

6 Online Resource

This section includes a description of the h_{thresh} selection process from section 2.2.2 as well as computational details regarding closeness and volumetric closeness metrics defined in section 2.2.3. Specifically, we describe in greater detail how the functions $d_{skel}(\cdot)$ and $d_{vol}(\cdot)$ are computed.

FIFA Quality Programme Test Manuals

In section 2.1, we reference certification tests performed under the FIFA Quality Programme for the optical limb-tracking system and instrumented ball. The details of these tests can be accessed as PDFs in the Online Resource.

Selecting h_{thresh} for the Decision Frame

As discussed in section 2.2.2, the decision frame is defined as the last peak in touch probability exceeding some threshold, notated h_{thresh} . We determine the optimal h_{thresh} through a coarse grid search across candidate values from 0.50 to 1.00 with a step size of 0.05, optimizing for accuracy of the rules-based closeness and volumetric closeness approaches (CA and VCA) across the entire duel dataset. The results of this process are illustrated in Figure 8. We note that using a threshold of 1 is actually equivalent to exclusively resorting to the $\arg \max_f \{S\}$ case in Definition 3, since the touch probability model never predicts values exactly equal to 1.

We observe a slight peak at $h_{thresh} = 0.75$ for both CA and VCA, which we prioritize because they are the best representations of the data-generating process we are trying to model. Threshold value differences yield significant but relatively small performance variations. As expected, NCA accuracy remains constant because it operates independently of the decision frame.

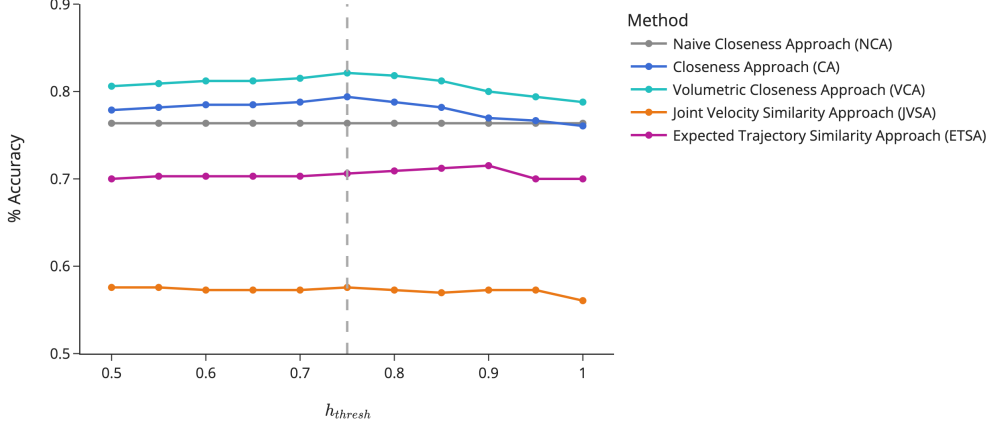


Fig. 8 Classification accuracy of rules-based approaches across different values of h_{thresh} for decision frame detection. The vertical dashed line represents peaks in CA and VCA accuracy observed at a value of 0.75. We observe that accuracies are not very sensitive to h_{thresh}

6.1 Mapping Limbs to Body Regions

The BSSC and BSMC approaches discussed in Section 2.2.4 involve segmenting the duels by the body region where the last touch most likely occurred. The body regions include the head, upper body, and lower body. Table 4 details which skeletal limbs map to which body regions for both skeletal and volumetric player representations.

Closeness

When measuring the closeness between the ball and a player p with limbs S_p , we compute the minimum Euclidean distance between a limb $s_i \in S_p$ and the ball, notated as $d_{skel}(\vec{s}_i)$. Let $\vec{r}_{ball} \in \mathbb{R}^3$ be the xyz-coordinate of the ball and let $\vec{s}_i^{start}, \vec{s}_i^{end} \in \mathbb{R}^3$ be the xyz-coordinates of the start and endpoints of the line segment representing the limb s_i . The point-to-line segment distance is computed as follows:

1. Compute the projection factor, or how far along the line segment the perpendicular projection of the ball falls:

$$t = \frac{(\vec{r}_{ball} - \vec{s}_i^{start}) \cdot (\vec{s}_i^{end} - \vec{s}_i^{start})}{\|\vec{s}_i^{end} - \vec{s}_i^{start}\|^2}$$

Table 4: Body Part to Region Mapping

Body Region	Skeletal Limb	Volumetric Body Part
Head (N=205)	- L/R Ear-to-Crown - L/R Ear-to-Nose - Ear-line - Eye-line - Mid-head - Throat	- Head
Upper (N=18)	- L/R Collar - L/R Upper-arm - L/R Forearm - L/R Lat - L/R Hip-line	- Torso - L/R Upper-arm - L/R Forearm
Lower (N=107)	- L/R Thigh - L/R Shin - L/R Achilles - L/R Outer-foot - L/R Inner-foot - L/R Toe-line	- L/R Thigh - L/R Shin - L/R Achilles - L/R Foot

2. If $0 \leq t \leq 1$, the perpendicular projection lies within the limb. In this case, compute the projected point

$$\vec{p} = \vec{s}_i^{\text{start}} + t(\vec{s}_i^{\text{end}} - \vec{s}_i^{\text{start}})$$

and then calculate the distance from the ball to the limb as

$$d_{skel}(s_i) = \|\vec{r}_{\text{ball}} - \vec{p}\|$$

3. If $t < 0$ or $t > 1$, the projection lies beyond the limb segment. The minimum distance is the smaller of the distances to the endpoints:

$$d_{skel}(s_i) = \min \left\{ \|\vec{r}_{\text{ball}} - \vec{s}_i^{\text{start}}\|, \|\vec{r}_{\text{ball}} - \vec{s}_i^{\text{end}}\| \right\}$$

Volumetric Closeness

When measuring the volumetric closeness between the ball and a player p with body part shapes B_p , we compute the minimum Euclidean distance between the surface boundary of $\vec{b}_i \in B_p$ and the ball, notated as $d_{vol}(\vec{b}_i)$. Let $\vec{r}_{ball} \in \mathbb{R}^3$ be the xyz-coordinate of the ball, which is a sphere of radius R_{ball} . We must consider three cases when computing each $d_{vol}(\vec{b}_i)$.

1. $\vec{b}_i$ represents a sphere.
2. $\vec{b}_i$ represents a cylinder.
3. $\vec{b}_i$ represents a triangular prism.

Case 1

Under case 1, where $\vec{b}_i$ is a sphere with center $\vec{r}_{b_i} \in \mathbb{R}^3$ and radius R_{b_i} , we can simply take the 3D Euclidean distance between the two sphere centers and subtract out the length of each radius.

$$d_{vol}(\vec{b}_i) = \max \left\{ 0, \left\| \vec{r}_{ball} - \vec{r}_{b_i} \right\| - (R_{ball} + R_{b_i}) \right\}.$$

The $\max\{0, \cdot\}$ operation ensures that if the two spheres intersect, the distance is set to zero rather than yielding a negative value, although we did not encounter this scenario.

Case 2

For case 2, where $\vec{b}_i$ represents a right circular cylinder with a central axis defined by endpoints $\vec{r}_{b_i}^{\text{start}}, \vec{r}_{b_i}^{\text{end}} \in \mathbb{R}^3$ and radius R_{b_i} , we compute the volumetric closeness between the ball and the cylinder as follows. First, determine the shortest Euclidean distance from $\vec{r}_{ball}$ to the line defined by the cylinder's central axis. The distance from the point $\vec{r}_{ball}$ to the line, notated as d_{axis} , can be computed using the same formula from Case 1. Then, the distance between the surfaces of the ball and the cylindrical limb is found by subtracting the sum of their radii from d_{axis} :

$$d_{vol}(\vec{b}_i) = \max \left\{ 0, d_{axis} - (R_{ball} + R_{b_i}) \right\}$$

The $\max\{0, \cdot\}$ operation ensures that the distance is set to zero if the objects overlap, which we did not encounter in this study.

Case 3

Case 3 occurs when $\vec{b}_i$ represents a triangular prism with four rectangular faces and two triangular faces defined by the set of vertices $\{v_i\}_{i=1}^6$. We compute $d_{vol}(\vec{b}_i)$ by finding the distance between the ball sphere and all six faces, and then taking the minimum value.

When the face is a triangle consisting of vertices $\vec{v}_1^{tri}, \vec{v}_2^{tri}, \vec{v}_3^{tri} \in \mathbb{R}^3$ and edges $\vec{e}_{12}^{tri}, \vec{e}_{23}^{tri}, \vec{e}_{31}^{tri} \in \mathbb{R}^3$, we compute four distances to the ball sphere. The first is the distance to the centroid of the triangle, and the next three are the point-to-line-segment distances between the edges and the ball. We compute the centroid as:

$$\vec{c}_{tri} = \frac{1}{3} \left(\vec{v}_1^{tri} + \vec{v}_2^{tri} + \vec{v}_3^{tri} \right)$$

and then calculate the distance from the ball center to the centroid and subtract the ball's radius:

$$d_{centroid} = \|\vec{r}_{ball} - \vec{c}_{tri}\| - R_{ball}$$

Next, for each of the three edges, compute the point-to-line-segment distance from the ball center to the edge, then subtract R_{ball} to obtain d_{12}, d_{23}, d_{31} . The distance to the triangular face is the minimum of these distances.

$$d_{tri} = \min \left\{ d_{centroid}, d_{12}, d_{23}, d_{31} \right\}$$

The computation for rectangular faces (d_{rect}) is identical, except we compute distances to the four edges as well as the two diagonals.

$$d_{rect} = \min \left\{ d_{centroid}, d_{12}, d_{23}, d_{34}, d_{41}, d_{13}, d_{24} \right\}$$

Finally, we compute $d_{vol}(\vec{b}_i)$ as the minimum of the six minima computed, one for each unique face. The final computation is:

$$d_{vol}(\vec{b}_i) = \max\left\{d_{tri}^1, d_{tri}^2, d_{tri}^3, d_{tri}^4, d_{rect}^5, d_{rect}^6\right\}$$