Supplementary Materials for

A Spatial Capture-Recapture Model with Hawkes-inspired Detection Rates to account for Animal Movement

A Bayesian Priors

Here we outline the priors are used for fitting our SESCR model in a data augmentation framework (see Section 2.5.2).

$$\begin{aligned}
s_{i1} &\sim U(x_{\min}, x_{\max}) \\
s_{i2} &\sim U(y_{\min}, y_{\max}) \\
\lambda_0 &\sim U(10^{-20}, 1) \\
\tfrac{1}{\beta} &\sim U(10^{-10}, 1000) \\
\log(\tfrac{1}{2\sigma^2}) &\sim U(a, b) \\
r &\sim U(10^{-10}, 1) \\
z_i &\sim \mathrm{Bern}(\psi) \\
\psi &\sim U(0, 1)
\end{aligned} \tag{1}$$

where i takes values $1, 2, ..., m$. The activity centers $\boldsymbol{s}_i$ have a uniform prior. The detectability multiplier λ_0 and self-excitement spatial size r also have uniform priors, whereas for the temporal self-excitement the reciprocal of β has a uniform prior. The prior for home range size σ is more complex, and has been previously used by Durbach et al. (2024) and Royle et al. (2014). In our simulation studies $a = \log_e(0.5)$ and $b = 10$, which means $0.00476 < \hat{\sigma} < 1$. Without this prior, we often observed exceptionally poor mixing of MCMC chains, resulting in misleading inference. The hyperparameter ψ is the probability that a member of the superpopulation exists.

B Randomly Located Traps Simulation Study

In addition to our parameter recovery study presented in Section 3.1, we performed another study that aimed to evaluate model performance with less regular and more realistic placements of traps. In this study, cameras were randomly located within the (0.25,0.75) square rather than being in a grid pattern. The choices for the remaining parameters were unchanged. A total of 520 simulations were performed, and groups of 10 simulations shared the same random trap array. One simulation is omitted from the results as no MLE was found. The number of cameras remained fixed at 25. Fitting was done in a frequentist framework only, as MCMC chains did not always mix well.

Table 1 below shows the results of this study. While SESCR gives poorer results, as measured by RMSE and median bias, than when the trap array is not a grid, its advantage relative to SCR is often greater. For example, the RMSE for N is 50% that of SCR, compared to 85% in Section 3.1. The total runtime was 2.1 hours for `secr` versus 13.7 hours for our model.

Parameter	Median Bias (%)	RMSE	CE (%)
Spatial Capture-Recapture: MLEs			
N	22.4	18.2	-
σ	-39.1	0.0650	-
λ_0	226	0.357	-
Semi-Complete Likelihood: MLEs			
N	-1.53	9.07	69.6
σ	2.68	0.0642	83.6
λ_0	-0.14	0.0101	98.1
β	0.00	0.286	-
r	-2.83	0.101	-

Table 1: Results of the parameter recovery study with randomly located cameras, aggregated across all combinations of true parameter values. Includes median biases and RMSE for both SESCR and SCR and the proportion of simulations for which SESCR gave a closer estimate (CE) than SCR. The RMSE for β excludes one very large estimate. Dashes indicate empty entries.

C SCR Simulation Study

We also performed a simulation study where the data was simulated according to a traditional SCR model. As SCR is a subset of our model, we hope that our model gives similar inference to SCR and that β estimates are high since self-exciting effects do not exist. We set $N = 50$ and chose μ_0 and σ to be uniformly distributed between the values used by the parameter recovery study in Section 3.1. A total of 256 simulations were performed.

The true value of β is ∞. Since this is on a boundary of the parameter space, we have reason to believe that maximum likelihood estimation will not give good inference. This was indeed observed in simulation studies; estimates of β were often close to 0, precisely the opposite of what we expect. We also observed that the profile likelihood with respect to β was often quite flat. Therefore, for this study we used data augmentation with the priors listed in Appendix A, except that the prior for β was $\beta \sim U(10^{-10}, 10)$.

Table 2 below shows the results of this study. SESCR and SCR (fit with the `secr` package in the same manner as Sections 3.1 and 3.2) give similar results. This is expected given that SCR is a special case of our SESCR model. Running time was 30 minutes for `secr` and 49.2 hours for model.

Parameter	Median Estimate	Median Bias (%)	RMSE	CE (%)
Spatial Capture-Recapture: MLEs				
N	50.1	0.262	5.74	-
σ	0.153	0.438	0.0189	-
λ_0	0.0541	0.372	0.116	-
Data Augmentation: Posterior Means				
N	50.8	1.51	5.98	46.1
σ	0.155	2.68	0.0241	47.7
λ_0	0.0528	-1.92	0.0115	49.2
β	6.89	-	-	-
r	0.792	-	-	-

Table 2: Results of the SCR simulation study. Includes median biases and RMSE for both SESCR and SCR, and the proportion of simulations for which SESCR gave a closer estimate (CE) than SCR. Dashes indicate empty entries.

D Martens Case Study

A camera trap survey of American martens (*Martes americana*) was carried out in New Hampshire, United States, and the data is presented by Panchaud et al. (2025). In this survey, 30 cameras were activated for a period of 11 days. The dataset consists of 74 detections of 9 unique individuals. We ensured that the survey region and mask were exactly the same as the original study. We refer once again to density (D) not population size for the reasons explained in Section 4.

Parameter	SCR Estimate	SCL Estimate	DA Estimate
D (/100 ha)	0.151 (0.0347)	0.128 (0.0360)	0.160 (0.0448)
σ (m)	516 (49.0)	975 (357)	809 (223)
λ_0 (/day)	1.60 (0.618)	0.118 (0.0682)	0.189 (0.0939)
β	-	1.90 (0.627)	2.00 (0.547)
r	-	0.208 (0.0543)	0.254 (0.0638)

Table 3: Parameter estimates for the Marten (*Martes americana*) dataset (Panchaud et al., 2025). Results shown are from SCR and SESCR fit in both semi-complete likelihood (SCL) and data augmentation (DA) frameworks. Standard errors are given in parentheses. Dashes indicate parameters that are not included in a model

For comparison, the Panchaud et al. (2025) model estimates the density as 0.132 individuals/100ha (12.57 individuals), σ as approximately 570m and β as 2.07. As this is a small dataset, it is not surprising that estimates for D and σ differ and have high standard errors. We cannot conclusively say that SCR may be biased in either direction for D, though it does appear that it may be underestimating σ. It is interesting however that the three estimates for β are all very similar, giving us confidence that there is real self-excitement or correlation that is being detected and modeled. The running time was 20 seconds and 21 minutes for the frequentist and data augmentation frameworks respectively.

E Further Simulation Study Results

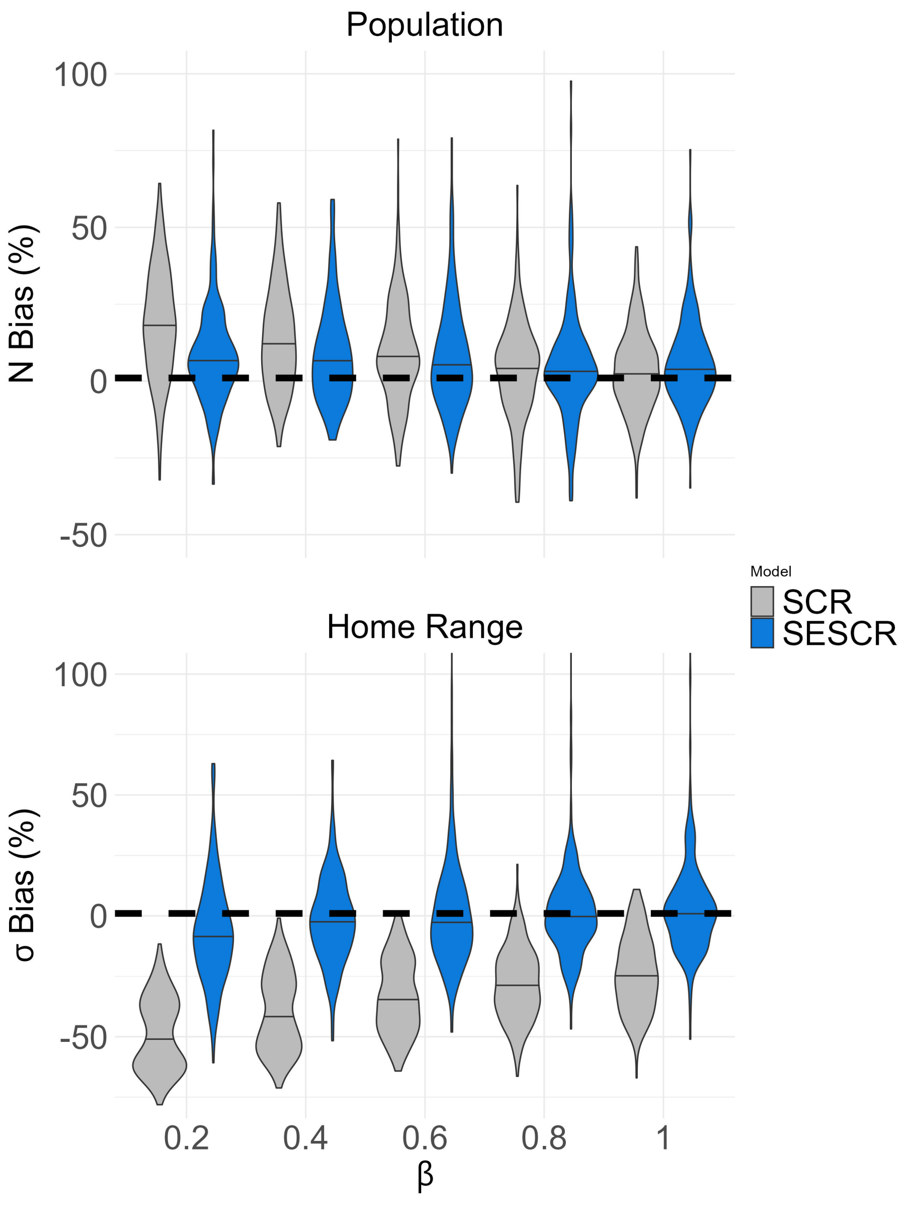

Figure 1: Bias against β for N (top) and σ (bottom), for the parameter recovery study fitted in a data augmentation framework. SESCR (blue) gives less biased estimates than SCR (grey), especially when β is low, meaning that self-exciting effects persist longer. Further results are in Table 2 of the main text.

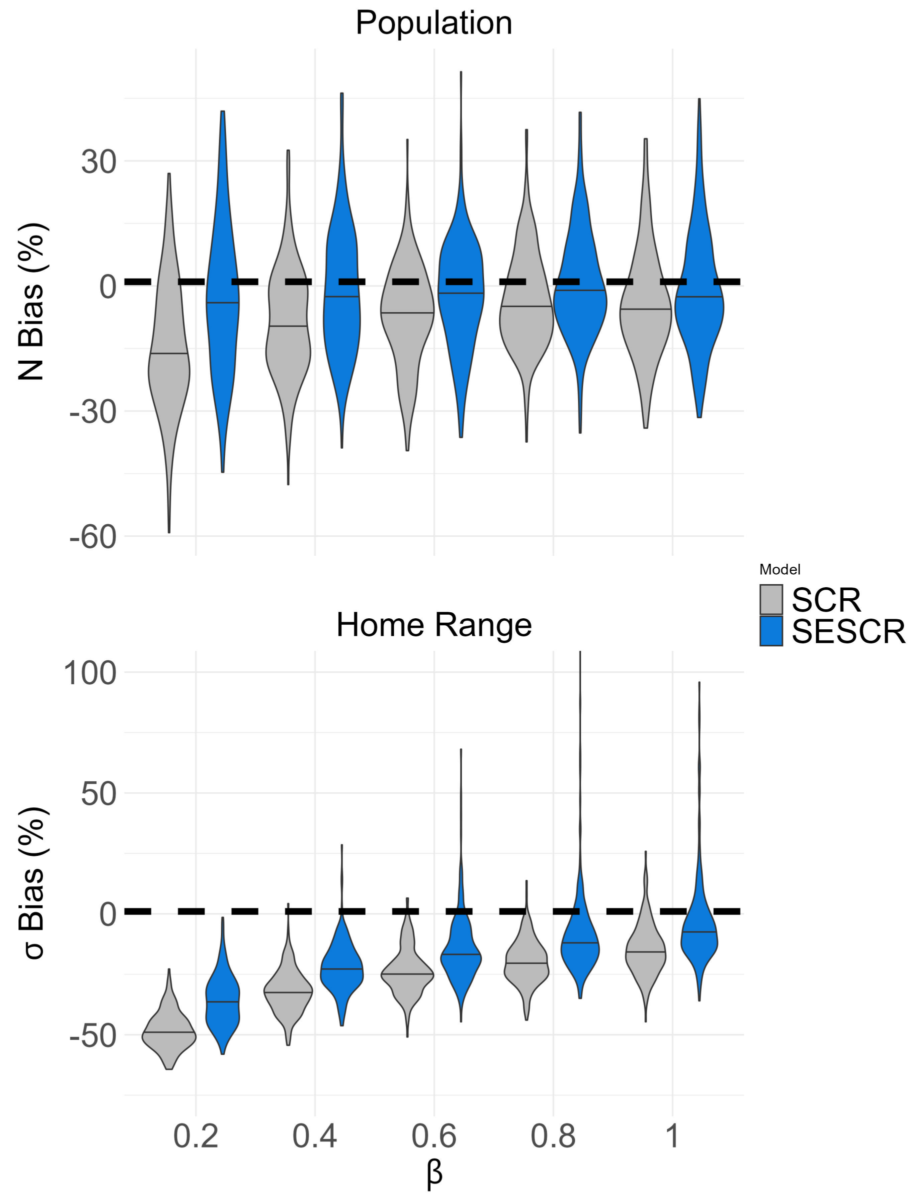

Figure 2: Bias against β for N (top) and σ (bottom), for the OU study fitted in a data augmentation framework. SESCR (blue) gives less biased estimates than SCR (grey), especially when survey duration β is low, meaning that self-exciting effects persist longer. Further results are in Table 3 of the main text.

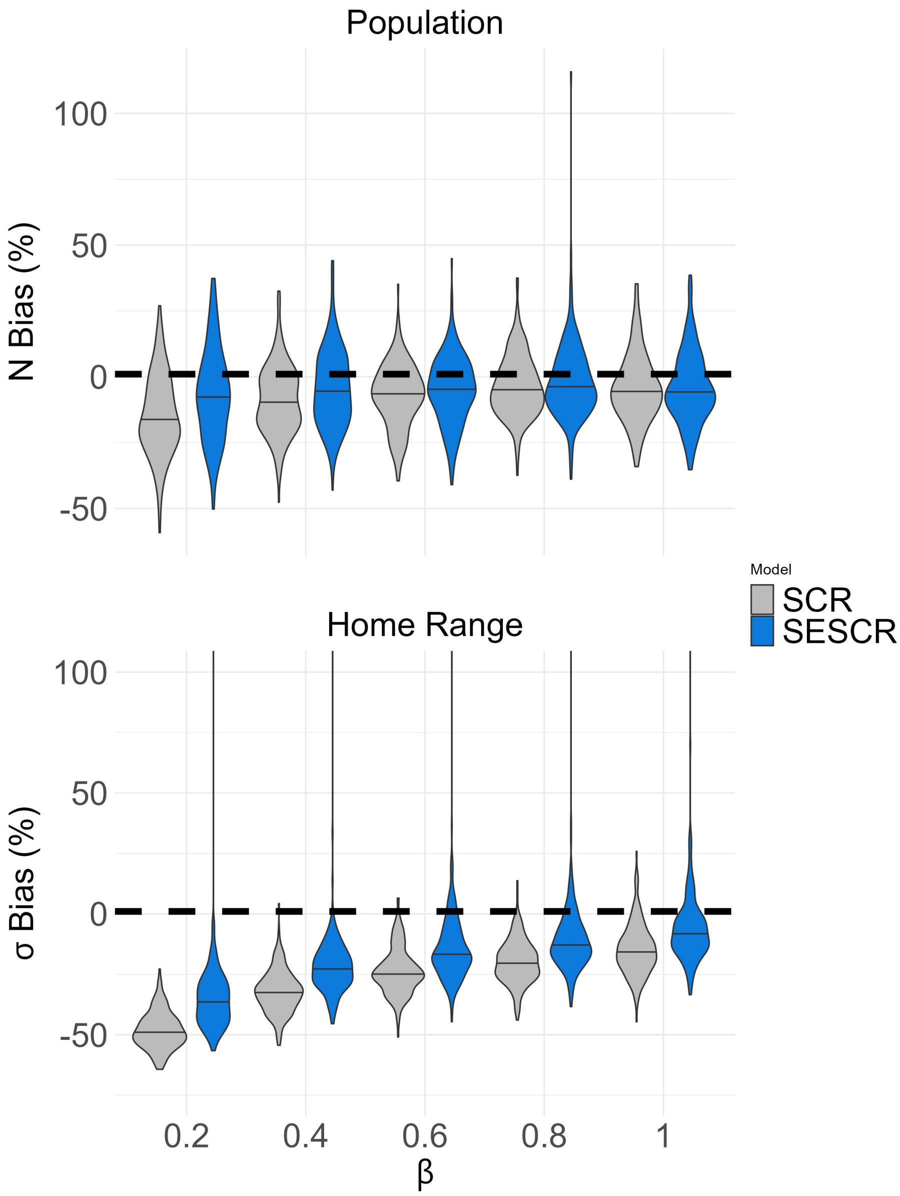

Figure 3: Bias against β for N (top) and σ (bottom), for the OU study fitted in a frequentist framework. SESCR (blue) gives less biased estimates than SCR (grey), especially when survey duration β is low, meaning that self-exciting effects persist longer. Further results are in Table 3 of the main text.

F Further Data

Raw results from the simulation studies, figures of the survey region and camera layout of the simulations and case studies, the data for the case studies, an animation of our model and our code for simulating and fitting our model can be found at `https://anonymous.4open.science/r/SESCR-3BCA/README.md`.

References

I. Durbach, R. Chopara, D. L. Borchers, R. Phillip, K. Sharma, and B. C. Stevenson. That's not the Mona Lisa! how to interpret spatial capture-recapture density surface estimates. *Biometrics*, 80: ujad020, 2024.

C. Panchaud, R. King, D. Borchers, H. Worthington, I. Durbach, and P. van Dam-Bates. Incorporating Memory into Continuous-time Spatial Capture-Recapture Models. [**preprint**] https://arxiv.org/pdf/2408.17278v2, 2025.

J. A. Royle, R. B. Chandler, R. Sollmann, and B. Gardner. *Spatial Capture-Recapture*. Academic Press, Waltham, Massachusetts, 2014.