}$$

In a final step the link strengths are normalised, following Neutel and Thorne, 2014, by dividing each matrix element by the absolute values of its corresponding diagonal term: $\bar{a_{ij}} = \frac{a_{ij}}{|a_{ii}|}$. The stability properties of matrices with varying diagonal elements are difficult to compare. The normalisation makes such comparisons possible, as it translates the matrices' diagonal structure into its off-diagonal structure and results in all diagonal terms being -1, just as in May-type random matrices (see also Thorne et al., 2021). As the normalisation procedure can only be applied if all diagonal elements are $\neq 0$, missing diagonal elements were replaced by small numbers. These were estimated as the mean link strengths, multiplied with -0.1(Koch et al., 2023). The number of missing diagonal elements varied strongly between assemblages. Again, robustness checks were performed by (Koch et al., 2023) which showed that the exact method of how missing values are replaced did not affect the outcome of the analysis.

2 Supplementary Note 2: Skewness in biomass distribution

The skewed distribution of link strengths we found in empirical data can be seen as a consequence of a colonisation-competition trade-off in bryozoan assemblages (Barnes and Conlan, 2007). Assemblages contain both strong colonisers who tend to dominate in biomass (Barnes and Clarke, 1998) but lose most contests, and strong competitors who are less abundant but win most contests.

Table 1 illustrates these strong differences in abundance within one data set. As shown in Figure 1 A for the same example system, this resulted in very skewed energy loss rates. Translating energy loss rates to link strengths then further increased the level of skewness because energy loss rates were divided by abundances, which are also skewed (Figure 1 B). Finally, the normalisation of link strengths slightly reduced the level of skewness again (Figure 1 C). Fig 1 D shows the skewness-kurtosis relationship of energy loss rates, raw and normalised link strengths in all data sets, demonstrating that what is shown in the example in Figure 1 A-C applies to all 30 networks. The skewness that we saw in the link strengths was thus an inherent property of the empirical data and was not imposed by the derivation of link strengths.

Table 1: Abundance [number of colonies] for each species in example data set Rothera 3

Species	Abundance
H	3
M	5
S	7
L	8
E	9
C	9
Ca	12
Ar	17
X	20
Ai	39
F	820

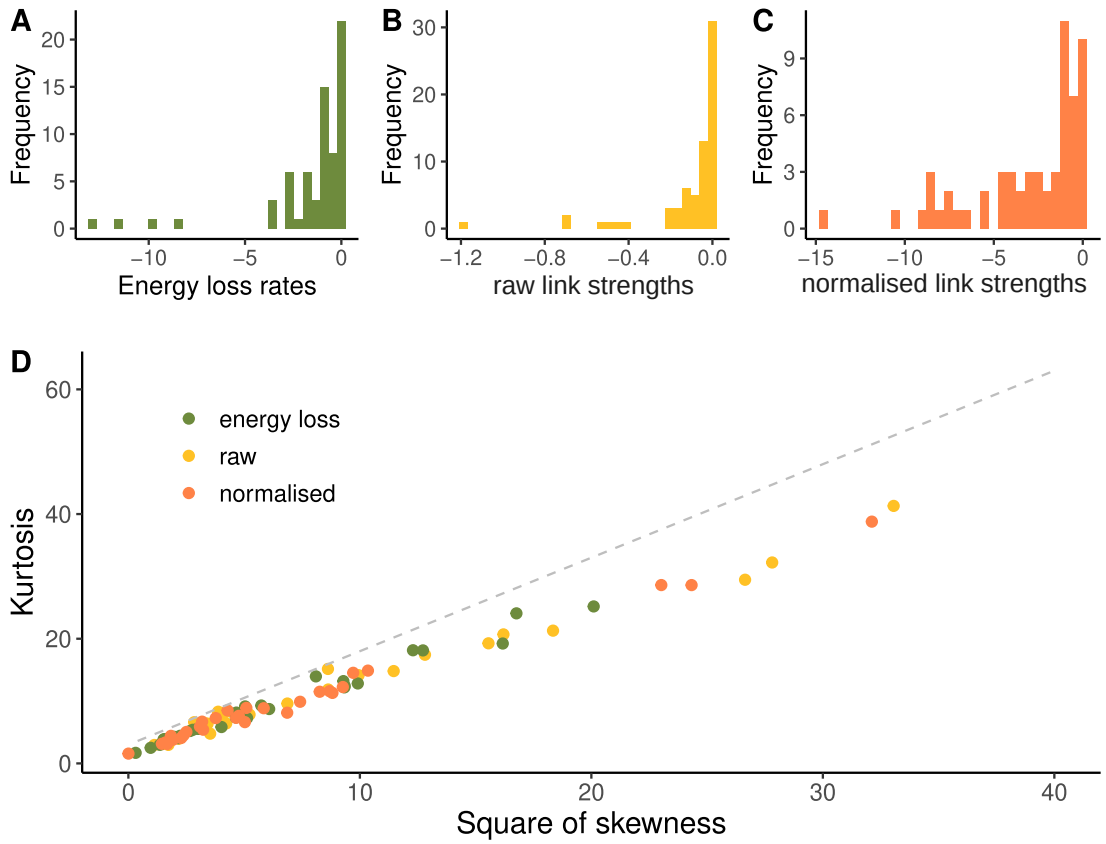


Figure 1: **Distribution of energy loss rates, raw and normalised link strengths**. (A-C) histograms showing the distribution of energy loss rates, raw link strengths and normalised link strengths for example data set *Rothera 3* (D) Skewness-kurtosis relationship of energy loss rates, raw and normalised link strengths for all data sets.

3 Supplementary Tables

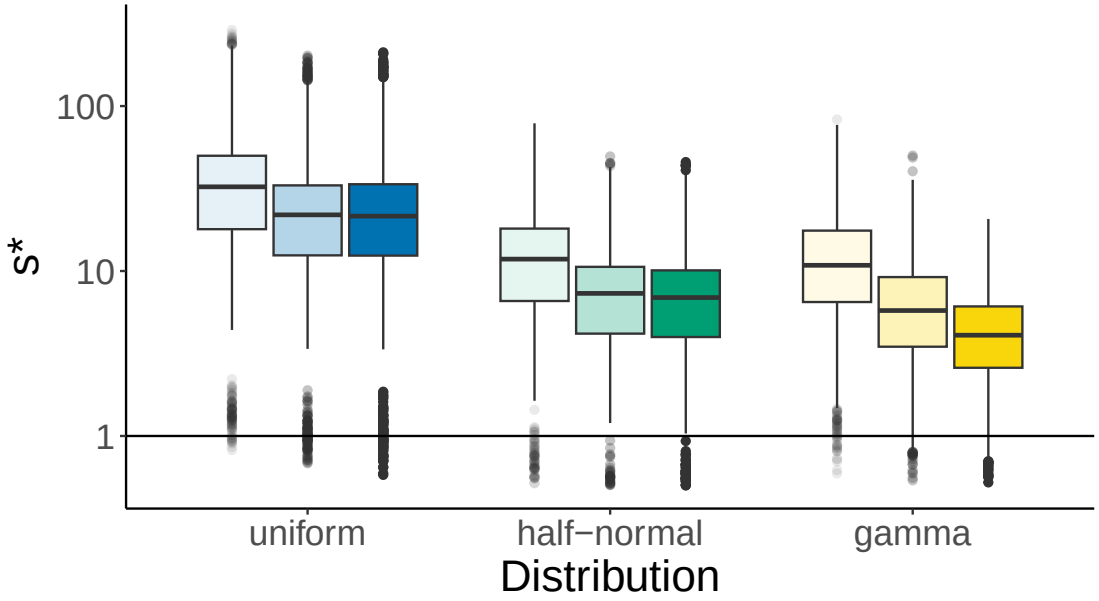
Supplementary Table 1: **Properties of empirical link strengths** For each data set we show the number of species, S, and the connectance C, which is the number of realised links relative to the number of possible links. Furthermore, distribution shape is characterised by their skewness $\hat{S}$, describing how asymmetric the distribution of link strengths is, and kurtosis $\hat{K}$ which describes how peaked the distribution is.

Name	Number of species S	Connectance C	Skewness $\hat{S}$	Kurtosis $\hat{K}$
Rothera_1	11	0.45	-4.80	28.61
Rothera_2	9	0.47	-1.51	4.08
Rothera_3	11	0.55	-1.36	4.42
Signy_1	8	0.89	-1.54	4.49
Signy_2	8	0.89	-1.78	6.70
Signy_3	8	0.89	-2.42	8.85
Signy_4	8	0.93	-3.22	14.88
Spitzbergen_H1_12_1	8	0.93	-3.12	14.53
Spitzbergen_H1_12_2	8	0.96	-2.16	7.31
Spitzbergen_H1_6_1	7	0.95	-2.62	8.13
Spitzbergen_H1_6_2	7	0.95	-4.93	28.60
Spitzbergen_H2_12_1	5	0.80	-0.06	1.57
Spitzbergen_H2_12_2	6	0.73	-2.72	9.89
Spitzbergen_H2_6_1	5	0.60	-1.30	3.12
Spitzbergen_H2_6_2	6	0.73	-2.95	11.58
Spitzbergen_H3_12_1	6	0.93	-1.20	3.14
Spitzbergen_H3_12_2	7	0.86	-1.58	5.04
Spitzbergen_H3_6_1	7	1.00	-1.76	5.91
Spitzbergen_H3_6_2	7	0.95	-2.18	7.63
Spitzbergen_K1_12_1	9	0.83	-5.67	38.78
Spitzbergen_K1_12_2	6	0.93	-1.94	7.26
Spitzbergen_K1_6_1	10	0.93	-2.07	8.49
Spitzbergen_K1_6_2	10	0.93	-2.26	8.90
Spitzbergen_K2_12_1	6	0.87	-3.04	12.25
Spitzbergen_K2_12_2	4	0.83	-2.24	6.62
Spitzbergen_K2_6_1	7	0.86	-2.87	11.50
Spitzbergen_K2_6_2	7	0.86	-1.39	3.79
Spitzbergen_K3_12_1	5	0.90	-2.97	11.31
Spitzbergen_K3_12_2	5	0.90	-1.27	3.37
Spitzbergen_K3_6_2	6	0.93	-1.80	5.41

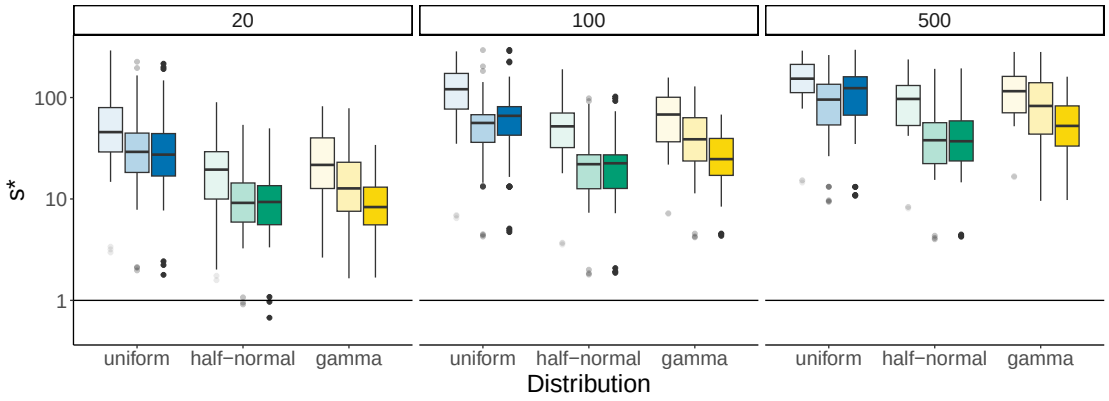
Supplementary Table 2: **Fitted parameter values for theoretical distributions** Theoretical distributions were fitted to each empirical set of link strengths using the R package *fitdistrplus*.

Name	uniform		half-normal	gamma	
	min	max	scale σ	shape α	rate λ
Rothera_1	-76.00	-0.0037	11.51	0.39	0.08
Rothera_2	-18.01	-0.0095	4.80	0.44	0.12
Rothera_3	-14.58	-0.0296	4.40	0.72	0.24
Signy_1	-9.24	-0.0402	2.85	0.74	0.36
Signy_2	-10.18	-0.0271	2.58	0.75	0.40
Signy_3	-13.19	-0.0545	3.21	0.78	0.36
Signy_4	-24.94	-0.0373	4.66	0.58	0.23
Spitzbergen_H1_12_1	-40.74	-0.0122	7.61	0.42	0.11
Spitzbergen_H1_12_2	-32.95	-0.0318	8.53	0.55	0.11
Spitzbergen_H1_6_1	-32.68	-0.0038	7.81	0.35	0.09
Spitzbergen_H1_6_2	-48.09	-0.0137	6.80	0.39	0.13
Spitzbergen_H2_12_1	-1.42	-0.0829	0.45	2.08	2.74
Spitzbergen_H2_12_2	-16.81	-0.1950	2.77	0.85	0.33
Spitzbergen_H2_6_1	-11.38	-0.0298	2.25	0.52	0.17
Spitzbergen_H2_6_2	-18.49	-0.0944	2.86	0.63	0.26
Spitzbergen_H3_12_1	-9.40	-0.0376	2.70	0.92	0.33
Spitzbergen_H3_12_2	-15.60	-0.0141	4.08	0.88	0.24
Spitzbergen_H3_6_1	-26.31	-0.0191	7.02	0.66	0.12
Spitzbergen_H3_6_2	-50.06	-0.0295	10.92	0.63	0.08
Spitzbergen_K1_12_1	-154.02	-0.0520	22.56	0.42	0.05
Spitzbergen_K1_12_2	-49.16	-0.0003	9.44	0.39	0.05
Spitzbergen_K1_6_1	-32.79	-0.0231	8.80	0.64	0.14
Spitzbergen_K1_6_2	-38.18	-0.0243	10.39	0.62	0.12
Spitzbergen_K2_12_1	-86.04	-0.0079	13.57	0.31	0.03
Spitzbergen_K2_12_2	-39.78	-0.0222	5.47	0.35	0.05
Spitzbergen_K2_6_1	-49.94	-0.0007	8.89	0.34	0.07
Spitzbergen_K2_6_2	-28.96	-0.0002	7.68	0.41	0.07
Spitzbergen_K3_12_1	-68.06	-0.0016	9.85	0.31	0.04
Spitzbergen_K3_12_2	-36.22	-0.0010	7.39	0.34	0.04
Spitzbergen_K3_6_2	-47.94	-0.0041	10.10	0.39	0.05

4 Supplementary Figures



Supplementary Figure 1: **Stabilising effect of asymmetries in matrices with varying distributions of link strengths and random topology.** Comparison of stability in random (lightest shade), pairwise asymmetric (medium shade) and community asymmetric (darkest shade) matrices with varying distributions of link strengths. Stability is measured as the critical amount of self-regulation s^*. Matrices are created exactly as those shown in Fig 3 A of the main analysis, but instead of using empirical topologies, the location of non-zero links is chosen at random.



Supplementary Figure 2: **Stabilising effect of asymmetries in larger matrices**. Comparison of stability in random (lightest shade), pairwise asymmetric (medium shade) and community asymmetric (darkest shade) matrices with varying distributions of link strengths and varying sizes. Matrices were created using the same distributions as in the main analysis. Connectance was chosen corresponding to the empirical matrices, but size was varied. Increasing the size of the matrices led to an overall increase in instability. In matrices with gamma- and normally distributed link strengths, the stability effects of patterning remained the same for all sizes: For gamma, community asymmetry was more stable than pairwise asymmetry, while in the normal distribution, we see no difference between the two. In the large uniformly distributed matrices, we observed that community asymmetry increases instability compared to pairwise asymmetry.

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