

Judgment post-stratified assessment combining ranking
information from multiple sources, with a field
phenotyping example

Supplementary Material

1 Simulation results for coverage probabilities for sample size $n = 15$

We provide here additional simulation results for the coverage probabilities of the confidence intervals of population mean. All simulation parameters are the same as the ones used in the main part of the paper. To see the impact of small sample sizes on the coverage probabilities of the confidence intervals, the sample size of $n = 15$ is taken. Tables S1 and S2 present the results of the simulations of 2000 replications for coverage probabilities for four types of distributions, symmetric and skewed to the right.

It is clear that the coverage probabilities are reasonably close to the nominal coverage probability 0.95 as long as the population does not have a strong skewness in its distribution. For example, the coverage probabilities for populations *I* (uniform), *II* (beta(4,4)) and *III* (beta(4,13)) are all close to the nominal coverage probability 0.95, while substantially smaller for population *IV* (exp(1)). Since the confidence intervals are approximate and rely on the

Table S1: Coverage probabilities of jackknife confidence intervals of population mean. The population and sample sizes are $N = 400, n = 30$, respectively. The population values are generated from $y_i = F_G^{-1}(i/(N + 1))$, where F_G is the CDF of the distribution in group G , $G = I, II, III, IV$. Simulation size is 2000.

			K=3						K=12					
			Design D_1			Design D_2			Design D_1			Design D_2		
Dist	H	M_i	$C_{E,1}$	$C_{S,1}$	$C_{J,1}$	$C_{E,2}$	$C_{S,2}$	$C_{J,2}$	$C_{E,1}$	$C_{S,1}$	$C_{J,1}$	$C_{E,2}$	$C_{S,2}$	$C_{J,2}$
I	3	M_1	0.949	0.950	0.949	0.958	0.958	0.955	0.941	0.942	0.944	0.959	0.957	0.953
	3	M_2	0.947	0.946	0.944	0.962	0.961	0.962	0.958	0.960	0.955	0.965	0.966	0.962
	3	M_3	0.946	0.946	0.946	0.962	0.962	0.957	0.952	0.952	0.949	0.964	0.964	0.948
	3	M_4	0.960	0.958	0.951	0.956	0.962	0.958	0.950	0.951	0.943	0.970	0.970	0.962
	3	M_5	0.951	0.951	0.948	0.956	0.958	0.955	0.963	0.964	0.955	0.957	0.962	0.957
	3	M_6	0.946	0.948	0.950	0.969	0.970	0.968	0.954	0.956	0.952	0.962	0.963	0.966
	6	M_1	0.956	0.955	0.948	0.959	0.958	0.954	0.955	0.956	0.945	0.962	0.963	0.952
	6	M_2	0.954	0.953	0.946	0.959	0.960	0.948	0.954	0.958	0.943	0.967	0.967	0.953
	6	M_3	0.960	0.961	0.950	0.970	0.971	0.957	0.960	0.961	0.938	0.979	0.978	0.962
	6	M_4	0.959	0.958	0.944	0.968	0.972	0.953	0.960	0.962	0.951	0.969	0.972	0.955
	6	M_5	0.949	0.959	0.939	0.967	0.970	0.958	0.942	0.949	0.948	0.970	0.976	0.951
	6	M_6	0.961	0.963	0.938	0.968	0.963	0.956	0.960	0.959	0.934	0.964	0.969	0.953
II	3	M_1	0.948	0.947	0.950	0.959	0.960	0.963	0.952	0.950	0.949	0.953	0.952	0.950
	3	M_2	0.951	0.949	0.947	0.966	0.966	0.962	0.954	0.952	0.950	0.967	0.967	0.960
	3	M_3	0.950	0.948	0.945	0.970	0.970	0.963	0.953	0.954	0.957	0.966	0.969	0.961
	3	M_4	0.952	0.954	0.951	0.966	0.969	0.967	0.952	0.954	0.941	0.964	0.963	0.961
	3	M_5	0.952	0.954	0.944	0.965	0.966	0.963	0.939	0.943	0.950	0.963	0.966	0.960
	3	M_6	0.947	0.947	0.939	0.968	0.968	0.965	0.947	0.946	0.947	0.963	0.965	0.959
	6	M_1	0.957	0.957	0.952	0.963	0.962	0.951	0.952	0.954	0.939	0.962	0.962	0.953
	6	M_2	0.960	0.959	0.948	0.960	0.962	0.954	0.946	0.946	0.944	0.962	0.966	0.963
	6	M_3	0.966	0.966	0.945	0.976	0.974	0.958	0.965	0.965	0.958	0.975	0.977	0.958
	6	M_4	0.962	0.962	0.941	0.968	0.971	0.953	0.962	0.958	0.953	0.967	0.971	0.953
	6	M_5	0.963	0.968	0.954	0.959	0.962	0.951	0.958	0.962	0.949	0.973	0.973	0.961
	6	M_6	0.960	0.961	0.940	0.962	0.966	0.954	0.946	0.949	0.941	0.964	0.969	0.962

Table S2: Coverage probabilities of jackknife confidence intervals of population mean. The population and sample sizes are $N = 400, n = 15$, respectively. The population values are generated from $y_i = F_G^{-1}(i/(N + 1))$, where F_G is the CDF of the distribution in group G , $G = I, II, III, IV$. Simulation size is 2000.

Dist	H	M_i	K=3						K=12					
			Design D_1			Design D_2			Design D_1			Design D_2		
			$C_{E,1}$	$C_{S,1}$	$C_{J,1}$	$C_{E,2}$	$C_{S,2}$	$C_{J,2}$	$C_{E,1}$	$C_{S,1}$	$C_{J,1}$	$C_{E,2}$	$C_{S,2}$	$C_{J,2}$
III	3	M_1	0.942	0.943	0.937	0.952	0.952	0.949	0.927	0.927	0.924	0.946	0.948	0.949
	2	M_2	0.944	0.938	0.941	0.951	0.947	0.945	0.947	0.949	0.934	0.949	0.951	0.946
	3	M_3	0.941	0.944	0.943	0.951	0.951	0.951	0.949	0.950	0.947	0.956	0.956	0.951
	4	M_4	0.940	0.939	0.933	0.950	0.947	0.943	0.940	0.941	0.936	0.949	0.950	0.944
	5	M_5	0.944	0.947	0.944	0.946	0.948	0.947	0.943	0.947	0.949	0.949	0.949	0.952
	6	M_6	0.936	0.936	0.937	0.950	0.952	0.948	0.935	0.940	0.941	0.948	0.946	0.943
	7	M_1	0.942	0.941	0.938	0.944	0.946	0.940	0.941	0.940	0.943	0.947	0.944	0.940
	8	M_2	0.941	0.945	0.932	0.954	0.958	0.945	0.947	0.942	0.946	0.945	0.946	0.944
	9	M_3	0.950	0.954	0.947	0.946	0.948	0.948	0.953	0.955	0.944	0.956	0.952	0.943
	10	M_4	0.937	0.941	0.933	0.953	0.950	0.949	0.950	0.949	0.954	0.953	0.957	0.952
	11	M_5	0.946	0.946	0.942	0.952	0.953	0.950	0.942	0.945	0.939	0.944	0.946	0.946
	12	M_6	0.942	0.943	0.943	0.946	0.951	0.956	0.951	0.951	0.939	0.942	0.943	0.946
IV	3	M_1	0.918	0.915	0.901	0.913	0.910	0.919	0.908	0.901	0.915	0.911	0.908	0.916
	2	M_2	0.913	0.913	0.912	0.908	0.908	0.911	0.902	0.910	0.907	0.918	0.916	0.915
	3	M_3	0.910	0.907	0.902	0.930	0.927	0.914	0.909	0.907	0.899	0.918	0.917	0.899
	4	M_4	0.912	0.907	0.899	0.921	0.920	0.922	0.910	0.912	0.905	0.925	0.923	0.916
	5	M_5	0.909	0.908	0.904	0.918	0.915	0.919	0.911	0.909	0.905	0.928	0.930	0.928
	6	M_6	0.919	0.917	0.903	0.922	0.922	0.932	0.924	0.919	0.917	0.912	0.911	0.912
	7	M_1	0.907	0.906	0.905	0.921	0.921	0.916	0.914	0.915	0.911	0.914	0.915	0.905
	8	M_2	0.911	0.914	0.901	0.919	0.921	0.907	0.913	0.916	0.906	0.921	0.919	0.907
	9	M_3	0.909	0.912	0.908	0.931	0.934	0.919	0.924	0.921	0.911	0.931	0.934	0.927
	10	M_4	0.915	0.915	0.916	0.926	0.930	0.923	0.908	0.907	0.904	0.923	0.926	0.922
	11	M_5	0.923	0.922	0.913	0.924	0.925	0.927	0.919	0.921	0.920	0.920	0.919	0.922
	12	M_6	0.916	0.916	0.908	0.919	0.916	0.916	0.914	0.911	0.908	0.917	0.916	0.926

central limit theorem, the sample size required to provide a good approximation depends on the skewness of the distribution. For heavily skewed distributions, such as an exponential distribution, the required sample size is larger than that required for symmetric distributions. For an exponential distribution, the sample size $n = 30$ appears to be sufficiently large.

2 Model-based inference under a super-population model and agreement weights estimator

We provide here some additional developments for the model-based inference and a weighted estimator using the agreement score among K different ranking methods. **** (2020) constructed estimators for the population mean and total using a super-population model. The super-population model assumes that the population values $y_1, \dots, y_N$ are just one realization of independent identically distributed (*i.i.d*) random variables $Y_1, \dots, Y_N$ from a super-population with mean μ and variance σ^2 .

$$M : Y_1, \dots, Y_N \text{ are } i.i.d \text{ from } F \text{ with mean } \mu \text{ and variance } \sigma^2. \quad (1)$$

Using this model, **** (2020) constructed a predictor for the population mean $\bar{Y}_N = \sum_{i=1}^N Y_i / N$ and provided the mean square prediction error. Under design D_2 , the predictor is given by

$$\bar{Y}_M = \sum_{h=1}^H \frac{I_{h,2} J_{h,2}}{d_{n,2}} \sum_{i=1}^n I(R_{i,2} = h) Y_i.$$

Even though $\bar{Y}_M$ has exactly the same form as $\bar{Y}_{JPS,2}$, its distributional properties are different due to the super-population model assumption.

The following result is adapted from **** (2020).

Theorem 1 *Let $(Y_i, R_i), i = 1, \dots, n$, be a JPS sample from a finite population $\mathcal{P}^N$. Under the super population model, the predictor $\bar{Y}_M$ is model unbiased, and the mean square prediction*

error of $\bar{Y}_M$ is given by

$$\begin{aligned}\sigma_M^2 &= MSPE(\bar{Y}_M) = \left[\frac{-1}{N} + HE \left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2} \right) \right] \sigma^2 \\ &+ \left[\frac{H}{H-1} var \left(\frac{I_{1,2}}{d_{n,2}} \right) - E \left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2} \right) \right] \sum_{h=1}^H (\mu_{[h],M} - \mu)^2,\end{aligned}\quad (2)$$

where $\mu_{[h],M}$ is the expected value of the h -th judgment order statistics in a comparison set of size H under Model (1).

For any set and sample sizes, an unbiased estimator of σ_M^2 is given by

$$\begin{aligned}\hat{\sigma}_M^2 &= \frac{(U_{1,2} + U_{2,2})}{2H^2} \left[\frac{-1}{N} + \frac{H^2}{H-1} var \left(\frac{I_{1,2}}{d_{n,2}} \right) \right] \\ &+ \frac{U_{2,2}}{2} \left[E \left(\frac{I_{1,2}^2 J_{1,2}}{d_{n,2}^2} \right) - \frac{H}{H-1} var \left(\frac{I_{1,2}}{d_{n,2}} \right) \right].\end{aligned}\quad (3)$$

Readers are referred to **** (2020) for further developments on these estimators.

Using the estimator $\hat{\sigma}_M^2$, we construct equally- and standard error-weighted combined estimators under Model (1)

$$\bar{Y}_{E,M} = \frac{1}{K} \sum_{k=1}^K \bar{Y}_{k,M} \text{ and } \bar{Y}_{S,M} = \frac{\sum_{k=1}^K \frac{\bar{Y}_{k,M}}{\hat{\sigma}_{k,M}^2}}{A_M}, \text{ where } A_2 = \sum_{k=1}^K \frac{1}{\hat{\sigma}_{k,M}^2}$$

and $\hat{\sigma}_{k,M}^2$ is the estimator in equation (3) for ranking method k , $k = 1, \dots, K$.

In addition to the standard error-weighted estimator, we now introduce another approach to combine the ranking information at each observation for both model- and design-based inferences. For each measured observation Y_i , we create agreement weights for each rank value h , $h = 1, \dots, H$. Let $A_{i,h,r}$ be the proportion of ranking methods that assign rank h on a measured observation Y_i for design D_r

$$A_{i,h,r} = \sum_{k=1}^K I(R_{i,k} = h) / K, \quad h = 1, \dots, H, \quad r = 1, 2, M.$$

The sum of $A_{i,h,r}$ over h is equal to 1 for each i . The value of $A_{i,h,r}$ represents the strength of agreement among the K ranking methods for rank h on response Y_i in design D_r . For

example, for a given design D_r if all K ranking methods assign rank $R_{i,k,r} = 1 (k = 1, \dots, K)$ for Y_i , then $A_{i,1,r} = 1$ and $A_{i,h,r} = 0$, for $h = 2, \dots, H$. We now construct our estimator using these agreement weights

$$\begin{aligned}\bar{Y}_{A,r} &= \frac{1}{d_{A,r}} \sum_{h=1}^H \frac{I_{h,r} J_{A,h,r}}{d_{A,r}} \sum_{i=1}^n A_{i,h,r} Y_i, \quad r = 1, 2, M, \\ J_{A,h,r} &= \begin{cases} \frac{1}{A_{h,r}} & A_{h,r} > 0 \\ 0 & A_{h,r} = 0, \end{cases}\end{aligned}\quad (4)$$

$A_{h,r} = \sum_{i=1}^n A_{i,h,r} = \sum_{k=1}^K n_{h,k,r}/K$; $I_{h,r} = I(A_{h,r} > 0)$; $d_{A,r} = \sum_{h=1}^H I_{h,r}$. One can interpret this type of estimator as a portioning (or partitioning) estimator since the value of each observation Y_i is partitioned into H different groups. Then the average of the group means yields the combined estimator. The expression $A_{h,r}$ ($A_{h,M}$) can be interpreted as the effective sample size for the h -th partitioned group in design D_r (super-population model M) estimator.

For the variance of the estimator $\bar{Y}_{A,1}$, we construct another estimator based on large sample approximation. Let $\bar{\mathbf{T}}_{n,1}$ be a $2H - 1$ dimensional vector $\bar{\mathbf{T}}_{n,1}^T = (\bar{T}_{1,n,1}, \dots, \bar{T}_{2H-1,n,1})$, where

$$\bar{T}_{h,n,1} = \begin{cases} \frac{1}{n} \sum_{i=1}^n A_{i,h,1} Y_i, & 1 \leq h \leq H \\ \frac{1}{n} \sum_{i=1}^n A_{i,h,1}, & H + 1 \leq h \leq 2H - 1. \end{cases} \quad (5)$$

We note that $\bar{\mathbf{T}}_{n,1}$ is the mean of $(2H - 1)$ -dimensional n independent random vectors. Using the central limit theorem we can establish that $\sqrt{n}(\bar{\mathbf{T}}_{n,1} - \boldsymbol{\theta})$ converges to a $2H - 1$ dimensional multivariate normal distribution with mean vector zero and variance covariance matrix $\boldsymbol{\Sigma}_1$, where $\boldsymbol{\theta}_1 = E\{\mathbf{T}_{n,1}\}$ and $\boldsymbol{\Sigma}_1 = Var(\bar{\mathbf{T}}_{n,1})$. We define a function $Q(\mathbf{T}_{n,1}) = \bar{Y}_{A,1}$,

$$\bar{Y}_{A,1} = Q(\mathbf{T}_{n,1}) = \sum_{h=1}^H \frac{I_{h,1}}{d_{A,1}} \frac{\bar{T}_{h,n,1}}{\bar{T}_{H+h,n,1}} \approx \sum_{h=1}^H \frac{1}{H} \frac{\bar{T}_{h,n,1}}{\bar{T}_{H+h,n,1}},$$

where $\bar{T}_{2H,n,1} = 1 - \sum_{h=1}^{H-1} \bar{T}_{H+h,n,1}$. The approximation in the above expression can be established since $I_{h,1}/d_{A,1} \approx 1/H$, $h = 1, \dots, H$, for large n . We now apply the same transformation to the parameter vector $\boldsymbol{\theta}_1$ and obtain $\eta_1 = Q(\boldsymbol{\theta}_1)$

$$\eta_1 = Q(\boldsymbol{\theta}_1) = \frac{1}{H} \sum_{h=1}^H \frac{\theta_{h,1}}{\theta_{H+h,1}},$$

where $\theta_{h,1}$ is the h -th component of $\boldsymbol{\theta}_1$ and $\theta_{2H,1} = 1 - \sum_{h=1}^{H-1} \theta_{H+h,1}$. Let $\mathbf{L}$ be the partial derivative vector of $Q(\boldsymbol{\theta}_1)$ with respect to $\boldsymbol{\theta}_1$, $\mathbf{L}_1 = \frac{d}{d\boldsymbol{\theta}_1}Q(\boldsymbol{\theta}_1)$. Using the continuity of Q we establish the asymptotic distribution of $\sqrt{n}(\bar{Y}_{A,1} - \eta_1)$. This result is summarized in Theorem 2.

Theorem 2 *Let $D_1 = \{Y_i, R_{i,k,1}\}, i = 1, \dots, n, k = 1, \dots, K$ be a JPS sample by design D_1 . For large sample size n , $\sqrt{n}(\bar{Y}_{A,1} - \eta_1)$ converges to a normal distribution with mean zero and variance $\sigma_{A,1}^2 = \frac{1}{H}\mathbf{L}_1^T \boldsymbol{\Sigma}_1 \mathbf{L}_1$, where $\mathbf{L}_1$ is a $2H - 1$ dimensional vector*

$$L_{h,1} = \begin{cases} \frac{1}{\theta_{H+h,1}} & h = 1, \dots, H \\ -\frac{\theta_{h,1}}{\theta_{H+h,1}^2} + \frac{\theta_{H,1}}{\theta_{2H,1}^2}, & h = 1, \dots, H - 1 \end{cases}$$

and $\eta_1 = \frac{1}{H} \sum_{h=1}^H \frac{\theta_{h,1}}{\theta_{H+h,1}}$ with $\theta_{2H,1} = 1 - \sum_{h=1}^{H-1} \theta_{H+h,1}$.

Based on theorem 2, we estimate $\sigma_{A,1}^2$ with a plug-in estimator. Let

$$\hat{\theta}_{h,1} = \bar{T}_{h,n,1}, \hat{\boldsymbol{\Sigma}}_1 = S_1^2 \text{ and } \hat{L}_{h,1} = \begin{cases} \frac{1}{\bar{\theta}_{H+h,1}} & h = 1, \dots, H \\ -\frac{\hat{\theta}_{h,1}}{\hat{\theta}_{H+h,1}^2} + \frac{\hat{\theta}_{H,1}}{\hat{\theta}_{2H,1}^2}, & h = 1, \dots, H - 1, \end{cases}$$

where S_1^2 is the sample variance-covariance matrix of $\mathbf{T}_{n,1}$ and $\hat{\theta}_{h,1} = \bar{T}_{h,n,1}$, $h = 1, \dots, 2H$. We then estimate $\sigma_{A,1}^2$ with

$$\bar{\sigma}_{A,1}^2 = \frac{1}{H} \hat{\mathbf{L}}_1^\top \hat{\boldsymbol{\Sigma}}_1 \hat{\mathbf{L}}_1.$$

Since sample D_2 is constructed without replacement, we can conjecture an approximate estimate for the variance of $\bar{Y}_{A,2}$ ($\sigma_{A,2}^2$) provided that the finite population correction factor (n/N) has a non-zero limit.

$$\bar{\sigma}_{A,2}^2 = (1 - n/N) \frac{1}{H} \hat{\mathbf{L}}_2^\top \hat{\boldsymbol{\Sigma}}_2 \hat{\mathbf{L}}_2.$$

where $\hat{\mathbf{L}}_2$ and $\hat{\boldsymbol{\Sigma}}_2$ are computed using the sample data generated from design D_2 .

For small sample sizes, using the notation in the the paper, the jackknife variance estimators of $\bar{Y}_{A,r}$ and $\bar{Y}_M$ are given by

$$\hat{\sigma}_{A,r}^2 = \sum_{i=1}^n (\bar{Y}_{A,r}^{(-i)} - \bar{J}_{A,r})^2 ((n-1)/n)^2, \quad r = 1, 2, M,$$

and

$$\hat{\sigma}_{S,M}^2 = \sum_{i=1}^n (Y_{S,M}^{*(-i)} - \bar{J}_{S,M})^2 ((n-1)/n)^2, \quad \hat{\sigma}_{E,M}^2 = \sum_{i=1}^n (Y_{E,M}^{*(-i)} - \bar{J}_{E,M})^2 ((n-1)/n)^2.$$

These variance estimates allow us to construct approximate $100(1 - \alpha)\%$ prediction and confidence intervals for the population mean and total,

$$\bar{Y}_{E,M} \pm t_{n-1, 1-\alpha/2} \hat{\sigma}_{E,M}, \quad \bar{Y}_{S,M} \pm t_{n-1, 1-\alpha/2} \hat{\sigma}_{S,M},$$

and

$$\bar{Y}_{A,r} \pm t_{n-1, 1-\alpha/2} \hat{\sigma}_{A,r}, \quad r = 1, 2, M,$$

where $t_{n-1, a}$ is the a -th upper quantile of the t-distribution with $n - 1$ degrees of freedom.

We performed two additional simulation studies to investigate the empirical properties of the estimators. The first simulation study investigates the efficiency of the agreement weight estimator, and the coverage probability of the related confidence interval. The simulation parameters are identical to those we used in the paper. Tables S3 and S4 present the relative efficiency of $\bar{Y}_{S,r}$ and $\bar{Y}_{A,r}$

$$RE_{S,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{S,r}), \quad RE_{A,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{A,r})$$

and the coverage probability $C_{A,r}$ of the confidence interval of the population mean based on the estimator $\bar{Y}_{A,r}$. We note that the simulation results in the paper showed that $\bar{Y}_{S,r}$ was the best estimator. Hence, we compare the estimator $\bar{Y}_{A,r}$ with $\bar{Y}_{S,r}$, and do not include the other estimators in this simulation. Tables S3 and S4 demonstrate that $RE_{S,r}$ and $RE_{A,r}$ values are approximately equal for models M_1 , M_2 and M_3 , but $RE_{S,r}$ is slightly larger than $RE_{A,r}$ for models M_4 , M_5 and M_6 for all populations and set sizes and the number of ranking methods $K = 3, 12$. Considering all ranking models, we may still conclude that the standard error weighted estimator is the best multi-ranker estimator.

The coverage probability of the confidence interval based on estimator $\bar{Y}_{A,r}$ behaves similarly to other estimators. Under these simulation settings, the coverage probabilities are reasonably close to the nominal coverage probability 0.95.

Table S3: Efficiency of weighted estimators relative to the SRS estimator, $RE_{S,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{S,r})$, $RE_{A,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{A,r})$, and the coverage probability ($C(A,r)$) of the confidence interval of population mean based on the agreement weight estimators $\bar{Y}_{A,r}$.

]		K=3							K=12						
		Design D_1				Design D_2			Design D_1			Design D_2			
Dist	H	M_i	$RE_{S,1}$	$RE_{A,1}$	$C_{A,1}$	$RE_{S,2}$	$RE_{A,2}$	$C_{A,2}$	$RE_{S,1}$	$RE_{A,1}$	$C_{A,1}$	$RE_{S,2}$	$RE_{A,2}$	$C_{A,2}$	
I	3	M_1	0.976	0.999	0.951	0.974	0.999	0.956	0.994	1.006	0.936	0.988	1.009	0.954	
	3	M_2	1.170	1.191	0.948	1.203	1.23	0.963	1.259	1.249	0.958	1.292	1.275	0.964	
	3	M_3	1.696	1.708	0.947	1.771	1.791	0.962	1.858	1.799	0.947	2.043	1.968	0.961	
	3	M_4	1.506	1.410	0.961	1.524	1.405	0.958	1.658	1.453	0.953	1.754	1.532	0.966	
	3	M_5	1.248	1.175	0.953	1.252	1.176	0.957	1.392	1.229	0.960	1.403	1.243	0.958	
	3	M_6	1.337	1.267	0.948	1.396	1.308	0.971	1.442	1.303	0.950	1.459	1.315	0.960	
	6	M_2	1.242	1.312	0.948	1.241	1.343	0.958	1.390	1.399	0.954	1.417	1.446	0.964	
	6	M_3	2.254	2.455	0.964	2.368	2.664	0.971	2.727	2.770	0.956	3.138	3.177	0.978	
	6	M_4	1.797	1.645	0.959	1.901	1.662	0.963	2.196	1.786	0.958	2.361	1.905	0.969	
	6	M_5	1.415	1.266	0.946	1.397	1.283	0.964	1.627	1.339	0.937	1.73	1.398	0.966	
	6	M_6	1.489	1.408	0.956	1.542	1.433	0.964	1.756	1.477	0.954	1.849	1.552	0.962	
II	3	M_1	0.957	0.987	0.948	0.975	1.000	0.958	0.996	1.008	0.952	0.992	1.009	0.953	
	3	M_2	1.162	1.187	0.950	1.224	1.242	0.965	1.270	1.260	0.952	1.313	1.292	0.964	
	3	M_3	1.619	1.640	0.949	1.737	1.747	0.971	1.913	1.857	0.950	1.941	1.898	0.965	
	3	M_4	1.478	1.376	0.952	1.595	1.430	0.962	1.631	1.458	0.948	1.692	1.487	0.962	
	3	M_5	1.258	1.188	0.952	1.307	1.213	0.96	1.379	1.221	0.943	1.397	1.240	0.963	
	3	M_6	1.303	1.241	0.946	1.361	1.276	0.962	1.414	1.288	0.942	1.442	1.316	0.960	
	6	M_1	0.944	0.991	0.945	0.904	0.993	0.964	0.975	1.009	0.952	0.979	1.010	0.963	
	6	M_2	1.227	1.331	0.965	1.226	1.325	0.967	1.381	1.390	0.946	1.432	1.442	0.960	
	6	M_3	2.099	2.254	0.961	2.284	2.510	0.970	2.656	2.694	0.964	2.972	3.029	0.972	
	6	M_4	1.789	1.631	0.955	1.84	1.657	0.968	2.079	1.752	0.958	2.229	1.856	0.964	
	6	M_5	1.372	1.247	0.957	1.466	1.306	0.954	1.602	1.333	0.950	1.710	1.367	0.963	
	6	M_6	1.425	1.370	0.956	1.558	1.413	0.958	1.7	1.483	0.942	1.803	1.546	0.960	

Table S4: Efficiency of weighted estimators relative to the SRS estimator, $RE_{S,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{S,r})$, $RE_{A,r} = MSE(\bar{Y}_{SRS,r})/MSE(\bar{Y}_{A,r})$, and the coverage probability ($C_{A,r}$) of the confidence interval of population mean based on the agreement weight estimators $\bar{Y}_{A,r}$.

Dist	H	M_i	K=3						K=12					
			Design D_1			Design D_2			Design D_1			Design D_2		
			$RE_{S,1}$	$RE_{A,1}$	$C_{A,1}$	$RE_{S,2}$	$RE_{A,2}$	$C_{A,2}$	$RE_{S,1}$	$RE_{A,1}$	$C_{A,1}$	$RE_{S,2}$	$RE_{A,2}$	$C_{A,2}$
III	3	M_1	0.970	0.998	0.942	0.971	0.996	0.952	0.999	1.005	0.944	1.006	1.009	0.958
	3	M_2	1.199	1.204	0.945	1.201	1.208	0.952	1.276	1.262	0.948	1.310	1.291	0.954
	3	M_3	1.624	1.641	0.946	1.683	1.698	0.957	1.840	1.800	0.944	1.917	1.887	0.965
	3	M_4	1.425	1.344	0.958	1.468	1.374	0.96	1.540	1.405	0.942	1.651	1.483	0.960
	3	M_5	1.262	1.177	0.954	1.278	1.176	0.958	1.341	1.207	0.946	1.368	1.231	0.961
	3	M_6	1.279	1.226	0.957	1.328	1.255	0.962	1.418	1.310	0.949	1.443	1.322	0.960
	6	M_1	0.945	0.989	0.948	0.937	1.003	0.954	0.982	1.013	0.948	0.991	1.014	0.952
	6	M_2	1.198	1.293	0.946	1.242	1.354	0.964	1.388	1.398	0.941	1.415	1.441	0.952
	6	M_3	2.078	2.263	0.956	2.289	2.545	0.972	2.488	2.545	0.950	2.924	2.981	0.972
	6	M_4	1.757	1.596	0.950	1.745	1.598	0.965	2.031	1.722	0.944	2.244	1.832	0.966
	6	M_5	1.401	1.275	0.960	1.424	1.316	0.954	1.561	1.332	0.950	1.635	1.353	0.952
	6	M_6	1.459	1.381	0.950	1.450	1.376	0.958	1.663	1.456	0.943	1.741	1.521	0.960
IV	3	M_1	0.996	1.012	0.928	0.997	1.005	0.940	1.008	1.010	0.923	1.010	1.010	0.930
	3	M_2	1.158	1.177	0.926	1.191	1.208	0.941	1.233	1.235	0.936	1.249	1.251	0.932
	3	M_3	1.464	1.495	0.938	1.498	1.518	0.944	1.568	1.574	0.922	1.637	1.640	0.942
	3	M_4	1.406	1.346	0.926	1.372	1.321	0.945	1.427	1.346	0.926	1.464	1.379	0.948
	3	M_5	1.189	1.149	0.927	1.210	1.158	0.934	1.290	1.195	0.937	1.294	1.207	0.936
	3	M_6	1.230	1.197	0.917	1.247	1.220	0.932	1.327	1.262	0.930	1.354	1.281	0.942
	6	M_1	0.933	0.995	0.922	0.919	0.986	0.942	0.995	1.011	0.932	0.990	1.014	0.926
	6	M_2	1.249	1.342	0.928	1.236	1.364	0.948	1.332	1.396	0.930	1.394	1.432	0.942
	6	M_3	1.800	1.991	0.934	1.939	2.150	0.952	2.126	2.221	0.939	2.214	2.337	0.949
	6	M_4	1.580	1.516	0.926	1.645	1.597	0.954	1.900	1.706	0.936	1.976	1.730	0.948
	6	M_5	1.313	1.261	0.934	1.331	1.277	0.944	1.447	1.311	0.930	1.525	1.354	0.943
	6	M_6	1.382	1.338	0.930	1.413	1.415	0.941	1.591	1.459	0.938	1.657	1.485	0.952

The second simulation study in this section considers the super-population model. We again used the same simulation parameters as in Section 5 of the paper, but the population values are constructed based on the super population model in equation (1). The population of size $N = 400$ is generated as a realization from group I, II, III or IV distributions at each repetition of the 2000 simulation replications. The judgment post-stratified samples

$$D_{M,k} = \{Y_i, R_{i,k,M}\}, i = 1, \dots, 30, k = 1, \dots, K,$$

are constructed from the realization of the 2000 finite population values using the same correlation models $M_j, j = 1, \dots, 6$. The sample size (n), set size (H) and the number of ranking methods (K) are set $n = 30, H = 3, 6$ and $K = 3, 12$, respectively. The efficiencies of all combined estimators are compared with the efficiency of $\bar{Y}_{SRS,M}$

$$RE_{E,M} = \frac{MSE(\bar{Y}_{E,M})}{\bar{Y}_{SRS,M}}, \quad RE_{S,M} = \frac{MSE(\bar{Y}_{S,M})}{\bar{Y}_{SRS,M}}, \quad RE_{J,M} = \frac{MSE(\bar{Y}_{JPS,M})}{\bar{Y}_{SRS,M}},$$

where $\bar{Y}_{SRS,M}$ is the SRS estimator under Model (1). Table S5 presents the efficiency values of the estimators for group I, II, III and IV populations. Under the super-population model, the efficiencies of the estimators follow patterns similar to those observed in Tables 1 and 2 in Section 5. When $K = 3, 12$, for models M_1, M_2 and M_3 , where ranking quality is the same for all ranking methods, the efficiency of $\bar{Y}_{S,M}$ is as good as or slightly better than other weighted estimators for all set sizes. On the other hand, if the ranking quality differs significantly among ranking methods (model M_4 and M_5 and M_6), there is a substantial efficiency gain over $\bar{Y}_{A,M}$ for set sizes $H = 3, 6$. All weighted estimators always perform better than the JPS estimator. The efficiency gains over JPS are substantial for the large set size, $H = 6$.

Table S6 presents the coverage probabilities of the prediction intervals for the population mean. The coverage probabilities are again reasonably close to the nominal coverage probability of 0.95.

Table S5: Efficiency of the multi-ranker combined estimators with respect to the estimator $\bar{Y}_{SRS,M}$, $RE_{E,M} = MSE(\bar{Y}_{SRS,M})/MSE(\bar{Y}_{E,M})$, under the super-population model (M). The population values are generated from $y_i = F_G^{-1}(i/(N+1))$ at each replication of the simulation, where F_G is the CDF of group G , $G = I(\text{uniform}), II(\text{beta}(4,4)), III(\text{beta}(4,13)), IV(\text{exp}(1))$. The population size is $N = 400$, sample size $n = 30$, simulation size is 2000.

		Population I						Population II					
		$K = 3$			$K = 12$			$K = 3$			$K = 12$		
H	M_i	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,2}$	$RE_{S,2}$	$RE_{J,2}$
3	M_1	0.977	0.966	0.913	0.999	0.990	0.913	0.991	0.988	0.935	1.004	1.000	0.928
3	M_2	1.225	1.218	1.061	1.299	1.301	1.066	1.217	1.216	1.051	1.265	1.271	1.052
3	M_3	1.747	1.74	1.370	2.021	2.051	1.412	1.838	1.829	1.448	1.967	1.992	1.405
3	M_4	1.503	1.532	1.393	1.677	1.787	1.407	1.500	1.542	1.388	1.629	1.730	1.419
3	M_5	1.304	1.356	1.481	1.328	1.417	1.476	1.256	1.300	1.431	1.323	1.399	1.418
3	M_6	1.350	1.391	1.419	1.413	1.482	1.373	1.296	1.320	1.313	1.395	1.459	1.379
6	M_1	0.923	0.904	0.776	0.994	0.982	0.794	0.926	0.912	0.788	0.987	0.978	0.794
6	M_2	1.314	1.297	0.998	1.427	1.426	0.959	1.248	1.234	0.933	1.432	1.446	0.971
6	M_3	2.364	2.297	1.440	3.203	3.232	1.515	2.231	2.183	1.446	2.87	2.913	1.537
6	M_4	1.813	1.936	1.617	2.130	2.407	1.548	1.712	1.811	1.412	2.033	2.302	1.547
6	M_5	1.345	1.458	1.562	1.527	1.761	1.455	1.313	1.407	1.441	1.490	1.703	1.467
6	M_6	1.432	1.509	1.526	1.639	1.826	1.471	1.444	1.532	1.514	1.618	1.797	1.490
		Population III						Population IV					
		$K = 3$			$K = 12$			$K = 3$			$K = 12$		
H	M_i	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,M}$	$RE_{S,M}$	$RE_{J,M}$	$RE_{E,2}$	$RE_{S,2}$	$RE_{J,2}$
3	M_1	0.976	0.972	0.924	1.005	1.003	0.932	0.988	0.990	0.944	1.007	1.014	0.927
3	M_2	1.217	1.217	1.051	1.269	1.282	1.068	1.175	1.184	1.019	1.239	1.260	1.026
3	M_3	1.721	1.711	1.400	1.891	1.909	1.390	1.574	1.566	1.312	1.63	1.629	1.226
3	M_4	1.480	1.517	1.349	1.555	1.625	1.283	1.374	1.392	1.283	1.459	1.510	1.269
3	M_5	1.236	1.279	1.357	1.313	1.391	1.419	1.212	1.243	1.290	1.248	1.289	1.303
3	M_6	1.289	1.311	1.349	1.376	1.439	1.385	1.221	1.241	1.188	1.311	1.356	1.302
6	M_1	0.917	0.904	0.793	0.984	0.980	0.782	0.942	0.929	0.820	0.989	0.991	0.761
6	M_2	1.226	1.214	0.993	1.409	1.415	0.978	1.213	1.187	0.974	1.375	1.391	0.925
6	M_3	2.340	2.321	1.557	2.769	2.782	1.416	1.981	1.890	1.334	2.332	2.230	1.347
6	M_4	1.713	1.827	1.454	2.003	2.235	1.361	1.536	1.546	1.325	1.848	1.947	1.346
6	M_5	1.296	1.422	1.519	1.490	1.698	1.508	1.288	1.340	1.458	1.414	1.553	1.392
6	M_6	1.434	1.514	1.501	1.601	1.765	1.481	1.37	1.413	1.356	1.544	1.659	1.260

Table S6: Coverage probabilities of jackknife confidence intervals of population mean under the super-population model (M). The population values are generated from $y_i = F_G^{-1}(i/(N + 1))$ at each replication of the simulation, where F_G is the CDF of group G , $G = I(\text{uniform}), II(\text{beta}(4,4)), III(\text{beta}(4,13)), IV(\text{exp}(1))$. The population size is $N = 400$, sample size $n = 30$, simulation size is 2000

		Population I						Population II					
		$K = 3$			$K = 12$			$K = 3$			$K = 12$		
H	M_i	$C_{E,M}$	$C_{S,1}$	$C_{J,M}$	$C_{E,M}$	$C_{S,2}$	$C_{J,M}$	$C_{E,M}$	$C_{S,M}$	$C_{J,M}$	$C_{E,M}$	$C_{S,M}$	$C_{J,M}$
3	M_1	0.947	0.946	0.946	0.954	0.956	0.955	0.949	0.949	0.95	0.947	0.947	0.951
3	M_2	0.952	0.954	0.94	0.960	0.962	0.950	0.960	0.960	0.952	0.961	0.964	0.955
3	M_3	0.956	0.957	0.946	0.954	0.953	0.936	0.956	0.955	0.948	0.963	0.964	0.959
3	M_4	0.962	0.963	0.948	0.958	0.969	0.95	0.949	0.952	0.951	0.955	0.958	0.946
3	M_5	0.95	0.955	0.948	0.964	0.966	0.956	0.950	0.951	0.949	0.954	0.958	0.95
3	M_6	0.956	0.959	0.951	0.957	0.959	0.948	0.96	0.959	0.950	0.961	0.962	0.952
6	M_1	0.961	0.960	0.951	0.954	0.954	0.948	0.962	0.964	0.951	0.95	0.949	0.939
6	M_2	0.963	0.965	0.949	0.956	0.955	0.948	0.963	0.964	0.942	0.959	0.962	0.946
6	M_3	0.963	0.964	0.940	0.974	0.974	0.937	0.958	0.963	0.94	0.969	0.969	0.942
6	M_4	0.968	0.964	0.945	0.966	0.969	0.944	0.957	0.962	0.94	0.961	0.964	0.951
6	M_5	0.950	0.952	0.943	0.968	0.976	0.939	0.956	0.957	0.945	0.953	0.963	0.941
6	M_6	0.964	0.970	0.946	0.957	0.966	0.942	0.967	0.968	0.947	0.953	0.958	0.936
		Population III						Population IV					
		$K = 3$			$K = 12$			$K = 3$			$K = 12$		
H	M_i	$C_{E,M}$	$C_{S,1}$	$C_{J,M}$	$C_{E,M}$	$C_{S,2}$	$C_{J,M}$	$C_{E,M}$	$C_{S,M}$	$C_{J,M}$	$C_{E,M}$	$C_{S,M}$	$C_{J,M}$
3	M_1	0.950	0.950	0.950	0.943	0.943	0.951	0.930	0.929	0.932	0.921	0.922	0.921
3	M_2	0.951	0.952	0.949	0.945	0.947	0.942	0.934	0.936	0.931	0.937	0.936	0.932
3	M_3	0.955	0.954	0.948	0.951	0.955	0.940	0.943	0.939	0.933	0.939	0.941	0.938
3	M_4	0.953	0.952	0.946	0.956	0.954	0.935	0.938	0.937	0.932	0.929	0.928	0.915
3	M_5	0.946	0.947	0.936	0.952	0.952	0.953	0.935	0.935	0.938	0.928	0.93	0.929
3	M_6	0.945	0.948	0.940	0.946	0.946	0.945	0.938	0.937	0.933	0.930	0.927	0.921
6	M_1	0.957	0.956	0.943	0.946	0.945	0.938	0.928	0.926	0.928	0.931	0.93	0.915
6	M_2	0.956	0.959	0.942	0.952	0.950	0.950	0.939	0.939	0.930	0.942	0.947	0.922
6	M_3	0.968	0.963	0.940	0.971	0.974	0.934	0.945	0.945	0.920	0.956	0.952	0.928
6	M_4	0.962	0.968	0.944	0.961	0.962	0.934	0.944	0.944	0.923	0.946	0.948	0.922
6	M_5	0.946	0.953	0.946	0.955	0.962	0.942	0.939	0.938	0.930	0.933	0.935	0.936
6	M_6	0.954	0.961	0.945	0.963	0.966	0.951	0.945	0.949	0.931	0.945	0.949	0.933

Reference

**** (2020). Model based inference using judgment post-stratified samples in finite populations. To appear in *Australian and New Zealand Journal of Statistics* .