Supplementary Information

1. **Figures**

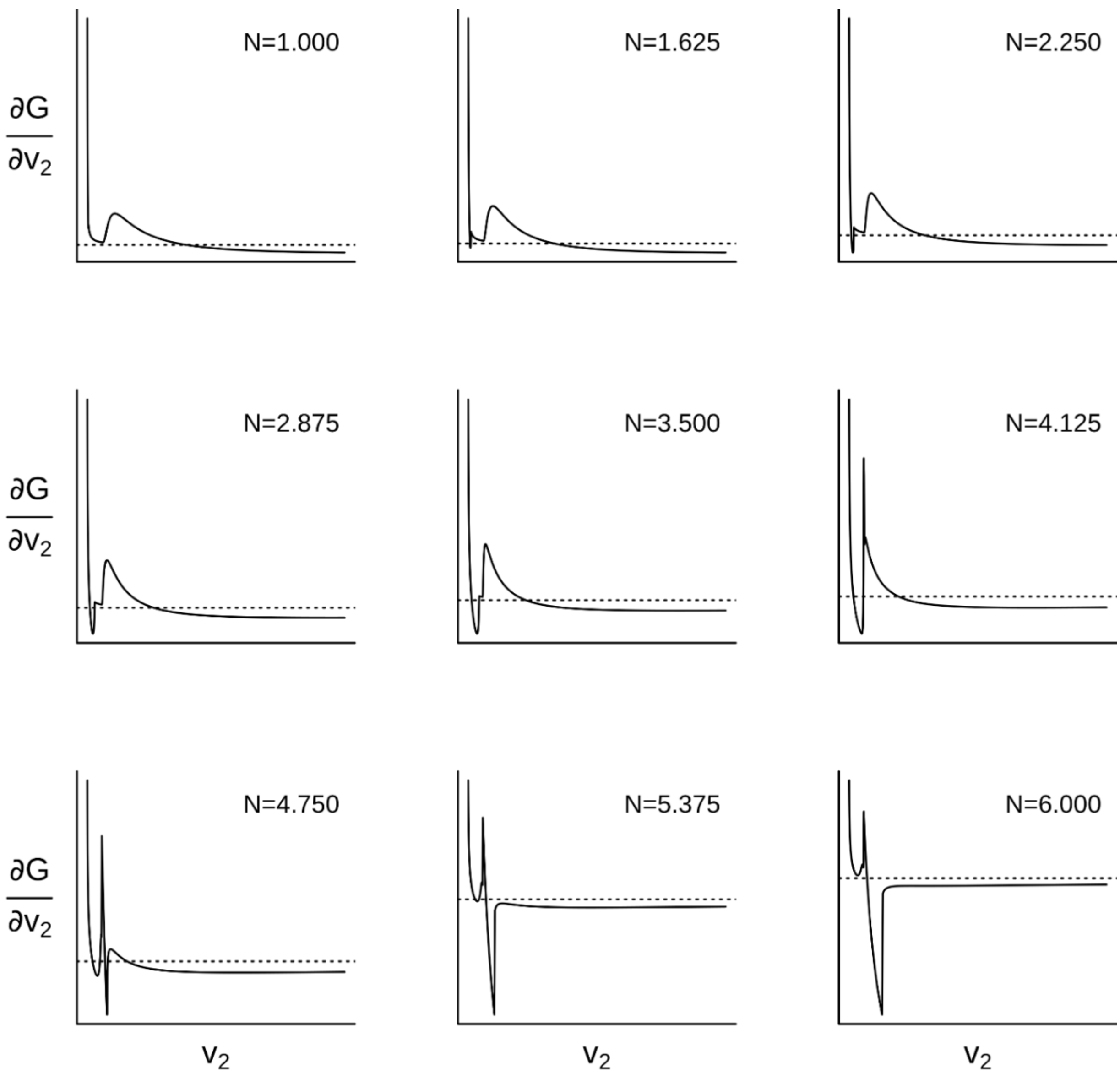

Figure S1 A plot of $\frac{\partial G}{\partial v_2}$ *vs.* v_2 *for various population densities (N). As N increases, we move from having only one optimal specialization point* ($N = 1$) *to having three points of optimal specialization* ($N = 1.625, 2.25, 2.875, 3.5, 4.125$) *to a maximum of five* ($N = 4.75$) *back down to three* ($N = 5.375$) *then one* ($N = 6$).

2. Data

Data and code used for the analysis can be found at 10.5061/dryad.pzgmsbcs7.

3. Further Analysis

We present our competition model again. Fitness of an organism is represented by the formula $G(\vec{v},\mathbf{u},\vec{N})\,|_{\vec{v}=\vec{u_i}} = \beta(\vec{v},\mathbf{u},\vec{N})\,|_{\vec{v}=\vec{u_i}} - d$ where $\beta(\vec{v},\mathbf{u},\vec{N})\,|_{\vec{v}=\vec{u_i}} = \sum_{j=1}^{n_R} \hat{\alpha}\,(z_j,\vec{v},\mathbf{u},\vec{N})\,|_{\vec{v}=\vec{u_i}}$ and where

$$\hat{\alpha}(z_j,\vec{v},\mathbf{u},\vec{N}) = \begin{cases} \alpha(z_j,\vec{v}) & ,\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N}) < R(z_j) \\ \dfrac{\alpha(z_j,\vec{v})}{\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N})}\cdot R(z_j) & ,\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N}) > R(z_j) \end{cases} \tag{1}$$

In our case, $R(z) = K_{max}e^{-\frac{z_j^2}{2\sigma_K^2}}$, $\alpha\left(z_j,\vec{v}\right) = v_2^{-n}e^{\frac{-(z_j-v_1)^2}{cv_2}}$, and $\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N}) = \alpha(z_j,\vec{v}) + \sum_{i=1}^{n_s}\alpha(z_j,\vec{u_i})N_i$.

Since each resource is independent of the other, we can say that $\frac{du_2}{dt} = \frac{\partial G(\vec{v},\mathbf{u},\vec{N})}{\partial v_2}\,|_{\vec{v}=\vec{u_i}} = \frac{\partial \beta(\vec{v},\mathbf{u},\vec{N})}{\partial v_2}\,|_{\vec{v}=\vec{u_i}} = \sum_{j=1}^{n_R}\frac{\partial\hat{\alpha}(z_j,\vec{v},\mathbf{u},\vec{N})}{\partial v_2}\,|_{\vec{v}=\vec{u_i}}$ where

$$\frac{\partial\hat{\alpha}(z_j,\vec{v},\mathbf{u},\vec{N})}{\partial v_2} = \begin{cases} \dfrac{\partial\alpha(z_j,\vec{v})}{\partial v_2} & ,\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N}) < R(z_j) \\ \dfrac{\partial\alpha(z_j,\vec{v})}{\partial v_2}\cdot\dfrac{\sum_{i=1}^{n_s}\alpha\,(z_j,\vec{u_i})N_i}{\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N})^2}\cdot R(z_j) & ,\alpha_T(z_j,\vec{v},\mathbf{u},\vec{N}) > R(z_j) \end{cases} \tag{2}$$

with $\frac{\partial\alpha(z_j,\vec{v})}{\partial v_2} = \frac{((z-v_1)^2 - cnv_2)}{cv_2^2}\alpha(z_j,\vec{v})$. Caluculating $\frac{\partial^2 G}{\partial v_1^2}$, we get

$$\frac{\partial^2 \hat{\alpha}(z_j, \vec{v}, \mathbf{u}, \vec{N})}{\partial v_1^2} = \begin{cases} \dfrac{\partial^2 \alpha(z_j, \vec{v})}{\partial v_1^2} & , \alpha_T(z_j, \vec{v}, \mathbf{u}, \vec{N}) < R(z_j) \\ \left(\dfrac{\partial^2 \alpha(z_j, \vec{v})}{\partial v_1^2} - \dfrac{2\left(\dfrac{\partial \alpha(z_j, \vec{v})}{\partial v_1}\right)^2}{\alpha_T(z_j, \vec{v}, \mathbf{u}, \vec{N})} \right) \dfrac{\sum_{i=1}^{n_s} \alpha\,(z_j, \overrightarrow{u_i}) N_i}{\alpha_T(z_j, \vec{v}, \mathbf{u}, \vec{N})^2} \cdot R(z_j) & , \alpha_T(z_j, \vec{v}, \mathbf{u}, \vec{N}) > R(z_j) \end{cases} \quad (3)$$

with $\frac{\partial \alpha(z_j, \vec{v})}{\partial v_1} = \frac{2(z - v_1)}{c v_2} \alpha(z_j, \vec{v})$ and $\frac{\partial^2 \alpha(z_j, \vec{v})}{\partial v_1^2} = \frac{4(z_j - v_1)^2 - 2 c v_2}{(c v_2)^2} \alpha(z_j, \vec{v})$.

As the number of resources n_R increases, $\beta(\vec{v}, \mathbf{u}, \vec{N}) \mid_{\vec{v}=\overrightarrow{u_i}}$ will go to infinity, causing similar extremes for $\frac{\partial \beta(\vec{u}, \mathbf{u}, \vec{N})}{\partial v_2} \mid_{\vec{v}=\overrightarrow{u_i}}$. If the number of resources is not increasing but instead being more finely divided, we can multiply each interaction by a decreasing weight w such that $\lim_{n_R \to \infty} w = 0$. We arrive at a new equation $\beta(\vec{v}, \mathbf{u}, \vec{N}) \mid_{\vec{v}=\overrightarrow{u_i}} = \sum_{j=1}^{n_R} w_j \cdot \hat{\alpha}(z_j, \vec{v}, \mathbf{u}, \vec{N}) \mid_{\vec{v}=\overrightarrow{u_i}}$. Similarly, $\frac{\partial \beta(\vec{u}, \mathbf{u}, \vec{N})}{\partial v_2} \mid_{\vec{v}=\overrightarrow{u_i}} = \sum_{j=1}^{n_R} w_j \cdot \frac{\partial \hat{\alpha}(z_j, \vec{v}, \mathbf{u}, \vec{N})}{\partial v_2} \mid_{\vec{v}=\overrightarrow{u_i}}$. Assuming w_j is constant ($w_j = w$ for all $j = 1, 2, \ldots, n_R$), it simply gets folded into the speed of the evolutionary and population dynamics and does not affect the equilibrium. This weight instead acts like a Reimann summation and therefore, the species' interactions with an infinitely fine discrete resource base can be calculated as an integral $\frac{du_2}{dt} = \int_{-\infty}^{\infty} \frac{\partial \hat{\alpha}(z, \vec{v}, \mathbf{u}, \vec{N})}{\partial v_2} \mid_{\vec{v}=\overrightarrow{u_i}} dz$.

To analyze our model, we must obtain the values of the resource attribute $z = \hat{z}$ such that $\alpha_T(\hat{z}, \vec{v}, \mathbf{u}, \vec{N}) = R(\hat{z})$ as the function is piecewise. Because α_T is the sum of multiple exponential functions and $R(z)$ is another exponential function, we cannot arrive at a general solution for evolutionary dynamics. Instead, we can arrive at a more limited solution using some key assumptions. Firstly, we can assume that there is only one population using a single strategy

$\vec{u}$ (or all species use the same strategy). Secondly, we only look to derive evolutionary dynamics when $\vec{v} = \vec{u}$. Doing so means that we numerically solve for the equilibrium. Because of these two assumptions, $\alpha_T(z, \vec{v}, \mathbf{u}, \vec{N}) \mid_{\vec{v}=\vec{u}} = (N+1)\alpha(z, \vec{u})$. Therefore, we now have an exponential function equalling an exponential function, making the evolutionary dyanmics solveable. Since $R(z)$ is symmetrical about 0, we can also assume that $u_1 = 0$. Altogether, solving for $\hat{z}$ yields

$$\hat{z} = \sqrt{\frac{-2\sigma_K^2 c u_2 log\left(\frac{K_{max} u_2^n}{N+1}\right)}{2\sigma_K^2 - cu_2}} \tag{4}$$

Within the $\hat{z}$, there is the potential for the square root of a negative number. Because $\sigma_K^2 c u_2$ is strictly positive, only $log\left(\frac{K_{max} u_2^n}{N+1}\right)$ and $2\sigma_K^2 - cu_2$ can affect the sign of $\hat{z}$. Hereafter, we refer to these terms as H and S respectively. Looking at the terms, we can view them as comparing the resource function $R(z)$ and the total utilization function $(N+1) \cdot \alpha(z, \vec{u})$. The first term $H = log\left(\frac{K_{max} \cdot u_2^n}{N+1}\right)$ compares the "height" of each curve at resource $z = 0$. If positive, then the amount of resource $z = 0$ is sufficient for all individuals $R(0) > (N+1) \cdot \alpha(0, (0, u_2))$; if negative, then the amount is insufficient $R(0) < (N+1) \cdot \alpha(0, (0, u_2))$. The second term compares $S = 2\sigma_K^2 - cu_2$ the "spread" of each curve. If positive, then at the furthest resources $\lim_{z\to-\infty,\infty}$ there will be sufficient resources for all individuals $\lim_{z\to-\infty,\infty} R(z) > (N+1) \cdot \alpha(z, (0, u_2))$; if negative, then insufficient $\lim_{z\to-\infty,\infty} R(z) < (N+1) \cdot \alpha(z, (0, u_2))$. For there to be a positive $\hat{z}$, the first term and the second term must be opposite signs. We can go on to break this down as such:

	$S > 0$	$S = 0$	$S < 0$

$H > 0$	Sufficient Amounts of All Resources	Sufficient Amounts of All Resources	Sufficient Amounts of Core Resources, Insufficient Amounts of Marginal Resources
$H = 0$	Sufficient Amounts of All Resources	Treated as Insufficient Amounts of All Resources	Insufficient Amounts of All Resources
$H < 0$	Insufficient Amounts of Core Resources, Sufficient Amounts of Marginal Resources	Insufficient Amounts of All Resources	Insufficient Amounts of All Resources

Table S1: Comparison between resource curve and total utilization curve

Here, core resources are the resources whose attributes fall between $-\hat{z}$ and $\hat{z}$, and marginal resources are the complement.

3.1. *The Relationship between Population Size and Death Rate*

Going back to our original function and looking at the table, we can note some key things. Firstly, there must be enough resources for at least one individual to overcome intrinsic mortality d. Consequently, $\int_{-\infty}^{\infty} R(z)dz > d$. This is a necessary but not sufficient condition for positive growth. One more key condition is that the utilization curve of a single individual must also overcome intrinsic mortality for some degree of specialization v_2. This is because the maximum amount of the resource an individual can capture based on its utilization curve is constrained within the resource distribution, and so $\int_{-\infty}^{\infty} R(z)dz \geq \int_{-\infty}^{\infty} \alpha(z, \vec{v})dz$. While these are key conditions, because all curves, resource and utilization, are positive, then there can be a d so small to guarantee positive fitness and positive population size.

Mathematically, we can show the fitness gains for an individual given some degree of specialization u_2. If there are sufficient amounts of resource for all individuals across all resource attributes which happens when $H > 0$ and $S > 0$, then the fitness gains are merely the integral of the individual's utilization curve $\int_{-\infty}^{\infty} \alpha(z, \vec{v}) dz \mid_{\vec{v}=\vec{u}} = \sqrt{\pi c u_2} u_2^{-n}$. If there are insufficient amounts of resource for all individuals across all resource attributes which happens when $H < 0$ and $S < 0$, then the fitness gains is the integral of the resource divided by the total population (since each individual has the same utilization curve, they cancel out) $\int_{-\infty}^{\infty} \frac{R(z)}{N+1} dz = \frac{K_{max}\sqrt{2\pi\sigma_K^2}}{N+1}$. If there are sufficient amounts for some resource attributes but insufficient for other, then there will be a $\hat{z}$ that divides the resources into core and marginal resources as seen earlier. Depending upon whether the sufficiency/insufficiency is in the core/marginal resources will determine fitness gains.

If core resources are sufficient and marginal resources are insufficient, $H > 0$ and $S < 0$, then we break the fitness gain into two parts. In the core resources, then the fitness gains equal the integral $\int_{-\hat{z}}^{\hat{z}} \alpha(z, \vec{v}) dz \mid_{\vec{v}=\vec{u}}$; the fitness gains for marginal resources is the integral $\int_{-\infty}^{-\hat{z}} \frac{R(z)}{N+1} dz$ plus the integral $\int_{\hat{z}}^{\infty} \frac{R(z)}{N+1} dz$, both of which are equal to each other due to symmetry. Solving for each component, we get:

- $\int_{-\hat{z}}^{\hat{z}} \alpha(z, \vec{v}) dz \mid_{\vec{v}=\vec{u}} = \sqrt{\pi c u_2} u_2^{-n} erf\left(\frac{\hat{z}}{\sqrt{c u_2}}\right)$
- $\int_{-\infty}^{-\hat{z}} \frac{R(z)}{N+1} dz = \int_{\hat{z}}^{\infty} \frac{R(z)}{N+1} dz = \frac{K_{max}\sqrt{\frac{\pi\sigma_K^2}{2}}}{N+1} erfc\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right)$

Total fitness gains now equal $\sqrt{\pi c u_2} u_2^{-n} erf\left(\frac{\hat{z}}{\sqrt{c u_2}}\right) + \frac{K_{max}\sqrt{2\pi\sigma_K^2}}{N+1} erfc\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right)$. We can do the same when core resources are insufficient and marginal resources are sufficient, $H < 0$ and $S > 0$. Doing so yields:

- $\int_{-\hat{z}}^{\hat{z}} \frac{R(z)}{N+1} dz = \frac{K_{max}\sqrt{2\pi\sigma_K^2}}{N+1} erf\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right)$
- $\int_{-\infty}^{-\hat{z}} \alpha(z,\vec{v})dz \mid_{\vec{v}=\vec{u}} = \int_{\hat{z}}^{\infty} \alpha(z,\vec{v})dz \mid_{\vec{v}=\vec{u}} = \frac{\sqrt{\pi c u_2} u_2^{-n}}{2} er f c\left(\frac{\hat{z}}{\sqrt{c u_2}}\right)$

Total fitness gains now equal $\sqrt{\pi c u_2} u_2^{-n} er f c\left(\frac{\hat{z}}{\sqrt{c u_2}}\right) + \frac{K_{max}\sqrt{2\pi\sigma_K^2}}{N+1} erf\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right)$.

Using these equations and some intuition, we can prove that increasing the population N always leads to decreases in fitness, and therefore a unique equilibrium N^*, for a certain death rate d given a fixed population specialization u_2. We start with a resource with a specific attribute. If there is a sufficient amount of that resource for all individuals in the population and an individual can gain positive fitness with some combination of $\vec{u}$, then an individual will have fitness gain from that resource of $\alpha(z,\vec{v}) \mid_{\vec{v}=\vec{u}}$. As long as there is a sufficient amount of resources across all attributes available to a population, then the members do not experience negative density dependence as the population grows. As soon as this resource turns insufficient, then density dependence occurs as species individuals take a smaller fraction of the insufficient resources, specifically $\frac{R(z)}{N+1}$. We also know that the fitness gain at this critical point where we go from sufficient resources to insufficient resources will be decreasing since insufficiency of resource is defined by the total consumption of that resource across all individuals being less than the resource amount $\alpha_T(z,\vec{v},\vec{u},N) \mid_{\vec{v}=\vec{u}} > R(z)$. Since $\alpha_T(z,\vec{v},\vec{u},N) \mid_{\vec{v}=\vec{u}} = (N+1) \cdot \alpha(z,\vec{v}) \mid_{\vec{v}=\vec{u}}$, then $\alpha(z,\vec{v}) \mid_{\vec{v}=\vec{u}} > \frac{R(z)}{N+1}$. This is compounded by the fact that as the population increases, more and more resources will become insufficient. As N increases, H decreases. If $S > 0$, $\hat{z}$ will turn positive and grow to infinity with increasing N, leading to more and more core resources to become insufficient and subject to density dependence. If $S < 0$, then $\hat{z}$ decreases with fewer and fewer core resources remaining sufficient, until all resources are insufficient and therefore subject to density dependence. And so, increasing N^* leads to a monotonic fitness decrease to zero. Therefore, given any value of d, there will always be a corresponding equilibrium population N^*. Taking the derivative of our equations for fitness gains in both scenarios with respect to population N should yield a constant negative number and confirm our result.

It must be noted that this argument relies on a fixed level of specialization throughout the growth of the population. Certainly, specialization may vary depending upon population size, which will affect whether there is sufficient resource of a given attribute. However, once all resources are insufficient, the population will not affect the optimal specialization as seen in the next section, making u_2 constant.

3.2. *Calculating Optimal Specialization*

To calculate optimal specialization for a population given a resource distribution, we let $f_1 = \frac{\partial\alpha(z,\vec{v},\mathbf{u},\vec{N})}{\partial v_2} |_{\vec{v}=\vec{u}}$ and $f_2 = \left(\frac{\partial\alpha(z,\vec{v})}{\partial v_2} \cdot \frac{\alpha(z,\vec{u})\cdot N}{\alpha_T(z,\vec{v},\vec{u},N)^2}\right) |_{\vec{v}=\vec{u}} \cdot R(z)$. If there are sufficient amounts of all resources, then $\frac{du_2}{dt} = \int_{-\infty}^{\infty} f_1 dz$. If there are insufficient amounts of all resources, then $\frac{du_2}{dt} = \int_{-\infty}^{\infty} f_2 dz$. If there are sufficient amounts of core resources and insufficient amounts of marginal resources, then $\frac{du_2}{dt} = \int_{-\infty}^{-\hat{z}} f_2 dz + \int_{-\hat{z}}^{\hat{z}} f_1 dz + \int_{\hat{z}}^{\infty} f_2 dz$. If there are insufficient amounts of core resources and sufficient amounts of marginal resources, then $\frac{du_2}{dt} = \int_{-\infty}^{-\hat{z}} f_1 dz + \int_{-\hat{z}}^{\hat{z}} f_2 dz + \int_{\hat{z}}^{\infty} f_1 dz$. Solving for each component reveals

- $\int_{-\infty}^{\infty} f_1 dz = \left(\frac{1}{2} - n\right) u_2^{-(n+1)} \sqrt{\pi c u_2}$
- $\int_{-\infty}^{\infty} f_2 dz = \sigma_K \sqrt{2\pi} \frac{K_{max} \cdot N}{c u_2 (N+1)^2} (\sigma_K^2 - n c u_2)$

- $\int_{-\hat{z}}^{\hat{z}} f_1 dz = \left(\frac{1}{2} - n\right) u_2^{-(n+1)} \sqrt{\pi c u_2} \cdot erf\left(\frac{\hat{z}}{\sqrt{c u_2}}\right) - \hat{z} e^{\frac{-\hat{z}^2}{c u_2}}$
- $\int_{-\infty}^{-\hat{z}} f_1 dz = \int_{\hat{z}}^{\infty} f_1 dz = \frac{1}{2}\left(\left(\frac{1}{2} - n\right) u_2^{-(n+1)} \sqrt{\pi c u_2} \cdot erfc\left(\frac{\hat{z}}{\sqrt{c u_2}}\right) - \hat{z} e^{\frac{-\hat{z}^2}{c u_2}}\right)$
- $\int_{-\hat{z}}^{\hat{z}} f_2 dz \ = \sigma_K \sqrt{2\pi} \frac{K_{max} \cdot N}{c u_2 (N+1)^2} (\sigma_K^2 - n c u_2) \cdot erf\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right) - 2\hat{z}\sigma_K^2 e^{\frac{-\hat{z}^2}{2\sigma_K^2}}$
- $\int_{-\infty}^{-\hat{z}} f_2 dz = \int_{\hat{z}}^{\infty} f_2 dz = \frac{1}{2}\left(\sigma_K \sqrt{2\pi} \frac{K_{max} \cdot N}{c u_2 (N+1)^2} (\sigma_K^2 - n c u_2) \cdot erfc\left(\frac{\hat{z}}{\sqrt{2\sigma_K^2}}\right) - 2\hat{z}\sigma_K^2 e^{\frac{-\hat{z}^2}{2\sigma_K^2}}\right)$

We now numerically search for optimal specialization u_2 such that $\frac{du_2}{dt} = 0$. We do this by first fixing the parameters $K_{max}, n, c,$ and σ_K. Then we calculate $\frac{du_2}{dt}$ depending upon the table and calculations for a chosen u_2. The bisection method for root solving is then used over the interval $u_2 = [0.001, 10{,}000]$ (or for multiple intervals within that range to determine multiple points of optimization) to arrive at optimal specialization.

3.3. *Calculating Selection on Niche Position*

The table from earlier remains the same. Similar to before, let $g_1 = \frac{\partial^2 \alpha(z_j, \vec{v})}{\partial v_1^2} |_{\vec{v}=\vec{u}}$ and

$$g_2 = \left(\left(\sum_{i=1}^{n_s} \alpha\left(z_j, \vec{u_i}\right) N_i \cdot \frac{\partial^2 \alpha(z_j,\vec{v})}{\partial v_1^2} - 2\left(\frac{\partial \alpha(z_j,\vec{v})}{\partial v_1}\right)^2 + \alpha(z_j, \vec{v}) \cdot \frac{\partial^2 \alpha(z_j,\vec{v})}{\partial v_1^2}\right) \frac{\sum_{i=1}^{n_S} \alpha(z_j, \vec{u_i}) N_i}{\alpha_T(z_j, \vec{v}, \mathbf{u}, \vec{N})^3}\right) |_{\vec{v}=\vec{u}} \cdot$$

$R(z)$. If there are sufficient amounts of all resources, then $\frac{\partial^2 G(\vec{v}, \mathbf{u}, \vec{N})}{\partial v_1^2} |_{\vec{v}=\vec{u}} = \int_{-\infty}^{\infty} g_1 dz$. If there are insufficient amounts of all resources, then $\frac{\partial^2 G(\vec{v}, \mathbf{u}, \vec{N})}{\partial v_1^2} |_{\vec{v}=\vec{u}} = \int_{-\infty}^{\infty} g_2 dz$. If there are sufficient amounts of core resources and insufficient amounts of marginal resources, then $\frac{\partial^2 G(\vec{v}, \mathbf{u}, \vec{N})}{\partial v_1^2} |_{\vec{v}=\vec{u}} = \int_{-\infty}^{-\hat{z}} g_2 dz + \int_{-\hat{z}}^{\hat{z}} g_1 dz + \int_{\hat{z}}^{\infty} g_2 dz$. If there are insufficient amounts of core resources and sufficient amounts of marginal resources, then $\frac{\partial^2 G(\vec{v},\mathbf{u},\vec{N})}{\partial v_1^2}|_{\vec{v}=\vec{u}} = \int_{-\infty}^{-\hat{z}} g_1 dz + \int_{-\hat{z}}^{\hat{z}} g_2 dz + \int_{\hat{z}}^{\infty} g_1 dz.$

Solving for each component reveals

- $\int_{-\infty}^{\infty} g_1 dz = 0$
- $\int_{-\infty}^{\infty} g_2 dz = -2K_{max}N\sqrt{\frac{2\pi\sigma_K^2}{cu_2}}\frac{cu_2(N+1)+2\sigma_K^2}{u_2^n\left((N+1)\sqrt{cu_2+2\sigma_K^2}\right)^3}$
- $\int_{-\hat{z}}^{\hat{z}} g_1 dz = -\frac{4\hat{z}}{cu_2^{n+1}}e^{-\frac{\hat{z}^2}{cu_2}}$
- $\int_{-\infty}^{-\hat{z}} g_1 dz = \int_{\hat{z}}^{\infty} g_1 dz = \frac{2\hat{z}}{cu_2^{n+1}}e^{-\frac{\hat{z}^2}{cu_2}}$
- $\int_{-\hat{z}}^{\hat{z}} g_2 dz = -\frac{2K_{max}N\sqrt{\frac{2\pi\sigma_K^2}{cu_2}}(cu_2(N+1)+2\sigma_K^2)}{u_2^n\left((N+1)\sqrt{cu_2+2\sigma_K^2}\right)} \cdot erf\left(\frac{\hat{z}\left(\sqrt{2\sigma_K^2}+\sqrt{cu_2}\right)}{\sqrt{2\sigma_K^2 cu_2}}\right) - \frac{8K_{max}N^2\hat{z}\sigma_K^2}{cu_2^{n+1}(cu_2+2\sigma_K^2)(N+1)^3}e^{\frac{\hat{z}^2(2\sigma_K^2+cu_2)}{2\sigma_K^2 cu_2}}$
- $\int_{-\infty}^{-\hat{z}} g_2 dz = \int_{\hat{z}}^{\infty} g_2 dz = -\frac{K_{max}N\sqrt{\frac{2\pi\sigma_K^2}{cu_2}}(cu_2(N+1)+2\sigma_K^2)}{u_2^n\left((N+1)\sqrt{cu_2+2\sigma_K^2}\right)} \cdot erfc\left(\frac{\hat{z}\left(\sqrt{2\sigma_K^2}+\sqrt{cu_2}\right)}{\sqrt{2\sigma_K^2 cu_2}}\right) - \frac{4K_{max}N^2\hat{z}\sigma_K^2}{cu_2^{n+1}(cu_2+2\sigma_K^2)(N+1)^3}e^{\frac{\hat{z}^2(2\sigma_K^2+cu_2)}{2\sigma_K^2 cu_2}}$

We now substitute all relevant parameters and optimal u_2 into the equations to calculate $\frac{\partial^2 G(\vec{v},\mathbf{u},\vec{N})}{\partial v_1^2}|_{\vec{v}=\vec{u}}$ and determine if selection is disruptive $\left(\frac{\partial^2 G(\vec{v},\mathbf{u},\vec{N})}{\partial v_1^2}|_{\vec{v}=\vec{u}} > 0\right)$ or stabilizing $\left(\frac{\partial^2 G(\vec{v},\mathbf{u},\vec{N})}{\partial v_1^2}|_{\vec{v}=\vec{u}} < 0\right)$.