Supplementary material for Bayesian modelling of non-negative data using dependency-extended two-part models

1 Web Appendix A

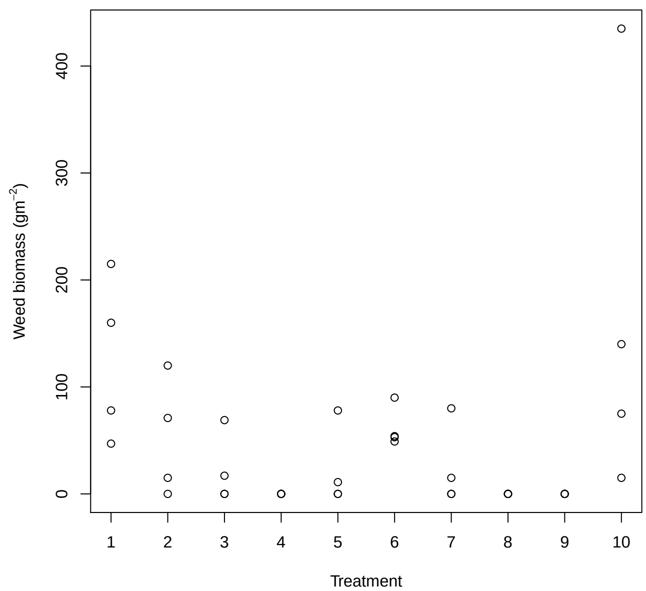

Figure A1: All observations ($N = 40$) of creeping thistle biomass (g m^{-2}), by treatment.

Table A1: The creeping thistle dataset

Replicate	Block	Treatment	Biomass (g m^{-2})
1	1	3	0
1	1	6	90
1	1	10	140
1	1	1	160
1	1	2	120
1	2	4	0
1	2	8	0
1	2	9	0
1	2	5	0
1	2	7	80
2	3	6	49
2	3	5	0
2	3	1	78
2	3	8	0
2	3	9	0
2	4	2	15
2	4	3	17
2	4	7	15
2	4	10	15
2	4	4	0
3	5	7	0
3	5	9	0
3	5	4	0
3	5	10	435
3	5	5	78
3	6	8	0
3	6	6	53
3	6	3	69
3	6	2	71
3	6	1	215
4	7	7	0
4	7	3	0
4	7	2	0
4	7	8	0
4	7	4	0
4	8	1	47
4	8	5	11
4	8	6	54
4	8	9	0
4	8	10	75

2 Web Appendix B

2.1 Full conditional posterior distributions of treatment and block effects and dispersion components

The Gibbs sampler algorithm is based on fully conditional posterior distributions. Full conditional posterior distributions can be identified directly from the joint posterior density, such as the latter is a function of the parameters of interest, while keeping all other parameters and data $\mathbf{y}$.

Let $\boldsymbol{\Theta}$ be the vector of parameters. Let $\boldsymbol{\Theta}^{(-)}$ be the vector of parameters without the parameter to which the full conditional posterior distribution is being considered.

2.1.1 Full conditional posterior distributions considering Model 0

Let $\boldsymbol{\Theta} = (\alpha_t, \beta_t, \sigma_b^2, \sigma_v^2, \phi)$ be the vector of parameters related to Model 0. The posterior full conditional distributions are given below.

$$
\begin{aligned}
p(\alpha_t|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{s \in S_t} [f(y_{st}|\exp(\alpha_t + b_s), \phi)]^{I_{\{y_{st}>0\}}} \\
&\times \mathrm{N}(\alpha_t; 0.01, 10), \ t = 1, \ldots, T
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
p(\beta_t|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{s \in S_t} \left[\exp(\beta_t + v_s)^{I_{\{y_{st}=0\}}}(1 + \exp(\beta_t + v_s))^{-1}\right] \\
&\times \mathrm{N}(\beta_t; 0.01, 10), \ t = 1, \ldots, T
\end{aligned} \tag{2}
$$

For a t such as $y_{st} = 0$ for all $s \in S_t$, the conditional posterior distribution of α_t and β_t in (1) and (2), respectively, are affected by vague prior distributions as there will be no gain in information from the data. Continuing, the full conditional posterior distribution of ϕ may be written as

$$
\begin{aligned}
p(\phi|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} [f(y_{st}|\exp(\alpha_t + b_s), \phi)]^{I_{\{y_{st}>0\}}} \\
&\times \mathrm{IG}(\phi; 0.01, 0.01).
\end{aligned}
$$

Finally, the full conditional posterior distribution of σ_b^2 and σ_v^2 are given by

$$
\begin{aligned}
p(\sigma_b^2|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} \left[\frac{1}{\sigma_b}\exp\left\{-\frac{1}{2\sigma_b^2}b_s^2\right\}\right]^{I_{\{y_{st}>0\}}} \\
&\times \mathrm{Cauchy}(\sigma_b^2; 0, 0.1) I_{\{\sigma_b^2>0\}}
\end{aligned}
$$

and

$$
\begin{aligned}
p(\sigma_v^2|\mathbf{y},\mathbf{\Theta}^{(-)}) &\propto \prod_{t=1}^{T}\prod_{s\in S_t}\left[\frac{1}{\sigma_v}\exp\left\{-\frac{1}{2\sigma_v^2}v_s^2\right\}\right]^{I_{\{y_{st}>0\}}} \\
&\times \text{Cauchy}(\sigma_v^2;0,0.1)I_{\{\sigma_v^2>0\}}
\end{aligned}
$$

respectively.

2.1.2 Full conditional posterior distributions considering Model 1

Let $\mathbf{\Theta}=(\alpha_t,\beta_t,\nu_1^2,\nu_1^2,\nu_2^2,\psi,\phi)$ be the vector of parameters related to Model 1. The full conditional posterior distributions are given below.

$$
\begin{aligned}
p(\alpha_t|\mathbf{y},\mathbf{\Theta}^{(-)}) &\propto \prod_{s\in S_t}[f(y_{st}|\exp(\alpha_t+b_s),\phi)]^{I_{\{y_{st}>0\}}} \\
&\times \text{N}(\alpha_t;0.01,10),\ t=1,\dots,T
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
p(\beta_t|\mathbf{y},\mathbf{\Theta}^{(-)}) &\propto \prod_{s\in S_t}\left[\exp(\beta_t+v_s)^{I_{\{y_{st}=0\}}}(1+\exp(\beta_t+v_s))^{-1}\right] \\
&\times \text{N}(\beta_t;0.01,10),\ t=1,\dots,T
\end{aligned} \tag{4}
$$

For a t such as $y_{st}=0$ for all $s\in S_t$, the conditional posterior distribution of α_t and β_t in (3) and (4), respectively, are affected by vague prior distributions as there will be no gain in information from the data. Continuing, the full conditional posterior distribution of ϕ may be written as

$$
\begin{aligned}
p(\phi|\mathbf{y},\mathbf{\Theta}^{(-)}) &\propto \prod_{t=1}^{T}\prod_{s\in S_t}[f(y_{st}|\exp(\alpha_t+b_s),\phi)]^{I_{\{y_{st}>0\}}} \\
&\times \text{IG}(\phi;0.01,0.01).
\end{aligned}
$$

The full conditional posterior distribution of the variance components ν_1,ν_2,ψ of $\mathbf{\Sigma}^*$ are given by

$$
\begin{aligned}
\pi(\nu_1^2|\mathbf{y},\mathbf{\Theta}^{(-)}) &\propto \prod_{t=1}^{T}\prod_{s\in S_t}\left[\frac{1}{(\nu_1^2)^{1/2}}\exp\Big\{-\frac{1}{2\nu_1^2}b_s^2\Big\}\right]\ (\nu_1^2)^{-(0.01+1)}\exp\Big\{-\frac{0.01}{\nu_1^2}\Big\} \\
&= (\nu_1^2)^{-(n/2+0.01)-1}\exp\Big\{-\frac{1}{\nu_1^2}\Big(\sum_{t=1}^{T}\sum_{s\in S_t}\frac{b_s^2}{2}+0.01\Big)\Big\},
\end{aligned}
$$

which represents the kernel of an IG distribution with parameters $(n/2+0.01)$ and $(\sum_{t=1}^{T}\sum_{s\in S_t}b_s^2/2+0.01)$, by

$$\pi(\nu_2^2|\mathbf{y},\mathbf{\Theta}^{(-)}) \propto \prod_{t=1}^{T}\prod_{s\in S_t}\left[\frac{1}{(\nu_2^2)^{1/2}}\exp\Big\{-\frac{1}{2\nu_2^2}(v_s-\psi b_s)^2\Big\}\right]\ (\nu_2^2)^{-(0.01+1)}\exp\Big\{-\frac{0.01}{\nu_2^2}\Big\}$$
$$= (\nu_2^2)^{-(n/2+0.01)-1}\exp\Big\{-\frac{1}{\nu_2^2}\Big(\sum_{t=1}^{T}\sum_{s\in S_t}\frac{(v_s-\psi b_s)^2}{2}+0.01\Big)\Big\},$$

which is the kernel of an IG distribution with parameters $(n/2+0.01)$ and $(\sum_{t=1}^{T}\sum_{s\in S_t}(v_s-\psi b_s)^2/2+0.01)$, and finally, by

$$\pi(\psi|\mathbf{y},\mathbf{\Theta}^{(-)}) \propto \prod_{t=1}^{T}\prod_{s\in S_t}\left[\exp\Big\{-\frac{1}{2\nu_2^2}(v_s-\psi b_s)^2\Big\}\right]\exp\Big\{-\frac{1}{2\,100}\psi^2.\Big\}$$
$$= \exp\left\{-\frac{(\sum_{t=1}^{T}\sum_{s\in S_t}b_s^2+\frac{1}{100})}{2\nu_2^2}\left[\psi-\frac{\sum_{t=1}^{T}\sum_{s\in S_t}v_s b_s}{(\sum_{t=1}^{T}\sum_{s\in S_t}b_s^2+\frac{1}{100})}\right]^2\right\},$$

which represents the kernel of a normal distribution with mean $\sum_{t=1}^{T}\sum_{s\in S_t}(v_s b_s)/(\sum_{t=1}^{T}\sum_{s\in S_t}b_s^2+\frac{1}{100})$ and variance $[(\sum_{t=1}^{T}\sum_{s\in S_t}b_s^2+\frac{1}{100})/\nu_2^2]^{-1}$.

2.1.3 Full conditional posterior distributions considering Model 2

Let $\mathbf{\Theta} = (p_t,\gamma_1,\gamma_2,\phi,\sigma_b^2)$ be the vector of parameters related to Model 2. The full conditional posterior distributions are given below.

$$p(p_t|\mathbf{y},\mathbf{\Theta}^{(-)}) \propto \prod_{s\in S_t}\left[p_t^{I_{\{y_{st}=0\}}}(1-p_t)^{I_{\{y_{st}>0\}}}\right]\times U(p_t;0,1)$$
$$\propto p_t^{\sum_{s\in S_t}I_{\{y_{st}=0\}}}(1-p_t)^{\sum_{s\in S_t}I_{\{y_{st}>0\}}}\times U(p_t;0,1),$$

which represents the kernel of a Beta distribution with parameters $\sum_{s\in S_t}I_{\{y_{st}=0\}}+1$ and $\sum_{s\in S_t}I_{\{y_{st}>0\}}+1$.

For parameters γ_1 and γ_2 the full conditional posterior distributions are given by

$$p(\gamma_1|\mathbf{y},\mathbf{\Theta}^{(-)}) \propto \prod_{t=1}^{T}\prod_{s\in S_t}[f(y_{st}|\exp(\gamma_1+\gamma_2\,\alpha_t+b_s),\phi)]^{I_{\{y_{st}>0\}}}$$
$$\times\ \mathrm{N}(\gamma_1;0.01,10)$$

and

$$p(\gamma_2|\mathbf{y},\mathbf{\Theta}^{(-)}) \propto \prod_{t=1}^{T}\prod_{s\in S_t}[f(y_{st}|\exp(\gamma_1+\gamma_2\,\alpha_t+b_s),\phi)]^{I_{\{y_{st}>0\}}}$$
$$\times\ \mathrm{N}(\gamma_2;0.01,10)I_{\{\gamma_2<0\}}$$

respectively.

The full conditional posterior distribution of dispersion parameters ϕ and σ_b^2 is given by

$$\begin{aligned} p(\phi|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} [f(y_{st}|\exp(\gamma_1 + \gamma_2\,\alpha_t + b_s), \phi)]^{I_{\{y_{st}>0\}}} \\ &\times \text{Inv-Gamma}(\phi; 0.01, 0.01) \end{aligned}$$

and

$$\begin{aligned} p(\sigma_b^2|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} \left[\frac{1}{\sigma_b}\exp\left\{-\frac{1}{2\sigma_b^2} b_s^2\right\}\right]^{I_{\{y_{st}>0\}}} \\ &\times \text{Cauchy}(\sigma_b^2; 0, 0.1) I_{\{\sigma_b^2>0\}} \end{aligned}$$

respectively.

2.1.4 Full conditional posterior distributions considering Model 3

Let $\boldsymbol{\Theta} = (\alpha_t, \gamma_1, \gamma_2, \phi, \sigma_b^2)$ be the vector of parameters related to Model 3. The posterior full conditional distributions are given below.

$$p(\alpha_t|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) \propto \prod_{t=1}^{T} \prod_{s \in S_t} [f(y_{st}|\exp(\alpha_t + b_s), \phi)]^{I_{\{y_{st}>0\}}} \times \text{N}(\alpha_t; 0.01, 10)$$

For parameters γ_1 and γ_2 the full conditional distribution is given by

$$\begin{aligned} p(\gamma_1|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} \left\{[\exp(\gamma_1 + \gamma_2 \log(\mu))]^{I_{\{y_{st}=0\}}} (1 + \exp(\gamma_1 + \gamma_2 \log(\mu)))^{-1}\right\} \\ &\times \text{N}(\gamma_1; 0.01, 10) \end{aligned}$$

and

$$\begin{aligned} p(\gamma_2|\mathbf{y}, \boldsymbol{\Theta}^{(-)}) &\propto \prod_{t=1}^{T} \prod_{s \in S_t} \left\{[\exp(\gamma_1 + \gamma_2 \log(\mu))]^{I_{\{y_{st}=0\}}} (1 + \exp(\gamma_1 + \gamma_2 \log(\mu)))^{-1}\right\} \\ &\times \text{N}(\gamma_2; 0.01, 10) \end{aligned}$$

The full conditional posterior distribution of dispersion parameters ϕ and σ_b^2 is given by

$$\begin{aligned} p(\phi|\mathbf{y}, \mathbf{\Theta}^{(-)}) &\propto \prod_{t=1}^{T}\prod_{s\in S_t}[f(y_{st}|\exp(\alpha_t + b_s), \phi)]^{I_{\{y_{st}>0\}}} \\ &\times \text{Inv-Gamma}(\phi; 0.01, 0.01) \end{aligned}$$

and

$$\begin{aligned} p(\sigma_b^2|\mathbf{y}, \mathbf{\Theta}^{(-)}) &\propto \prod_{t=1}^{T}\prod_{s\in S_t}\left[\frac{1}{\sigma_b}\exp\left\{-\frac{1}{2\sigma_b^2}b_s^2\right\}\right]^{I_{\{y_{st}>0\}}} \\ &\times \text{Cauchy}(\sigma_b^2; 0, 0.1) I_{\{\sigma_b^2>0\}} \end{aligned}$$

respectively.

3 Web Appendix C

The tables of this appendix present posterior means for varying sets of prior distributions.

3.1 Sets of prior distributions

Table A2: Sets of prior distributions for Model 0

Set	α_t	β_t	σ_b^2	σ_v^2	ϕ
1	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)
2	N(0.01, 1)	N(0.01, 1)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)
3	N(0.01, 100)	N(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)
4	N(0.01, 10)	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	Cauchy(0, 0.1)$I_{\sigma_v^2>0}$	IG(0.01, 0.01)
5	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.1, 0.1)
6	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.001, 0.001)
7	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	U(0, 50)
8	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	U(0, 100)

Table A3: Sets of prior distributions for Model 1

Set	α_t	β_t	σ_b^2	σ_v^2	ϕ	ξ
1	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	N(0.01, 100)
2	N(0.01, 1)	N(0.01, 1)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	N(0.01, 100)
3	N(0.01, 100)	N(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	N(0.01, 100)
4	N(0.01, 10)	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	Cauchy(0, 0.1)$I_{\sigma_v^2>0}$	IG(0.01, 0.01)	N(0.01, 100)
5	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.1, 0.1)	N(0.01, 100)
6	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.001, 0.001)	N(0.01, 100)
7	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	U(0, 50)	N(0.01, 100)
8	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	U(0, 100)	N(0.01, 100)
9	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	N(0.01, 10)
10	N(0.01, 10)	N(0.01, 10)	IG(0.01, 0.01)	IG(0.01, 0.01)	IG(0.01, 0.01)	N(0.01, 100)$I_{\xi<0}$

Table A4: Sets of prior distributions for Model 2

Set	γ_1	γ_2	σ_b^2	ϕ	p
1	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(1, 1)
2	N(0.01, 1)	N(0.01, 1)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(1, 1)
3	N(0.01, 100)	N(0.01, 100)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(1, 1)
4	N(0.01, 10)	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(1, 1)
5	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	IG(0.01,0.01)	IG(0.01, 0.01)	Beta(1, 1)
6	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.1 ,0.1)	Beta(1, 1)
7	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.001, 0.001)	Beta(1, 1)
8	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	U(0, 50)	Beta(1, 1)
9	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	U(0, 100)	Beta(1, 1)
10	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(0.5, 0.5)
11	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)	Beta(2, 2)

Table A5: Sets of prior distributions for Model 3

Set	γ_1	γ_2	α_t	σ_b^2	ϕ
1	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
2	N(0.01, 1)	N(0.01, 1)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
3	N(0.01, 100)	N(0.01, 100)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
4	N(0.01, 10)	N(0.01, 10)	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
5	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 1)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
6	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 100)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.01, 0.01)
7	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	IG(0.01,0.01)	IG(0.01, 0.01)
8	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.1 ,0.1)
9	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	IG(0.001, 0.001)
10	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	U(0, 50)
11	N(0.01, 10)	N(0.01, 10)$I_{\gamma_2<0}$	N(0.01, 10)	Cauchy(0, 0.1)$I_{\sigma_b^2>0}$	U(0, 100)

3.2 Gamma distribution

Table A6: Posterior means using Model 0 and gamma distribution for sets of priors specified in Table A2

	Prior distribution set							
	1	2	3	4	5	6	7	8
ϕ	10.04	10.29	9.94	9.95	8.41	10.04	12.69	12.50
σ_b	1.26	4.52	1.08	1.81	1.37	1.32	1.36	1.25
σ_v	1.19	0.84	2.81	1.96	1.20	1.18	1.17	1.14
α_1	4.19	0.82	4.71	3.72	4.04	4.08	4.09	4.22
α_2	3.69	0.34	4.20	3.23	3.54	3.58	3.57	3.72
α_3	3.66	0.34	4.17	3.20	3.52	3.56	3.55	3.70
α_4	0.07	0.01	-0.32	0.00	0.08	0.07	-0.14	0.13
α_5	2.32	-1.01	2.84	1.84	2.17	2.20	2.17	2.31
α_6	3.59	0.23	4.11	3.12	3.44	3.48	3.46	3.61
α_7	3.70	0.38	4.19	3.25	3.55	3.58	3.59	3.73
α_8	0.08	0.03	-0.23	-0.05	0.03	-0.10	-0.09	-0.01
α_9	0.11	0.02	-0.55	-0.03	0.05	-0.01	0.10	0.09
α_{10}	3.99	0.61	4.51	3.52	3.86	3.88	3.86	4.01
β_1	-3.58	-1.04	-10.17	-3.47	-3.51	-3.37	-3.41	-3.39
β_2	-1.36	-0.52	-2.66	-1.57	-1.29	-1.37	-1.32	-1.35
β_3	0.08	0.08	0.21	0.15	0.12	0.14	0.11	0.08
β_4	3.73	1.08	10.15	3.71	3.60	3.55	3.58	3.51
β_5	-0.02	-0.05	-0.11	0.04	-0.01	0.02	0.06	-0.03
β_6	-3.44	-1.00	-9.84	-3.31	-3.47	-3.50	-3.37	-3.42
β_7	-0.17	-0.08	-0.34	-0.21	-0.15	-0.11	-0.17	-0.14
β_8	3.47	1.10	9.69	3.63	3.61	3.50	3.61	3.48
β_9	3.59	1.12	9.71	3.72	3.52	3.58	3.58	3.59
β_10	-3.43	-0.99	-9.55	-3.32	-3.35	-3.44	-3.42	-3.48

Table A7: Posterior means using Model 1 and gamma distribution for sets of priors specified in Table A3

	Prior distribution set									
	1	2	3	4	5	6	7	8	9	10
ϕ	9.61	10.27	9.54	9.89	8.50	10.03	12.25	12.30	9.86	10.04
σ_b	1.17	4.64	1.17	1.62	1.28	1.29	1.25	1.25	1.19	1.50
σ_v	1.93	1.32	3.69	2.66	1.91	1.92	1.94	1.94	1.92	1.85
ρ	0.47	-0.51	0.61	0.21	0.43	0.41	0.48	0.48	0.45	-0.54
α_1	4.45	0.87	4.88	4.06	4.34	4.33	4.37	4.42	4.39	3.94
α_2	3.94	0.36	4.38	3.54	3.82	3.82	3.86	3.89	3.88	3.43
α_3	3.88	0.36	4.35	3.50	3.79	3.77	3.84	3.87	3.85	3.39
α_4	-0.10	0.02	0.02	-0.05	-0.08	-0.02	0.04	-0.03	0.13	-0.11
α_5	2.62	-0.96	3.07	2.17	2.50	2.48	2.49	2.53	2.54	2.07
α_6	3.84	0.28	4.27	3.44	3.73	3.72	3.76	3.80	3.79	3.35
α_7	3.92	0.43	4.37	3.55	3.81	3.78	3.85	3.89	3.87	3.46
α_8	-0.03	0.02	-0.23	0.08	0.28	-0.05	-0.01	0.04	-0.23	0.14
α_9	0.21	0.05	-0.20	-0.13	0.07	-0.05	-0.10	0.13	0.03	-0.07
α_{10}	4.26	0.67	4.72	3.84	4.16	4.14	4.17	4.21	4.21	3.74
β_1	-3.43	-0.86	-9.47	-3.46	-3.57	-3.54	-3.45	-3.41	-3.61	-3.03
β_2	-1.71	-0.45	-2.94	-1.77	-1.74	-1.64	-1.73	-1.67	-1.75	-1.41
β_3	0.00	0.18	0.25	0.20	-0.06	0.01	-0.03	-0.10	0.04	0.23
β_4	3.74	1.23	10.73	3.77	3.59	3.59	3.66	3.72	3.58	3.78
β_5	-0.26	0.16	-0.09	-0.10	-0.31	-0.17	-0.33	-0.25	-0.25	0.41
β_6	-3.45	-0.87	-9.45	-3.41	-3.55	-3.47	-3.42	-3.50	-3.56	-3.19
β_7	-0.42	0.05	-0.64	-0.45	-0.52	-0.48	-0.44	-0.55	-0.44	0.08
β_8	3.32	1.22	9.72	3.65	3.32	3.41	3.34	3.27	3.39	3.78
β_9	3.47	1.31	9.68	3.62	3.53	3.54	3.52	3.51	3.48	3.99
β_{10}	-3.57	-0.85	-9.66	-3.44	-3.65	-3.57	-3.67	-3.65	-3.63	-2.96

Table A8: Posterior means using Model 2 and gamma distribution for sets of priors specified in Table A4

	Prior distribution set										
	1	2	3	4	5	6	7	8	9	10	11
ϕ	7.39	8.42	7.45	7.72	7.86	7.15	7.60	10.03	10.03	6.98	7.94
σ_b	0.83	0.99	0.84	0.85	0.94	0.82	0.87	0.89	0.89	0.82	0.86
γ_1	3.76	3.09	3.84	3.76	3.76	3.74	3.76	3.78	3.78	3.74	3.85
γ_2	-0.41	-0.59	-0.41	-0.43	-0.43	-0.42	-0.44	-0.47	-0.47	-0.32	-0.59
p_1	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.06	0.18
p_2	0.31	0.25	0.31	0.30	0.29	0.31	0.30	0.31	0.31	0.27	0.36
p_3	0.41	0.33	0.41	0.40	0.39	0.41	0.41	0.37	0.37	0.42	0.43
p_4	0.84	0.83	0.83	0.83	0.83	0.83	0.84	0.84	0.84	0.90	0.75
p_5	0.69	0.64	0.68	0.69	0.69	0.68	0.68	0.72	0.72	0.66	0.68
p_6	0.25	0.23	0.26	0.26	0.26	0.26	0.25	0.28	0.28	0.18	0.33
p_7	0.42	0.35	0.42	0.41	0.41	0.44	0.41	0.38	0.38	0.41	0.43
p_8	0.83	0.83	0.83	0.82	0.83	0.83	0.83	0.83	0.83	0.90	0.74
p_9	0.83	0.84	0.84	0.83	0.83	0.84	0.83	0.84	0.84	0.90	0.76
p_{10}	0.12	0.13	0.13	0.13	0.14	0.13	0.13	0.14	0.14	0.07	0.20

Table A9: Posterior means using Model 3 and gamma distribution for sets of priors specified in Table A5

	Prior distribution set										
	1	2	3	4	5	6	7	8	9	10	11
ϕ	8.80	9.69	7.82	8.51	9.50	8.40	9.04	7.18	8.66	11.52	11.61
σ_b	1.24	1.29	1.17	1.13	3.55	1.02	1.48	1.10	1.09	1.29	1.26
γ_1	2.99	1.37	4.20	2.93	3.16	2.28	2.81	3.07	2.97	2.86	2.79
γ_2	-1.13	-0.70	-1.45	-1.10	-1.04	-0.91	-1.09	-1.14	-1.10	-1.10	-1.08
α_1	4.19	4.16	4.13	4.28	1.39	4.66	4.01	4.32	4.32	4.11	4.16
α_2	3.62	3.62	3.55	3.70	0.89	4.08	3.46	3.72	3.73	3.56	3.61
α_3	3.45	3.52	3.33	3.53	0.78	3.91	3.31	3.55	3.54	3.47	3.48
α_4	-2.26	-2.87	-2.20	-2.25	-1.04	-8.26	-2.35	-2.13	-2.15	-2.33	-2.38
α_5	2.31	2.26	2.33	2.42	-0.38	2.81	2.12	2.47	2.49	2.20	2.27
α_6	3.59	3.57	3.56	3.69	0.83	4.08	3.43	3.73	3.74	3.51	3.55
α_7	3.50	3.58	3.39	3.57	0.91	3.94	3.38	3.61	3.61	3.48	3.53
α_8	-2.09	-2.75	-1.99	-2.05	-1.00	-7.56	-2.33	-2.08	-1.96	-2.08	-2.10
α_9	-2.49	-2.98	-2.25	-2.38	-1.25	-8.38	-2.56	-2.37	-2.26	-2.45	-2.52
α_{10}	4.01	3.96	4.01	4.12	1.24	4.50	3.81	4.17	4.17	3.92	3.96

3.3 Lognormal distribution

Table A10: Posterior means using Model 0 and lognormal distribution for sets of priors specified in Table A2

	Prior distribution set							
	1	2	3	4	5	6	7	8
ϕ	0.36	0.35	0.36	0.36	0.36	0.36	0.39	0.39
σ_b	1.24	4.41	1.05	1.60	1.32	1.46	1.27	1.27
σ_v	1.16	0.79	2.82	1.96	1.16	1.17	1.14	1.14
α_1	4.12	0.79	4.62	3.89	4.01	3.87	4.09	4.09
α_2	3.68	0.36	4.16	3.45	3.55	3.39	3.62	3.62
α_3	3.66	0.34	4.15	3.41	3.54	3.38	3.60	3.60
α_4	-0.06	0.03	0.04	-0.10	-0.03	0.08	-0.01	-0.01
α_5	2.32	-0.97	2.81	2.04	2.17	2.03	2.28	2.28
α_6	3.54	0.23	4.04	3.32	3.43	3.28	3.51	3.51
α_7	3.68	0.39	4.17	3.46	3.57	3.42	3.63	3.63
α_8	0.01	0.05	0.87	0.24	-0.12	-0.13	-0.05	-0.05
α_9	0.04	0.01	-0.19	-0.03	0.19	0.06	-0.02	-0.02
α_{10}	3.97	0.62	4.47	3.72	3.85	3.69	3.93	3.93
β_1	-3.34	-1.07	-9.58	-3.50	-3.40	-3.30	-3.33	-3.33
β_2	-1.30	-0.46	-2.43	-1.51	-1.29	-1.38	-1.29	-1.29
β_3	0.11	0.06	0.48	0.25	0.07	0.05	0.16	0.16
β_4	3.56	1.06	10.22	3.69	3.61	3.61	3.48	3.48
β_5	0.00	-0.01	-0.01	-0.04	-0.04	-0.08	0.00	0.00
β_6	-3.37	-1.01	-9.41	-3.47	-3.49	-3.43	-3.42	-3.42
β_7	-0.17	-0.06	-0.25	-0.27	-0.14	-0.14	-0.19	-0.19
β_8	3.55	1.05	9.79	3.58	3.51	3.47	3.41	3.41
β_9	3.65	1.12	9.85	3.57	3.54	3.59	3.39	3.39
β_{10}	-3.44	-1.00	-9.19	-3.23	-3.38	-3.33	-3.25	-3.25

Table A11: Posterior means using Model 1 and lognormal distribution for sets of priors specified in Table A3

	Prior distribution set									
	1	2	3	4	5	6	7	8	9	10
ϕ	0.37	0.35	0.36	0.35	0.37	0.37	0.40	0.40	0.37	0.37
σ_b	1.19	4.64	1.18	1.69	1.31	1.16	1.27	1.27	1.19	1.33
σ_v	2.07	1.34	3.64	2.62	1.87	1.89	2.01	2.01	2.07	1.93
ρ	0.49	-0.52	0.59	0.23	0.40	0.54	0.50	0.50	0.49	-0.48
α_1	4.29	0.81	4.75	3.92	4.22	4.39	4.26	4.26	4.29	4.05
α_2	3.84	0.38	4.29	3.49	3.76	3.93	3.81	3.81	3.84	3.58
α_3	3.81	0.39	4.28	3.45	3.73	3.88	3.77	3.77	3.81	3.55
α_4	-0.01	-0.03	-0.02	-0.11	0.12	0.17	0.08	0.08	-0.01	-0.01
α_5	2.53	-0.96	2.95	2.12	2.45	2.60	2.51	2.51	2.53	2.22
α_6	3.73	0.27	4.15	3.36	3.65	3.81	3.69	3.69	3.73	3.47
α_7	3.81	0.44	4.29	3.49	3.75	3.92	3.77	3.77	3.81	3.59
α_8	0.01	-0.02	0.26	0.20	-0.06	-0.05	0.06	0.06	0.01	0.11
α_9	-0.01	-0.05	0.03	0.06	0.07	-0.03	0.11	0.11	-0.01	0.18
α_{10}	4.16	0.64	4.60	3.77	4.08	4.24	4.13	4.13	4.16	3.88
β_1	-3.51	-0.83	-9.38	-3.50	-3.46	-3.45	-3.51	-3.51	-3.51	-3.25
β_2	-1.79	-0.44	-3.00	-1.74	-1.65	-1.67	-1.78	-1.78	-1.79	-1.42
β_3	-0.01	0.17	0.32	0.10	-0.02	-0.12	-0.10	-0.10	-0.01	0.19
β_4	3.77	1.18	10.30	3.63	3.64	3.60	3.73	3.73	3.77	3.77
β_5	-0.23	0.20	-0.24	-0.12	-0.25	-0.33	-0.20	-0.20	-0.23	0.30
β_6	-3.51	-0.87	-9.70	-3.46	-3.55	-3.52	-3.62	-3.62	-3.51	-3.22
β_7	-0.54	0.03	-0.65	-0.49	-0.49	-0.46	-0.55	-0.55	-0.54	0.02
β_8	3.39	1.17	9.85	3.40	3.46	3.35	3.34	3.34	3.39	3.73
β_9	3.54	1.35	10.08	3.71	3.52	3.48	3.57	3.57	3.54	3.91
β_{10}	-3.74	-0.87	-9.85	-3.45	-3.55	-3.71	-3.60	-3.60	-3.74	-3.10

Table A12: Posterior means using Model 2 and lognormal distribution for sets of priors specified in Table A4

	Prior distribution set										
	1	2	3	4	5	6	7	8	9	10	11
ϕ	0.41	0.40	0.41	0.42	0.41	0.41	0.42	0.48	0.48	0.43	0.41
σ_b	0.86	1.00	0.84	0.83	0.95	0.84	0.83	0.80	0.80	0.80	0.84
γ_1	3.69	2.99	3.72	3.68	3.68	3.68	3.66	3.72	3.72	3.68	3.76
γ_2	-0.44	-0.57	-0.44	-0.44	-0.44	-0.44	-0.44	-0.39	-0.39	-0.30	-0.56
p_1	0.11	0.11	0.11	0.11	0.12	0.12	0.11	0.11	0.11	0.06	0.19
p_2	0.30	0.26	0.31	0.31	0.29	0.31	0.30	0.30	0.30	0.25	0.35
p_3	0.39	0.34	0.39	0.40	0.39	0.39	0.39	0.42	0.42	0.39	0.42
p_4	0.83	0.83	0.83	0.83	0.83	0.83	0.84	0.83	0.83	0.90	0.74
p_5	0.68	0.64	0.70	0.69	0.70	0.70	0.67	0.68	0.68	0.70	0.69
p_6	0.27	0.23	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.19	0.34
p_7	0.42	0.34	0.41	0.41	0.40	0.40	0.41	0.44	0.44	0.40	0.42
p_8	0.83	0.84	0.84	0.83	0.83	0.84	0.83	0.83	0.83	0.90	0.75
p_9	0.83	0.82	0.84	0.83	0.83	0.83	0.83	0.84	0.84	0.90	0.75
p_{10}	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.13	0.13	0.08	0.21

Table A13: Posterior means using Model 3 and lognormal distribution for sets of priors specified in Table A5

	Prior distribution set										
	1	2	3	4	5	6	7	8	9	10	11
ϕ	0.42	0.37	0.56	0.58	0.37	0.38	0.42	0.44	0.43	0.70	0.70
σ_b	1.20	1.13	0.90	0.82	3.53	1.11	1.44	1.04	1.14	0.84	0.84
γ_1	2.92	0.89	4.77	5.04	3.04	2.15	3.74	3.05	2.89	5.92	5.92
γ_2	-1.13	-0.58	-1.69	-1.75	-1.04	-0.91	-1.38	-1.17	-1.13	-2.03	-2.03
α_1	4.10	4.52	4.25	4.30	1.35	4.47	3.91	4.28	4.14	4.21	4.21
α_2	3.56	4.01	3.60	3.60	0.89	3.96	3.38	3.72	3.60	3.50	3.50
α_3	3.37	3.95	3.33	3.35	0.79	3.84	3.20	3.51	3.41	3.13	3.13
α_4	-2.30	-8.87	-1.96	-1.84	-1.05	-7.96	-2.21	-2.13	-2.22	-1.86	-1.86
α_5	2.27	2.68	2.51	2.57	-0.38	2.63	2.05	2.45	2.35	2.53	2.53
α_6	3.54	3.94	3.71	3.76	0.81	3.90	3.35	3.71	3.60	3.73	3.73
α_7	3.41	3.98	3.36	3.39	0.89	3.88	3.25	3.58	3.49	3.18	3.18
α_8	-2.16	-9.39	-1.78	-1.70	-1.01	-7.83	-1.92	-1.89	-2.06	-1.54	-1.54
α_9	-2.43	-9.11	-2.15	-2.02	-1.26	-8.24	-2.45	-2.33	-2.46	-1.94	-1.94
α_{10}	3.93	4.36	4.10	4.14	1.21	4.29	3.73	4.10	4.00	4.06	4.06